Government Size versus Government Efficiency in a Model of Economic Growth

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Abstract

In this paper we develop a Solow type growth model where firms produce a single homogenous good using labor, private capital and a public good - such as highways or sewers. The "amount" of public good available depends on current government spending and government quality, which is the result of the accumulation of public capital. The government charges distortionary taxes and uses them to provide the public good and investing in "quality" by accumulating public capital. We analyze how the composition of government spending between current expenditures and quality affects the equilibrium level of income and the growth rate in steady state. We aim to understand the difference in terms of steady state levels between leviathan, quality driven or even benevolent governments.

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JEL: H, O

1 Introduction

Economists have long been preoccupied with the causes and consequences of the size of government. One of the reasons is surely the little room for additional increases in public spending and the need to accommodate demand by a rise in the efficiency of provision. Since [16], there is a solid perception of a relationship between per capita income and the share of government spending in Gross Domestic Product (GDP). [15], for instance, report a rise in general government spending in OECD countries from 13 to 46 percent of GDP between 1913 and 1996. In the last two decades, the share of total government spending in GDP has increased by 10 percent in OECD countries, according to [19], the continuation of a long trend in government growth in the twentieth century. Given this high weight of government expenditures on GDP, the emphasis has naturally shifted towards the efficiency of government provision. This paper studies precisely the normative issues facing a government that decides how much to spend and whether and how much to improve the efficiency of that spending. The investigation in the framework of an exogenous growth model so that it can be more closely linked with the existing literature.

There are at least three good reasons to conduct such an exercise. First, the burgeoning literature on corruption (or "bureaucracy") and its determinants and consequences, has put forward interesting models and results.[1] However, it is still lacking as to the distinction between corruption and government efficiency, especially at the analytical level.[2] Second, whereas the micro determinants of government efficiency have been widely studied, at least since seminal work, more needs to be done at the macroeconomic level. Thirdly, there is a substantial body of theoretical work on the relationship between government spending and growth,[3] but rather less on government quality and practically nothing on the joint choice of size and quality that

[1] [11] , for instance, finds evidence that economic growth and private investment are negatively affected by the extent of corruption

[2] [14] has shown that the two concepts, though empirically related, are distinct. Kaufmann questions how fundamental are "good governance and controlling corruption" for development? "[9].

[3] See, for instance, [4] and [5].

governments face. After a review of the literature we are left with several questions, among them a preeminent one: Why are some countries stuck in a "bad" equilibrium with low growth rates, low level of income and inefficient governments while others have more efficient governments and wealthier economies?

Our paper is a contribution to the shortcomings in the literature. It develops a growth model where firms produce a single homogenous good using labor, private capital and a public good - such as highways or sewers. The "amount" of public good available depends on current government spending and government quality, which is the result of the accumulation of public capital. The government charges distortionary taxes and uses them to provide the public good and investing in "quality" by accumulating public capital. We analyze how the composition of government spending between current expenditures and quality affects the equilibrium level of income and the growth rate in steady state. As in [7] we are not interested in taking government decisions as given. Although in the context of a Solow model we can not analyze decision making we can see the impact of different scenarios in terms of government options.

The paper is structured as follows. After this introduction, section 2 will present some empirical evidence concerning the way governments manage their budgets. In section 3 we outline the model. In section 4 we will drive the growth rates for the variables of interest. In section 5 we find the steady state levels and in section 6 we will see how the government can make its decisions in order to maximize the steady state levels of each of the variables. In section 7 we will study the stability of equilibrium and in section 8 we conclude.

2 Empirical Evidence

Countries have had for a long time different perspectives about the role of governments and in consequence about the way governments should spend their money. Not only we have countries that devote much of their budget to some sectors that other countries neglect but also we have countries that spend a lot on current expenses while others are clearly more concern with long term investment.

In this section we will try to show that even countries with some resemblance in terms of wealth (we will consider only 21 OECD countries) have considerable different attitudes towards the way they manage their budget.

We will look at three variables: per capita gdp, current expenses (not including social security expenses), and investment in public capital.

In the tables below we order the countries by decreasing order of each of these variables and single out the seven most important ones in each category[4].

[4]The data on GDP was obtained from OECD dataset available online and refers to the last year available.

The data on Expenditures was obtained from IMF government financial statistics and referes to the average of the last 10 years.

The data on public capital was obtained from **??** and referes to the average of the last 10 years

Table 1	GDPpc	Expenditures	Capital
Norway	**40 533.6**	24.58	50.21
United States	**37 962.6**	15.34	56.87
Ireland	**36 701.5**	31.94	68.63
Iceland	**34 776.2**	24.09	51.88
Switzerland	**34 017.9**	11.69	49.92
Netherlands	**32 610.6**	33.38	73.30
Sweden	**32 609.6**	22.79	42.35
Austria	32 049.7	20.67	71.25
Canada	31 746.3	14.26	41.66
Denmark	31 468.3	22.71	64.49
Australia	31 342.5	18.45	48.93
Finland	31 050.7	21.31	47.97
Belgium	30 463.3	29.79	43.13
United Kingdom	30 150.4	26.52	54.39
Japan	28 320.7	13.88	100.79
Germany	28 145.6	15.58	55.25
France	27 338.2	23.90	55.32
Italy	26 473.2	29.31	48.38
Spain	24 164.5	16.99	41.22
Greece	23 955.5	26.01	49.41
Portugal	17 754.2	29.33	33.40

Table 2	GDPpc	Expenditures	Capital
Netherlands	32 610.6	**33.38**	73.30
Ireland	36 701.5	**31.94**	68.63
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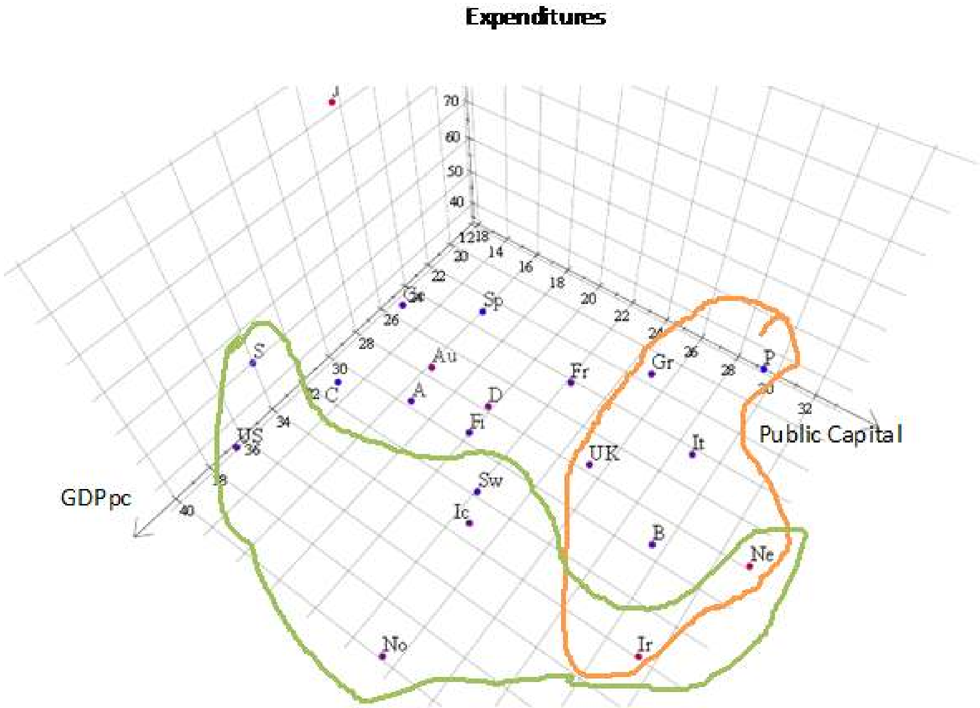
The countries in red are the top seven in terms of GDPpc, the ones shaded with green are in the top seven with respect to public expenditures and the underline ones are the ones that invest most in public capital.

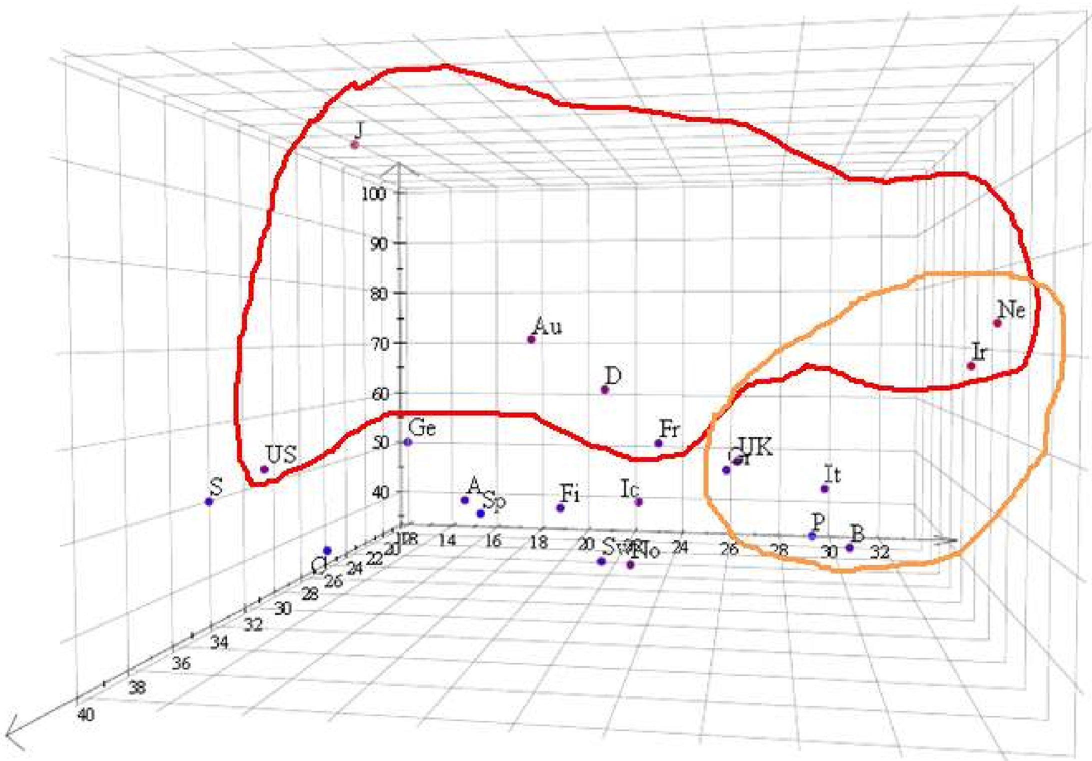
In the top seven most in one category and outside in all other categories, we have:

- High Spender: Belgium, Portugal, Greece and Italy
- High Investors in public capital: Japan, Austria and Denmark
- Wealthier nations: Norway, Iceland and Switzerland

  The most balanced countries seem to be the Netherlands and Ireland (both of them are in the top seven in the three variables) and the United States that are in the top seven in wealth and public capital. We can also see that only 6 out of 21 countries are not top seven in something and only 8 out of 21 are not in the top seven something and positioned lower than the seventh raking in everything else.

To Help get a clear view of this ranking we can look at the following charts:





We can state with confidence that there is very distinct behaviors among countries. What exactly determines this behaviors? What are the consequences in terms of the way governments work? Do this extreme positions determine the efficiency or quality of public service? What we intend to do is to provide the theoretical fundamentals behind some of these questions.

3 The Model

3.1 The Government

In our model the government divides its resources between a productive public good, H_t - public spending - and investment in public capital, K_{gt}, which affects the efficiency of public spending in the future. Public capital is "transformed" into an indicator of the efficiency of public spending q_t, or the quality of government. Thus, quality as the characteristics of capital good, with delayed but persistent effects on output. We think this captures the nature of the quality versus the "size" of government. As [14] demonstrates for a cross-section of OECD countries, there is a robust positive relationship between measures of public capital and indicators of the efficiency and quality of government. This is the relationship we capture here. Relative to public spending, the efficiency of public spending takes time to build through capital accumulation in the public sector.

The following equations describe public spending, public investment and quality.

$$H_t = \theta \tau Y_t \tag{1}$$

$$\overset{\bullet}{K}_{gt} + \delta_g K_{gt} = (1-\theta)\, \tau Y_t \tag{2}$$

$$q_t = (K_{gt})^{\psi} \tag{3}$$

Where H_t stands for government spending, which is financed through an income tax, τ, where income is Y_t. A fraction θ of tax revenues is used to finance government spending so that τ and θ are policy decisions. A fraction $1-\theta$ of tax revenues finances investment in public capital, which is then transformed into quality as q_t. The parameter ψ represents the elasticity of quality q_t with respect to K_{gt}. Introducing this production function for quality allows for diminishing returns on public capital, corresponding to the assumption that $\psi \leqslant 1$. Thus, Y_t is the aggregate production of the economy and δ_g is the depreciation rate of public capital.

Before going any further let's look a little bit closer to the variables H_t and q_t. The first variable can be thought of as the amount of public expenditures associated with the size of government. In fact the weight of government is given by τ, the tax rate on income but a fraction $1-\theta$ has only a lagged effect on productivity through the accumulation of public capital. We can think of $H_t = \theta\tau Y_t$ as the amount of public spending, which has the same factor composition as output. As for q_t , in order to be sustained or increased, quality requires an investment effort from the government. This layout allows us to study the impact of different government objective functions, namely a government that maximizes H_t - a leviathan -, and a government that worries about output per capita and optimizes the size and quality mix, in other words, τ and θ.

3.2 Production

There is only one production good, which is also the consumption good. We define the aggregate production function as follows:

$$Y_t = \left(K_{pt}^{\alpha} L_t^{1-\alpha}\right)^{\beta} \left(H_t q_t\right)^{1-\beta} \tag{4}$$

Where K_{pt} stands for private capital and L_t represents labor. The first part of the production function is a Cobb-Douglas production function with capital and labor as complements, each with diminishing returns on Yt. Public spending and its quality matter for production, with the same weight so that quality "qualifies" the amount of spending: it is the volume of public goods times its quality that enters as productive inputs to the private sector.

The private capital law of motion is given by :

$$\dot{K}_{pt} = s\left(1-\tau\right) Y_t - \delta_p K_{pt} \tag{5}$$

where s is the saving rate out of the net income that we consider exogenous and δ_p stands for the depreciation rate of private capital. We assume that s is constant, even though later its level may become a choice variable for the private sector. Henceforth we will assume, with no loss of generality, that the labor force, L, is constant and normalize it to 1:

$$Y_t = \left(K_{pt}^{\alpha}\right)^{\beta} \left(H_t q_t\right)^{1-\beta} \tag{6}$$

Replacing H_t for its expression above we find that:

$$Y_t = K_{pt}^{\alpha\beta} \left(\theta\tau q_t\right)^{\frac{1-\beta}{\beta}} \tag{7}$$

Finally, replacing quality, q_t for its expression above:

$$Y_t = K_{pt}^{\alpha} \left(\theta\tau K_{gt}^{\psi}\right)^{\frac{1-\beta}{\beta}} \tag{8}$$

and we are left with a production function in where income depends only on private and public capital.

4 Growth Rates

4.1 Private Capital

We have already defined the law of motion for private capital in (5):

$$\dot{K}_p = s(1-\tau)Y_t - \delta_p K_{pt} \tag{9}$$

And conclude that:

$$\frac{\dot{K}_p}{K_{pt}} = s(1-\tau)K_{pt}^{\alpha-1}(\theta\tau K_{gt}^{\psi})^{\frac{1-\beta}{\beta}} - \delta_p \tag{10}$$

so that the growth rate of private capital depends on its level, on the saving rate net of taxes and on the level of public capital.

4.2 Public Capital

We have defined the accumulation of public capital in (2):

$$\dot{K}_g = (1-\theta)\tau y_t - \delta_g K_{gt} \tag{11}$$

Which, through appropriate transformation, gives us:

$$\frac{\dot{K}_g}{K_{gt}} = (1-\theta)\tau K_{pt}^{\alpha}(\theta\tau)^{\frac{1-\beta}{\beta}} K_{gt}^{\frac{\psi(1-\beta)}{\beta}-1} - \delta_g \tag{12}$$

Similar to what we had on (10) we have that the growth rate of public capital depends on its own level and on the level of private capital.

5 The steady state

In the steady state we must have $\dot{K}_g = \dot{K}_p = 0$. Assuming that $\delta_p = \delta_g = \delta$, i.e., the nature of public and private capital is similar as far as depreciation, we can rewrite the above equations:

$$\begin{cases} s(1-\tau)K_{pt}^{\alpha-1}(\theta\tau K_{gt}^{\psi})^{\frac{1-\beta}{\beta}} = \delta \\ (1-\theta)\tau K_{pt}^{\alpha}(\theta\tau)^{\frac{1-\beta}{\beta}} K_{gt}^{\frac{\psi(1-\beta)}{\beta}-1} = \delta \end{cases} \tag{13}$$

Which in turn leads us to the following steady state ratio:

$$\frac{K_{pt}}{K_{gt}} = \frac{s(1-\tau)}{(1-\theta)\tau} \tag{14}$$

Some basic conclusions can be drawn from the relation described in (14):

If the private saving rate increases, this ratio will also increase so that there will be relatively more private capital. This seem intuitive given the positive relation between private capital accumulation and the saving rate. On the other hand, if the percentage of public resources that are directed towards the accumulation of public capital (which we can call public saving rate), $(1-\theta)$, increase, the ratio of private to public capital decreases. Lastly, if the tax rate increases the ratio of private to public capital decreases. This is due to two different reasons. First, if the tax rate increases then the income available to the private sector decreases which causes a slower accumulation of private capital (for a constant saving rate). Secondly, if the tax rate increases then there will be more available resources to the public sector which allows a greater investment on public capital, for given θ.

Using data from [8] and from de World Development Indicators 2004 we estimated 14. We used OLS estimation with a pooled data set of 536 observations.[5] The following regression[6] allows us to confirm that private saving rate has a positive effect on the ratio of private to public capital, the public saving rate has a negative impact on the same ratio and the ratio of

[5] We used 21 OECD countries and the maximum number of years available between 1972 and 2000.

[6] t-stat between brackets

taxes ($\frac{1-\tau}{\tau}$) has a positive effect on the amount of private capital (as we could already guess from 14).

$$\ln\left(\frac{\hat{K_{pt}}}{K_{gt}}\right) = \underset{(7.661913)}{0.238}\ln s + \underset{(8.656481)}{0.231}\ln\left(\frac{1-\tau}{\tau}\right) \underset{(-4.529)}{-0.1056}\ln(1-\theta) \tag{15}$$

We will now solve for the steady state level of the main variables so that we can investigate the relation between those variables and the policy variables.

5.1 Private Capital

We have already seen that:

$$s(1-\tau)K_{pt}^{\alpha-1}(\theta\tau K_{gt}^{\psi})^{\frac{1-\beta}{\beta}} = \delta \tag{16}$$

replacing K_{gt} using the relation given by (14), and solving for K_{pt} we are left with:

$$K_p^{ss} = \left\{\frac{\tau^{(1-\beta)(1+\psi)}(1-\theta)^{\psi(1-\beta)}\left[s(1-\tau)\right]^{\beta-\psi(1-\beta)}\theta^{(1-\beta)}}{\delta^{\beta}}\right\}^{\frac{1}{\beta(1-\alpha)-\psi(1-\beta)}} \tag{17}$$

Some conclusions can now be drawn from the above equation concerning the relation between the model parameters, the policy variables and private capital .

The depreciation rate should have a negative impact on the steady state level of private capital so that $\frac{\partial k_p^{ss}}{\partial \delta} \leq 0$. This implies that:

$$\frac{\beta}{\beta(1-\alpha)-\psi(1-\beta)} \geq 0 \tag{18}$$

This ratio will be zero only in the case where $\beta = 0$. This would mean that the private inputs have no relevance to production whatsoever, which is rightly excluded here. This leads to the requirement that:

$$\beta(1-\alpha) - \psi(1-\beta) > 0 \tag{19}$$

or

$$\alpha + \psi\frac{1-\beta}{\beta} < 1 \tag{20}$$

In other words, we need diminishing returns on private and public capital together.

On the other hand the private saving rate should have - up to a given point - a positive effect on the equilibrium level of k_p^{ss}:

$$\frac{\partial k_p^{ss}}{\partial s} \geq 0 \iff \frac{\beta - \psi(1-\beta)}{\beta(1-\alpha) - \psi(1-\beta)} \geq 0 \tag{21}$$

this partial derivative will be zero if $\psi = \frac{\beta}{1-\beta}$ meaning that changes in the saving rate will have no impact on the steady state level of private capital. We exclude this hypothesis here and we are left with two possibilities: an increase in the saving rate leads to a decrease in the equilibrium level of K_p^{ss} or to an increase in the same variable. The last option seems more reasonable and, under (21), implies that:

$$\beta - \psi(1-\beta) > 0 \tag{22}$$

or

$$\beta > \psi(1-\beta) \tag{23}$$

that is, on the margin private capital is "more productive" than public capital.

5.2 Income

Making use of (8), (14) and finally of (17) we can, after some algebra, present the steady state level of per capita output as:

$$Y^{ss} = \left\{ \frac{\tau^{(1-\beta)(1+\psi)}(1-\theta)^{\psi(1-\beta)}\left[s(1-\tau)\right]^{\alpha\beta}\theta^{(1-\beta)}}{\delta^{\alpha\beta+\psi(1-\beta)}} \right\}^{\frac{1}{\beta(1-\alpha)-\psi(1-\beta)}} \tag{24}$$

5.3 Consumption

Using the expression for consumption

$$C^{ss} = (1-\tau)(1-s)Y^{ss} \tag{25}$$

where C^{ss} stands for steady state level of consumption, we can replace Y_t using equation (24) and we obtain:

$$C^{ss} = (1-s)\left[\frac{\tau^{(1-\beta)(1+\psi)}(1-\theta)^{\psi(1-\beta)}s^{\alpha\beta}(1-\tau)^{\beta-\psi(1-\beta)}\theta^{(1-\beta)}}{\delta^{\alpha\beta+\psi(1-\beta)}}\right]^{\frac{1}{\beta(1-\alpha)-\psi(1-\beta)}} \tag{26}$$

5.4 Quality

Making use of relation (3) and (14) we can write the following:

$$q^{ss} = \left[\frac{(1-\theta)\tau}{s(1-\tau)}k_p^{ss}\right]^{\psi} \tag{27}$$

and replacing K_p^{ss} by (17) we get:

$$q^{ss} = \left\{\frac{\tau^{\psi(1-\beta\alpha)}\theta^{(1-\beta)\psi}(1-\theta)^{\psi\beta(1-\alpha)}\left[s(1-\tau)\right]^{\beta\alpha\psi}}{\delta^{\beta\psi}}\right\}^{\frac{1}{\beta(1-\alpha)-\psi(1-\beta)}} \tag{28}$$

5.5 Public Good

Using the relation between income and the amount of public good provided, we have:

$$H^{ss} = \theta\tau H^{ss} \iff \tag{29}$$

$$H^{ss} = \left\{\frac{\tau^{\beta(1-\alpha)+1-\beta}(1-\theta)^{\psi(1-\beta)}\left[s(1-\tau)\right]^{\alpha\beta}\theta^{\beta(1-\alpha)+(1-\beta)(1-\psi)}}{\delta^{\alpha\beta+\psi(1-\beta)}}\right\}^{\frac{1}{\beta(1-\alpha)-\psi(1-\beta)}} \tag{30}$$

6 Government Decisions

In the previous section we have presented the steady state levels for the main variables in the model. We now turn to the government decisions on the size of government and the composition of government spending, τ and θ the decisions the government can make in order to maximize this equilibrium levels.

We will consider alternative scenarios considering the government main concern.

We will start with a benevolent government, which desires to maximize steady-state consumption, and later will consider a *leviathan* government, which maximizes current spending, and a *quality-drive* government, which maximizes the level of quality in steady-state.

6.1 Benevolent Government

In this section we will try and see the choices the government will undertake if interested in maximizing the steady state level of consumption. In the appendix we consider briefly the objective of maximizing output.

6.1.1 Consumption

We can take the partial derivative of steady-state consumption, C^{ss} , with respect to θ and obtain:

$$\frac{\partial C^{ss}}{\partial \theta} = (1-\tau)(1-s)\frac{\partial y^{ss}}{\partial \theta} \tag{31}$$

$$\theta = \frac{1}{1+\psi} \tag{32}$$

This is like a modified golden-rule, which can give us the saving rate that maximizes steady-state consumption. It is easy to understand that the level of θ that maximizes C^{ss} is exactly the same as the one that maximizes Y^{ss}, all the conclusions are therefore the same. The distribution of public resources between its two possible uses affects the equilibrium level of consumption in the exact same way it affects the equilibrium level of per capita income (and consequently of per capita available income).

Now the tax rate that maximizes the steady state level of consumption is some what different.

$$\frac{\partial C_t^{ss}}{\partial \tau} = 0 \iff \tau = (1-\beta)(1+\psi) \tag{33}$$

If $0 < \beta < 1$ and if $0 \leq \psi \leq \frac{\beta}{1-\beta}$, then $0 < \tau < 1$. In this case, the equilibrium level of consumption will be maximized by a tax rate that depends negatively on the weight of the private sector on the production function and positively on the marginal productivity of public capital (for a given ratio of $\frac{K_{gt}}{q_t}$). As the weight of the private sector increases a benevolent policy maker will be interested in increasing the available income, and hence in decreasing the tax rate, in order to increase C^{ss}. On the other hand if public capital becomes more productive, accumulating it will have a positive effect on available income (and consequently on consumption) even if it means taking more resources out of the private sector. If $\beta = 0$ or if $\psi > \frac{\beta}{1-\beta}$ we would have $\tau > 1$, this would mean that we would have a negative level of consumption which is not possible. If $\beta = 1$ then $\tau = 0$, in this case the public sector would have no relevance in the production function and the way to maximize C^{ss} would be by not taking any resources from the private sector.

So far we have seen the case of a benevolent policy maker that chooses θ or τ in order to maximize the steady state levels of private quantities. What if the policy maker wanted to maximize its own quantities? The state could be interested in maximizing the amount of public good rendered or he could be by nature an investor and hence be interested in maximizing quality.

6.2 Self Interested Government

We now consider a government that is concerned with its own policies, either a *leviathan* type that maximizes public consumption or a *quality-driven* that maximizes the level of public capital.

6.2.1 Government Quality

Let us first consider a *quality-driven* government and recall equation [28] above. We can obtain the maxima for the level of government quality through the expression, namely

$$\frac{\partial q^{ss}}{\partial \theta} = 0 \iff \theta = \frac{1-\beta}{1-\beta\alpha} \tag{34}$$

and:

$$\frac{\partial q^{ss}}{\partial \tau} = 0 \iff \tau = 1-\beta\alpha \tag{35}$$

The product of θ and τ, that is, the size of government is given by

$$\theta\tau = 1-\beta \tag{36}$$

These results can be read as follows: The percentage of total resources of the economy that should be devoted to the public sector (τ) in order to maximize its quality, depends negatively on the weight of the private sector in the production function (β) and on the sensitivity of the product to variations in the private capital (α). As the share of the private sector increases in the production function (β) or the marginal productivity of private capital decreases (α) the *quality-driven* government will choose to increase the share of public resources going into the accumulation of public capital .

6.2.2 Quantity of Public Good

In this section we are interested in the choices of a *leviathan* government interested in maximizing the size of the public sector. Taking the partial derivative of public consumption with respect to θ, we find the θ that maximizes the steady state quantity of the public good as:

$$\frac{\partial H^{ss}}{\partial \theta} = 0 \iff \theta = \frac{\beta(1-\alpha)+(1-\beta)(1-\psi)}{\beta(1-\alpha)+1-\beta} \tag{37}$$

As we can see when the weight of private capital in the production function goes to 0 the percentage of public resources devoted to public good in order to maximize the equilibrium value of this public good will go to $1-\psi$., a value still lower than 1 since the optimal investment in quality is not zero, even if the objective is to maximize public consumption. The percentage of public resources that go into the production of public good will only be 1 if public capital is totally unproductive.

Turning to the tax rate we now have:

$$\frac{\partial H^{ss}}{\partial \tau} = 0 \iff \tau = \beta(1-\alpha) + 1 - \beta = 1 - \beta\alpha \tag{38}$$

We can observe that, if $\beta = 0$ we will have $\tau = 1$. In other words, if private inputs do not contribute to production then the government will be interested in taking all the resources from the private sector in order to maximize the amount of public goods. If $\beta = 1$, $\tau = 1 - \alpha$, that is, even if public inputs are of no interest to production, the policy maker will still be interested in collecting taxes at a positive rate since its objective is to maximize public consumption.

7 The Equilibrium Path

Having analyzed the steady state levels for the several variables in the model, and the government decisions to maximize the different variables, we are now interested in analyzing the stability of the equilibrium. To do that we will discuss the phase diagram of the two state variables in the model: private capital and public capital (presented in Figure 1). Recall equations (10) (12) and assuming that capital depreciates at the same rate, irrespective of its private or public nature, we can write:

$$\dot{K}_p = s(1-\tau)K_p^{\alpha}(\theta\tau k_g^{\psi})^{\frac{1-\beta}{\beta}} - \delta K_p \tag{39}$$

and also:

$$\dot{K}_g = (1-\theta)\tau K_p^{\alpha}(\theta\tau)^{\frac{1-\beta}{\beta}} K_g^{\frac{\psi(1-\beta)}{\beta}} - \delta K_g \tag{40}$$

Setting both expressions to zero and assuming that expression (20) holds, we can draw figure1.

The concave line represents $\dot{K}_g = 0$ and the other line represents $\dot{K}_p = 0$. The interception point of the curves gives us the steady state ratio describe

Figure 1:

above in (14). If we increase K_p from the $\dot{K}_g = 0$ line - moving to a point below the curve - this will lead to $\dot{K}_g > 0$ so that, below $\dot{K}_g = 0$, K_g tends to increase. On the other hand if we increase k_g from the $\dot{k}_p = 0$ line - go to a point above this curve - we are left with $\dot{k}_p > 0$ so that, above $\dot{K}_p = 0$, K_p will tend to increase. There are four distinct regions in figure 1:

- Above both lines - Region A
- Between both lines and below $\dot{K}_g = 0$ - Region B
- Below both lines - Region C
- Between both lines but above $\dot{K}_g = 0$ - Region D

Let us now see what can happen if we are located in each of these four regions. If we are in region A we have K_p growing and K_g diminishing. One of three things might happen: we could reach immediately the equilibrium, we can first reach the line $\dot{K}_g = 0$ or we can first reach the line $\dot{K}_p = 0$. In the first scenario the problem would be automatically solved. In the second one we would be in a situation where K_g does not grow and K_p is still growing we would then go to a point inside region B. In the third scenario K_p does not grow but K_g is decreasing, we would fall inside region D.

If we start at point in region B we can also reach one of the two lines. If we reach the line $\dot{K}_g = 0$ we would be in a situation where K_g does not grow but K_p grows. We would be driven towards the interior of region B again but this time closer to the equilibrium. On the other hand we can fall into the line $\dot{K}_p = 0$ first. In this case because K_p is not growing but K_g is, we would fall back into region B but once again closer to the equilibrium. This situation would repeat itself until we reach the interception point.

If we start at a point located at region D and reach the line $\dot{K}_g = 0$ we will fall back inside region D because K_g will not grow and K_p will decrease. If we reach $\dot{K}_p = 0$ then, because K_g is decreasing and K_p is not growing, we will go back to region D. In both cases we are again inside region D but we are one step closer to the equilibrium and sooner or later we will reach it.

The last hypothesis is to be start from a point in region C. Once again we can reach each of the lines. If we reach $\dot{K}_g = 0$ and because K_p is now decreasing we will get inside region D leading us to the equilibrium. If we reach $\dot{K}_p = 0$ we know that K_p is not growing but K_g is. This will lead us to a point inside region B. Once inside region B we will eventually reach the equilibrium point.

We can conclude that independently of the starting point we will follow a path to equilibrium.

8 Conclusions

In this paper we present a model where the government is a decision maker and public spending contributes to production through current public spending that enhances government quality, a function of public capital and thus of public investment. Per capita output depends on private capital and public capital per capita. We assume diminishing returns on both types of capital and draw several conclusions.

There is a stable steady state level of public and private capital that does not depend on the starting levels. In equilibrium the ratio of private capital to public capital will be a decreasing function of the tax rate and an increasing function of the percentage of resources devoted to current expenses.

In equilibrium income per capita has an inverted U-curve relationship with the tax rate (which is consistent with the findings of [3]). The level of the tax rate that maximizes the steady state level of per capita income is a negative function of α and of β and a positive function of ψ. α gives us the elasticity of production in respect to per capita private capital so that, if income is more sensitive to variations in per capita private capital, an increase in the tax rate will have a strong negative impact on output because it will diminish private capital. The relationship with β is explained in the same manner as this parameter represents the weight of the private sector in the production function. ψ is a measure of the sensitivity of government quality to public capital. If government quality is quite sensitive then a small increase of public capital could have a strong impact on quality and hence on production so it is reasonable that the tax rate that maximizes income per capita should be a positive function on this parameter.

In equilibrium income has an inverted U-curve relationship with the public expenses composition. There is an optimal level of resources devoted to current expenses. This optimum level depends only on the elasticity of quality in respect to public capital. If quality is inelastic then level of θ that maximizes the equilibrium per capita income will be larger than in the case of a more elastic quality production function. It is interesting to see that the steady state level of quality also has an inverted U-curve relationship with both θ and τ.

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Appendix A - A Benevolent Government That Mazimizes Steady-state Output

Here we consider a government that, as an alternative to maximizing private consumption, maximizes steady-state output, y_t. The levels of θ and of τ that maximize per capita income are obtained from:

$$\frac{\partial y^{ss}}{\partial \theta} = 0 \iff \theta = \frac{1}{1+\psi} \tag{A1}$$

with respect to the composition of government spending, which is equivalent to

$$(1-\theta) = \frac{\psi}{1+\psi} \tag{A2}$$

and

$$\frac{\partial y^{ss}}{\partial \tau} = 0 \iff \tau = \frac{(1-\beta)(1+\psi)}{\alpha\beta + (1-\beta)(1+\psi)} \tag{A3}$$

with respect to the tax rate.

We are now able to draw some conclusions. The percentage of public resources devoted public capital accumulation, $(1-\theta)$, that maximizes the steady state level of output is an increasing function of ψ,the elasticity of quality with respect to public capital. This makes perfect sense: the more sensitive quality is to increases in the amount of public capital, the larger should be the amount of resources allocated to investment in public capital.

As to level of the tax rate, we can check that $\tau = 0$ if $\beta = 1$. In other words, if only private capital matters for production, there is no point in taking resources from the private to the public sector. On the other extreme, if $\beta = 0$ then $\tau = 1$, meaning that if the production function depends only on the public sector then the government will want to collect as much resources as possible. As β increases from 0 to 1, τ will go from 1 to 0. The higher the weight of private inputs in the production function, the fewer the resources transferred from the private sector if the aim is maximizing the steady-state level of per capita income. In addition, the tax rate that maximizes the equilibrium level of per capita output is a decreasing function of α. As the marginal product of private capital increases, the optimal tax rate will decrease. The optimal tax rate is a positive function of ψsince, as the elasticity of quality with respect to public capital increases, the steady-state output level is maximized at higher tax rates.