# Redistributive taxation vs. education subsidies: Fostering equality and social mobility in an intergenerational model

**Andrea Schneider***
Helmut Schmidt University, 22043 Hamburg, Germany

**Abstract**
Redistributive taxation and education subsidies are common policies intended to alleviate the access to education for poor children. However, this paper shows that in an intergenerational framework these policies can raise social mobility only for some investment situations but not in general. I also study the impact of both policies on the skill ratio and inequality. While redistributive taxation raises social mobility but never reduces inequality at the same time, education subsidies can, under some conditions, achieve both targets simultaneously. Unfortunately, these conditions necessarily require a population in which the skill ratio is already quite high.

*Keywords:* Redistributive taxation; Education subsidies; Intergenerational mobility; Inequality

*JEL classification:* D91; H23; H24; I21; J24; J62; O15

# 1 Introduction

Education decisions determine a great part of future income[1] and therefore potential inequality within and across generations. The wage gap, needed to induce investment, implies that it is much easier for rich parents than for poor ones to invest in the education of their children. In this context, several policy interventions that foster investment incentives of the poor and therefore equalize the education

*Corresponding author. Tel.: +49-40-6541-3725; Fax +49-40-6541-3726; E-mail: andrea.schneider@hsu-hh.de.

[1]One of the earliest studies that shows a positive effect of schooling on earning is by Mincer (1958). There is also evidence that the return to schooling has increased over the last decades (Blackburn and Neumark 1993).

distribution are possible. The present paper analyzes the impact of two of them - redistributive taxation and education subsidies[2] - on the aggregate proportion of educated people as well as on social mobility and inequality.[3]

The paper is related to a great number of intergenerational models focusing on the number of steady states (SS), inequality, and social mobility. This body of literature starts with Gary S. Becker. He shows in a paper with Nigel Tomes that there is a unique equilibrium which is characterized by social immobility and inequality (Becker and Tomes 1979). Here, wages of the skilled and unskilled are exogenous and not by the measures of both occupation types. Inequality in this model is mainly driven by luck. Some other papers assuming endogenously determined wages and homogenous agents find a continuum of SSs which mostly are also characterized by inequality and the absence of social mobility (Banerjee and Newman 1993; Galor and Zeira 1993; Freeman 1996; Mookherjee and Ray 2003).[4] In these models the equilibrium outcome is determined by the conditions at the beginning, i.e. there is great historical dependence. But according to Maoz and Moav (1999), Mookherjee and Napel (2006), and Napel and Schneider (2007) these results are strongly connected to the assumption of homogeneous agents.[5] If children are heterogeneous with respect to their inherent talent it becomes possible that a poor parent invests in his highly talented child and also that a rich parent refuses investment in his low-talented child. Thus, steady states with social mobility (SSM) are fostered by the heterogeneity of talents. In Mookherjee and Napel (2006) steady states are characterized by inequality and social mobility. They are locally unique and under some conditions global uniqueness is provided.

Although there are many intergenerational models neither of them investigates the impact of different policy interventions on the wage gap and the degree of social mobility. Thus, this paper tries to fill the gap.

In a basic overlapping generational model where parents decide if their children that differ in their talent get education or not it can be shown that generally neither redistributive taxation nor education subsidies can both decrease inequality and increase social mobility. Depending on the type of SSM, i.e. the equilibrium

[2]In the present context subsidies are transfer payments to the households that invest in education.

[3]A higher degree of social mobility benefits intergenerational equity. Inequality within a generation is measured as the difference between skilled and unskilled wages.

[4]Galor and Zeira (1993) and Mookherjee and Ray (2003) find equal and unequal SSs.

[5]While Mookherjee and Napel (2006) assume that talent is independently and identically distributed Napel and Schneider (2007) show that the results are robust if the child's talent depends on the talent of the parent. Maoz and Moav (1999) focus on the qualitative features of the convergency process that leads to a steady state. They also find that redistributive policy has a negative effect on growth in developed economies but a positive effect in developing countries.

investment decisions of all parents without any public intervention, the impact of both policies on inequality and social mobility is analyzed. While redistributive taxation and subsidization have similar outcomes for some types of SSMs, they have different effects on the skill ratio for other types of SSMs. Under most circumstances there is a trade-off between the reduction of inequality and the increase of social mobility. However, the paper shows that in a situation where unskilled parents are indifferent in their investment decision for a child with low costs education subsidies can reach both targets at the same time. Unfortunately, this result only holds for a high-skilled population.

The paper is organized as follows. I present the basic intergenerational model without policy intervention in section 2. Section 3 studies the impact of redistributive taxation on the skill ratio as well as on inequality and social mobility. Section 4 does the same for education subsidies. A conclusion is given in section 5.

# 2 Model

Assume an overlapping generational model that consists of a unit mass of families. In each point in time a family consists of a parent and a child. The parent can work as a skilled ($s$) or an unskilled ($n$) worker. The skill ratio of the population at time $t$ is denoted by $\lambda_t$. However, skilled work requires education while unskilled work does not. Education costs depend on the talent[6] of the child and must be financed by parent's current income. There are two possible types of talent with corresponding education costs $x^l$ for a highly talented child and $x^h$ for a low-talented child respectively. I assume child's talent to be private information of the parent. The fractions of both types of talent are exogenously given and fix over time and the talent of the child depends on the talent of his parent in a Markov way. Thus, for $i, j \in \{l, h\}$ the conditional probability $p_{i \to j}$ denotes the probability that a parent with education costs $x^i$ has a child with education costs $x^j$.

The economy produces a single consumption good with the Cobb-Douglas production function $H = \lambda_t^{\gamma}(1-\lambda_t)^{1-\gamma}$ with $\gamma \in (0,1)$. Wages are given by the marginal productivities. Thus, in equilibrium wages are

$$w_t^s \equiv \gamma(\frac{1-\lambda_t}{\lambda_t})^{1-\gamma} \tag{1}$$

and

$$w_t^n \equiv (1-\gamma)(\frac{\lambda_t}{1-\lambda_t})^{\gamma}. \tag{2}$$

[6]Here "talent" should be perceived as "potential to benefit from education" similar as in De Fraja (2005).

Investment in education requires $w_t^s > w_t^n$ and therefore $\lambda_t < \gamma$ in equilibrium. The bequest motive is assumed to be altruistic. Therefore, parents maximize

$$U(c_t, w_{t+1}^k) = \ln(w_t^k - Dx) + \delta \ln(w_{t+1}^k) \tag{3}$$

where $c_t$ denotes parent's own consumption, $w_t^k$ and $w_{t+1}^k$ with $k \in \{s, n\}$ are the incomes of the parent and the child respectively, $x$ denotes the child's education costs and the parameter $\delta \in (0, 1)$ weights the motive of altruism. The binary $D$ is 1 in the case of investment and 0 otherwise. Thus, in the case of non-investment the parent can consume his whole income while his child only gets the lower wage of an unskilled worker. In the case of investment the parent can only consume his income minus education costs $x$ but the child's income is given by the skilled wage.
Given the utility function (3) the subjective benefit $B(\cdot)$ and the subjective costs $C^k(\cdot)$ from investment are

$$B(\lambda_{t+1}) \equiv \delta\Big(\ln w_{t+1}^s - \ln w_{t+1}^n\Big) \tag{4}$$

and

$$C^k(\lambda_t, x) \equiv \ln w_t^k - \ln(w_t^k - x). \tag{5}$$

Subjective benefit from investment is determined by the skill ratio of the child's working period and is independent of the occupation type , whereas subjective costs depend on the skill ratio in the parent's working period, on the occupation type of the parent and on the child's talent. It is clear that a parent invests (does not invest) in the education of his child with education costs $x$ whenever the subjective benefit is higher (lower) than the subjective costs. If subjective benefit equals subjective costs skilled (unskilled) parents invest with market clearing probability $\alpha$ ($\beta$).
Let us define $I^k(\lambda)$, $k \in \{s, n\}$, as the curves of indifference for the skilled and unskilled respectively, i.e.

$$I^k(\lambda) \equiv \Big(1 - \Big(\frac{w^n(\lambda)}{w^s(\lambda)}\Big)^{\delta}\Big) w^k(\lambda). \tag{6}$$

Thus, $I^k(\lambda)$ denotes - depending on the skill ratio - the education costs of a child that makes his parent with occupation $k$ just indifferent in his investment decision. All combinations of $(\lambda, x)$ below $I^k$ describe a situation where parents with occupation $k$ invest, while they do not invest if $(\lambda, x)$ is located above $I^k$. Figure 1 illustrates a situation where unskilled parents invest in a child with education costs $x^l$ for $\lambda \in (\lambda_1, \lambda_3)$ and never invest in a child with education costs

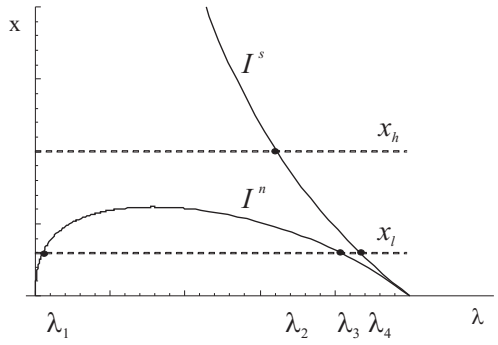


Figure 1: Indifference curves of the skilled ($I^s$) and unskilled ($I^n$)

$x^h$. Analogously, skilled parents invest in the low cost type for $\lambda \in (0, \lambda_4)$ and in the high cost type for $\lambda \in (0, \lambda_2)$.

The situation of the population can be described by the occupation and cost distribution that is denoted by

$$\pi(t) \equiv \{\pi_{s^l}(t), \pi_{s^h}(t), \pi_{n^l}(t), \pi_{n^h}(t)\} \tag{7}$$

where $\pi_{k^i}(t)$ is the fraction of agents with occupation $k \in \{s, n\}$ and education costs $x^i$, $i \in \{l, h\}$ at time $t$. Thus, the skilled fraction of the population is given by $\lambda_t = \pi_{s^l}(t) + \pi_{s^h}(t)$.
The dynamics of the model depends on the skilled fraction since it determines the investment decisions of the parents and therefore the transition matrix. To make it clearer, e.g.

$$\pi(t) \cdot \begin{pmatrix} p_{l\to l} & \alpha \cdot p_{l\to h} & 0 & (1-\alpha) \cdot p_{l\to h} \\ p_{h\to l} & \alpha \cdot p_{h\to h} & 0 & (1-\alpha) \cdot p_{h\to h} \\ p_{l\to l} & 0 & 0 & p_{l\to h} \\ p_{h\to l} & 0 & 0 & p_{h\to h} \end{pmatrix} = \pi(t+1). \tag{8}$$

describes a situation where all parents at time $t$ invest in a child with education costs $x^l$ and skilled parents additionally are indifferent in the investment decision for a child with education costs $x^h$. In such a child skilled parents invest with probability $\alpha$. Summing up, the dynamic can be described by a heterogenous Markov-chain.

Whenever the current skill ratio $\lambda_t$ and the expectations about the next period $\lambda^e_{t+1}$ induce a total skill ratio $\lambda_{t+1} = \lambda^e_{t+1}$ the sequence $\lambda_{t=0,1,2,\ldots}$ describes a

competitive equilibrium.
In my analysis I only focus on equilibria with stationary skill ratios (SS), i.e. $\lambda_t = \lambda_{t+1} \equiv \lambda^*$. In this case the transition matrix is stationary and the Markov chain becomes homogeneous. Since a situation without mobility is at odds with reality I restrict the analysis to steady states with mobility (SSM), i.e. equilibria with stationary skill ratios in which the measure of unskilled investors is positive and equals the number of skilled non-investors.
Since investment of the skilled - caused by the wage difference - always requires investment of the unskilled and investment in a child with education costs $x^h$ always requires investment in a child with education costs $x^l$ there are four different types of SSMs that are summarized in table 1. In the table *yes* denotes strict investment, *no* denotes strict non-investment and $\alpha$ ($\beta$) denotes that skilled (unskilled) parents are indifferent and invest with market clearing probabilities.

| | Type I | Type II | Type III | Type IV |
|---|---|---|---|---|
| Skilled invest in $x^l$-type | yes | yes | yes | yes |
| Unskilled invest in $x^l$-type | yes | $\beta$ | yes | $\beta$ |
| Skilled invest in $x^h$-type | $\alpha$ | $\alpha$ | no | no |
| Unskilled invest in $x^h$-type | no | no | no | no |

Table 1: Four possible types of SSMs

Since a SSM of type II is unstable in the sense that it diminishes whenever there is a small change in the mobility flows I will not consider this type of SSM for the remaining analysis of the impact of policy interventions. As the following analysis of the policy interventions is based on changes in the upward ($u(\cdot)$) and downward ($d(\cdot)$) social mobility flows I define both flows in the interval where SSMs can appear at this point. It is

$$u(\lambda) \equiv \beta(\pi_{n^l} p_{l\to l} + \pi_{n^h} p_{h\to l}) \tag{9}$$

with $\beta = 1$ if the unskilled invest with strict preferences in the cost type $x^l$ (see SSM types I and III) and $\beta \in (0,1)$ otherwise (see SSM types II and IV). Analogously it is

$$d(\lambda) \equiv (1-\alpha)(\pi_{s^l} p_{l\to h} + \pi_{s^h} p_{h\to h}) \tag{10}$$

with $\alpha = 0$ if the skilled strictly do not invest in the cost type $x^h$ (see SSM types III and IV) and $\alpha \in (0,1)$ otherwise (see SSM types I and II).

According to equation (9) the upflow is characterized by one upward jump and the corresponding downward jump. Between upward and downward jump the function strictly decreases since the total number of unskilled decreases. However, the downflow by (10) is an increasing function with one upward jump. It can easily be seen and is also mentioned in Mookherjee and Napel (2006) that there is the possibility for a limited multiplicity of SSMs. More precisely, two SSMs can simultaneously occur (e.g. see figure 2).

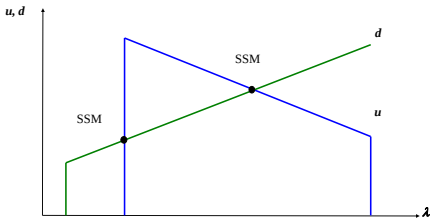


Figure 2: Mobility flows for a case with two SSMs

# 3 Redistributive taxation

In this section I analyze the effect of redistributive taxation on an existing SSM. Due to redistributive taxation the number and types of SSMs can change. To simplify matters I only focus on the marginal impact of redistributive taxation. Therefore, I introduce a tax rate that is marginally higher than zero. The main target of this analysis is to understand how the incentives of skilled and unskilled parents change and how these changes affect the skill ratio in the steady state, inequality and social mobility.
The introduction of a small tax rate $\tau$ results in post-tax wages

$$w^s_{\tau,t} \equiv w^s_\tau(\lambda_t) = (1-\tau)w^s_t + \tau\Big(\lambda_t w^s_t + (1-\lambda_t)w^n_t\Big) \tag{11}$$

and

$$w^n_{\tau,t} \equiv w^n_\tau(\lambda_t) = (1-\tau)w^n_t + \tau\Big(\lambda_t w^s_t + (1-\lambda_t)w^n_t\Big) \tag{12}$$

with $w^s_t$ and $w^n_t$ defined as in equations (1) and (2). Thus, redistributive taxation is just an unconditional transfer from the skilled to the unskilled. While this policy

increases the wage of unskilled workers it decreases the wage of skilled workers. This directly gives lemma 1.

LEMMA **1** *Subjective benefit from investment ($B_\tau$) is reduced by redistributive taxation. Subjective costs from investment are lowered for the unskilled ($C_\tau^n$) but raised for the skilled ($C_\tau^s$).*

Thus, for skilled parents investment incentives shrink due to increased costs and decreased benefit. However, for unskilled parents two counteracting effects appear. On the one hand investment becomes easier as a result of decreased costs (*cost effect*), on the other hand return of investment in human capital drops due to a lowered wage gap (*human capital effect*). Let $\hat{\lambda} \in (0, \gamma)$ be defined as the solution of

$$w^n(\lambda) - \frac{w^n(\lambda)^{\delta+1}}{w^s(\lambda)^\delta} = w_\tau^n(\lambda) - \frac{w_\tau^n(\lambda)^{\delta+1}}{w_\tau^s(\lambda)^\delta}, \tag{13}$$

i.e. $\hat{\lambda}$ is the skill ratio at which the indifference curves of the unskilled with and without redistributive taxation intersect.[7] At $\hat{\lambda}$ redistributive taxation has no influence on the investment incentives of the unskilled agents. Up to $\hat{\lambda}$ it raises investment incentives of the unskilled while for all $\lambda \in (\hat{\lambda}, \gamma)$ the reverse is true. The change in the indifference curve of the unskilled is illustrated in figure 3.

The change in the investment incentives directly implies that the downflow does not decrease due to redistributive taxation. The upflow never decreases if the pre-tax SSM satisfies $\lambda^* \in (0, \hat{\lambda})$ but it never increases if $\lambda^* \in (\hat{\lambda}, \gamma)$ holds. Thus, the social mobility upflow can only be raised due to redistributive taxation if the population before the policy intervention is characterized by a low skill ratio. Proposition 1 summarizes the results for the change in the skill ratio.

PROPOSITION **1** *The skill ratio decreases due to redistributive taxation in the case of a type I or IV SSM. It is unchanged in the case of a type III SSM.*

Proof: Let $\lambda^*$ denote the SSM before taxation while $\lambda_\tau^*$ is the stationary skill ratio after taxation. Focusing on a SSM of type I the downflow is increased by the

[7] Existence and uniqueness of $\hat{\lambda}$ are proofed by a single point of intersection of $C^n(\lambda) - C_\tau^n(\lambda)$ and $B(\lambda) - B_\tau(\lambda)$. While the difference in costs is a convex function in $\lambda$ the difference in benefit has a s-form, i.e. it is concave for $\lambda < \frac{1-2\gamma+2\tau\gamma}{2\tau}$ and convex otherwise. This directly results from the second order derivative. Considering, that at the beginning of the investment interval the cost difference is higher than the difference in benefit and that both differences are zero at the end of the investment interval, i.e. at $\lambda = \gamma$, both functions intersect at a unique $\hat{\lambda}$.

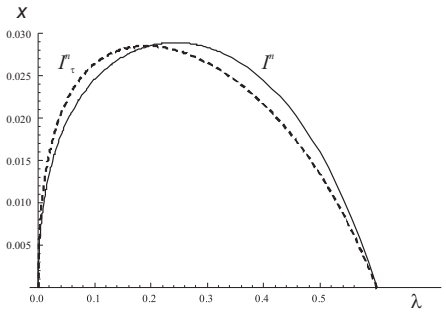


Figure 3: Indifference curves of the unskilled with and without redistributive taxation

policy while the upflow is not influenced. This implies that the downflow jumps to a positive value at $\lambda^*_\tau < \lambda^*$. Thus, the stationary skill ratio decreases. Note, at the pre-tax SSM $\lambda^*$ the downflow is higher than the upflow after introducing the policy. A SSM of type IV can appear when the upflow jumps from zero to a positive value, i.e. when $\lambda^* < \hat{\lambda}$ holds, or when the upflow jumps from a positive value to zero, i.e. when $\lambda^* > \hat{\lambda}$ holds. With redistributive taxation the upflow jumps to a positive value (and also back to zero) at a smaller skill ratio than without taxation. Therefore, $\lambda^*_\tau < \lambda^*$ holds, i.e. the stationary skill ratio is smaller with than without taxation. For a SSM of type III, i.e. a SSM where all agents have strict investment incentives, a small tax rate $\tau$ does not change up- and downflow. Therefore, stationary skill ratios before and after taxation are equal. The results for all cases are illustrated in figure 4. While solid lines illustrate mobility flows without taxation dashed lines illustrate mobility flows with redistributive policy. □

Thus, redistributive taxation is no recommendable policy to increase the skill ratio. However, the change in the skill ratio is not only interesting for itself but also determines the change in inequality. Lemma 2 describes the impact of a change in the skill ratio on the level of inequality.

Lemma **2** *A constant or increased skill ratio reduces intragenerational inequality while a decreased skill ratio lowers and maybe overcompensates the direct tax effect and therefore can increase inequality.*

Proof: For an unchanged skill ratio redistributive taxation increases the wage of the unskilled and decreases the wage of the skilled. Thus, for an unchanged skill ratio inequality is reduced. I call this effect *direct tax effect*. Since an increased skill ratio ceteris paribus leads to a reduction of the wage gap, the *direct tax effect* is

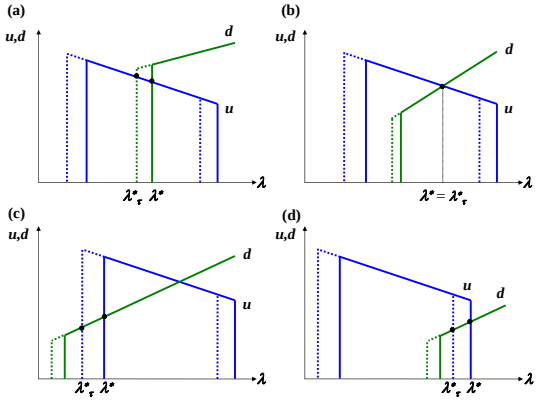


Figure 4: Change in skill ratio due to taxation for (a) SSM type I, (b) SSM type III, and (c) and (d) SSM type IV

enhanced if the skill ratio increases. Analogously, the *direct tax effect* is weakened and maybe overcompensated due to a decreased skill ratio.[8] □

Summing up, for a type I or IV SSM redistributive taxation may increase inequality while it is definitely reduced in case of a type III SSM that is characterized by strict investment incentives of all agents.

The second point of interest is the change in social mobility due to redistributive taxation.

PROPOSITION **2** *Due to redistributive taxation social mobility increases if the SSM is of type I while it decreases if the SSM is of type IV. For a SSM of type III social mobility is not influenced by redistributive taxation.*

Proof: For a SSM of type I the skill ratio is decreased by redistributive taxation (see prop. 1). This implies - since the upflow is a strictly decreasing function - an increased upward mobility and therefore in the SSM also an increased downward mobility. For a SSM of type IV the skill ratio also increases due to redistributive taxation (see prop. 1). However, the strictly decreasing downflow in this situation implies a reduction in the donward mobility and therefore in the SSM also a

[8]This *indirect tax effect* that occurs due to a change in the skill ratio is already mentioned by Dur and Teulings (2001) and also plays a crucial role when Konrad and Spadaro (2006) show that not only low-talented but also highly talented agents may support redistribution.

decrease in the upward mobility. For a SSM of type III the skill ratio does not change due to redistributive taxation (see prop. 1). Therefore up- and downflow do not change at $\lambda^*$. The results for the different types of SSM can also be inferred by figure 4.□

Summarizing, redistributive taxation is in general not a good policy to reduce inequality and increase social mobility. However, it reduces inequality - for a constant level of social mobility - if the pre-tax SSM is of type III and it increases social mobility - with an ambiguous effect on inequality - if the SSM is of type I.

# 4 Education subsidies

One may argue that redistributive taxation is in general no good policy intervention because it is an unconditional transfer. However, this section shows that education subsidies as a conditional transfer also can not decrease inequality and increase social mobility generally. In the analysis the focus again is on the marginal effect of subsidization and therefore a subsidization rate $\theta$ that is marginally higher than zero is assumed. Otherwise the number and types of SSMs could change completely.
Assume that education subsidies are available to all parents that choose to acquire education and are independent of the parent's occupation type. They are paid proportional to the education costs of the child and are financed by a linear income tax levied on the general public (similar to Bovenberg and Jacobs (2005)). In equilibrium the aggregate amount of subsidization $\Theta(\lambda)$ for an exogenous and small subsidization rate $\theta$ is

$$\Theta(\lambda) = \theta\lambda(\rho_l x^l + \alpha\rho_h x^h) + \theta(1-\lambda)\beta\rho_l x^l \tag{14}$$

Considering a tax rate $\tau_{\text{sub}}$ the aggregate tax amount in equilibrium is $\tau_{\text{sub}}(\lambda w^s + (1-\lambda)w^n)$.[9] Thus, for a subsidization rate $\theta$, that is exogenously assessed, the tax rate $\tau_{\text{sub}}$ is determined by the government's budget restriction as

$$\tau_{\text{sub}} \equiv \frac{\theta\lambda(\rho_l x^l + \alpha\rho_h x^h) + \theta(1-\lambda)\beta\rho_l x^l}{\lambda w^s + (1-\lambda)w^n}. \tag{15}$$

For the remaining analysis $\theta > \tau$ is assumed to hold. This assumption is necessary to foster investment incentives of the agents[10] and can be guaranteed for at least

[9] All parameters that refer to the case of education subsidies are indexed with 'sub'.

[10] Subjective benefit is not influenced by the policy (see lemma 3) and, easily checked again, only for $\tau_{\text{sub}} < \theta$ subjective costs are smaller with than without education subsidies.

the small exogenous subsidization rate.[11]

Considering the described policy intervention, equilibrium wages are

$$w^s_{\text{sub}} = (1 - \tau_{\text{sub}})w^s \quad \text{and} \quad w^n_{\text{sub}} = (1 - \tau_{\text{sub}})w^n \tag{16}$$

with $w^s$ and $w^n$ given as in equations (1) and (2). Replacing $w^s$ and $w^n$ in equations (4) and (5) by $w^s_{\text{sub}}$ and $w^n_{\text{sub}}$ gives subjective benefit and costs after subsidization.

LEMMA **3** *Subjective benefit from investment ($B_{\text{sub}}$) does not change due to education subsidies. Subjective costs of the skilled ($C^s_{\text{sub}}$) and unskilled ($C^n_{\text{sub}}$) are reduced.*[12]

The proof of the lemma directly results by the change in wages. Caused by the unchanged benefit and the reduced costs the investment incentives of all parents increase. Therefore, the upflow is never decreased due to the described policy while the downflow is never increased. The change in the mobility flows then gives proposition 3.

PROPOSITION **3** *Education subsidies that are financed by a linear income tax levied on the general public increase the skill ratio if the SSM is of type I and they do not change the skill ratio if the SSM is of type III. If the SSM is of type IV subsidization decreases the skill ratio in a low-skilled population, i.e. if $\lambda^* < \hat{\lambda}$ holds, but it increases the skill ratio in a high-skilled population, i.e. if $\lambda^* > \hat{\lambda}$ holds.*

Proof: A SSM of type I is characterized by the upward jump of the downflow. With subsidization the downflow jumps upward at a higher skill ratio than without policy intervention and therefore the skill ratio increases due to subsidization. For a SSM of type III up- and downflow do not change at $\lambda^*$. Thus, the skill ratio is not influenced. A SSM of type IV is characterized by the indifference in the investment incentives of the unskilled. Since investment incentives of the unskilled are increased due to subsidization the skill ratio at which the upflow jumps from

[11] Considering equation (15), $\rho_l x^l + \alpha\rho_h x^h \big( < (\rho_l + \alpha\rho_h)x^h \big) < w^s$ and $\beta\rho_l x^l < w^n$ are sufficient conditions to provide $\tau_{\text{sub}} < \theta$. As $w^s \geq (1-\theta)x^h$ and $w^n \geq (1-\theta)x^l$ are necessary conditions for investment we can rewrite the sufficient conditions as $(1-\theta) \geq \rho_l + \alpha\rho_h$ and $(1-\theta) \geq \beta\rho_l$. As the second condition always holds if the first condition is fulfilled and the right-hand side of the first condition is smaller than 1 there exists a $\theta > 0$ so that the first condition and therefore $\theta > \tau_{sub}$ holds.

[12] The fact that subjective benefit is not influenced by subsidization depends on the special form of the utility function. If the utility function is e.g. $u(c) = \frac{c^{1-\phi}}{1-\phi}$ subsidization causes an increase in the subjective benefit.

zero to a positive value is smaller with than without subsidization and the skill ratio at which the upflow jumps from a positive value to zero is higher with than without subsidization. Thus, if $\lambda^* < \hat{\lambda}$ holds education subsidies decrease the skill ratio while they increase the skill ratio if $\lambda^* > \hat{\lambda}$ holds. The changes in the skill ratio for all types of SSMs are illustrated in figure 5. Again, solid lines represent the case without policy intervention while dashed lines are the social mobility flows with education subsidies. □

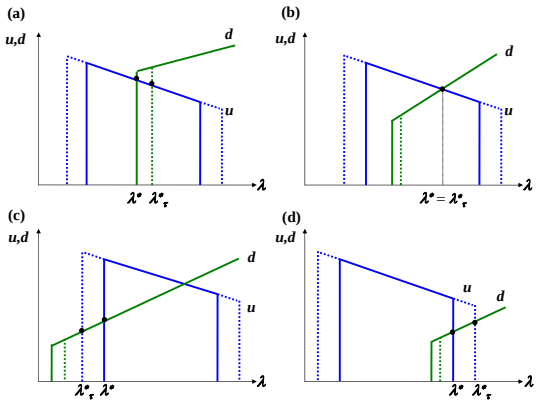


Figure 5: Change in skill ratio caused by education subsidies for (a) SSM type I, (b) SSM type III, and (c) and (d) SSM type IV

As wages $w^s_{\text{sub}}$ and $w^n_{\text{sub}}$ do not depend on the subsidization rate $\theta$ but only on the taxation rate $\tau_{\text{sub}}$ eudcation subsidies that are financed by a linear income tax affect inequality in the same way as redistributive taxation does. Thus, lemma 2 also holds for education subsidies. Summarizing, education subsidies are an appropriate policy to reduce inequality under most circumstances, i.e. if the SSM is of type I, III or in a high-skilled population of type IV. However, if the population is characterized by a low skill ratio and unskilled are indifferent in their investment decision with respect to the cost type $x^l$ the impact on the wage gap is ambiguous.

Focusing on the impact of education subsidies on social mobility I get the proposition 4.

PROPOSITION **4** *Due to education subsidies social mobility decreases if the SSM is of type I. It is not influenced if the SSM is of type III. If the SSM is of type IV social mobility is decreased by subsidization in a low-skilled population, i.e. if* $\lambda^* < \hat{\lambda}$ *holds, but it is increased in a high-skilled population, i.e. if* $\lambda^* > \hat{\lambda}$ *holds.*

Proof: A SSM of type I is characterized by the upward jump of the downward mobility flow. Since the skill ratio at which the downflow jumps upward is higher with than without education subsidies and the upward mobility flow is strictly decreasing social mobility decreases due to subsidization. For a SSM of type III both mobility flows are not influenced at $\lambda^*$. Thus, social mobility does not change due to the policy intervention. In a low-skilled population the skill ratio decreases for a SSM type IV (see proposition 3). Considering the increasing form of the downflow this implies a reduction in the social mobility. In a high-skilled population the skill ratio increases for a SSM type IV (see proposition 3). Therfeore, the increasing downflow results in an increase of the social mobility. All results can also be inferred by figure 5. □

Thus, education subsidies increase social mobility only in a high-skilled population where the unskilled are indifferent in their investment decision with respect to the cost type $x^l$.

Comparing both types of policy intervention, redistributive taxation and education subsidies affect inequality and social mobility in the same way when the SSM is of type III or the SSM is of type IV and $\lambda^* < \hat{\lambda}$ holds but they have different effects in all other cases. Redistributive taxation can in no case reduce inequality and at the time increase social mobility, However, education subsidies can achieve both targets simultaneously in a high-skilled population where the unskilled are indifferent in their investment decision having a child with low education costs.

# 5 Concluding remarks

This paper shows that neither redistributive taxation nor education subsidies are generally a recommendable policy to help poor children become educated when parents decide on the education of their children and wages are endogenous. The impact of both policy interventions on the skill ratio as well as on inequality and social mobility depends on the type of the SSM, i.e. on the investment decisions of all parents before any public intervention. Under some circumstances both policies can definitely reduce inequality and under other circumstances increase social mobility. However, only education subsidies can ensure both at the same time for at least one situation. Unfortunately, this situation requires that the population without policy intervention is already characterized by a high skill ratio. Thus one can say, the simultaneous improvement of intra- and intergenerational equity can only be reached in developed countries. Finally, although both policies should increase social mobility I provide conditions for both analyzed interventions under which social mobility is reduced due to public provisions. Only when the

government is aware of the specific situation can it implement the 'right' policy to raise the number of poor children that get educated. However, in a low-skilled population where unskilled parents are indifferent with respect to an investment in a child with low education costs (SSM of type IV) neither redistributive taxation nor education subsidies yield an increase in social mobility.
However, in the present model a child's future wage only depends on the education decision of its parents but not on the inherent talent of the child. Thus, this model does not consider a possible positive effect in aggregate productivity or growth if the most talented agents get educated (see e.g. Hassler and Rodríguez Mora (1998) and De Fraja (2002) for models that consider this effect). Additionally, the influence of the analyzed policies on social welfare could be an interesting starting point for further research. Nevertheless, such an approach seems to be complicated because the special forms of the utility and production functions do not allow to quantify the exact change in the skill ratio but at most to determine the direction of change.

# References


Banerjee, A. and A. Newman (1993). Occupational choice and the process of development. *Journal of Political Economy 101*(2), 274–298.

Becker, G. and N. Tomes (1979). An equilibrium theory of the distribution of income and intergenerational mobility. *Journal of Political Economy 87*(6), 1153–1189.

Blackburn, M. and D. Neumark (1993). Omitted-ability bias and the increase in the return to schooling. *Journal of Labor Economics 11*(3), 521–544.

Bovenberg, A. and B. Jacobs (2005). Redistribution and education subsidies are Siamese twins. *Journal of Public Economics 89*(11-12), 2005–2035.

De Fraja, G. (2002). The design of optimal education policies. *Review of Economic Studies 69*(239), 437–466.

De Fraja, G. (2005). Reverse discrimination and efficiency in education. *International Economic Review 46*(3), 1009–1031.

Dur, R. and C. Teulings (2001). Education and efficient redistribution. *CESifo Working Paper 592*.

Freeman, S. (1996). Equilibrium income inequality among identical agents. *Journal of Political Economy 104*(4), 1047–1064.

Galor, O. and J. Zeira (1993). Income distribution and macroeconomics. *Review of Economic Studies 60*(1), 35–52.

Hassler, J. and J. Rodríguez Mora (1998). IQ, social mobility and growth. *C.E.P.R. Discussion Papers 1827*.

Konrad, K. and A. Spadaro (2006). Education, redistributive taxation and confidence. *Journal of Public Economics 90*(1-2), 171–188.

Maoz, Y. and O. Moav (1999). Intergenerational mobility and the process of development. *Economic Journal 109*(458), 677–697.

Mincer, J. (1958). Investment in human capital and personal income distribution. *Journal of Political Economy 66*(4), 281–302.

Mookherjee, D. and S. Napel (2007). Intergenerational mobility and macroeconomic history dependence. *Journal of Economic Theory 137*(1), 49–78.

Mookherjee, D. and D. Ray (2003). Persistent inequality. *Review of Economic Studies 70*(2), 369–393.

Napel, S. and A. Schneider (2008). Intergenerational talent transmission, inequality, and social mobility. *Economics Letters 99*(2), 405–409.