Pricing and Information Disclosure in Markets with Loss-Averse Consumers *

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- Preliminary -

Abstract

Suppose consumers are loss-averse and fully informed about match value and price at the time they make their purchasing decision. A share of consumers is initially uncertain about their tastes and forms a reference point consisting of an expected match value and an expected price distribution, while the other consumers are perfectly informed all the time. In a duopoly with asymmetric firms, we show that firms' prices exhibit more price variation the larger the share of ex ante uninformed consumers. Furthermore, firms may price more aggressively in such a case. We also derive implications for firm strategy and public policy concerning the firms' incentives to "educate" consumers about their own tastes. In particular, we show that private incentives to disclose information early on may be socially insufficient. We show that whenever firms have conflicting incentives with respect to information disclosure, the more efficient firm gains from information disclosure.

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1 Introduction

Consumer information about price and match value of products is a key ingredient in determining market outcomes. Previous work has emphasized the role of consumer information at the moment of purchase.[1] If consumers are loss-averse information prior to the moment of purchase matters: Product information plays an important role at the stage at which loss-averse consumers form expectations about future transactions. Our analysis applies to inspection goods with the feature that consumers readily observe prices in the market but have to inspect products before knowing the match value between product characteristics and consumer tastes.

Loss-aversion in consumer choice has been widely documented in a variety of laboratory and field settings starting with Kahneman and Tversky (1979). Loss-averse consumers have to form expectations about product performance. We postulate that, to make their consumption choices, loss-averse consumers form their probabilistic reference point based on expected future transactions which are confirmed in equilibrium. Here, a consumer's reference point is her probabilistic belief about the relevant consumption outcome held between the time she first focused on the decision determining the consumption plan—i.e., when she heard about the products, was informed about the prices for the products on offer, and formed her expectations—and the moment she actually makes the purchase. [2]

We distinguish between "informed" and "uninformed" customers at the moment consumers form their reference point. Informed consumers know their taste ex ante and will perfectly foresee their equilibrium utility from product characteristics. Therefore they will not face a loss or gain in product satisfaction beyond their intrinsic valuation.

Uninformed consumers, by contrast, are uncertain about their ideal product characteristic: they form expectations about the difference between ideal and actual product characteristic which will serve as a reference point when evaluating a product along its taste or match value dimension. They will also face a gain or a loss relative to their expected distributions of price after learning the taste realization. Since all consumers become fully

[1] See e.g. Varian (1980), Janssen and Moraga-González (2004), and Armstrong and Chen (2008).

[2] For evidence that expectation-based counterfactuals can affect the individual's reaction to outcomes, see Breiter, Aharon, Kahneman, Dale, and Shizgal (2001), Medvec, Madey, and Gilovich (1995), and Mellers, Schwartz, and Ritov (1999). The general theory of expectation-based reference points and the notion of personal equilibrium have been developed by Koszegi and Rabin (2006) and Koszegi and Rabin (2007).

informed before they have to make their purchasing decision, we isolate the effect of consumer loss aversion on consumption choices and abstract from the effects of differential information at the moment of purchase.[3]

In this paper, we study the competitive effects of firm asymmetry and consumer loss aversion in duopoly markets. Consumers are loss-averse with respect to prices and match value and have rational expectations about equilibrium outcomes to form their reference point, as in Heidhues and Koszegi (2008). Firms are asymmetric due to deterministic cost differences and this is common knowledge among the firms when the game starts.[4] Firms compete in prices for differentiated products. Prices are deterministic and possibly asymmetric. Consumers observe equilibrium prices before forming their reference point. Note that if prices are asymmetric, uninformed consumers will face either a loss or a gain in the price dimension depending on which product they buy. Hence, an (ex ante) uninformed consumer's realized net utility depends not only on the price of the product she buys but also on the price of the product she does not buy.

Our theory applies to a number of inspection good industries in which some consumers form expectations before knowing the match value a particular product offers. Let us provide some examples. First, prices of clothing and electronic devices are easily accessible (and are often advertised) in advance while, for inexperienced consumers, the match quality between product and personal tastes is impossible or difficult to evaluate before actually seeing or touching the product. A related example is high-end hifi-equipment and, in particular, loudspeakers. Price tags are immediately observed but it may take several visits to the retailers (on appointment) or even trials at home to figure out the match value of the different products under consideration—for example, because people differ with respect to the sound they like. In these markets potential cost differences may arise from size differences of producers and product-specific costs (or, as we allow in our extension, from different ex ante observable quality differences). Second, the housing market has the feature that the price is listed (and, in some countries, not negotiable) whereas the match value is only found out after visiting the flat. Third, price information on products sold over the internet—for example, CDs of a particular classical concert—is immediately available, while the match value is often determined only after listening to some of the material that is provided online. Fourth, competing services such as long-distance bus rides and flights are differentiated by departure times. Here consumers are perfectly aware of the product characteristics ex ante—i.e.,price and departure time—but learn their pref-

[3] Our model can alternatively be interpreted as one in which consumers know their ideal taste ex ante but are exposed to uncertainty about product characteristics when they form their reference point.

[4] In the extension section we show that our analysis also applies to products of different qualities.

erence concerning their ideal point of departure only at some later stage (after forming their probabilistic reference point but before purchase).

Our first main result is that, in asymmetric markets, price variation is increased, relative to the scenario without loss-averse consumers. This is in stark contrast to the focal price result by Heidhues and Koszegi (2008).[5]

Our second main result is that loss aversion—or, more precisely, the presence of more ex ante uninformed, loss averse consumers—may lead to lower prices. Hence, the standard result that more informed consumers (or more consumers without a behavioral bias) lead to lower prices is challenged in our model when firms are strongly asymmetric. The driving force behind this result is that loss aversion in the price dimension has a pro-competitive effect while the effect of loss aversion in the taste dimension is anti-competitive.[6] The pro-competitive effect dominates the anti-competitive effect if the size of loss aversion in the price dimension becomes sufficiently large. This occurs if the price difference is large, which is caused by strong cost asymmetries. In this situation uninformed consumers are very reluctant to buy the expensive product and rather accept a large reduction in match value when buying the low-price product.

This paper contributes to the understanding of the effect of consumer loss aversion in market environments and is complementary to Heidhues and Koszegi (2008). More broadly, it contributes to the analysis of behavioral biases in market settings, as in Eliaz and Spiegler (2006), Gabaix and Laibson (2006), and Grubb (2008, forthcoming). An important issue in our paper, as also in Eliaz and Spiegler (2006), is the comparative statics effects in the composition of the population. However, whereas in their models this composition effect is behavioral in the sense that the share of consumers with a behavioral bias changes, we do not need to resort to this interpretation, although our analysis is compatible with it: We stress the composition effect to be informational in the sense that the arrival of information in the consumer population is changed (while the whole population is subject to the same behavioral bias).

The informational interpretation lends itself naturally to address questions about the effect of early information disclosure to additional consumers. We analyze information

[5]In a related setting to ours, Heidhues and Koszegi (2008) show that consumer loss aversion can explain the empirical observation that firms often charge the same price in differentiated product markets even if they have different costs. One of the distinguishing features of our model is that realized costs are public information and consumers observe prices before forming their reference point.

[6]Note that this is different from Heidhues and Koszegi (2008) where loss aversion has an anti-competitive effect in both dimensions.

disclosure policies by firms and public authorities in the context of a behavioral industrial organization framework. We thus demonstrate the possible use of behavioral models to address policy questions in industrial organization. As stated above, our model has the feature that, absent behavioral bias, information disclosure policies are meaningless. Thus the behavioral bias is essential in our model to address these issues. In particular, we show that private and social incentives to disclose information early on are not aligned. We also show that the more efficient and thus larger firm discloses information if firms have conflicting interests.

Our analysis contributes to the literature on the economics of advertising (see Bagwell (2007) for an excellent survey). It uncovers the role of advertising as consumer expectation management. Note that at the point of purchase consumers are fully informed so that there is no role for informative advertising. However, since consumers are loss-averse, educating consumers about their preferences or, alternatively, about product characteristics, makes these consumers informed in our terminology. Advertising thus can remove the uncertainty consumers face when forming their reference point. This form of advertising can be seen as a hybrid form of informative and persuasive advertising because it changes preferences at the point of purchase—this corresponds to the persuasive view of advertising—, albeit due to information that is received ex ante—this corresponds to the informative view of advertising. It also points to the importance of the timing of advertising: for expectation management it is important to inform consumers early on.

Other marketing activities can also be understood as making consumers informed at the stage when consumers form their reference point. For instance, test drives for cars or lending out furniture, stereo equipment, and the like make consumers informed early on. Arguably, in reality uncertainty would otherwise not be fully resolved even at the purchasing stage. However, to focus our minds, we only consider the role of marketing activities on expectation formation before purchase. In short, in our model firms may use marketing to manage expectations of loss-averse consumers at an early stage.[7]

Our paper can be seen as complementary to the work on consumer search in product markets (see e.g. Varian (1980), Anderson and Renault (2000), Janssen and Moraga-González (2004), Armstrong and Chen (2008)). Whereas that literature focuses on the effect of differential information (and consumer search) at the purchasing stage, our paper abstracts from this issue and focuses on the effect of differential information at the expectation formation stage which is relevant if consumers are loss aversion.

[7] For a complementary view see Bar-Isaac, Caruana, and Cunat (2007).

We will discuss the connections to a number of the above cited contributions in more detail in the main text. The plan of the paper is as follows. In Section 2, we present the model. Here, we have to spend some effort to determine the demand of uninformed consumers. In Section 3, we establish equilibrium uniqueness and equilibrium existence. Our existence proof requires to bound the parameters of our model, in particular, the two firms cannot be too asymmetric for equilibrium existence to hold. In Section 4, we obtain comparative statics results. First, we characterize equilibrium under cost symmetry and, secondly, analyze the impact of the degree of asymmetry on equilibrium outcomes. Thirdly and most importantly, we analyze the effect changing the share of ex ante informed consumers on market outcomes. In Section 5 we provide two extensions. Section 6 concludes.

2 The Model

2.1 Setup

Consider a market with two asymmetric firms, A and B, and a continuum of loss-averse consumers of mass 1. The firms' asymmetry consists of differences in marginal costs. Here, the more efficient firm is labeled to be firm A—i.e., $c_A \leq c_B$. Firms are located on a circle of length 2 with maximum distance, $y_A = 0$, $y_B = 1$. Firms announce prices p_A and p_B and product locations to all consumers. Consumers of mass one are uniformly distributed on the circle of length 2. A consumer's location x, $x \in [0, 2)$, represents her taste parameter. Her taste is initially, i.e., before determining her reference point, known only to herself if she belongs to the set of informed consumers. Note that consumers' differential information here applies to the date at which consumers determine their reference point and not to the date of purchase: at the moment of purchase all consumers are perfectly informed about product characteristics, prices, and tastes. However, a fraction $(1 - \beta)$ of loss-averse consumers, $0 \leq \beta \leq 1$, is initially uninformed about their taste. As will be detailed below, they endogenously determine their reference point and then, before making their purchasing decision, observe their taste parameter (which is private information of each consumer). All consumers have reservation value v for an ideal variety and have unit demand. Their utility from not buying is $-\infty$ so that the market is fully covered.

Two remarks about our modeling choice are in order: First, we could alternatively work with the Hotelling line. Results directly carry over to the Hotelling model in which consumers are uniformly distributed on the $[0, 1]$-interval. Second, the circle model allows for an alternative and equivalent interpretation about the type of information some consumers initially lack: at the point in time consumers form their reference point distribution, they all know their taste parameters but only a fraction $(1-\beta)$ does not know the location of the high- and the low-cost firm. These uninformed consumers only know that the two firms are located at maximal distance and that one is a high- whereas the other is a low-cost firm.

To determine the market demand faced by the two firms, let the informed consumer type in $[0,1]$ who is indifferent between buying good A and good B be denoted by $\hat{x}_{in}(p_A, p_B)$. Correspondingly, the indifferent uninformed consumer is denoted by $\hat{x}_{un}(p_A, p_B)$. Since market shares on $[0,1]$ and $[1,2]$ are symmetric, the firms' profits are:

$$\pi_A(p_A, p_B) = (p_A - c_A)[\beta \cdot \hat{x}_{in}(p_A, p_B) + (1-\beta) \cdot \hat{x}_{un}(p_A, p_B)]$$
$$\pi_B(p_A, p_B) = (p_B - c_B)[\beta \cdot (1 - \hat{x}_{in}(p_A, p_B)) + (1-\beta) \cdot (1 - \hat{x}_{un}(p_A, p_B))].$$

The timing of events is as follows:

Stage 0.) Marginal costs (c_A, c_B) realize (and become common knowledge among firms)

Stage 1.) Firms simultaneously set prices (p_A, p_B)

Stage 2.) All consumers observe prices and

a) informed consumers observe their taste x (for them uncertainty is resolved)

b) uninformed consumers form reference point distributions over purchase price and match value, as detailed below

Stage 3.) Inspection stage: Entering the shop also uninformed consumers observe their taste x (uncertainty is resolved for **all** consumers)

Stage 4.) Purchase stage: Consumers decide which product to buy:

a) informed consumers make rational purchase decision ($\equiv$ benchmark case)

b) uninformed consumers compare price and match value with reference point distributions

At stage 1 we solve for subgame perfect Nash equilibrium, where firms foresee that uninformed consumers play a personal equilibrium at stage 2b. Personal equilibrium in our context simply means that consumers hold rational expectation about their final purchasing decision; for the general formalization see Koszegi and Rabin (2006). Without loss of generality we consider realizations $c_A \leq c_B$.

2.2 Demand of informed consumers

Let us first consider informed consumers. They ex ante observe prices and their taste parameter and therefore do not face any uncertainty when forming their reference point. Hence, their behavior is the same as the behavior of unboundedly rational consumers in a classical Salop model. For prices p_A and p_B an informed consumer located at x obtains the following indirect utility from buying product i

$$u_i(x, p_i) = v - t|y_i - x| - p_i,$$

where t scales the disutility from distance between ideal and actual taste on the circle. The expression $v - t|y_i - x|$ then captures the match value of product i for consumer of type x. Denote the indifferent (informed) consumer between buying from firm A and B on the first half of the circle by $\hat{x}_{in} \in [0, 1]$ and solve for her location given prices. The informed indifferent consumer is given by

$$\hat{x}_{in}(p_A, p_B) = \frac{(t + p_B - p_A)}{2t}. \tag{1}$$

Symmetrically, a second indifferent (informed) consumer type is located at $2-\hat{x}_{in}(p_A, p_B) \in [1, 2]$. Without loss of generality we focus on demand of consumers between 0 and 1 and multiply by 2. Cost differences influence the location of indifferent consumers via prices: If asymmetric costs lead to asymmetric prices in equilibrium, then the indifferent informed consumer will also be located apart from $1/2$ (resp. $3/2$), the middle between A and B.[8]

2.3 Demand of uninformed consumers

Uninformed consumers do not know their ideal taste x ex ante. Since they cannot judge which product they will buy before they inspect products and learn their ideal taste x, they ex ante face uncertainty about their match value and purchase price (although they know firms' prices already). With regard to this uncertainty uninformed consumers form reference point distributions over match value and purchase price. Following Heidhues and Koszegi (2008) they will experience gains or losses in equilibrium depending on their realized taste and their purchase decision. These gains and losses occur in two dimensions, in a taste dimension (as determined by the fit between idiosyncratic taste and product

[8]E.g. if there are only informed consumers, $\hat{x}_{in} = 1/2 + (c_B - c_A)/(6t)$ in equilibrium . This is closer to B for $c_B > c_A$. Thus, the low-cost firm serves a larger market share.

characteristics) and in a price dimension. In both dimensions losses are evaluated at a rate λ and gains at a rate 1 with $\lambda > 1$. This reflects widespread experimental evidence that losses are evaluated more negatively than gains. Three properties of this specification are worthwhile pointing out. First, consumers have gains or losses not about net utilities but about each product "characteristic", where price is then treated as a product characteristic. This is in line with much of the experimental evidence on the endowment effect; for a discussion see e.g. Koszegi and Rabin (2006). Second, consumers evaluate gains and losses *across* products.[9] This appears to be a natural property for consumers facing a discrete choice problem: they have to compare the merits of the two products to each other. In other words, consumers view the purchasing decision with respect to these two problems as a single decision problem. Third, to reduce the number of parameters, we assume that the gain/loss parameters are the same across dimensions. This appears to be the natural benchmark.

While our setting is related to Heidhues and Koszegi (2008) (see also Heidhues and Koszegi (2005) for a related monopoly model) our model has three distinguishing features. First, firms' deterministic costs are known by their competitor. This property is in line with a large part of the industrial organization literature on imperfect competition and is approximately satisfied in markets in which firms are well-informed not only about their own costs but also about their relative position in the market. Second, prices are already set before consumers form their reference point.[10] This property applies to markets in which consumers are from the start well-informed about the price distribution they face in the market. This holds in markets in which firms inform consumers about prices (but consumers are initially uncertain about the match value and thus their eventual purchasing decision) or in which prices are publicly posted.[11] Third, there is a fraction of $(1-\beta)$ of uninformed consumers who face uncertainty about their ideal taste x and a fraction of β informed consumers who know their ideal taste ex ante. As motivated in the introduction, various justifications for differential information at the ex ante stage can be given. Consumers differ by their experience concerning the relevant product feature. Alternatively, a share of consumers know that they will be subject to a taste shock between forming

[9] Gains and losses also matter in the price dimension because, even though prices are deterministic, they are different across firms. Hence, a consumer who initially does not know her taste parameter is uncertain at this point in time about the price at which she will buy.

[10] This is particularly appropriate in market environments in which price information has been provided from the outset, while uninformed (or inexperienced) consumers observe the match value only when physically or virtually inspecting the product.

[11] Note that in an asymmetric market firms set different prices. Hence, although prices are deterministic, a consumer who does not know her taste parameter is uncertain about the price she will pay for her preferred product.

their reference point and making their purchasing decision. These consumers then do not condition their reference point on the ex ante taste parameter, whereas those belonging to the remaining share do.

Consider an uninformed consumer who will be located at x after her ideal taste is realized. Suppose firms set prices p_A and p_B in equilibrium. Then the uninformed consumer will buy from firm A if $x \in [0, \hat{x}_{un}(p_A, p_B)] \cup [2 - \hat{x}_{un}(p_A, p_B), 2]$, where $\hat{x}_{un}(p_A, p_B)$ is the location of the indifferent (uninformed) consumer we want to characterize. Hence, the uninformed consumer at x will pay p_A in equilibrium with $Prob[x < \hat{x}_{un}(p_A, p_B) \vee x > 2 - \hat{x}_{un}(p_A, p_B)]$ and p_B with $Prob[\hat{x}_{un}(p_A, p_B) < x < 2 - \hat{x}_{un}(p_A, p_B)]$. Since x is uniformly distributed on $[0, 2]$ we obtain that $Prob[x < \hat{x}_{un}(p_A, p_B) \vee x > 2 - \hat{x}_{un}(p_A, p_B)] = \hat{x}_{un}(p_A, p_B)$, i.e., from an ex ante perspective p_A is the relevant price with probability $\text{Prob}[p = p_A] = \hat{x}_{un}$. Correspondingly, the purchase at price p_B occurs with probability $\text{Prob}[p = p_B] = 1 - \hat{x}_{un}$.

The reference point with respect to the match value is the reservation value v minus the expected distance between ideal and actual product taste times the taste parameter t. The distribution of the expected distance is denoted by $G(s) = Prob(|x - y_\sigma| \leq s)$, where $s \in [0, 1]$, the location of the firm $y_\sigma \in \{0, 1\}$, and the consumer x's purchase strategy in equilibrium for given prices is denoted by $\sigma \in \{A, B\}$, $\sigma \in \arg\max_{j \in \{A,B\}} u_j(x, p_j, p_{-j})$.

Since $c_A \leq c_B$, we restrict attention to the case $\hat{x}_{un} \geq 1/2$, i.e., firm A has a weakly larger market share than firm B also for uninformed consumers. Given that some uninformed consumers will not buy from their nearest firm, $G(s)$ will be kinked. This kink is determined by the maximum distance $|x - y_B|$that consumers are willing to accept buying product B, $s = 1 - \hat{x}_{un}$ because $s \leq 1 - \hat{x}_{un}$ holds for consumers close to either A or B, while $s > 1 - \hat{x}_{un}$ only holds for the more distant consumers of A. Hence, the distribution of s is

$$G(s) = \begin{cases} 2s & \text{if } s \in [0, 1 - \hat{x}_{un}] \\ s + (1 - \hat{x}_{un}) & \text{if } s \in (1 - \hat{x}_{un}, \hat{x}_{un}] \\ 1 & \text{otherwise.} \end{cases}$$

Note that if the indifferent uninformed consumer is located in the middle between A and B, $\hat{x}_{un} = 1/2$, the expected distance between ideal and actual product taste, $\mathbb{E}[s]$, is minimized and equal to $1/4$.

Following Koszegi and Rabin (2006), after uncertainty is resolved consumers experience a gain-loss utility: the reference distribution is split up for each dimension at the value of realization in a loss part with weight $\lambda > 1$ and a gain part with weight 1. In the loss part the realized value is compared to the lower tail of the reference distribution; in the gain part it is compared to the upper tail of the reference distribution.

Consider the gain-loss utility of an uninformed consumer located at x, at the moment she decides whether to purchase the product. Recall that at this point she knows her taste parameter x. The initially uninformed consumer now decides which product to buy taking into account her intrinsic utility from a product and her gain-loss utility when she compares the price-taste combination of a product with her two-dimensional reference point distribution.

First, consider the utility of an uninformed consumer from a purchase of product A when this consumer is located at $x \in (1 - \hat{x}_{un}, 1]$.[12]

$$\begin{aligned} u_A(x, p_A, p_B) \quad =&(v - tx - p_A) - \lambda \cdot \text{Prob}[p = p_A](p_A - p_A) + \text{Prob}[p = p_B](p_B - p_A) \\ &- \lambda \cdot t \int_0^x (x - s)dG(s) + t \int_x^1 (s - x)dG(s), \end{aligned} \tag{2}$$

where the first term is the consumer's intrinsic utility from product A. The second term is the loss in the price dimension from not facing a lower price than p_A. This term is equal to zero because p_A is the lowest price offered in the market place. The third term is the gain from not facing higher price than p_A, which is positive. The last two terms correspond to the loss (gain) from not facing a smaller (larger) distance in the taste dimension than x. An uninformed consumer's utility from a purchase of product B is derived analogously,

$$\begin{aligned} u_B(x, p_A, p_B) \quad =& \underbrace{v - t(1 - x) - p_B}_{\text{Intrinsic utility}} \underbrace{- \lambda \cdot \text{Prob}[p = p_A](p_B - p_A)}_{\text{Loss from facing a higher } p \text{ than } p_A} \\ & \underbrace{- \lambda \cdot t \int_0^{1-x} ((1 - x) - s)dG(s)}_{\text{Loss from facing larger distance than 0}} + \underbrace{t \int_{1-x}^1 (s - (1 - x))dG(s)}_{\text{Gain from facing smaller distance than 1}} \end{aligned} \tag{3}$$

This allows us to determine the location of the indifferent uninformed consumer $\hat{x}_{un}$.

[12]The indifferent uninformed consumer will be located at $x = \hat{x}_{un}$, therefore $(1 - \hat{x}_{un}, 1]$ is the relevant interval for determining $\hat{x}_{un}$.

Lemma 1. *Suppose that* $\hat{x}_{un} \in [1/2, 1)$. *Then* $\hat{x}_{un}$ *is given by*

$$\hat{x}_{un}(\Delta p) = \frac{\lambda}{(\lambda-1)} - \frac{\Delta p}{4t} - \underbrace{\sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\Delta p + \frac{(\lambda+1)^2}{4(\lambda-1)^2}}}_{\equiv S(\Delta p)}. \tag{4}$$

where $\Delta p \equiv p_B - p_A$.

Proof. Using the properties of the reference distributions, we rewrite the utility function further,

$$\begin{aligned} u_A(x, p_A, p_B) =& (v - tx - p_A) + (1 - \hat{x}_{un})(p_B - p_A) \\ & - \lambda \cdot t\Big(\int_0^{1-\hat{x}_{un}} 2(x-s)\,ds + \int_{1-\hat{x}_{un}}^{x} (x-s)\,ds\Big) + t\Big(\int_x^{\hat{x}_{un}} (s-x)\,ds\Big) \\ =& (v - tx - p_A) + (1 - \hat{x}_{un})(p_B - p_A) \\ & - \lambda \cdot \frac{t}{2}\Big(x^2 + 2x(1-\hat{x}_{un}) - (1-\hat{x}_{un})^2\Big) + \frac{t}{2}(\hat{x}_{un} - x)^2 \end{aligned} \tag{5}$$

$$\begin{aligned} u_B(x, p_A, p_B) =& (v - t(1-x) - p_B) - \lambda \cdot \hat{x}_{un}(p_B - p_A) - \lambda \cdot t \int_0^{1-x} 2((1-x) - s)\,ds \\ & + t\Big(\int_{1-x}^{1-\hat{x}_{un}} 2(s - (1-x))\,ds + \int_{1-\hat{x}_{un}}^{\hat{x}_{un}} (s - (1-x))\,ds\Big) \\ =& (v - t(1-x) - p_B) - \lambda \cdot \hat{x}_{un}(p_B - p_A) - \lambda \cdot t(1-x)^2 \\ & + t\Big((x - \hat{x}_{un})^2 + (\frac{1}{2} - x - \hat{x}_{un} + 2x\hat{x}_{un})\Big). \end{aligned} \tag{6}$$

Next, we find the location of the indifferent uninformed consumer $x = \hat{x}_{un}$ by setting $u_A = u_B$, where

$$\begin{aligned} u_A(\hat{x}_{un}, p_A, p_B) =& \; v - t\hat{x}_{un} - p_A + (1-\hat{x}_{un})(p_B - p_A) - \lambda \cdot \frac{t}{2}\Big(1 - 2(1-\hat{x}_{un})^2\Big) \\ u_B(\hat{x}_{un}, p_A, p_B) =& \; v - t(1-\hat{x}_{un}) - p_B - \lambda \cdot \hat{x}_{un}(p_B - p_A) - \lambda \cdot t(1-\hat{x}_{un})^2 + 2t(\frac{1}{2} - \hat{x}_{un})^2 \end{aligned}$$

If she buys product A the indifferent uninformed consumer will experience no gain but the maximum loss in the taste dimension. If she buys product B she will experience a gain and a loss because distance could have been smaller or larger than $1 - \hat{x}_{un}$. With respect to the price dimension the indifferent uninformed consumer (like all other consumers) faces only a loss when paying price p_B and only a gain when paying price p_A.

$u_A(\hat{x}_{un}, p_A, p_B) = u_B(\hat{x}_{un}, p_A, p_B)$ can be transformed to the following quadratic equation in $\hat{x}_{un}$,

$$0 = \quad 2t(\lambda - 1) \cdot \hat{x}_{un}^2 - \Big((\lambda - 1)(p_B - p_A) - 4t\lambda\Big) \cdot \hat{x}_{un} + \Big(2(p_B - p_A) + \frac{t}{2}(3\lambda + 1)\Big) \tag{7}$$

Solving this quadratic equation w.r.t. $\hat{x}_{un}$ leads to the expression given in the lemma. □

The square root, $S(\Delta p)$, is defined for $\Delta p \in [0, \Delta\overline{p}]$ with

$$\Delta\overline{p} \equiv \frac{2t}{(\lambda - 1)}\Big(2(\lambda + 2) - \sqrt{(2(\lambda + 2))^2 - (\lambda + 1)^2}\Big), \tag{8}$$

which is strictly positive for all $\lambda > 1$. It can be shown that for $\lambda \geq 3 + 2\sqrt{5} \approx 7.47$, $\hat{x}_{un}(\Delta p) \in [1/2, 1]$ for all $\Delta p \in [0, \Delta\overline{p}]$. Given monotonicity $\hat{x}_{un}(\Delta\overline{p})$ expresses the upper bound on firm A's demand from uninformed consumers for $\beta = 0$. If the degree of loss aversion is smaller, $\lambda < 3 + 2\sqrt{5}$, $\hat{x}_{un}(\Delta\overline{p})$ rises above one. Hence, we define another upper bound on the price difference, $\Delta\tilde{p}$, with $\Delta\tilde{p} < \Delta\overline{p}$ by the solution to $\hat{x}_{un}(\Delta\tilde{p}) = 1$. We can solve explicitly,

$$\Delta\tilde{p} = \frac{(\lambda + 3)t}{2(\lambda + 1)}. \tag{9}$$

The location of the indifferent uninformed consumer, $\hat{x}_{un}$, has a number of properties. Clearly, $\hat{x}_{un}(0) = 1/2$, i.e. market splits equally under symmetric prices. Another obvious property is that $\hat{x}_{un}(\Delta p)$ is equal to the demand of firm A if only a measure zero set of consumers is informed, i.e. $\beta = 0$.

It can be shown that the first derivative of $\hat{x}_{un}(\Delta p)$ with respect to Δp, $\hat{x}'_{un}(\Delta p)$, is strictly positive for all $\Delta p \in [0, \Delta\overline{p}]$:

$$\hat{x}'_{un}(\Delta p) = -\frac{1}{4t} - \frac{1}{2 \cdot S(\Delta p)} \cdot \left(\frac{\Delta p}{8t^2} - \frac{(\lambda + 2)}{2t(\lambda - 1)}\right)$$

At $\Delta p = 0$ the first derivative of $\hat{x}_{un}(\Delta p)$ is equal to

$$\hat{x}'_{un}(0) = -\frac{1}{4t} + \frac{(\lambda + 2)}{2t(\lambda + 1)}.$$

$\hat{x}'_{un}(0)$ is approaching $1/(2t)$ from below for $\lambda \to 1$ and $1/(4t)$ from above for $\lambda \to \infty$. This implies that, evaluated at $\Delta p = 0$, demand of uninformed consumers reacts less sensitive to price changes than demand of uninformed consumers—we return to this property in the following section. Moreover, $\hat{x}_{un}(\Delta p)$ is strictly convex for all $\Delta p \in [0, \Delta\overline{p}]$.

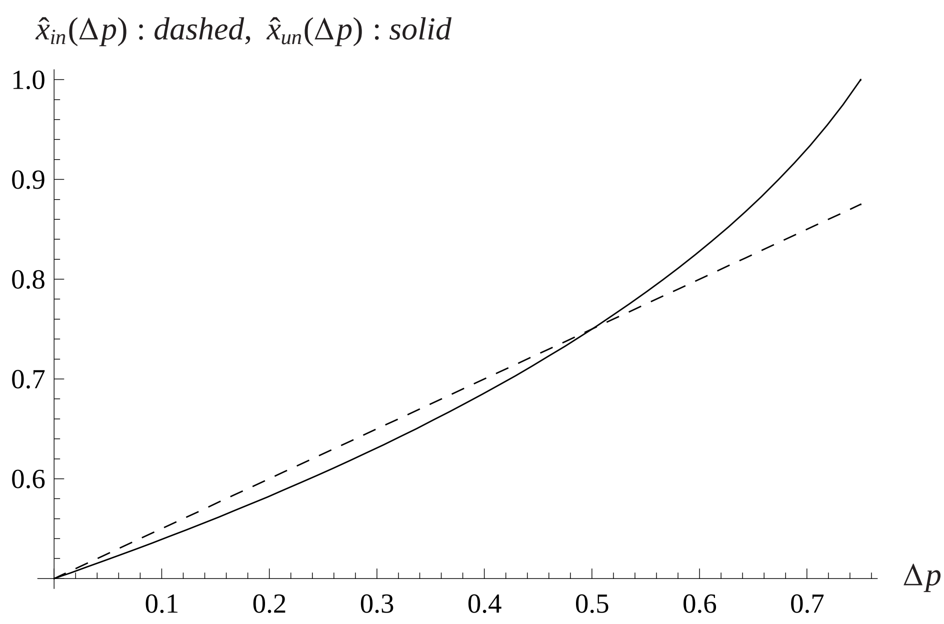
$$\hat{x}''_{un}(\Delta p) = \frac{(3+\lambda)(5+3\lambda)}{64t^2 \cdot (S(\Delta p))^3} > 0$$

Finally, it can be shown that the level of convexity of $\hat{x}_{un}(\Delta p)$ is strictly increasing in λ.

2.4 Demand comparison between informed and uninformed consumers

In this subsection we establish a number of properties when comparing market demand for uninformed relative to informed consumers, i.e. we compare $\hat{x}_{un}(\Delta p)$ and $\hat{x}_{in}(\Delta p)$ with one another.

The first property is a continuity property. For $\lambda \to 1$, the indirect utility function of uninformed consumers differs from the one of informed consumers only by a constant (this can be called a level effect). Equation (7) collapses to a linear equation and we receive $\hat{x}_{un}(\Delta p) = \hat{x}_{in}(\Delta p)$ as a solution in this case. This means that if consumers put equal weights on gains and losses, the effect of comparing expectations with realized values exactly cancels out when a choice between two products is made.



The Figureshows the location of the indifferent consumer (= demand of firm A) for informed and uninformed consumers as a function of price difference Δp for parameter values of $t = 1$ and $\lambda = 3$: $\Delta\overline{p} = 0.8348$, $\Delta\tilde{p} = 3/4$ and $\Delta\hat{p} = 0.2789$.

Figure 1: Demand of informed and uninformed consumers

The next properties refer to the sensitivity of demand with respect to price. The first derivative of $\hat{x}_{in}(\Delta p)$ w.r.t. Δp is equal to $1/(2t)$ for all Δp. Therefore $\hat{x}'_{in}(0)$ is strictly larger than $\hat{x}'_{un}(0)$. This implies that the demand of uninformed consumers, evaluated at equal prices reacts less sensitive to price changes than the demand of informed consumers.

Evaluated at large price differences, this relationship is possibly reversed: for $\Delta p \rightarrow \Delta \overline{p}$ the square root, $S(\Delta p)$, becomes zero and $\hat{x}'_{un}(\Delta p)$ rises to infinity. Thus, $\hat{x}'_{un}(\Delta \overline{p}) > \hat{x}'_{in}(\Delta \overline{p}) = 1/(2t)$. Demand of uninformed consumers, evaluated at a large price difference reacts more sensitive to an increase in the price difference than the demand of informed consumers. (This property is satisfied if the indifferent consumer at this price difference is strictly interior; otherwise some more care is needed, as is done in the following section.)

Due to monotonicity of $\hat{x}'_{un}(\Delta p)$ and applying the mean value theorem, there exists an intermediate price difference $\Delta \hat{p} \in [0, \Delta \overline{p}]$ such that $\hat{x}'_{un}(\Delta \hat{p}) = \hat{x}'_{in}(\Delta \hat{p}) = 1/(2t)$. This critical price difference can be explicitly calculated as

$$\Delta \hat{p} = \frac{t\left(2\sqrt{2} \cdot (2(\lambda+2)) - 3 \cdot \sqrt{(2(\lambda+2))^2 - (\lambda+1)^2}\right)}{\sqrt{2}(\lambda-1)},$$

which is strictly positive for all $\lambda > 1$ since $\Delta \hat{p}(\lambda = 1) = 0$ and $\Delta \hat{p}'(\lambda) > 0$.

Hence, we find that the demand of uninformed (or loss-averse) consumers is less price sensitive than the demand of informed consumers if price differences are small, $\Delta p < \Delta \hat{p}$. The underlying intuition is that for small price differences loss-averse consumers are harder to attract by price cuts because their gain from lower prices is outweighed by their loss in the taste dimension if they change producers. Thus, demand of loss-averse consumers reacts less sensitive to price in this range. For large price differences, however, their gain from lower prices starts to dominate their loss in the taste dimension if consumers switch to the cheaper producer. Therefore loss-averse consumers are more price-sensitive than informed (or classical Hotelling) consumers for $\Delta p > \Delta \hat{p}$. In section 4 it becomes apparent that this demand characteristic is a driving force for our comparative static results.

3 Market Equilibrium

In this section we focus on the market equilibrium of the firms' price-setting game. We derive market conditions under which equilibrium exists and under which it is unique. We start by showing some properties of market demand which will be needed later to prove some of the results. We then give an equilibrium characterization before turning to uniqueness and existence.

3.1 Properties of market demand

For notational convenience we first define an upper bound for the price difference (which depends on the parameters t and λ):

$$\Delta p^{max} \equiv \begin{cases} \Delta\tilde{p}, & \text{if } 1 < \lambda \leq \lambda^c; \\ \Delta\overline{p}, & \text{if } \lambda > \lambda^c. \end{cases} \tag{10}$$

with $\lambda^c \equiv 3 + 2\sqrt{5} \approx 7.47$. Note that $\Delta\tilde{p} \in [t \cdot (\sqrt{5} - 1)/2, t) \approx [0.618t, t)$ for $1 < \lambda \leq \lambda^c$ and $\Delta\overline{p} \in (t \cdot 2(\sqrt{3} - 2), t \cdot (\sqrt{5} - 1)/2) \approx (0.536t, 0.618t)$ for $\lambda > \lambda^c$. Using results from Section 2.4, we define the upper bound of firm A's demand of uninformed consumers as[13]

$$\hat{x}_{un}(\Delta p^{max}) \equiv \begin{cases} \hat{x}_{un}(\Delta\tilde{p}) = 1, & \text{if } 1 < \lambda \leq \lambda^c, \\ \hat{x}_{un}(\Delta\overline{p}) < 1, & \text{if } \lambda > \lambda^c. \end{cases} \tag{11}$$

Combining (1) and (4), we obtain the market demand of firm A as the weighted sum of the demand by informed and uninformed consumers,

$$\begin{aligned} q_A(\Delta p; \beta) &= \beta \cdot \hat{x}_{in}(\Delta p) + (1 - \beta) \cdot \begin{cases} \hat{x}_{un}(\Delta p), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ 1, & \text{if } t \geq \Delta p \geq \Delta p^{max} \end{cases} \\ &= \begin{cases} \frac{1}{2} - \frac{1}{4t}(1 - 3\beta)\Delta p + (1 - \beta)\frac{(\lambda+1)}{2(\lambda-1)} - (1 - \beta)S(\Delta p), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ \beta \cdot \frac{t+\Delta p}{2t} + (1 - \beta), & \text{if } t \geq \Delta p \geq \Delta p^{max} \end{cases} \\ &\equiv \begin{cases} \phi(\Delta p; \beta), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ \beta \cdot \frac{t+\Delta p}{2t} + (1 - \beta), & \text{if } t \geq \Delta p \geq \Delta p^{max} \end{cases} \end{aligned} \tag{12}$$

where

$$S(\Delta p) = \sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\lambda + 2)}{2t(\lambda - 1)}\Delta p + \frac{(\lambda + 1)^2}{4(\lambda - 1)^2}}.$$

The demand of firm A is a function in the price difference Δp, which is kinked at Δp^{max} and for $\Delta p^{max} = \Delta\overline{p}$ additionally discontinuous at Δp^{max}. It approaches one for $\Delta p = t$.[14] Firm B's demand is determined analogously by $q_B(\Delta p; \beta) = 1 - q_A(\Delta p; \beta)$. In the following

[13] $\hat{x}_{un}(\Delta\overline{p}) = \frac{\lambda}{\lambda-1} - \frac{2(\lambda+2) - \sqrt{4(\lambda+2)^2 - (\lambda+1)^2}}{2(\lambda-1)} \in (\sqrt{3}/2, 1)$ for $\lambda > \lambda^c$, i.e. $\hat{x}_{un}(\Delta\overline{p})$ is lower than one for $\lambda > \lambda^c$. This leads to a jump in demand of uninformed consumers at $\Delta\overline{p}$ from $\hat{x}_{un}(\Delta\overline{p})$ to one (see the definition of $q_A(\Delta p; \beta)$), as $\hat{x}'_{un}(\Delta\overline{p}) \to \infty$.

[14] At $\Delta p = t$ firm A serves also all distant informed consumers which are harder to attract than distant uninformed consumers because the latter face a loss in the price dimension if buying from the more expensive firm B. For $\Delta p > t$ demand of firm A shows a second kink. This region we ignore since we are interested in cases in which both firms face a positive demand.

we are interested in interior equilibria in which products are bought by a positive share of uninformed consumers, i.e. Δp is lower than Δp^{max}.[15] We next state properties of $\phi(\Delta p;\beta)$, the demand of firm A in this case:[16]

Lemma 2. *For $0 \leq \Delta p < \Delta p^{max}$, the demand of firm A, $q_A(\Delta p;\beta) = \phi(\Delta p;\beta)$ is strictly increasing and convex in Δp.*

Proof.

$$\begin{aligned}\phi' &= \frac{\partial q_A(\Delta p;\beta)}{\partial \Delta p} = -\frac{\partial q_A(\Delta p;\beta)}{\partial p_A} = -\frac{\partial q_B(\Delta p;\beta)}{\partial \Delta p} = -\frac{\partial q_B(\Delta p;\beta)}{\partial p_B} \\ &= \beta \cdot \hat{x}'_{in}(\Delta p) + (1-\beta)\cdot \hat{x}'_{un}(\Delta p) \\ &= -\frac{1}{4t}(1-3\beta) - \frac{(1-\beta)}{2(S(\Delta p))}\underbrace{\left(\frac{\Delta p}{8t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\right)}_{\ominus} > 0\end{aligned}$$

$\phi' > 0 \quad \forall \Delta p$ feasible and $\forall \beta$. At the boundaries we have

$$\begin{aligned}\phi'(0;\beta) &= -\frac{1}{4t}(1-3\beta) + (1-\beta)\frac{(\lambda+2)}{2t(\lambda-1)} > 0 \\ \phi'(\Delta p \to \Delta \bar{p}; \beta < 1) &\to \infty \quad \text{since } S(\bar{p}) = 0.\end{aligned}$$

For $0 \leq \Delta p < \Delta p^{max}$ the demand of A is convex in Δp. At the boundaries we have

$$\phi''(\Delta p;\beta) = (1-\beta)\cdot \hat{x}''_{un}(\Delta p) = (1-\beta)\cdot \frac{(3+\lambda)(5+3\lambda)}{64t^2 \cdot (S(\Delta p))^3} \geq 0$$

$\phi'' > 0 \quad \forall \Delta p$ feasible and $\forall \beta < 1$ since $S(\Delta p) \geq 0$:

$$\begin{aligned}\phi''(0;\beta) &= (1-\beta)\cdot \frac{(3+\lambda)(5+3\lambda)}{32t^2 \cdot \frac{(\lambda+1)^3}{(\lambda-1)^3}} > 0 \\ \phi''(\Delta p \to \Delta \bar{p}; \beta < 1) &\to \infty.\end{aligned}$$

□

We note that also the third derivative, ϕ''', is greater than zero. However, we do not have a sign restriction for the derivative of ϕ with respect to β. The first derivative of the demand of A w.r.t. β is the difference of the demand of informed and uninformed consumers:

$$\frac{\partial \phi(\Delta p;\beta)}{\partial \beta} \equiv \phi_\beta = \hat{x}_{in}(\Delta p) - \hat{x}_{un}(\Delta p) = \frac{3}{4t}\Delta p - \frac{\lambda+1}{2(\lambda-1)} + S(\Delta p) \gtrless 0$$

[15]This corresponds to industries in which firms are not too asymmetric.
[16]We will use ϕ as a short-hand notation for $\phi(\Delta p;\beta)$.

with $\phi_\beta = 0$ at $\Delta p = 0$ and $\Delta p = t/2$.

This expression is of ambiguous sign, as has been pointed out in the previous section. We also note that cross derivative of the demand of A w.r.t. Δp and β,

$$\frac{\partial \phi'}{\partial \beta} \equiv \phi'_\beta = \hat{x}'_{in}(\Delta p) - \hat{x}'_{un}(\Delta p) = \frac{3}{4t} + \frac{1}{2S(\Delta p)} \cdot \left(\frac{\Delta p}{8t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\right),$$

is of ambiguous sign. This derivative has the boundary behavior that $\phi'_\beta = 0$ at $\Delta \hat{p}$. and $\phi'_\beta \to \infty$ for $\Delta p = \Delta \overline{p}$; the latter holds because $S(\Delta \overline{p}) = 0$.

3.2 Equilibrium characterization

We next turn to the equilibrium characterization. At the first stage firms foresee consumers' purchase decisions and set prices simultaneously to maximize profits. This yields the following first-order conditions:

$$\frac{\partial \pi_i}{\partial p_i} = \quad q_i + (p_i - c_i)\tfrac{\partial q_i}{\partial p_i} = 0 \quad \forall i \in \{A, B\}$$

If that the solution has the feature that demand of each group of consumers, informed and uninformed, is positive. Then first-order conditions can be expressed by

$$\frac{\partial \pi_A}{\partial p_A} = \phi - (p_A - c_A)\phi' = 0 \qquad (FOC_A)$$

$$\frac{\partial \pi_B}{\partial p_B} = (1-\phi) - (p_B - c_B)\phi' = 0. \qquad (FOC_B)$$

In this case concavity of the profit functions would assure that the solution characterizes an equilibrium.

$$\frac{\partial^2 \pi_A}{\partial p_A^2} = -2\phi' + (p_A - c_A)\phi'' < 0 \qquad (SOC_A)$$

$$\frac{\partial^2 \pi_B}{\partial p_B^2} = -2\phi' - (p_B - c_B)\phi'' < 0. \qquad (SOC_B)$$

Given the properties of ϕ —particularly that ϕ is strictly increasing and convex for $\beta < 1$— SOC_B holds globally, while SOC_A is not necessarily satisfied. Using that $(p_A - c_A) = \phi/\phi'$ by FOC_A, SOC_A can be expressed as follows

$$-2(\phi')^2 + \phi\phi'' < 0. \qquad (13)$$

It can be shown that (13) is satisfied for small Δp (and λ) while it is violated for $\Delta p \to \Delta\overline{p}$ as ϕ'' goes faster to infinity in Δp than $(\phi')^2$.[17] This violation reflects that firm A has an increasing interest to non-locally undercut prices to gain the entire demand of uninformed consumers when Δp is large. The driving force behind this is that loss aversion in the price dimension dominates loss aversion in the taste dimension if price differences are large. Moreover, large losses in the price dimension if buying the expensive product B makes far-distant consumers of A more willing to opt for product A.

We will discuss the issue of non-interior solutions and non-existence in Proposition 2, but focus next on interior solutions. We denote an equilibrium with prices (p_A^*, p_B^*) that is determined by an interior solution as an interior equilibrium.

Lemma 3. *In an interior equilibrium with equilibrium prices (p_A^*, p_B^*), the price difference $\Delta p^* = p_B^* - p_A^*$ satisfies*

$$\Delta p^* \quad = \quad \Delta c + f(\Delta p^*; \beta) \quad \forall \beta \in [0,1], \Delta p \text{ feasible}, \tag{14}$$

with $\Delta c = c_B - c_A$ and $f(\Delta p; \beta) = (1 - 2\phi)/\phi'$.

Proof. Combining (FOC_A) and (FOC_B) yields the required equilibrium condition as a function of price differences. □

Thus, (14) implicitly defines the optimal Δp as a correspondence of Δc, β, λ, and t.[18]

3.3 Equilibrium uniqueness

In Proposition 1 we state conditions under which an interior equilibrium is unique. Given parameters λ and t, the condition states that the cost asymmetry between firms is not too large.

Proposition 1. *An interior equilibrium is unique if*

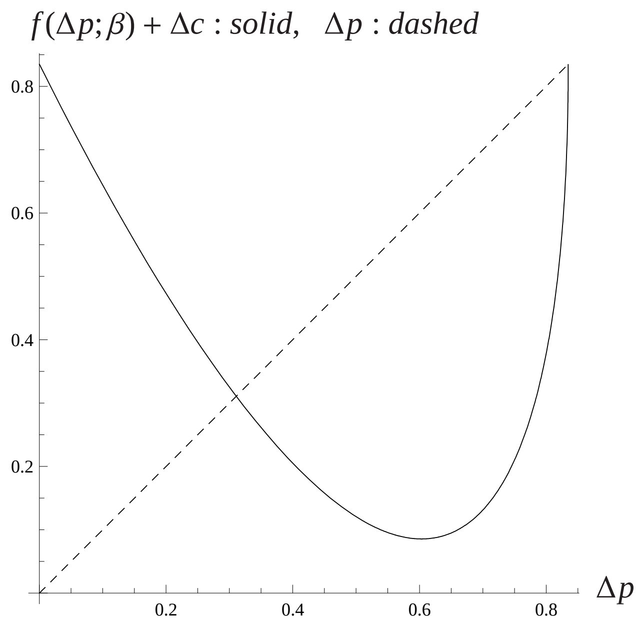
$$\Delta c < \Delta\overline{p} = \frac{2t}{(\lambda - 1)}\left(2(\lambda + 2) - \sqrt{(2(\lambda + 2))^2 - (\lambda + 1)^2}\right), \tag{15}$$

where $\Delta\overline{p}$ depicts the critical value of Δp such that the $S(\Delta p)$ in $\hat{x}_{un}(\Delta p)$ is equal to zero.[19]

[17] This implies that π_A is not globally concave. We will show later that it is neither globally quasi-concave. Moreover, the non-concavity of π_A becomes more severe as Δp (resp. $-p_a$) increases.

[18] Besides β the latter two parameters affect the functional form of f via ϕ.

[19] Cf. equation (8).



The Figureshows the equilibrium condition (14) at $\Delta c = \Delta\overline{p}$ for parameter values of $\beta = 0$, $t = 1$, and $\lambda = 3$: $\Delta\tilde{p} = 0.75$, $\Delta\overline{p} = 0.8348$.

Figure 2: Two potential interior equilibria

Proof. We first consider the case of $\lambda > \lambda^c$. We can derive a number of useful properties of $f(\Delta p;\beta) = (1-2\phi)/\phi'$:

$f(0;\beta) = 0/\phi'(0) = 0\,\forall\beta$, $f(\Delta\overline{p};\beta) \to 0$ since $\phi'(\Delta\overline{p}) \to \infty\,\forall\beta < 1$, and $f(\Delta\overline{p},1) = -2\Delta\overline{p} < 0$.

$$f'(\Delta p;\beta) = \frac{-2(\phi')^2 - \phi''(1-2\phi)}{(\phi')^2} = -\left(2 + \frac{\phi''(1-2\phi)}{(\phi')^2}\right) \lesseqgtr 0,$$

$f'(0;\beta) = -2 < 0 \quad \forall\beta$ and $f'(\Delta\overline{p};\beta) \to -(2 + \frac{-\infty^{3/2}}{\infty^1}) \to \infty \quad \forall\beta < 1$, and $f'(\Delta p, 1) = -2 \quad \forall\Delta p$.

It has to be shown that $f(\Delta p;\beta)$ is strictly convex in Δp for $\beta < 1$. We find that

$$f''(\Delta p;\beta) = -\frac{(\phi'\phi''' - 2(\phi'')^2)(1-2\phi) - 2(\phi')^2}{(\phi')^3} > 0.$$

If $\beta < 1$ by continuity of $f(\Delta p)$, $f(0;\beta) = 0$, $f(\Delta\overline{p};\beta) \to 0$, $f'(0;\beta) < 0$, $f'(\Delta\overline{p};\beta) \to \infty > 1$, and strict convexity of $f(\Delta p)$ for $\beta < 1$, we know that for $\Delta c = \Delta\overline{p}$ there are two potential interior equilibria. This is illustrated in Figure 2. The second equilibrium arises because $\Delta\overline{p}$ depicts a second solution to $\Delta p = f(\Delta p;\beta < 1) + \Delta\overline{p}$ since $f(\Delta\overline{p};\beta < 1) = 0$. Moreover, by continuity of $f(\Delta p)$ two potential equilibria occur for $\Delta c > \Delta\overline{p}$ (if any) because $\Delta\overline{p} < f(\Delta\overline{p};\beta < 1) + \Delta c$. For values of Δc lower than $\Delta\overline{p}$, $f(\Delta\overline{p};\beta < 1) + \Delta c$ is always smaller than $\Delta\overline{p}$ and no second equilibrium can arise.

If $\beta = 1$, $f(\Delta p;\beta)$ is strictly decreasing for all Δp and at most one intersection between $f(\Delta p; 1) + \Delta c$ and Δp exists (standard Hotelling case).[20]

Secondly, in the case of $1 \leq \lambda < \lambda^c$ there are corner solutions if $\Delta p > \Delta\tilde{p}$ because firm A's demand of uninformed consumers is bounded at one. This reduces firm A's incentives to set a very low p_A in equilibrium (that leads to $\Delta p > \Delta\tilde{p}$) because that would decrease the profit margin for all its consumers while only increasing firm A's demand of informed consumers. It can be shown that a Δp above $\Delta\tilde{p}$ is not optimal if the optimal price difference for informed consumers $\Delta p^* = \Delta c/3$ lies below $\Delta\tilde{p}$.[21] Thus, there exists no second equilibrium in this case. For $\Delta p^* = \Delta c/3 > \Delta\tilde{p}$ a higher price difference than $\Delta\tilde{p}$ can arise in equilibrium because attracting further informed consumers is profitable in this situation. But then $\Delta p^* = \Delta c/3$ describes the only potential equilibrium which is driven by the demand of informed consumers (standard Hotelling case). Hence, the uniqueness condition (15) also suffices to rule out second equilibria for $\lambda \in (1, \lambda^c]$. □

3.4 Equilibrium existence

The next proposition clarifies the issue of equilibrium existence. It deals with the non-concavity of firm A's profit function by determining critical levels for firm A's incentive to non-locally undercut prices. Moreover, it is shown that non-interior equilibria fail to exist.

Proposition 2. *An interior equilibrium with prices (p_A^*, p_B^*)* ***exists*** *if and only if*

1. *Δc satisfies*

$$\Delta c \leq \Delta c^{nd} \equiv \max\{\Delta p^{nd} - f(\Delta p^{nd};\beta), 0\}, \tag{16}$$

with Δp^{nd} being implicitly determined by the following non-deviation condition

$$\Delta p^{nd} = \left\{\Delta p \mid \Delta p = \Delta p^{max} - \frac{\phi \cdot \Big(\phi(\Delta p^{max};\beta) - \phi\Big)}{\phi' \cdot \phi(\Delta p^{max};\beta)}, \Delta p \neq \Delta p^{max}\right\}, \tag{17}$$

where $\phi(\Delta p^{max};\beta) = \beta \cdot \hat{x}_{in}(\Delta p^{max}) + (1-\beta) \leq 1$,

[20] An analytical solution for (14) can be determined in this case: $\Delta p^* = \Delta c/3$.

[21] Under (15) Δc is weakly lower than $\Delta\overline{p}$ which can rise above $3\Delta\tilde{p}$ for $\lambda \to 1$.

2. *and if* $\Delta p^{nd} < 0$, β *additionally satisfies*

$$\beta \geq \beta^{crit}(\lambda), \tag{18}$$

with $\beta^{crit}(\lambda)$ *being an increasing function in* λ *which is expressed by*

$$\beta^{crit}(\lambda) \equiv \begin{cases} 0, & \text{if } \lambda \in (1, 1 + 2\sqrt{2}]; \\ \beta_0^{crit}(\lambda) \in (0, 0.349], & \text{if } \lambda \in (1 + 2\sqrt{2}, \lambda^c]; \\ \beta_1^{crit}(\lambda) \in (0.349, 0.577), & \text{if } \lambda > \lambda^c. \end{cases} \tag{19}$$

Moreover, any equilibrium is interior.

Before turning to the proof, let us comment on this proposition. The result shows that an equilibrium exists if firm A has no incentive to non-locally undercut prices. In fact, the incentive to undercut prices increases the more asymmetric industry or the more loss-averse consumers. For a low degree of loss aversion ($1 < \lambda < 1 + 2\sqrt{2} \approx 3.828$) equilibrium exists if the cost difference between firms is not too large (see (16)).[22] In this case, an equilibrium exists for all values of β. However, if the degree of loss aversion rises further, equilibria only exist if there is a sufficiently large share of informed consumers. Such a large share of informed consumers reduces the undercutting incentive of firm A. The possible non-existence due to undercutting even holds for symmetric industries. Again, if the share of informed consumers is sufficiently large, an equilibrium exists; e.g. if 60% (which is greater than 57.7%) of the consumers are informed then an equilibrium exists in symmetric industries for any level of loss aversion $\lambda > 1$.

In the proof we first provide the critical level of Δc for which the equilibrium condition in (14) is satisfied for *potentially* interior equilibria. We next identify the set of interior equilibria which locally satisfy the SOC's and which are robust to non-local price deviations of firm A. Finally, the existence of non-interior equilibria is refuted.

Proof. 1. To find an upper bound on Δc for which the equilibrium condition (14) is satisfied we determine the point at which $f(\Delta p; \beta)$ is a tangent on the Δp-line.

Tangent condition:

$$f'(\Delta p; \beta) = 1 \qquad \Leftrightarrow \qquad 3(\phi')^2 + \phi''(1 - 2\phi) = 0 \tag{20}$$

[22] Note that according to experimental work on loss aversion λ takes the value of approximately 3, which is within this range.

An analytical solution to $3(\phi')^2 + \phi''(1-2\phi) = 0$ can be found for $\beta = 0$.[23] Denote this critical price difference as $\Delta p^{ta}(\lambda, t)$.[24]

Then, the equilibrium condition in (14) can be fulfilled if and only if Δc satisfies the following condition

$$\Delta c \leq \Delta c^{ta} \equiv \Delta p^{ta}(\lambda, t) - f(\Delta p^{ta}(\lambda, t); \beta = 0). \tag{21}$$

2. We next rule out some *potentially* interior equilibria . First suppose $\Delta p'$ does not satisfy SOC_A, then $\Delta p'$ depicts a profit minimum for firm A. $\Delta p'$ cannot be an equilibrium. Moreover, comparing (13) and (20) shows that the critical price difference for locally satisfying SOC_A is always lower than Δp^{ta}. Hence, a non-empty set of *potentially* interior equilibria is ruled out by local non-concavity.

   Secondly, if a *potentially* interior equilibrium locally satisfies SOC_A but SOC_A is locally violated for some larger Δp, the profit function of firm A is strictly convex for a sufficiently large non-local price decrease p_A. If the convexity is sufficiently large the profit of firm A is increasing for large non-local price decreases. Thus, a non-local deviation becomes profitable for firm A.[25] Given the non-decreasing convexity of π_A in $-p_A$ the optimal deviation of firm A is such that firm A serves the entire demand of uninformed consumers, i.e. p_A^d s.t. $\Delta p^d = \Delta p^{max}$. Decreasing p_A^d further is not profitable since firm A only attracts informed consumers while its profit margin goes down for informed and uninformed consumers.[26] In the following we can restrict our attention to price deviations by firm A that steal the entire demand of uninformed consumers.

   In such a situation firm A sets $p_A^d = p_B^* - \Delta p^{max}$. For $\beta = 0$ the firm A's deviation profit, π_A^d, is equal to $(p_A^d - c_A)\cdot 1$ while for $\beta \in (0, 1]$ it is equal to $(p_A^d - c_A)\cdot\phi(\Delta p^{max}; \beta)$ with $\phi(\Delta p^{max}; \beta) \equiv \beta \cdot \hat{x}_{in}(\Delta p^{max}) + (1-\beta)\cdot 1$. Using that $p_A^d = p_B^* - \Delta p^{max}$ we receive

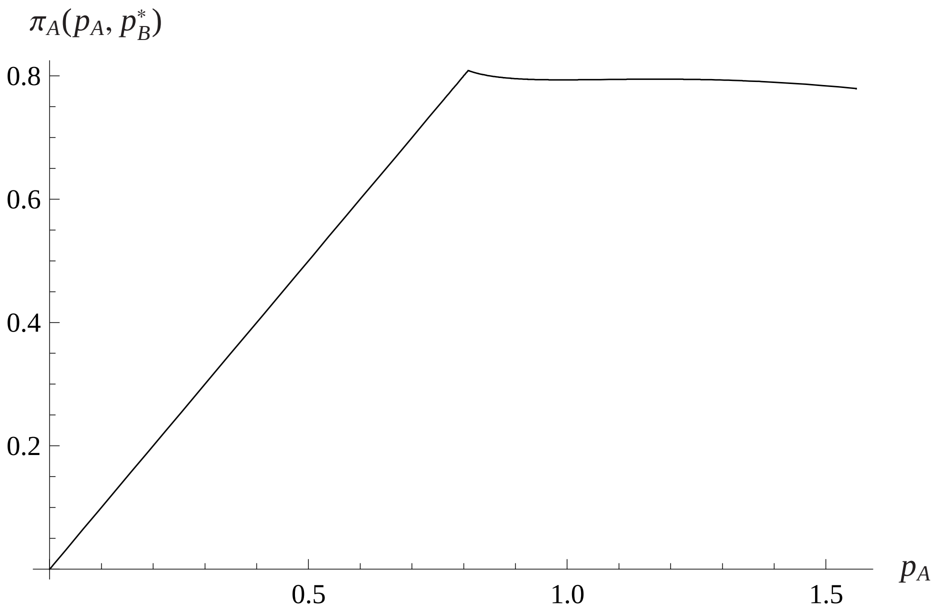
$$\begin{aligned}
\pi_A^d &= \left(p_B^* - \Delta p^{max} - c_A\right)\cdot \phi(\Delta p^{max}; \beta) \\
&= \left(\frac{1-\phi}{\phi'} + \Delta c - \Delta p^{max}\right)\cdot \phi(\Delta p^{max}; \beta) \qquad \text{by } FOC_B
\end{aligned}$$

[23]This is sufficient since $\beta = 0$ is the most critical case w.r.t. existence and uniqueness. The reason for this is that for $\beta > 0$ there is a positive weight on the demand of informed consumers which is purely linear.

[24]$\Delta p^{ta}(\lambda, t)$ is decreasing in λ.

[25]Figure 3 shows an example of an *potentially* interior equilibrium in which deviating by firm A is profitable.

[26]For situations with $\lambda \to 1$, in which $\Delta p^* > \Delta p^{max}$ can arise, it can be shown that non-concavity of π_A is not a problem.



The Figureshows the profit of firm A, $\pi_A(p_A, p_B^*)$, as a function of its own price given $p_B = p_B^*$ for $\Delta c = 1$ ($c_A = 0, c_B = 1$) and parameter values of $\beta = 0$, $t = 1$, and $\lambda = 3$: $p_A^* = 1.17309$, $p_B^* = 1.55863$, $p_A^d = 0.80863$, $\Delta p^* = 0.385537$, and $\Delta p^{max} = \Delta\tilde{p} = 3/4$.

Figure 3: Non-existence

$$= \left(\Delta p^{nd} + \frac{\phi}{\phi'} - \Delta p^{max}\right) \cdot \phi(\Delta p^{max}; \beta) \quad \text{by (14)} \tag{22}$$

For non-deviation, firm A's profit is equal to $\pi_A(\Delta p^*) = (p_A^* - c_A)\phi$, which is equivalent to ϕ^2/ϕ' by FOC_A.

Thus, deviation of firm A is not profitable if and only if $\pi_A(\Delta p^*) \geq \pi_A^d$.[27] Rearranging yields the required non-deviation condition

$$\Delta p \leq \Delta p^{nd} \equiv \Delta p^{max} - \frac{\phi \cdot (\phi(\Delta p^{max}; \beta) - \phi)}{\phi' \cdot \phi(\Delta p^{max}; \beta)}.$$

In Lemma 4 in the appendix we show that Δp^{nd} is uniquely determined by this non-deviation condition if $\Delta p^{nd} \neq \Delta p^{max}$ and that the set of non-negative Δp^{nd} is non-empty.

Combining this with the equilibrium condition (14) we get that existence of interior equilibria is ensured for non-negative Δp^{nd} if and only if $\Delta c \leq \Delta c^{nd} \equiv \Delta p^{nd} - f(\Delta p^{nd})$. However, Δp^{nd} can become negative if the degree of loss aversion becomes too high. Here deviation is profitable even for symmetric settings

[27] We assume that firm A does not deviate from an interior strategy if it is indifferent between deviating and playing the interior best-response.

($\Delta c = 0$). But an upper limit on the amount of uninformed consumers can reinforce existence of symmetric equilibria in this case. In the second part of Lemma 4 the critical level of loss aversion for which Δp^{nd} becomes negative is determined and the critical level of β as a function of λ for $\Delta c = 0$, $\beta^{crit}(\lambda)$, is defined. As we argue above the non-deviation condition implies local concavity of the firms' profit function. Here we see that the inverse is not true. We therefore receive $\forall \beta$ that $\Delta p^{nd} < \Delta p^{SOC_A} (< \Delta p^{ta} < \Delta p^{max})$.

3. Any equilibrium is interior because discontinuity of firm A's best response function rules out non-interior equilibria.

□

We conclude this section by a numerical example. For $\lambda = 3$, $t = 1$ and $\beta = 0$, the following price differences arise $\Delta p^{nd} = 0.27889$, $\Delta p^{ta} = 0.69532$, $\Delta p^{max} = \Delta \tilde{p} = 3/4$, and $\Delta \overline{p} = 0.83485$.[28] Moreover, Δc^{nd} is equal to $(\Delta p^{nd} - f(\Delta p^{nd}; 0)) = 0.75963$, i.e. an equilibrium exists for $\Delta c < 0.75963$. Compare table 3 and 4 in the appendix with $\Delta c = 0.25$ and 0.75 at $\beta = 0$. For non-existence at $\beta = 0$ consider Figure 2 and 3 with $\Delta c = \Delta \overline{p}$ and 1.

Table 1 depicts the critical level of price differences and cost differences for non-deviation for $\beta \geq 0$ and $\lambda \geq 3$. It can be seen that a sufficiently large share of informed consumers dampens firm A's incentive to deviate even if the degree of loss aversion becomes high.[29]

Finally, the critical β for existence of symmetric equilibria ($\beta \geq \beta^{crit}(\lambda)$) is depicted in Figure 4.

4 Comparative Static Analysis

In this section we focus on comparative static properties of the equilibrium. As a starting point, we analyze comparative statics properties of symmetric markets, i.e., markets in which $c_A = c_B$. We then investigate the role of cost asymmetries and then turn to the role of the degree of initial information disclosure (captured by the share of informed consumers) in asymmetric markets. Finally, we investigate the effect of various demand characteristic on equilibrium outcomes.

[28] Figure 7 in the appendix depicts the determination of Δp^{nd} for these parameter values.

[29] Note that for $\Delta c^{nd}(\beta) > \Delta \overline{p}$ potential second equilibria can arise (=second intersection of Δp and $\Delta c + f(\Delta p; \beta)$, compare Figure 2). However, those equilibria can be ruled out by the non-deviation condition since $\Delta p^{**} > \Delta p^{nd}(\beta)$. This means that by combining uniqueness and existence conditions equilibrium uniqueness can be granted for a broader class of industries.

Table 1: Non-deviation condition

The table shows the variation of Δp^{nd} and Δc^{nd} in β and λ.

	$\lambda = 3$		$\lambda = 6$		$\lambda = 9$	
β	$\Delta p^{nd}(\beta)$	$\Delta c^{nd}(\beta)$	$\Delta p^{nd}(\beta)$	$\Delta c^{nd}(\beta)$	$\Delta p^{nd}(\beta)$	$\Delta c^{nd}(\beta)$
1.0	-	-	-	-	-	-
0.8	0.648337	1.75869	0.372669	1.07069	0.294726	0.857815
0.6	0.543254	1.45317	0.23824	0.686206	0.150303	0.440498
0.4	0.459237	1.22329	0.107415	0.314749	0.000320	0.000959
0.2	0.377489	1.00993	-0.0719496	-	-0.229582	-
0.0	0.278889	0.75963	-0.521395	-	-1.0704	-

4.1 Symmetric Market

In contrast to Heidhues and Koszegi (2008) our framework allows us to explicitly solve for equilibrium markup in our model. The following result characterizes the symmetric equilibrium.

Proposition 3. *For $\Delta c = 0$, any equilibrium is unique and symmetric. Equilibrium prices are given by*

$$p_i^* = c_i + \frac{t}{1 - \frac{(1-\beta)}{2}\frac{(\lambda-1)}{(\lambda+1)}}, i = A, B. \tag{23}$$

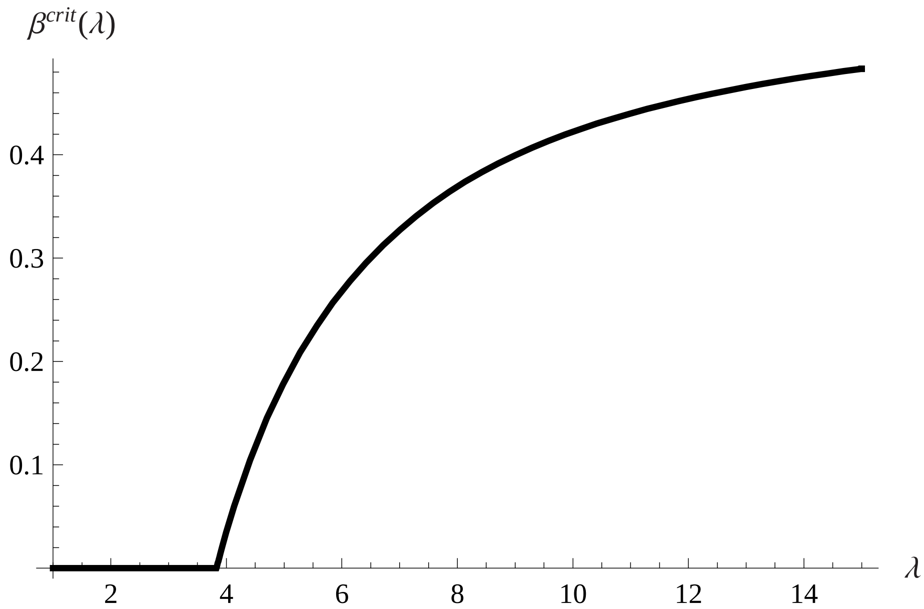
Proof. For $\Delta c = 0$ we get by (14), (15), and $f(0;\beta) = 0$ that $\Delta p^*(\beta) = 0$ is the unique equilibrium $\forall \beta \in [0,1]$ (provided it exists). Rearranging (FOC_i) and applying that $\phi(0,\beta) = 1/2$ for all β yields

$$p_i^* - c_i = \frac{\frac{1}{2}}{\phi'(0;\beta)} \quad \forall i \in \{A, B\},$$

where

$$\phi'(0;\beta) = -\frac{1}{4t}(1-3\beta) - \frac{(1-\beta)}{2(S(0))}\left(0 - \frac{(\lambda+2)}{2t(\lambda-1)}\right)$$

$$= -\frac{1}{4t}(1-3\beta) + \frac{(1-\beta)}{2\frac{\lambda+1}{2(\lambda-1)}}\left(\frac{(\lambda+2)}{2t(\lambda-1)}\right)$$



The Figureshows the critical amount of informed consumers, $\beta^{crit}(\lambda)$, for which symmetric equilibria exist as a function of the degree of loss aversion $\lambda > 1$. Parameter values are $\Delta c = 0$ and $t = 1$: $\Delta p^{nd}(\Delta c = 0, \beta = \beta^{crit}(\lambda)) = 0$. Non-deviation for $\beta \geq \beta^{crit}(\lambda)$.

Figure 4: Non-deviation for symmetric industries

$$= -\frac{1}{4t}(1 - 3\beta) + \frac{(1-\beta)(\lambda+2)}{2t(\lambda+1)}$$

$$= \frac{1}{4t(\lambda+1)}\Big(2(\lambda+1) - (1-\beta)(\lambda-1)\Big).$$

This gives rise to (23). □

For $\Delta p^*(\beta) = 0$ loss aversion about prices is irrelevant even for uninformed consumers. In this situation uninformed consumers exclusively try to avoid losses in the taste dimension. This reduces the attractiveness of a lower-priced firm and thus the price elasticity of demand. This can be exploited by the firms the higher the degree of loss aversion and the higher the share of uninformed consumers. Since firms apply a markup over marginal costs equilibrium profits are independent of the level of marginal costs.[30]

Three comparative statics results are immediate.

Corollary 1. *For $\Delta c = 0$ and $\lambda > 1$, equilibrium markup is decreasing in the share of uninformed consumers β.*

[30]This is a standard property of models with demand aggregated over the two products that is perfectly price inelastic (more specifically of spatial models with full coverage).

This follows directly from differentiating (23) with respect to β and means that as the share of informed consumers increases the firms' markup goes down. In other words, informed consumers exert a positive externality on uninformed consumers. This prediction is in line with alternative models from the search literature, where a larger share of consumers who do not know some products exert a negative externality on those who do. Nevertheless our framework is substantially different since all consumers are fully informed at the moment of purchase. Here, an externality also arises due to uncertainty at the moment consumers form their reference points. With respect to recent work with behavioral biases, our result is of interest in the light of claims that better informed consumers are cross-subsidized at the cost of less informed consumers. This, for instance, holds in Gabaix and Laibson (2006) where only a fraction of consumers are knowledgeable about their future demand of an "add-on service", while other consumers are "naively" unaware of this. This shows that the particular type of behavioral bias is central to understand the competitive effect of changes in the composition of the consumer population.

Our first comparative statics result in the symmetric setting implies that firms do not have an incentive to inform consumers at an early stage. However, there is a potential role of public authorities to inform consumers about their match value at an early point in time so that all uncertainty is resolved early on. This increases competitive pressure and thus lead to higher consumer surplus. As we already pointed out in the introduction, it is not required that public authorities aim at eliminating the behavioral bias directly (and thus to manipulate consumer preferences) but rather to disclose information at an early stage. This neutralizes the behavioral bias (but does not change the consumers' utility function). This insight provides a novel rational for information disclosure by public authorities due to behavioral biases in the consumer population.

Second, equilibrium markup is increasing in the degree of loss aversion, λ. For $\lambda \to 1$ firms receive the standard Hotelling markup of t. Third, equilibrium markup is increasing in the inverse measure of industry competitiveness, t. For $t \to 0$ firms face full Bertrand competition and markups converge zero for all levels of loss aversion. This shows that consumer loss aversion does not affect market outcomes in perfectly competitive environments and our results rely on the interaction of imperfect competition and behavioral bias. The second and third comparative statics results are rather obvious but still noteworthy.

Table 2 shows the variation of equilibrium markups in the share of informed consumers β and the degree of loss aversion λ for fully symmetric markets ($\Delta c = 0$). We make the following observations: (1) The highest markup is reached when all consumers are uninformed and the degree of loss-aversion approaches its critical level for existence in

Table 2: Symmetric Equilibrium: Equilibrium Markups

The table shows the variation of $m_i^*(\Delta c = 0, \beta, \lambda) \equiv p_i^*(\Delta c = 0, \beta, \lambda) - c_i$ for all $i \in \{A, B\}$ in β and λ.

β | λ	1	2	3	3.8284	5	7	9	∞
1	1	1	1	1	1	1	1	1
0.8	1	1.03448	1.05263	1.06222	1.07143	1.08108	1.08696	1.11111
0.6	1	1.07143	1.11111	1.1327	1.15385	1.17647	1.19048	1.25
0.4	1	1.11111	1.17647	1.2132	1.25	1.29032	1.31579	-
0.2	1	1.15385	1.25	1.30602	1.36364	-	-	-
0	1	1.2	1.33333	1.41421	-	-	-	-

symmetric markets $\lambda = 1 + 2\sqrt{2} \approx 3.82843$.[31] (2) If the share of informed consumers is sufficiently large (above 57.7%) symmetric equilibria exist for all $\lambda > 1$. With such a large share of informed consumers the equilibrium markup is below its maximum level since the demand of informed consumers is more elastic and thus dampens the firms' incentives to set higher prices.

4.2 The role of cost asymmetries

In this subsection we take a first look at comparative statics properties of the asymmetric market. Here we focus on the degree of cost asymmetry, i.e. the level of $\Delta c = c_B - c_A$.

Proposition 4. *In equilibrium, the price difference $\Delta p^*(\Delta c, \beta)$ is an increasing function in the cost asymmetry between firms Δc. Moreover, $\Delta p^*(\Delta c, \beta) \geq 1/3$.*

Proof.

$$\frac{d\Delta p^*(\Delta c)}{d\Delta c} = -\frac{(\phi')^2}{3(\phi')^2 + \phi''(1-2\phi)} \cdot (-1) = \frac{(\phi')^2}{3(\phi')^2 + \phi''(1-2\phi)} \tag{24}$$

Since ϕ' is strictly positive and denominator of $d\Delta p^*(\Delta c)/d\Delta c$ is equivalent to the tangent condition (20). We obtain that

$$\frac{d\Delta p^*(\Delta c)}{d\Delta c} > 0 \tag{25}$$

[31] Compare Figure 4.

if $\Delta p < \Delta p^{ta}(\lambda, t)$. Moreover, since $\phi''(1-2\phi) = 0$ for $\Delta c = 0$ (i.e. $\Delta p = 0$, compare symmetric equilibrium) and $\phi''(1-2\phi) \leq 0$ for $\Delta c > 0$ it holds true that $d\Delta p^*(\Delta c)/d\Delta c \geq 1/3$. □

This result says that the more pronounced the cost asymmetry the larger the price difference between high-cost and low-cost firm. This result shows that standard comparative statics result with respect to cost difference are qualitatively robust to consumers being loss averse. However, in our model the marginal effect of an increase in cost differences on price variation is much stronger if some consumers are loss averse. To see this, note that $d\Delta p^*(\Delta c)/d\Delta c$ is equal to $1/3$ for $\beta = 1$, i.e. if all consumers are informed. This coincides with the standard Hotelling case. By contrast, for $\beta < 1$ our model predicts exacerbated price variation in markets with cost asymmetries.

This is in stark contrast to Heidhues and Koszegi (2008) who found that price variation is reduced in markets with loss-averse consumers. This difference arises because in our model prices are set early and become transparent before consumers form their reference point distributions. Consumers in our setup therefore incorporate the realized level of price variation into their reference point distribution instead of forming expectations about the future level of price variation: they do not form beliefs about firms' price setting strategy but only about their own product choice for given observed prices. This product choice is uncertain due to the uncertainty about ideal tastes. Consumers therefore correctly identify high-price firms before forming their reference point distributions. This affects firm behavior. They condition their price-setting behavior on the cost difference since they are informed about own and rival's costs. It follows that high-cost firms have less incentives to pool with more efficient firms in our setup than in Heidhues and Koszegi (2008).

Let us now look at the individual prices set by the two firms. For comparative statics we use markups $m_i^* \equiv p_i^* - c_i$, $i \in \{A, B\}$ instead of prices because markups are net of individual costs and depend solely on cost differences.[32] At the same time we could use individual prices but focus on changes in rival's costs only.

First, we observe that the low-cost firm's markup is increasing or decreasing depending on the degree of market asymmetries (=cost differences) and the share of uninformed consumers in the market.

[32] This follows directly from firms' first-order conditions. Δc affects $p_i - c_i = \phi(\Delta p)/\phi'(\Delta p)$ via Δp.

Proposition 5. *For $\beta < 1$ and $\lambda > 1$, the equilibrium markup charged by the low-cost firm $m^*_A(\Delta c) \equiv p^*_A(\Delta c, c_A) - c_A$ is either first monotonously increasing and then decreasing in the cost difference if the share of informed consumers β is high, or always monotonously decreasing if β is sufficiently low. For $\beta = 1$ or $\lambda \to 1$, $m^*_A(\Delta c)$ is always monotonously increasing.*

In the latter case when all consumers are informed or the behavioral bias vanishes we receive the standard Hotelling result that the low-cost firm faces a larger markup in more asymmetric markets.

Proof.

$$\frac{dm^*_A(\Delta p^*(\Delta c))}{d\Delta c} = \frac{\partial m^*_A}{\partial \Delta p^*} \cdot \frac{\partial \Delta p^*}{\partial \Delta c},$$

where by (FOC_A)

$$\frac{\partial m^*_A}{\partial \Delta p^*} = \frac{\partial p^*_A}{\partial \Delta p^*} = \frac{(\phi')^2 - \phi'' \cdot \phi}{(\phi')^2} \gtrless 0, \tag{26}$$

which may be positive or negative for $\beta < 1$. Firm A's markup is increasing in the price difference if the price difference is rather low and the share of uninformed consumers is not too high. It is decreasing for large price differences and/or if the share of uninformed consumers is high. Using (24) we receive that

$$\frac{dm^*_A(\Delta p^*(\Delta c))}{d\Delta c} = \frac{(\phi')^2 - \phi'' \cdot \phi}{3(\phi')^2 + \phi''(1 - 2\phi)} \gtrless 0. \tag{27}$$

Hence m^*_A is not strictly increasing in Δp^*. Firm A's markup decreases in the price difference if the price difference, i.e. if the cost asymmetries in the industry, and/or the share of uninformed consumers become too large. (Compare markup of B.) □

Note that, for $\beta = 1$, $dm^*_A/d\Delta c$ collapses to $1/3$. This implies that in the standard Hotelling world without behavioral biases ($\beta = 1$) the markup of the more efficient firm is increasing in the cost difference. The proposition thus shows that a local increase of the cost difference may have the reverse effect under consumer loss aversion ($\beta < 1$, $\lambda > 1$). If the degree of loss aversion and the share of uninformed consumers are high, firms obtain much higher markups under symmetric costs than in the standard Hotelling world (compare table 2). This leads to a level effect due to high markups if cost differences increase: Firm A decreases its markup to gain more consumers already in slightly asymmetric markets. It does so although in these markets price sensitivity of demand is lower than in the standard Hotelling world due to the dominating loss in the taste dimension. Here, the effect of a high markup level dominates the effect of a low price sensitivity of demand. For intermediately and strongly asymmetric markets firm A decreases its markup even further since in these markets the price sensitivity of demand becomes even larger than in the standard Hotelling world due to the dominating loss in the price dimension. Under very large cost differences firm A's markup might even fall below its level in the standard Hotelling case (compare Figure 5).

Second, we consider the markup of firm B.

Proposition 6. *The equilibrium markup charged by the high-cost firm* $m_B^*(\Delta c) \equiv p_B^*(\Delta c, c_B) - c_B$ *is always decreasing in the cost difference.*

Proof.

$$\frac{dm_B^*(\Delta p^*(\Delta c))}{d\Delta c} = \frac{\partial m_B^*}{\partial \Delta p^*} \cdot \frac{\partial \Delta p^*}{\partial \Delta c},$$

where by (FOC_B)

$$\frac{\partial m_B^*}{\partial \Delta p^*} = \frac{\partial p_B^*}{\partial \Delta p^*} = \frac{-(\phi')^2 - \phi'' \cdot (1-\phi)}{(\phi')^2} < 0, \tag{28}$$

which is always negative for all β. Using (24) we obtain that

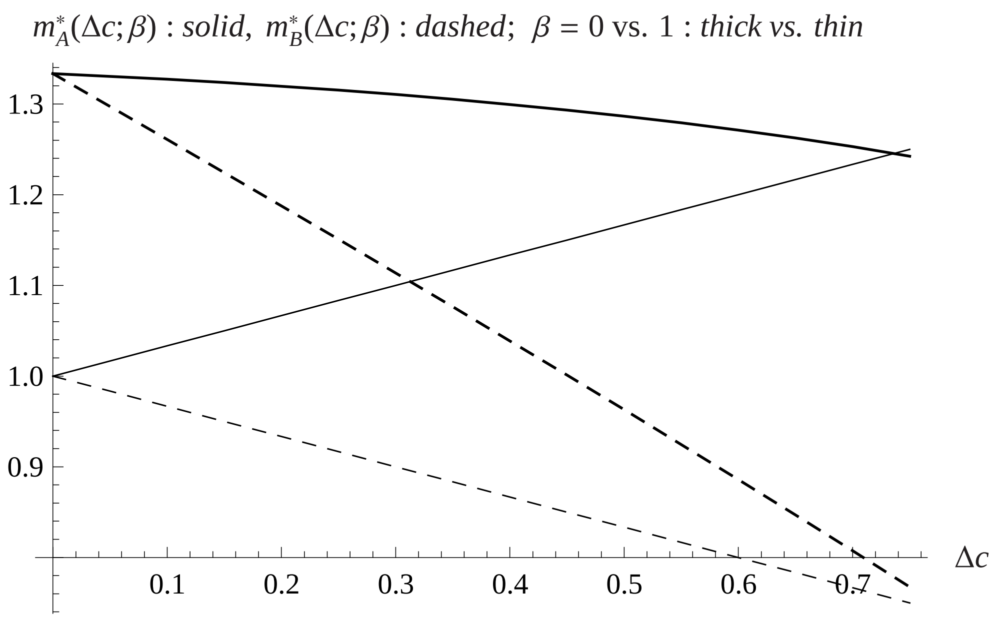
$$\frac{dm_B^*(\Delta p^*(\Delta c))}{d\Delta c} = -\frac{(\phi')^2 + \phi'' \cdot (1-\phi)}{3(\phi')^2 + \phi''(1-2\phi)} < 0. \tag{29}$$

□

Note that for $\beta = 1$, $dm_B^*/d\Delta c$ is equal to $-1/3$. Thus the qualitative finding that the equilibrium markup of the high-cost firm is decreasing in the cost difference is preserved under consumer loss aversion. Due to a level effect of high markups we find that firm B's markup is decreasing more strongly than in the standard Hotelling world without behavioral bias. However, the critical market asymmetry for which its markup drops below its Hotelling level has to be larger than for firm A. This is presented in Figure 5.

4.3 The role of information

In this subsection we focus on comparative statics results with respect to β, the share of initially informed consumers. These results are relevant to evaluate information disclosure policies by public authorities and firms. The latter provide new insights into the



The Figure shows the equilibrium markups of firm A and B for markets in which either all consumers are uninformed ($\beta = 0$) or informed (=benchmark case, $\beta = 1$) as a function of cost differences Δc for parameter values of $t = 1$ and $\lambda = 3$: $\Delta c^{nd}(\beta = 0) = 0.75963$.

Figure 5: Equilibrium markup of both firms

firms' advertising and marketing activities. Our first result concerns the equilibrium price difference.

Proposition 7. *The equilibrium price difference $\Delta p^*(\beta)$ is decreasing in β.*

Proof. Recall that the equilibrium is implicitly characterized by

$$\Delta p - \Delta c - \frac{1 - 2\phi(\Delta p;\beta)}{\phi'(\Delta p;\beta)} = 0$$

The equilibrium price difference then satisfies

$$\begin{aligned}
\frac{d\Delta p^*(\beta)}{d\beta} &= -\left(1 - \frac{-2(\phi')^2 - \phi''(1-2\phi)}{(\phi')^2}\right)^{-1}\left(-\frac{-2\phi'\frac{\partial\phi}{\partial\beta} - \frac{\partial\phi'}{\partial\beta}(1-2\phi)}{\phi'^2}\right) \\
&= -\frac{(\phi')^2}{3(\phi')^2 + \phi''(1-2\phi)} \cdot \left(\frac{2\phi'\phi_\beta + \phi'_\beta - 2\phi'_\beta\phi}{(\phi')^2}\right) \\
&= -\frac{2\phi'\phi_\beta + \phi'_\beta(1-2\phi)}{3(\phi')^2 + \phi''(1-2\phi)}
\end{aligned}$$

We show that the numerator of $\frac{d\Delta p^*(\beta)}{d\beta}$, denoted by $N(\Delta p^*;\beta) = -(2\phi'\phi_\beta + \phi'_\beta(1-2\phi))$ is negative: For all Δp with $0 \leq \Delta p \leq \Delta p^{max}$ and for all $\beta \in [0,1]$, we can rewrite

$$\begin{aligned}
N(\Delta p;\beta) &= -2\phi'\phi_\beta - \phi'_\beta(1-2\phi) = 2((1-\beta)\hat{x}'_{un} + \beta\frac{1}{2t})\cdot(\hat{x}_{un} - \hat{x}_{in}) \\
&\quad +(\hat{x}'_{un} - \frac{1}{2t})(1 - 2(1-\beta)\hat{x}_{un} - 2\beta\hat{x}_{in}) \\
&= \frac{1}{t}(\hat{x}_{un} - \hat{x}_{in}) + (\hat{x}'_{un} - \frac{1}{2t})(1 - 2\hat{x}_{in}) \\
&= \frac{1}{t}\hat{x}_{un} + (\hat{x}'_{un})(1 - 2\hat{x}_{in}) - \frac{1}{2t} \\
&= \frac{1}{t}(\hat{x}_{un} + \frac{1}{2}) - \hat{x}'_{un}(2\hat{x}_{in} - 1) \\
&= -2t\hat{x}'_{un}\cdot(\hat{x}_{in} - \frac{1}{2}) + 1(\hat{x}_{un} - \frac{1}{2}) \\
&= -2t\hat{x}'_{un}(\Delta p)(\hat{x}_{in}(\Delta p) - \frac{1}{2}) + (\hat{x}_{un}(\Delta p) - \frac{1}{2})
\end{aligned}$$

Since $N(0;\beta) = 0$ and

$$\begin{aligned}
\frac{\partial N(\Delta p;\beta)}{\partial \Delta p} &= -\frac{1}{t}\Big(2t\hat{x}''_{un}(\Delta p)(\hat{x}_{in}(\Delta p) - \frac{1}{2}) + 2t(\hat{x}'_{un}(\Delta p))(\hat{x}'_{in}(\Delta p)) - \hat{x}'_{un}(\Delta p)\Big) \\
&= -\frac{1}{t}\Big(2t\hat{x}_{un}(\Delta p)(\hat{x}_{in}(\Delta p) - \frac{1}{2}) + 0 - 0\Big) < 0
\end{aligned}$$

it holds that $N(\Delta p^*;\beta) < 0$ for all admissible $\Delta p, \beta$.

Consider now the denominator of $\frac{d\Delta p^*(\beta)}{d\beta}$, denoted by $D(\Delta p^*;\beta) = 3(\phi')^2 + \phi''(1-2\phi)$. We show that on the relevant domain of price differences $D(\Delta p^*;\beta)$ is strictly positive. We have that

$$\begin{aligned}
D(0;\beta) &= 3(\phi'(0;\beta))^2 + \phi''(0;\beta)\cdot 0 \\
&= 3(\phi'(0;\beta))^2 > 0
\end{aligned}$$

The sign of the derivative is of ambiguous sign:

$$\begin{aligned}
\frac{\partial D(\Delta p;\beta)}{\partial \Delta p} &= 6\phi'\phi'' + \phi'''(1-2\phi) - 2\phi''\phi' \\
&= 4\phi'\phi'' + \phi'''(1-2\phi)
\end{aligned}$$

Thus $D(\Delta p^*;\beta)$ is not necessarily non-negative. However, since $D(\Delta p^*;\beta)$ is equivalent to the tangent condition (20) which approaches zero at $\Delta p = \Delta p^{ta}(\lambda, t)$ we conclude that

$$\frac{d\Delta p^*(\beta)}{d\beta} < 0 \tag{30}$$

for $\Delta p < \Delta p^{ta}(\lambda, t)$, which is the relevant domain for equilibrium existence. □

The above proposition says that prices become more equal as the share of initially informed consumers increases, or, in other words, that the population average becomes less loss-averse. Put differently, more loss-averse consumers lead to larger price differences. This is in stark contrast to one of the main findings in Heidhues and Koszegi (2008) who show in their setting that consumers loss aversion is a rationale for focal prices compared to a setting without behavioral biases in which firms would set different prices (using our terminology they compare a setting with mass 1 of uninformed consumers, i.e. $\beta = 0$, to a setting with mass 0 of uninformed consumers, which corresponds to a world without behavioral bias). Their message is that consumer loss aversion tends to lead to the (more) equal prices; our finding says that consumer loss aversion leads to larger price differences of asymmetric firms.

Let us now look at the individual prices set by the two firms. We first observe that the low-cost firm's price is monotone or inverse U-shaped in β depending on the parameter constellation.

Proposition 8. *The equilibrium price charged by the low-cost firm $p_A^*(\beta)$ may be increasing or decreasing in the share of informed consumers β: $p_A^*(\beta)$ is monotonously increasing, monotonously decreasing or first increasing and then decreasing in β. It tends to be decreasing for small and increasing for large cost differences.*

Proof.

$$\frac{dp_A^*(\Delta p^*(\beta);\beta)}{d\beta} = \frac{\partial p_A^*}{\partial \Delta p^*} \cdot \frac{\partial \Delta p^*}{\partial \beta} + \frac{\partial p_A^*}{\partial \beta},$$

where

$$\frac{\partial p_A^*}{\partial \Delta p^*} = \frac{(\phi')^2 - \phi'' \cdot \phi}{(\phi')^2} \gtrless 0,$$

which may be positive or negative. Hence p_A^* is not strictly increasing in Δp^*. Firm A's prices goes down in the price difference if the price difference becomes too large, i.e. if the cost asymmetries in the industry or the share of uninformed consumers becomes too large. (Compare price of B.)

$$\begin{aligned}
\frac{\partial p_A^*}{\partial \beta} &= \frac{\phi'\phi_\beta - \phi'_\beta \phi}{(\phi')^2} \\
&= -\Big[((1-\beta)\hat{x}'_{un} + \beta\hat{x}'_{in})(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}'_{un} - \hat{x}'_{in}) \cdot ((1-\beta)\hat{x}_{un} + \beta\hat{x}_{in})\Big] \cdot \frac{1}{\phi'^2} \\
&= -\Big[(1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) - (1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in})\frac{1}{2t}(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}_{un} - \frac{1}{2t})\hat{x}_{in}\Big] \cdot \frac{1}{\phi'^2}
\end{aligned}$$

$$= -\left[\frac{1}{2t}\hat{x}_{un} - \hat{x}'_{un}\hat{x}_{in}\right] \cdot \frac{1}{\phi'^2}$$

The numerator of $\frac{\partial p_A^*}{\partial \beta}$ is independent of β.

$$\frac{\partial p_A^*}{\partial \beta}(\Delta p = 0) = -\frac{1}{2}\left(\frac{1}{2t} - \hat{x}'_{un}(0)\right) \cdot \frac{1}{\phi'(0)^2} < 0$$

$$\frac{\partial p_A^*}{\partial \beta}(\Delta p = \Delta\bar{p} - \epsilon) = -\frac{\left(\frac{1}{2t}\hat{x}_{un} - \hat{x}'_{un}\hat{x}_{in}\right)}{\phi'^2} > 0$$

for ϵ small because the numerator is positive for Δp slightly less than $\Delta\bar{p}$. This implies that $\frac{\partial p_A^*}{\partial \beta} = 0$ for some $\Delta p \in (0, \Delta p^{max}), \forall\beta$. □

The critical price difference (which implies the critical cost difference) at which price locally does not respond to β (c.p. Δp, i.e. partial effect) can be solved analytically for. The critical Δp, which is a function of λ and t and is independent of β:

$$\Delta p^{crit\,\partial p_A/\partial\beta}(\lambda, t) = \frac{t}{4(3+5\lambda)}\left((9 - (26 - 15\lambda)\lambda) + \sqrt{3} \cdot |-1 + 5\lambda| \sqrt{(2(\lambda+2))^2 - (\lambda-1)^2}\right)$$

For example, for parameters $\lambda = 3$ and $t = 1$ the critical price difference, at which the price of the low-cost firm reaches its maximum, satisfies $\Delta p^{crit\,\partial p_A/\partial\beta}(3, 1) = 0.2534$. It is also insightful to evaluate the derivative in the limes as β turns to 1. In this case we can also solve analytically for a critical Δp at which the total derivative of p_A is zero, i.e. $\frac{dp_A^*(\Delta p^*(\beta);\beta)}{d\beta} = 0$:

$$\Delta p^{crit\,dp_A/d\beta}(\lambda, t) = t\frac{3(\lambda(31\lambda + 42) - 41) - \sqrt{21} \cdot |7 - 11\lambda| \sqrt{(\lambda+3)(3\lambda+5)}}{2(\lambda-3)(9\lambda-1)} \quad \text{at } \beta = 1$$

For example, $\Delta p^{crit\,dp_A/d\beta}(3, 1) = 7/26 = 0.2692$ at $\beta = 1$. This means that, given parameters $\lambda = 3$ and $t = 1$, if we observe $\Delta p^*(1) = A < 0.2692$ a small increase in the share of informed consumers leads to a lower price of the more efficient firm, $dp_A/d\beta < 0$ (this confirms our numerical results in table 3 and 4), while for $\Delta p^*(1) > 0.2692$ the opposite holds, i.e. $dp_A/d\beta > 0$. (this confirms our numerical results in 5).

The previous proposition implies that consumers who end up buying from the low-cost firm may actually be worse off when additional consumers become informed ex ante. Consider a change in policy from β to β' with $\beta' > \beta$. This parameterizes the market environment. Some consumers buy from the low-price firm in both market environments.

For a sufficiently large cost asymmetry, the equilibrium price of the low-cost firm is locally increasing for all environments between β and β'. Hence, all those consumers of the low-cost firm whose ex ante information is constant across the two market environments are worse off from information disclosure to a share of $\beta' - \beta$ of consumers. This tends to occur in markets in which the initial share of informed consumers is small and in which the asymmetry (i.e. cost difference) between firms is large.

What is the effect on the price of the high-cost firm? Here our result is qualitatively similar: The price tends to be decreasing in β for small cost differences and increasing for large cost differences.

Proposition 9. *In equilibrium, the price of the high-cost firm $p_B^*(\beta)$ may be increasing or decreasing in the share of informed consumers β: $p_B^*(\beta)$ is monotonously increasing, monotonously decreasing or first increasing and then decreasing in β. It tends to be decreasing for small cost differences and increasing for large cost differences.*

Proof.

$$\frac{dp_B^*(\Delta p^*(\beta);\beta)}{d\beta} = \frac{\partial p_B^*}{\partial \Delta p^*}\cdot\frac{\partial \Delta p^*}{\partial \beta} + \frac{\partial p_B^*}{\partial \beta},$$

$$\text{where} \quad \frac{\partial p_B^*}{\partial \Delta p^*} = \frac{-(\phi')^2 - \phi''(1-\phi)}{(\phi')^2} = -\left(1 + \frac{\phi''(1-\phi)}{(\phi')^2}\right) < 0$$

In contrast to A, the price of B is always decreasing in $\Delta p^*(\beta)$.

$$\begin{aligned}
\frac{\partial p_B^*}{\partial \beta} &= \frac{-\phi'\phi_\beta - \phi'_\beta(1-\phi)}{(\phi')^2} \\
&= -\Big[-((1-\beta)\hat{x}'_{un} + \beta\frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}'_{un} - \frac{1}{2t})(1-(1-\beta)\hat{x}_{un} - \beta\hat{x}_{in})\Big]\cdot\frac{1}{(\phi')^2} \\
&= -\Big[-(1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) + (1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) \\
&\quad -\frac{1}{2t}(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}'_{un} - \frac{1}{2t})(1 - \hat{x}_{in})\Big]\cdot\frac{1}{(\phi')^2} \\
&= -\Big[-\frac{1}{2t}(\hat{x}_{un}) - (\hat{x}'_{un} - \frac{1}{2t}) + \hat{x}'_{un}\hat{x}_{in}\Big]\cdot\frac{1}{(\phi')^2} \lesseqgtr 0
\end{aligned}$$

□

We can also solve for critical values at which the comparative statics effect changes sign:

$$\Delta p^{crit\,\partial p_B/\partial\beta}(\lambda,t) = \frac{t}{2(\lambda+1)(\lambda+7)}\left((-23 + (\lambda-10)\lambda) + |5-\lambda|\sqrt{(2(\lambda+2))^2 - (\lambda-1)^2}\right)$$

For instance, $\Delta p^{crit\,\partial p_B/\partial\beta}(3,1) = 0.3201$. At $\beta = 1$ we can also solve analytically for a critical Δp at which the total derivative of p_B is zero, i.e. $\frac{dp_B^*(\Delta p^*(\beta);\beta)}{d\beta} = 0$:

$$\Delta p^{crit\,dp_B/d\beta}(\lambda,t) = \frac{t\left(3(\lambda(17\lambda+6)-55) - \sqrt{15}\cdot|11-7\lambda|\sqrt{(\lambda+3)(3\lambda+5)}\right)}{4\lambda(3\lambda-11)}$$

For instance, $\Delta p^{crit\,dp_B/d\beta}(3,1) = 1/2\cdot(5\sqrt{35}-29) = 0.2902$ at $\beta = 1$. This means that for $\Delta p^*(1) < 0.2902$ we expect $dp_B/d\beta < 0$ at $\beta = 1$ (compare table 3 and 4), while for $\Delta p^*(1) > 0.2902$ we expect $dp_A/d\beta > 0$ at $\beta = 1$ (compare table 5). Thus, for this set of parameter values the overall effect of a marginal increase in β can indeed become positive if price differences (resp. cost asymmetries) become large enough.

Let us distinguish consumer groups by the product they consume. We observe that $\Delta p^{crit\,dp_B/d\beta}(\lambda,t) > \Delta p^{crit\,dp_A/d\beta}(\lambda,t)\ \forall\lambda,t$. Hence, for a larger range of cost parameters the price of the high-cost firm is locally decreasing (compared to the low-cost firm). This implies that, focusing on the consumers whose ex ante information remains unchanged, there exists an intermediate range of values of β under which consumers of the low-cost product lose whereas consumers of the high cost product gain from an increase in β. This means that in such cases additional information in the population benefits those consumers who purchase the high-cost product. Since the high-cost product only serves a niche market we may call these consumers niche consumers. Hence, informed niche consumers are more likely to benefit from an increase in β than the other informed consumers.[33]

The above observation helps us to shed some light on information acquisition by consumers. A particular application are consumer clubs that provide early information on match value to its members. Whether existing club members have an incentive to attract additional members depends on the market environment. Our above observation also indicates, that consumer clubs may be more likely to be formed by niche consumers. We also note that a forward-looking club may be willing to cope with increasing prices for a while with the understanding that, as the club further increases in size (reflected by an increase in β) prices will eventually fall.

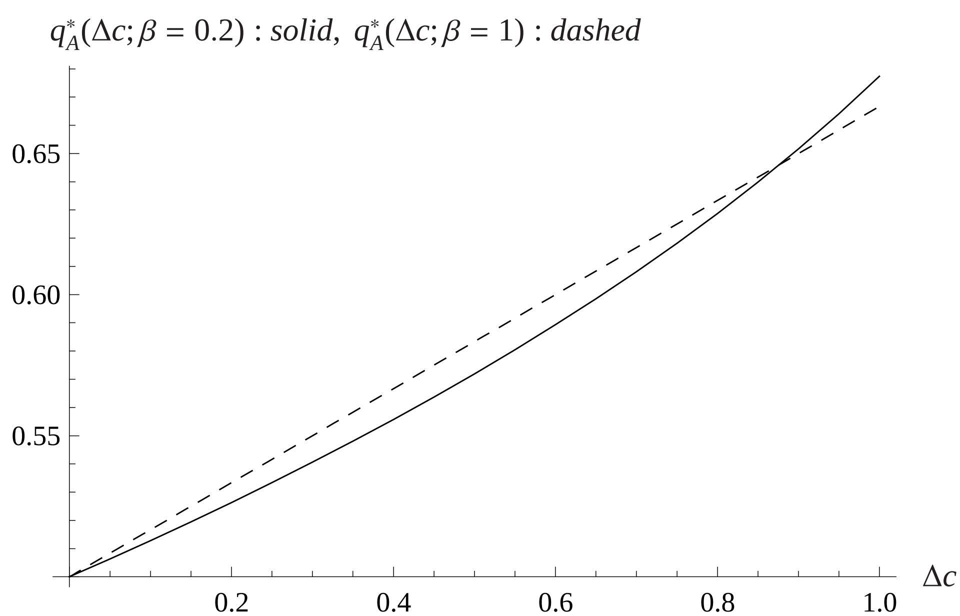
With respect to equilibrium demand our model generates the following predictions.

$$\frac{dq_A(\Delta p^*(\beta);\beta)}{d\beta} = \beta\frac{d\hat{x}_{in}(\Delta p^*)}{d\Delta p^*}\cdot\frac{d\Delta p^*}{d\beta} + \hat{x}_{in}(\Delta p^*) + (1-\beta)\frac{d\hat{x}_{un}(\Delta p^*)}{d\Delta p^*}\cdot\frac{d\Delta p^*}{d\beta} - \hat{x}_{un}(\Delta p^*)$$

[33]The effect on uninformed consumers is ambiguous from an ex ante perspective since they buy the low-cost and the high-cost product with positive probability.

$$= \underbrace{\frac{\partial q_A(\Delta p^*)}{\partial \Delta p^*}}_{\oplus} \cdot \underbrace{\frac{d\Delta p^*}{d\beta}}_{\oplus} + (\hat{x}_{in}(\Delta p^*) - \hat{x}_{un}(\Delta p^*)) \gtrless 0,$$

which is positive for small cost (resp. price) differences and negative for large cost (resp. price) differences (consider also Figure 6). Hence, in rather symmetric markets the demand of the more efficient firm rises, as the share of informed consumers increases (compare Table 3 in the appendix). This implies that with consumer loss aversion (and a positive share of uninformed consumers) firm A's equilibrium demand is lower than in the standard Hotelling case.[34] Our result is reversed in strongly asymmetric markets in which the demand of the more efficient firm decreases in the share of informed consumers (compare Table 5 in the appendix).



The Figure shows the equilibrium demand of firm A for markets with either many uninformed consumers ($\beta = 0.2$) or only informed consumers (=benchmark case, $\beta = 1$) as a function of cost differences Δc for parameter values of $t = 1$ and $\lambda = 3$: $\Delta c^{nd}(\beta = 0.2) = 1.00993$.

Figure 6: Equilibrium demand of firm A

What about private incentives to disclose information? To address this question we will have to investigate the effect on profits. Here private information disclosure can be seen as the firms' management of consumer expectations (i.e. reference points). Note that in our simple setting information disclosure by one firm fully discloses the information of both firms since consumers make the correct inferences from observing the match value

[34] This is qualitatively in line with Heidhues and Koszegi (2008) who predict equal splits of demand between firms in asymmetric markets.

for one of the two products.[35]

$$\frac{d\pi_A(\Delta p^*(\beta), p_A^*(\beta); \beta)}{d\beta} = \frac{dp_A^*(\Delta p^*; \beta)}{d\beta} \cdot q_A(\Delta p^*; \beta) + \left(p_A^*(\Delta p^*; \beta) - c_A\right) \cdot \frac{dq_A(\Delta p^*; \beta)}{d\beta} \lesseqgtr 0$$

$$\begin{aligned}\frac{d\pi_B(\Delta p^*(\beta), p_B^*(\beta); \beta)}{d\beta} &= \frac{dp_B^*(\Delta p^*; \beta)}{d\beta} \cdot \left(1 - q_A(\Delta p^*; \beta)\right) \\ &\quad -\left(p_B^*(\Delta p^*; \beta) - c_B\right) \cdot \frac{dq_A(\Delta p^*; \beta)}{d\beta} \lesseqgtr 0\end{aligned}$$

It is of interest to compare the size of the price effect to the size of the quantity effect for different degrees of market asymmetry. Numerical simulations suggest that the price effect dominates the quantity effect for all $\lambda > 1$. Thus, profits closely follow prices. Here, we confine attention to a single numerical example. The critical value of Δp such that $d\pi_A(.)/d\beta = 0$ at $\beta = 1$ and $\lambda = 3$ and $t = 1$, $c_A = 0.25$, and $c_B = 1$ is $\Delta p = 0.2581$. The critical values of Δp s.t. $d\pi_B(.)/d\beta = 0$ at the same values as above is $\Delta p = 0.2870$.[36] For comparison, we take a look at table 4 in the appendix: The critical value at $\beta = 1$ is $\Delta p^*(1) = 0.25$. Hence, the critical values of Δp at $\beta < 1$ are larger than $\Delta p^*(1)$. Moreover, $\Delta p_B^{crit} > \Delta p_A^{crit}$.

Our numerical example also suggests that increasing the initial share of ex ante informed consumers first none, then one and then both firms gain from information disclosure. In case of conflicting interests it is the more efficient firm which locally gains from information disclosure as an expectation management tool.

Our numerical finding has direct implication for the observed advertising strategy of the firm. Our model predicts that it is rather more efficient firms that advertise product features and price and run promotions that allow consumers test-drives etc. This means that one should observe a positive correlation between efficiency level and advertising and marketing activities of the above mentioned form. We would like to stress that although all consumers will be fully informed at the moment of purchase, advertising content and price matters for firms if consumers are loss-averse. Without this behavioral bias it would be irrelevant whether or not a firm advertises price and characteristics.

[35]This is due to our assumption that firms necessarily locate at distance 1 from each other. It applies to either the setting in which uninformed consumers do not know their type before forming their reference point or they do not know the locations of firms in the product space.

[36]Note that we have problems to obtain an analytical solution as a function of λ and t or c_B even for the special case $\beta = 1$.

How are the different consumer groups doing after an increase of the share of informed consumers? Let us first consider informed consumers. Their change in consumer surplus is simply a weighted average of price changes. To show this we next derive the aggregate consumer surplus for informed consumers.

$$CS_{in}(p_A(\beta), p_B(\beta)) = \int_0^{\hat{x}_{in}(\Delta p(\beta))} u_A(x, p_A(\beta))dx + \int_{\hat{x}_{in}(\Delta p(\beta))}^1 u_B(x, p_B(\beta))dx$$

We thus receive

$$\begin{aligned} \frac{dCS_{in}}{d\beta} &= \int_0^{\hat{x}_{in}(\Delta p(\beta))} \underbrace{\frac{\partial u_A(x, p_A(\beta))}{\partial p_A(\beta)}}_{=-1} \cdot \frac{dp_A}{d\beta} \cdot dx + \int_{\hat{x}_{in}(\Delta p(\beta))}^1 \underbrace{\frac{\partial u_B(x, p_B(\beta))}{\partial p_B(\beta)}}_{=-1} \cdot \frac{dp_B}{d\beta} \cdot dx \\ &= -\hat{x}_{in}(\Delta p)\frac{dp_A}{d\beta} - (1 - \hat{x}_{in}(\Delta p))\frac{dp_B}{d\beta} \gtrless 0. \end{aligned}$$

Consumer surplus of informed consumers may increase or decrease in the share of informed consumers. The sign of the derivative is determined by the weighted marginal price changes $dp_i/d\beta$ of the two products. If the two prices respond in different directions some informed consumers are better off whereas others are worse off in response to a increase in the share of informed consumers.

Evaluating the ex ante effect on uninformed consumers is more involved because gains and losses relative to their reference point have to be taken into account.

$$\begin{aligned} CS_{un}(p_A(\beta), p_B(\beta)) = & \Big(\int_0^{1-\hat{x}_{un}(\Delta p(\beta))} \tilde{u}_A(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta)))dx \\ & + \int_{1-\hat{x}_{un}(\Delta p(\beta))}^{\hat{x}_{un}(\Delta p(\beta))} u_A(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta)))dx \Big) \\ & + \int_{\hat{x}_{un}(\Delta p(\beta))}^1 u_B(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta)))dx, \end{aligned}$$

where $u_A(x, .)$ and $u_B(x, .)$ represent uninformed consumers' gain/loss utility for distant consumers of A and nearby consumers of B derived in (5) and (6), and

$$\begin{aligned} \tilde{u}_A(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta))) =& (v - tx - p_A) + (1 - \hat{x}_{un})(p_B - p_A) \\ & - \lambda \cdot tx^2 + \frac{t}{2}\Big((1 - \hat{x}_{un})^2 - 2(1 - x)x + \hat{x}_{un}^2\Big), \end{aligned}$$

which demonstrates the gain/loss utility for nearby uninformed consumers of A. $\tilde{u}_A(x, .)$ differs from $u_A(x, .)$ only in the taste dimension of the gain/loss utility.

In contrast to intrinsic utility the gain/loss utility also depends on reference point distributions which require knowledge of all prices and the location of the indifferent uninformed consumer. Taking derivatives with respect to β we obtain

$$\begin{aligned}\frac{dCS_{un}}{d\beta} = &\int_0^{\hat{x}_{un}(\Delta p(\beta))}\left(\frac{\partial u_A(x,.)}{\partial p_A}\cdot\frac{dp_A}{d\beta}+\frac{\partial u_A(x,.)}{\partial p_B}\cdot\frac{dp_B}{d\beta}\right)\cdot dx\\ &+\Bigg(\int_0^{1-\hat{x}_{un}(\Delta p(\beta))}\left(\frac{\partial\tilde{u}_A(x,.)}{\partial\hat{x}_{un}}\cdot\frac{d\hat{x}_{un}(\Delta p)}{d\Delta p}\cdot\frac{d\Delta p}{d\beta}\right)\cdot dx\\ &+\int_{1-\hat{x}_{un}(\Delta p(\beta))}^{\hat{x}_{un}(\Delta p(\beta))}\left(\frac{\partial u_A(x,.)}{\partial\hat{x}_{un}}\cdot\frac{d\hat{x}_{un}(\Delta p)}{d\Delta p}\cdot\frac{d\Delta p}{d\beta}\right)\cdot dx\Bigg)\\ &+\int_{\hat{x}_{un}(\Delta p(\beta))}^{1}\left(\frac{\partial u_B(x,.)}{\partial p_A}\cdot\frac{dp_A}{d\beta}+\frac{\partial u_B(x,.)}{\partial p_B}\cdot\frac{dp_B}{d\beta}+\frac{\partial u_B(x,.)}{\partial\hat{x}_{un}}\cdot\frac{d\hat{x}_{un}(\Delta p)}{d\Delta p}\cdot\frac{d\Delta p}{d\beta}\right)\cdot dx.\end{aligned}$$

Beside consumers' intrinsic utility a price change also affects consumers' gains/losses with respect to the price dimension via the varying price difference. A change of the location of the indifferent uninformed consumer $\hat{x}_{un}$ has an impact on consumers' gains/losses in both dimensions. The taste dimension is affected since an increase of $\hat{x}_{un}$ shifts mass of the reference point distribution to the upper tail.[37] An impact on the price dimension occurs since the probability of buying at a specific price depends on the location at which consumers are indifferent between two products. The equation of $dCS_{un}/d\beta$ can be further simplified to

$$\begin{aligned}\frac{dCS_{un}}{d\beta} = &-\hat{x}_{un}\cdot\frac{dp_A}{d\beta}-(1-\hat{x}_{un})\cdot\frac{dp_B}{d\beta}\\ &+\left((\lambda-1)\hat{x}_{un}(1-\hat{x}_{un})+\Delta p\left(\hat{x}_{un}+\lambda(1-\hat{x}_{un})\right)\cdot\frac{d\hat{x}_{un}}{d\Delta p}\right)\cdot(-\frac{d\Delta p}{d\beta})\\ &-t\left(\frac{1}{2}(2\hat{x}_{un}-1)\Big((\lambda-1)(2\hat{x}_{un}-1)+2\Big)\right)\cdot\frac{d\hat{x}_{un}}{d\Delta p}\cdot(-\frac{d\Delta p}{d\beta})\gtrless 0,\end{aligned}\tag{31}$$

where the first line shows marginal effect of β on intrinsic utility (compare CS_{in}). This effect is positive in markets with small cost differences in which prices decrease in the share of informed consumers ($dp_i/d\beta<0$) and negative in markets with large cost differences in which the reverse is true.

In the second line of equation (31) the marginal effect of β on the price dimension of consumers' gain/loss utility is depicted. An increase of the share of informed consumers has a positive overall impact on CS_{un}. This holds true for two reasons. Firstly, from Proposition 7 we obtain that the price difference is a decreasing function in the share of informed consumers. It turns out that a lower price difference (=seize of gains and losses

[37] It can be easily shown that $G(s|\hat{x}'_{un})$ first-order stochastically dominates $G(s|\hat{x}_{un})$ for all $\hat{x}'_{un}>\hat{x}_{un}$ feasible.

in the price dimension) always reduces the losses for B consumers more in total terms than the gains for A consumers (consider the first term in second line). Secondly, a downward shift of the location of the indifferent uninformed consumer (caused by an reduction of the price difference) makes uninformed consumers of both firms better off with respect to gains/losses in the price dimension since the reference point distribution becomes skewed towards gains. This means that the probability of facing a loss in the price dimension decreases (for B consumers), while the probability of facing a gain in the price dimension increases (for A consumers).

The third line shows that the marginal effect of β on the match value dimension of consumers' gain/loss utility is always negative. A downward shift of the location of the indifferent uninformed consumer (caused by an increase in β) decreases the probability of large taste differences ($s \in (1 - \hat{x}_{un}, \hat{x}_{un}]$) keeping the probability of small taste differences ($s \in [0, 1 - \hat{x}_{un}]$) constant.[38] Since remaining uninformed consumers of firm B are located on the interval with small taste differences, they feel the same losses but lower gains. They are clearly worse off with respect to the the match value dimension of their gain/loss utility. The same holds true for nearby uninformed consumers of firm A. On top of lower gains, more distant consumers of A experience higher losses due to the downward shifted reference point distribution for the taste dimension. Thus, the overall effect of β on the taste dimension of consumers' gain/loss utility must be negative indeed.

The overall effect of β on CS_{un} is positive in rather symmetric markets since the effect of β on individual prices p_i is negative in these markets (compare CS_{in} and the tables in the appendix). By the same argument, the effect is negative in more asymmetric markets. Hence, the result from informed consumers qualitatively carries over to uninformed consumers. The reason for this that the sign of the effect of β on both dimensions of consumers' gain/loss utility does not change in market asymmetries. Moreover, it can be shown that for all $\lambda > 1$ and Δc feasible the sum of the second and the third line of (31) is negative, i.e. the marginal effect of β on the taste dimension dominates its effect on the price dimension of consumers' gain/loss utility. Unfortunately, this does not suffice to predict that the sign of $dCS_{un}/d\beta$ is changing for a higher level of β in intermediately asymmetric markets since the price changes, which determine the sign change of consumer surplus, are weighted by different means between informed and uninformed consumers. Table 4 demonstrates the effect of the weight difference dominates the negative effect of β on the both dimensions of consumers' gain loss utility, i.e. the critical β at which the marginal consumer surplus of uninformed consumers switches sign is lower

[38] This argument also relies on the FOSD property of $G(s|\hat{x}_{un})$.

than the critical β for informed consumers.

To determine the overall effect of β on aggregate consumer surplus of both consumer groups, an additional decomposition effect has to be taken into account. This effect reflects the consumer surplus of the group of formerly uninformed consumers which become informed. The overall effect of β on aggregate consumer surplus is determined by the first derivative of $CS(\beta) = \beta \cdot CS_{in}(p_A(\beta), p_B(\beta)) + (1-\beta) \cdot CS_{un}(p_A(\beta), p_B(\beta))$ with respect to β, which yields the following expression

$$\begin{aligned} \frac{dCS}{d\beta} &= \beta \cdot \frac{dCS_{in}}{d\beta} + CS_{in} + (1-\beta) \cdot \frac{dCS_{un}}{d\beta} - CS_{un} \\ &= \beta \cdot \frac{dCS_{in}}{d\beta} + (1-\beta) \cdot \frac{dCS_{un}}{d\beta} + (CS_{in} - CS_{un}). \end{aligned}$$

It can be shown that the decomposition effect represented by $(CS_{in} - CS_{un})$ is always strictly positive, which is intuitive since the group of uninformed consumers faces a lower average utility level due to the higher weight on losses than on gains. Although some uninformed consumers which receive high match value at low price are better off than their informed counterparts, the average utility of uninformed consumers is lower due to the losses in the taste dimension of consumers located apart from the product they purchase and the losses in the price dimension of B consumers (consider the tables in the appendix). It turns out that the decomposition effect always dominates the group-specific effect of β on consumer surplus. This means that the group of consumers who becomes informed is so much better off that its surplus increase always dominates the surplus change of the remaining uninformed consumers and the old informed consumers. This holds even in strongly asymmetric markets in which remaining uninformed and old informed consumers are worse off if the share of informed consumers increases.

5 Extensions

5.1 Relative weight on gain-loss utility

Consider next consumer preferences for which the intrinsic utility is weighted by one, while the gain-loss utility has a weight of $\alpha > 0$.[39] It could now be asked whether a change of the relative weight on the gain-loss utility has a different influence on the location of

[39] For $\alpha = 0$ we are obviously situated in a standard Salop world.

the indifferent uninformed consumer than a change in the degree of loss aversion λ. The next proposition shows that this is not the case.

Proposition 10. *Suppose the utility function of uninformed consumers shows an additional weight, $\alpha > 0$, on the gain-loss utility, i.e. all terms except for the intrinsic utility term in (5) (resp. (6)) are pre-multiplied by α.*
Then, $\forall \lambda' > 1, \alpha' > 0 \; \exists \lambda > 1$ such that

$$\hat{x}_{un}(\Delta p; \lambda, \alpha = 1) = \hat{x}_{un}(\Delta p; \lambda', \alpha'), \tag{32}$$

where $\hat{x}_{un}(\Delta p; \lambda, \alpha)$ is the location of the indifferent uninformed consumer given α-extended preferences. Moreover, $\lambda \geq \lambda'$ for $\alpha' \geq 1$ and $\lambda < \lambda'$ for $\alpha' < 1$.

Proof of Proposition 10. The derivation of the indifferent uninformed consumer with α-extended preferences is analogous to the derivation of the indifferent uninformed consumer for $\alpha = 1$ provided in the proof of Lemma 1. With α-extended preferences the location equals

$$\hat{x}_{un}(\Delta p; \lambda, \alpha) = \frac{1 + \alpha(2\lambda - 1)}{2\alpha(\lambda - 1)} - \frac{\Delta p}{4t} - \sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\alpha(2\lambda + 1) + 3)}{4\alpha t(\lambda - 1)}\Delta p + \frac{(\alpha\lambda + 1)^2}{4\alpha^2(\lambda - 1)^2}}. \tag{33}$$

By solving for λ in equation (32) we receive

$$\lambda(\lambda', \alpha') = \frac{1 + \alpha'(2\lambda' - 1)}{1 + \alpha'}. \tag{34}$$

Since $\lambda(\lambda', \alpha' = 1) = \lambda'$ and $\partial\lambda/\partial\alpha' = 2(\lambda' - 1)/(1 + \alpha')^2 > 0$, λ shows the required properties. □

The previous proposition points out that for any change of the relative weight on gain-loss utility apart from one, there is an equivalent change of the degree of loss aversion, λ, which shows the same sign.

5.2 Asymmetric product quality

Our model is easily extended to allow for differences in product quality which are known to consumers at the beginning of the game. An informed consumer's utility function is $u_i(x, p_i) = (v_i - p_i) - t|y_i - x|$. We then distinguish between a quality-adjusted price dimension, which includes easily communicated product characteristics which are of unambiguous value to consumers and a taste dimension which includes those product characteristics whose value depends on the consumer type. We define quality-adjusted (or hedonic) prices $\tilde{p}_i = p_i - v_i$, $i \in \{A, B\}$ for all consumers and consider those to be relevant for consumers' purchase decision. The main difference arises for uninformed consumers when building their reference point distribution with respect to prices. Here, only the gain/loss in quality-adjusted prices $\Delta\tilde{p} = \Delta p - \Delta v$ matters, $\Delta v \equiv v_B - v_A$. We label firms such that $\Delta c - \Delta v > 0$ and call firm A the more efficient firm. In the following proposition we show that any market with asymmetric quality is equivalent to a market with symmetric quality and more asymmetric costs.

Proposition 11. *For any market with asymmetric quality represented by a vector* $(\Delta v, \Delta c)$ *with* $\Delta c - \Delta v > 0$ *there exists a market with symmetric quality represented by a vector* $(\Delta v', \Delta c')$ *with* $\Delta v' = 0$, $\Delta c' > 0$ *such that market equilibria of both markets are the same, i.e.* $\Delta p^* - \Delta v = \Delta p'^*$. *Moreover,* $\Delta c' = \Delta c - \Delta v$.

As a special case, it can be thought of all asymmetry in the first market being generated by quality differences. This means that firm A delivers higher quality in a market with symmetric costs, $\Delta v < 0$ and $\Delta c = 0$. Then, the costs asymmetry in the second market shows the same size in absolute terms as the quality difference in the first market, $\Delta c' = -\Delta v$.
In the proof we show that the optimization problems of the two consumer groups and the firms are the same in both markets.

Proof of Proposition 11. First consider informed consumers' utility: We find $u_i(x, p_i) = (v_i - p_i) - t|y_i - x| = -\tilde{p}_i - t|y_i - x|$ for all $i \in \{A, B\}$ in the first market and $u_i(x, p'_i) = (v'_i - p'_i) - t|y_i - x|$ for all $i \in \{A, B\}$ in the second market. Since in the second market quality levels are identical ($\Delta v' = 0$), it holds true that $\hat{x}_{in}(\Delta\tilde{p}) = \hat{x}_{in}(\Delta p')$ for $\Delta p' = \Delta p - \Delta v$. If uninformed consumers use quality-adjusted prices for determining their reference point distribution in the price dimension we also receive $\hat{x}_{un}(\Delta\tilde{p}) = \hat{x}_{un}(\Delta p')$ for $\Delta p' = \Delta p - \Delta v$ by the same argument. Finally, compare firms' maximization problem for both markets. Firm A solves

$$\max_{\tilde{p}_A} \pi_A(\tilde{p}_A, \tilde{p}_B) = (\tilde{p}_A + v_A - c_A)[\beta \cdot \hat{x}_{in}(\tilde{p}_B - \tilde{p}_A) + (1 - \beta) \cdot \hat{x}_{un}(\tilde{p}_B - \tilde{p}_A)] \quad \text{and}$$
$$\max_{p'_A} \pi_A(p'_A, p'_B) = (p'_A - c'_A)[\beta \cdot \hat{x}_{in}(p'_B - p'_A) + (1 - \beta) \cdot \hat{x}_{un}(p'_B - p'_A)].$$

Firm A's equilibrium prices are identical iff markups in both markets are identical, i.e. $\tilde{p}_A + v_A - c_A = p'_A - c'_A$, and both demand functions are identical, i.e. $\Delta p' = \Delta p - \Delta v$.

Analogously, for firm B this holds true iff $\tilde{p}_B + v_B - c_B = p'_B - c'_B$ and $\Delta p' = \Delta p - \Delta v$. Finally, taking markup differences between firms we get $\Delta\tilde{p} + \Delta v - \Delta c = \Delta p - \Delta c$ in first market and $\Delta p' - \Delta c'$ in the second market. For $\Delta p' = \Delta p - \Delta v$ both markup differences are the same iff $\Delta c' = \Delta c - \Delta v$. □

6 Conclusion

This paper has studied the impact of consumer loss aversion on market outcomes in asymmetric imperfectly competitive markets. Consumer loss aversion only makes a difference compared to a market in which consumers lack this behavioral bias if they are uncertain about product characteristics or associated match value at an initial stage where they form expectations. Early information disclosure can thus be interpreted as expectation management. Such information disclosure can be achieved through advertising campaigns and promotional activities which do not generate additional information at the moment of purchase (at this point consumers would be informed in any case) but make consumers informed much in advance of their actual purchasing decision.

We followed Heidhues and Koszegi (2008) and modeled the market as a Salop circle. Our framework, however, has notable differences to their work: consumers and firms know the market environment; in particular, they know the actual (asymmetric) cost realizations. Consumers also observe prices from the outset. Our model is enriched by considering a heterogenous population which differs according to their knowledge of their preference point at the initial point when they form their (probabilistic) reference point. Our model delivers remarkably different results compared to Heidhues and Koszegi (2008): while they obtained focal pricing as a consequence of the presence of loss-aversion in the population, we have shown, by contrast, that the price difference *increases* in the share of uninformed loss averse consumers. We also show that prices and profits *decrease* if the cost asymmetry is large.

Our results have implications for public policy and the firms' advertising strategies. There are instances in which consumers would gain from more information whereas both firms would refrain from early information disclosure, namely when the market is symmetric or moderately asymmetric. Thus, our model predicts that advertising and other marketing instruments that allow for early information disclosure about match value should be more prevalent in markets characterized by large asymmetries between firms. Public information disclosure (which allows consumers to learn the products' match values) then would increase consumer surplus. In more asymmetric markets one or both firm gain from information disclosure because this leads to higher prices. Whenever firms have conflicting interests with respect to information disclosure, it is the more efficient firm that discloses information.

We have analyzed industries that are characterized by cost asymmetries. Alternatively, asymmetries with respect to observed product quality may be introduced. Since there is a one-to-one relationship between these two models our insights are directly applicable to a model in which firms differ in observed product quality.

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7 Appendix

Existence result completed

Lemma 4. *1. For $\lambda \in (1, 1+2\sqrt{2}]$, $\Delta p^{nd} \geq 0$ is uniquely determined by the non-deviation condition in* (17)*,*

$$\Delta p^{nd}(\Delta c \geq 0, \beta = 0) = \left\{\Delta p \mid \Delta p = \Delta p^{max} - \frac{\phi \cdot \left(\phi(\Delta p^{max};\beta) - \phi\right)}{\phi' \cdot \phi(\Delta p^{max};\beta)}, \Delta p \neq \Delta p^{max}\right\},$$

2. For $\lambda > 1+2\sqrt{2}$, $\exists \beta^{crit}(\lambda) \geq 0$ s.t. $\Delta p^{nd}(\Delta c = 0, \beta = \beta^{crit}(\lambda)) = 0$.

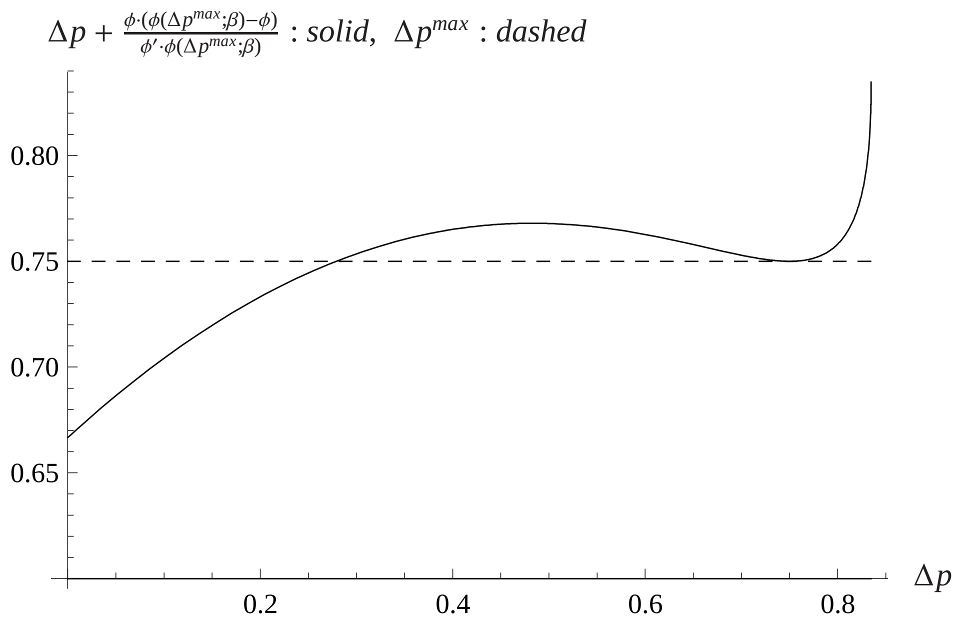
Proof. First note that the non-deviation condition is trivially satisfied at $\Delta p = \Delta p^{max}$ (see Figure 7 below for a graphical illustration of the non-deviation condition). It can be shown that $\Delta p + \frac{\phi\cdot(\phi(\Delta p^{max};\beta)-\phi)}{\phi'\cdot\phi(\Delta p^{max};\beta)}$ approaches Δp^{max} from above for $\Delta p < \Delta p^{max}$. At $\Delta p = 0$, $\Delta p + \frac{\phi\cdot(\phi(\Delta p^{max};\beta)-\phi)}{\phi'\cdot\phi(\Delta p^{max};\beta)}$ is strictly increasing and strictly concave. Moreover, $\Delta p + \frac{\phi\cdot(\phi(\Delta p^{max};\beta)-\phi)}{\phi'\cdot\phi(\Delta p^{max};\beta)}$ is continues and exhibits at most one saddle point for $\Delta p \leq \Delta p^{max}$. Taken together, there exists a unique $\Delta p < \Delta p^{max}$ at which the non-deviation condition is satisfied. Denoting this Δp by Δp^{nd}, $\Delta p^{nd} \leq 0$ if and only if at $\Delta p = 0$, $\Delta p + \frac{\phi\cdot(\phi(\Delta p^{max};\beta)-\phi)}{\phi'\cdot\phi(\Delta p^{max};\beta)} \leq \Delta p^{max}$. It can be shown that $\forall t > 0$ and $\beta = 0$ this holds if and only if $\lambda \in (1, 1+2\sqrt{2}]$.

(* Establish continuity and monotonicity of non-deviation condition in β *)
For $\lambda > 1+2\sqrt{2}$ the non-deviation condition can be reinforced if $\beta > 0$. Solving for $\beta^{crit}(\lambda)$ in $\Delta p^{nd}(\Delta c = 0, \beta = \beta^{crit}(\lambda) > 0) = 0$ yields

$$\beta_0^{crit}(\lambda) \equiv 1 - \frac{-\lambda(5\lambda+14) + \sqrt{(3\lambda+5)(\lambda(11\lambda(\lambda+5)+113)+77)} - 13}{2(\lambda-1)(\lambda+3)}, \tag{35}$$

for $\lambda \in (1+2\sqrt{2}, \lambda^c]$ (i.e. $\Delta p^{max} = \Delta\tilde{p}$) and

$$\beta_1^{crit}(\lambda) \equiv 1 - \frac{37\lambda^3 - 21\Lambda\lambda^2 + 177\lambda^2 - 54\Lambda\lambda + 247\lambda - 21\Lambda - \Omega + 83}{2\left(12\lambda^3 - 7\Lambda\lambda^2 + 46\lambda^2 - 10\Lambda\lambda + 8\lambda + 17\Lambda - 66\right)} \tag{36}$$



The Figureshows the non-deviation condition of firm A, as a function of the price difference Δp for $\Delta c = 0.25$ ($c_A = 0.25, c_B = 0.5$) and parameter values of $\beta = 0$, $t = 1$, and $\lambda = 3$: $\Delta p^{nd} = 0.27889$, $\Delta c^{nd} = (\Delta p^{nd} - f(\Delta p^{nd}; 0)) = 0.75963$, $\Delta p^{max} = \Delta\tilde{p} = 3/4$, and $\Delta\overline{p} = 0.83485$. Non-deviation for $\Delta p \leq \Delta p^{nd} = 0.27889$.

Figure 7: Non-deviation for asymmetric industries

with $\Omega \equiv (4\lambda^6 - 2\Lambda\lambda^5 + 1596\lambda^5 - 918\Lambda\lambda^4 + 19848\lambda^4 - 9316\Lambda\lambda^3 + 91384\lambda^3 - 31228\Lambda\lambda^2 + 197268\lambda^2 - 42618\Lambda\lambda + 201868\lambda - 20366\Lambda + 78880)^{1/2}$
and $\Lambda \equiv \sqrt{3\lambda^2 + 14\lambda + 15}$ for $\lambda > \lambda^c$ (i.e. $\Delta p^{max} = \Delta\overline{p}$). For $\lambda \to \infty$ it holds that $\beta_1^{crit}(\lambda) \to 1 - \frac{-37+21\sqrt{3}+\sqrt{4-2\sqrt{3}}}{-24+14\sqrt{3}} \approx 0.577$. Compare Figure 4. □

Table 3: Small Cost Differences:

The table shows the analytical solution of the market equilibria for parameter values of $t = 1$, $\lambda = 3$, $c_A = 0.25$, $c_B = 0.5$:

β	$p_A^*(\beta)$	$p_B^*(\beta)$	$\Delta p^*(\beta)$	$q_A(\Delta p^*)$	$\hat{x}_{in}(\Delta p^*)$	$\hat{x}_{un}(\Delta p^*)$	π_A^*	π_B^*	CS^*	CS_{in}^*	CS_{un}^*
1.0	1.33333	1.41667	0.0833333	0.541667	0.541667	0.532453	0.586806	0.420139	1.37674	1.37674	1.16648
0.8	1.37274	1.45643	0.0836887	0.539995	0.541844	0.532597	0.606272	0.439961	1.29508	1.33717	1.12672
0.6	1.41524	1.49932	0.0840806	0.538326	0.54204	0.532755	0.627281	0.461361	1.21022	1.29448	1.08382
0.4	1.46121	1.54572	0.0845149	0.536662	0.542257	0.532931	0.650008	0.484522	1.12178	1.24832	1.03742
0.2	1.51103	1.59603	0.0849986	0.535002	0.542499	0.533127	0.674653	0.509652	1.02934	1.19828	0.987112
0.0	1.56518	1.65072	0.0855405	0.533347	0.54277	0.533347	0.701446	0.536986	0.932421	1.14388	0.932421

Table 4: Intermediate Cost Differences

The table shows the analytical solution of the market equilibria for parameter values of $t = 1$, $\lambda = 3$, $c_A = 0.25$, $c_B = 1$:
Prices of both firms are first increasing and then decreasing in β.

β	$p_A^*(\beta)$	$p_B^*(\beta)$	$\Delta p^*(\beta)$	$q_A(\Delta p^*)$	$\hat{x}_{in}(\Delta p^*)$	$\hat{x}_{un}(\Delta p^*)$	π_A^*	π_B^*	CS^*	CS_{in}^*	CS_{un}^*
1.0	1.5	1.75	0.25	0.625	0.625	0.605992	0.78125	0.28125	1.14063	1.14063	0.834921
0.8	1.5039	1.758	0.254109	0.62324	0.627054	0.60798	0.781477	0.285586	1.07357	1.13519	0.827071
0.6	1.50553	1.76414	0.25861	0.621651	0.629305	0.61017	0.780502	0.289112	1.00758	1.13188	0.821115
0.4	1.50448	1.76803	0.263546	0.62026	0.631773	0.612585	0.778104	0.29165	0.942908	1.13111	0.81744
0.2	1.50029	1.76925	0.26896	0.619097	0.63448	0.615251	0.774048	0.293008	0.879835	1.13332	0.816464
0.0	1.49248	1.76737	0.274896	0.618194	0.637448	0.618194	0.768092	0.292988	0.818625	1.13897	0.818625

Table 5: Large Cost Differences:

The table shows the analytical solution of the market equilibria for parameter values of $t = 1$, $\lambda = 3$, $c_A = 0.25$, $c_B = 1.25$: Non-existence for $\beta = 0$ (see Figure 3). $q_A(\Delta p^*)$ is decreasing in β, i.e. uninformed consumers are easier to attract than informed consumers. Reason: Due to large price differences loss aversion in price dimension dominates loss aversion in taste dimension. Uninformed consumers are more willing to buy the less expensive product.

β	$p_A^*(\beta)$	$p_B^*(\beta)$	$\Delta p^*(\beta)$	$q_A(\Delta p^*)$	$\hat{x}_{in}(\Delta p^*)$	$\hat{x}_{un}(\Delta p^*)$	π_A^*	π_B^*	CS^*	CS_{in}^*	CS_{un}^*
1.0	1.58333	1.91667	0.333333	0.666667	0.666667	0.648371	0.888889	0.222222	1.02778	1.02778	0.673468
0.8	1.5623	1.90417	0.341863	0.66734	0.670931	0.652973	0.875753	0.217615	0.974147	1.04598	0.686806
0.6	1.5361	1.88738	0.351282	0.668631	0.675641	0.658117	0.859926	0.211208	0.923306	1.06911	0.7046
0.4	1.5043	1.86596	0.361666	0.670654	0.680833	0.663868	0.841199	0.202865	0.87537	1.09757	0.727236
0.2	1.46663	1.83971	0.373075	0.673535	0.686538	0.670284	0.819444	0.192519	0.830299	1.13163	0.754968
0.0	-	-	-	-	-	-	-	-	-	-	-