

# US Volatility Cycles of Output and Inflation, 1919-2004: A Money and Banking Approach to a Puzzle\*

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## Abstract

The post-1983 moderation coincided with an ahistorical divergence in the money aggregate growth and velocity volatilities away from the downward trending GDP and inflation volatilities. Using an endogenous growth monetary DSGE model, with micro-based banking production, enables a contrasting characterization of the two great volatility cycles over the historical period of 1919-2004, and enables this puzzle to be addressed more easily. The volatility divergence is explained by the upswing in the credit volatility that kept money supply variability from translating into inflation and GDP volatility.

**Keywords:** Volatility, money and credit shocks, growth, inflation

**JEL:** E13, E32, E44

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# 1 Introduction

Explaining changes in real output and in inflation has been done by focusing on short run factors. For example the standard is to estimate the effect of current monetary policy shocks with the shock restricted to be only in the short run. Yet studies continue to find that trend inflation is Granger caused by money, such as in Crowder (1998) for the US, and Assenmacher-Wesche and Gerlach (2008) for the Euro area in which the inflation-money causality is found for the medium and longer run. Econometric studies find a long run negative effect of inflation on growth, such as Fountas, Karanasos, and Kim (2006), who find this for the US, UK and Japan with Granger causality from inflation to growth. This suggests that money shocks may well have persistence that affects real variables over long periods of time. And as Muller and Watson (2008) conclude:

Most macroeconomic series and relationships thus exhibit pronounced non-trivial dynamics below business-cycle frequencies...this underlies the importance of understanding the sources and implications of such low frequency volatility changes (p.1008).

Ignoring the long run impact of monetary policy also excludes a reputable inflation tax literature that starts with Bailey, 1956, and goes up to the inflation-induced goods to leisure substitution that decreases the endogenous output growth rate (Gomme 1993, Gillman and Kejak 2005). Benk et al. (2008) apply these Lucas (1988) -based growth extensions to Cooley and Hansen (1989) so as to include key likely shocks that affect the fluctuations of velocity: goods productivity, money supply, and credit shocks as in Benk, Gillman, and Kejak (2005).

This paper includes long run features of the data and applies the Benk et al. (2008) framework to explain a puzzle: why the annual US volatility of inflation and output diverged downwards after 1983 away from the volatilities of velocity and money supply growth which moved upwards post-1983, after all four of these volatilities had moved together historically from 1919-1983 (Section 2). Rather than filtering out long run features of the shocks on the basis that they are unimportant to volatility, a minimal 86 year Christiano and Fitzgerald (2003) filter is used to obtain stationarity series, with windows for the short run, business cycle and long run as defined in Levy and

Dezhbakhsh (2003), at 0-2 years, 3-8 years, and 8+ years respectively; the latter window is similar to the long run of Muller and Watson (2008).

Money shocks are found to have a significant effect on the volatility of endogenous growth rate of output and of inflation, as are the credit and goods productivity shocks, across the full frequency spectrum (Section 5). As in Ingram, Kocherlakota, and Savin (1994) and Benk et al. (2008), the parsimonious set of shocks are constructed from the equilibrium solution of the economy and from actual filtered data on equilibrium variables, including the state variable. These shocks are found to explain, on average across the four subperiods covering 1919-2004, about 50% of the output growth variability and 72% of the inflation variability, and with variation taking place in all three frequencies.

The explanation of the puzzle is that the model's implied credit shock volatility rose at the same time as velocity and money supply volatilities, suggesting that a greater volatility of credit during the financially deregulated period insulated the economy from inflation and GDP volatility. As a corollary, in contrast during the Depression period when credit was constrained by virtue of the bank failures, inflation and velocity variability were much higher than credit variability and monetary shocks could more easily translate into inflation and GDP shocks.

The puzzle's explanation is supported by simulation results that show a good ability of the model in explaining RBC and monetary relative volatilities and correlations, as compared to the data over the period (Section 4). Also supportive is that the model's credit shock correlation with the goods productivity shock changes from an historically negative sign, during the subperiods occurring from 1919-1983, to a positive sign during the Great Moderation subperiod of 1984-2004. And the standard deviation of the money shock is found to vary little across the four subperiods of the two cycles, indicating that indeed the money supply shock process can be viewed historically as part of a continuous monetary policy process in which shocks arise as part of a stable variance structure (Section 6).

The financial deregulation approach to the puzzle is not inconsistent with role of finance contributing to the Great Moderation as in Jermann and Quadrini (2006) or Perri and Quadrini (2008). And the long run contribution of the money shock to volatility is consistent with what Chari, Kehoe, and McGrattan (2008) argue needs to be a part of the monetary policy process in order to explain interest rate empirics. Ignoring this component, as well

as ignoring the distortions along the labor-leisure margin, they argue result in a "dubiously" specified set of shocks with non-robust policy prognoses. In our economy, with the money supply as part of the shocks of the economy, and with credit productivity shocks associated with changes in banking laws, these shocks can affect the long run inflation rate, nominal interest rate and the leisure-labor margin as well as provide potentially a policy-related way in which important volatilities can rise and fall. Fluctuations in money-induced inflation taxes and in implicit taxes from banking regulation can affect the economy's margins.

## 2 Historical Trends

Viewing the historical volatility cycles reveals a volatility puzzle. US GDP and inflation rate volatilities rose steadily from the 1950s through the mid-1980s, and then subsequently decreased during the "Great Moderation", thereby creating a full volatility cycle. Preceding this volatility cycle was a larger rise and decline in these two volatilities in the period from 1919-1954, encompassing the Great Depression and WWII. Figures 1-4 show that inflation, its volatility, the money supply growth rate, money velocity volatility and GDP volatility all moved roughly together from 1919-1983. Post-1983, inflation and GDP volatility moved downwards together while money supply growth and velocity volatility diverged upwards.<sup>1</sup>

1) The absolute value of the inflation rate level and its volatility move together, as can be seen in Figure 1. That inflation is positively related to inflation uncertainty is supported in Fountas and Karanasos (2007) for the G7 countries.

2) The M1 money supply growth rate tracks inflation, as seen in Figure 2, although with prominent deviations post 1983. Here a 5-year moving average is used for money growth so as to focus on the trend.

3) Inflation volatility, GDP volatility, and GDP growth rate volatility moved together closely (except WWII); see Figure 3. There was a volatility cycle after WWI, that went up and down from the 1920s to the 1950s (with a double hump for GDP including WWII); there is another lower magnitude

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<sup>1</sup>Volatility is calculated as the standard deviation of the variable over a certain window. For annual data it is a 7 year window; the formula is

$$volatility(x_t) = SD(x_{t-k}, x_{t-k+1}, \dots, x_t, \dots, x_{t+k-1}, x_{t+k}), \text{ where } k = 3.$$

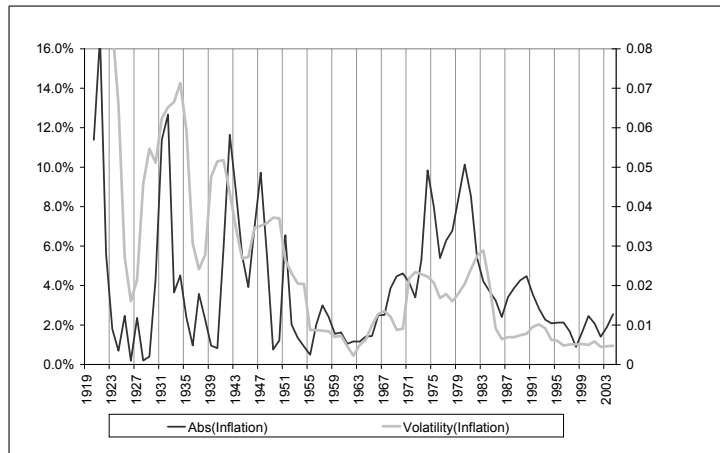


Figure 1: Absolute Value of Inflation and its Volatility, 1919-2004

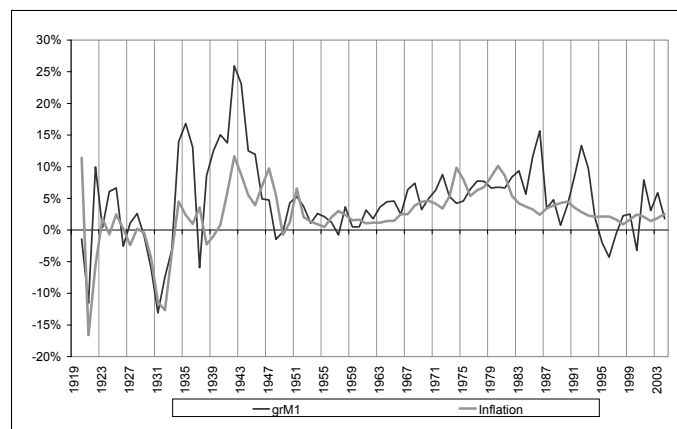


Figure 2: M1 Money Supply Growth (5-Yr MA) and Inflation, 1919-2004

cycle, up and down, from the 1950s to 2000.<sup>2</sup>

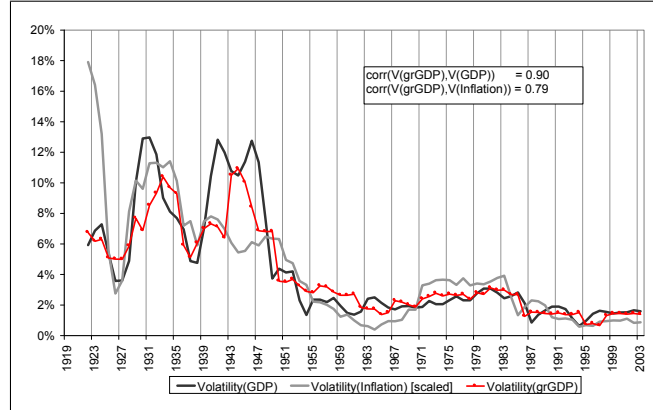


Figure 3: Volatilities of Inflation, GDP, and GDP Growth, 1919-2004

4) Lesser known, money velocity volatility and M1 growth rate volatility moved together and broadly followed inflation and GDP volatility up until 1983, when they together sharply diverged from the other two.

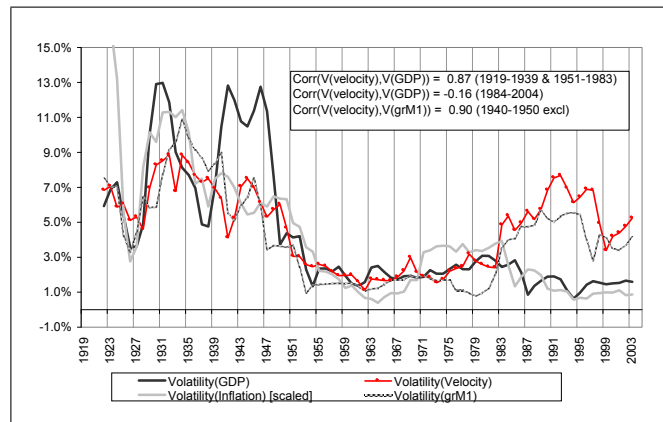


Figure 4: Volatilities of GDP, Inflation, Velocity and M1 Growth 1919-2004

Together these facts suggest as historically plausible a priori the proposition that the money supply growth may partly cause inflation and its volatility, which is correlated with GDP volatility, while allowing for the possibility

<sup>2</sup>Inflation volatility in Figures 3 and 4 is scaled by multiplying it by the average proportion difference between it and the GDP volatility for the 1919-2004 period. Standard data is used for GDP (described in Appendix A.1); alternative experimentation with the Miron and Romer (1990) data found a larger volatility of GDP during the 1919-1939 period.

that the puzzling divergent increase in the volatility of money supply growth and money velocity post-1983 could be entangled with new credit instruments that enabled facile inflation avoidance after financial deregulation.

### 3 Stochastic Endogenous Growth with Banking

The representative agent economy is extended from Benk, Gillman, and Kejak (2008) by decentralizing the bank sector that produces credit. By combining the business cycle with endogenous growth, stationary inflation lowers the output growth rate as supported empirically for example in Gillman, Harris, and Matyas (2004) and Fountas, Karanasos, and Kim (2006). Over the business cycle, shocks cause changes in growth rates and in stationary ratios. The shocks to the goods sector productivity and the money supply growth rate are standard, while the third shock to the credit sector productivity exists by virtue of the model's endogeneity of money velocity via a micro-based production of exchange credit.

The shocks occur at the beginning of the period, observed by the consumer before the decision process, and follow a vector first-order autoregressive process. For goods sector productivity,  $z_t$ , the money supply growth rate,  $u_t$ , and bank sector productivity,  $v_t$  :

$$Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt}, \quad (1)$$

where the shocks are  $Z_t = [z_t \ u_t \ v_t]'$ , the autocorrelation matrix is  $\Phi_Z = \text{diag}\{\varphi_z, \varphi_u, \varphi_v\}$  and  $\varphi_z, \varphi_u, \varphi_v \in (0, 1)$  are autocorrelation parameters, and the shock innovations are  $\varepsilon_{Zt} = [\varepsilon_{zt} \ \varepsilon_{ut} \ \varepsilon_{vt}]' \sim N(\mathbf{0}, \Sigma)$ . The general structure of the second-order moments is assumed to be given by the variance-covariance matrix  $\Sigma$ . These shocks affect the economy as described below.

#### 3.1 Consumer Problem

A representative consumer has expected lifetime utility from consumption of goods,  $c_t$ , and leisure,  $x_t$ ; with  $\beta \in (0, 1)$  and  $\theta > 0$ , this is given by

$$U = E_0 \sum_{t=0}^{\infty} \beta \frac{(c_t x_t^\Psi)^{1-\theta}}{1-\theta}. \quad (2)$$

Output of goods,  $y_t$ , and increases in human capital, are produced with physical capital and effective labor each in Cobb-Douglas fashion; the bank sector produces exchange credit using labor and deposits as inputs. Let  $s_{Gt}$  and  $s_{Ht}$  denote the fractions of physical capital that the agent uses in the goods production ( $G$ ) and human capital investment ( $H$ ), whereby

$$s_{Gt} + s_{Ht} = 1. \quad (3)$$

The agent allocates a time endowment of one amongst leisure,  $x_t$ , labor in goods production,  $l_t$ , time spent investing in the stock of human capital,  $n_t$ , and time spent working in the bank sector, denoted by  $f_t$ :

$$l_t + n_t + f_t + x_t = 1. \quad (4)$$

Output of goods can be converted into physical capital,  $k_t$ , without cost and so is divided between consumption goods and investment, denoted by  $i_t$ , net of capital depreciation. Thus, the capital stock used for production in the next period is given by:

$$k_{t+1} = (1 - \delta_k)k_t + i_t = (1 - \delta_k)k_t + y_t - c_t. \quad (5)$$

The human capital investment is produced using capital  $s_{Ht}k_t$  and effective labor  $n_t h_t$  (King and Rebelo 1990):

$$H(s_{Ht}k_t, n_t h_t) = A_H (s_{Ht}k_t)^{1-\eta} (n_t h_t)^\eta. \quad (6)$$

And the human capital flow constraint is:

$$h_{t+1} = (1 - \delta_h)h_t + H(s_{Ht}k_t, n_t h_t). \quad (7)$$

With  $w_t$  and  $r_t$  denoting the real wage and real interest rate, the consumer receives nominal income of wages and rents,  $P_t w_t (l_t + f_t) h_t$  and  $P_t r_t (s_{Gt} + s_{Qt}) k_t$ , a nominal transfer from the government,  $T_t$ , and dividends from the bank.

The consumer buys shares in the bank by making deposits of income at the bank. Each dollar deposited buys one share at a fixed price of one, and the consumer receives the residual profit of the bank as dividend income in proportion to the number of shares (deposits) owned. Denoting the real quantity of deposits by  $d_t$ , and the dividend per unit of deposits as  $R_{Qt}$ , the consumer receives a nominal dividend income of  $P_t R_{Qt} d_t$ . The consumer also pays to the bank a fee for credit services, whereby one unit of credit service

is required for each unit of credit that the bank supplies the consumer for use in buying goods. With  $P_{Qt}$  denoting the nominal price of each unit of credit, and  $q_t$  the real quantity of credit that the consumer can use in exchange, the consumer pays  $P_{Qt}q_t$  in credit fees.

With other expenditures on goods, of  $P_t c_t$ , and physical capital investment,  $P_t k_{t+1} - P_t(1 - \delta_k)k_t$ , and on investment in cash for purchases, of  $M_{t+1} - M_t$ , the consumer's budget constraint is

$$\begin{aligned} & P_t w_t (l_t + f_t) h_t + P_t r_t s_{Gt} k_t + P_t R_{Qt} d_t + T_t \\ & \geq P_{Qt} q_t + P_t c_t + P_t k_{t+1} - P_t(1 - \delta_k)k_t + M_{t+1} - M_t. \end{aligned} \quad (8)$$

The consumer can purchase the goods by using either money  $M_t$  or credit services. With the lump sum transfer of cash  $T_t$  coming from the government at the beginning of the period, and with money and credit equally usable to buy goods, the consumer's exchange technology is

$$M_t + T_t + P_t q_t \geq P_t c_t. \quad (9)$$

Since all cash comes out of deposits at the bank, and credit purchases are paid off at the end of the period out of the same deposits, the total deposits are equal to consumption. This gives the constraint that

$$d_t = c_t. \quad (10)$$

Given  $k_0$ ,  $h_0$ , and the evolution of  $M_t$  ( $t \geq 0$ ) as given by the exogenous monetary policy in equation (18) below, the consumer maximizes utility subject to the budget, exchange and deposit constraints (8)-(10).

### 3.2 Banking Firm Problem

The bank produces credit that is available for exchange at the point of purchase. The bank determines the amount of such credit by maximizing its dividend profit subject to the labor and deposit costs of producing the credit. The production of credit uses a constant returns to scale technology with effective labor and deposited funds as inputs. This follows the "financial intermediation approach" (Matthews and Thompson 2008) that is dominant in the banking literature, which was started by Clark (1984) and Hancock (1985). In particular, with  $A_F > 0$  and  $\gamma \in (0, 1)$ ,

$$q_t = A_F e^{v_t} (f_t h_t)^\gamma d_t^{1-\gamma}, \quad (11)$$

where  $A_F e^{v_t}$  is the stochastic factor productivity.<sup>3</sup>

Subject to the production function in equation (11), the bank maximizes profit  $\Pi_{Q_t}$  with respect to the labor  $f_t$  and deposits  $d_t$ :

$$\Pi_{Q_t} = P_{Q_t} q_t - P_t w_t f_t h_t - P_t R_{Q_t} d_t. \quad (12)$$

Equilibrium implies that

$$\left(\frac{P_{Q_t}}{P_t}\right) \gamma A_F e^{v_t} \left(\frac{f_t h_t}{d_t}\right)^{\gamma-1} = w_t; \quad (13)$$

$$\left(\frac{P_{Q_t}}{P_t}\right) (1-\gamma) A_F e^{v_t} \left(\frac{f_t h_t}{d_t}\right)^\gamma = R_{Q_t}. \quad (14)$$

These indicate that the marginal cost of credit,  $\left(\frac{P_{Q_t}}{P_t}\right)$ , is equal to the marginal factor price divided by the marginal factor product, or  $\frac{w_t}{\gamma A_F e^{v_t} \left(\frac{f_t h_t}{d_t}\right)^{\gamma-1}}$ , and that the zero profit dividend yield paid on deposits is equal to the fraction of the marginal cost given by  $\left(\frac{P_{Q_t}}{P_t}\right) (1-\gamma) \left(\frac{q_t}{d_t}\right)$ .

### 3.3 Goods Producer Problem

The firm maximizes profit given by  $y_t - w_t l_t h_t - r_t s_{G_t} k_t$ , subject to a standard Cobb-Douglas production function in effective labor and capital:

$$y_t = A_G e^{z_t} (s_{G_t} k_t)^{1-\alpha} (l_t h_t)^\alpha. \quad (15)$$

The first order conditions for the firm's problem yield the following expressions for the wage rate and the rental rate of capital:

$$w_t = \alpha A_G e^{z_t} \left(\frac{s_{G_t} k_t}{l_t h_t}\right)^{1-\alpha}, \quad (16)$$

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<sup>3</sup>This "banking time" model can be interpreted as a special case of the shopping time model: substituting  $q_t$  from equation (11) into equation (9), and for  $d_t$  from equation (10), and solving for the effective banking time as  $f_t h_t = \left(\frac{c_t - m_t}{A_F e^{v_t} c_t^{1-\gamma}}\right)^{1/\gamma}$ , with  $(M_t + T_t)/P_t \equiv m_t$ , then  $f_t h_t = g(m_t, c_t)$ , with  $g_1 < 0$  and  $g_2 > 0$ , as in a shopping time model. However there is no Feenstra (1986) equivalence to a standard money-in-the-utility function model because then  $h_t$  would enter the utility function, as seen by solving for the raw bank time  $f_t = \hat{g}(m_t, c_t) h_t$ , substituting for  $f_t$  in the allocation of time constraint (4), solving for  $x_t$  from this time constraint and substituting into the utility function.

$$r_t = (1 - \alpha)A_G e^{z_t} \left( \frac{s_G k_t}{l_t h_t} \right)^{-\alpha}. \quad (17)$$

### 3.4 Government Money Supply

It is assumed that the government policy includes sequences of nominal transfers which satisfy:

$$T_t = \Theta_t M_t = (\Theta^* + e^{u_t} - 1)M_t, \quad \Theta_t = [M_t - M_{t-1}]/M_{t-1}. \quad (18)$$

where  $\Theta_t$  is the growth rate of money and  $\Theta^*$  is the stationary gross growth rate of money.

### 3.5 Definition of Competitive Equilibrium

The representative agent's optimization problem can be written recursively as:

$$V(s) = \max_{c, x, l, n, f, s_G, q, d, k', h', M'} \left\{ \frac{(c_t x_t^\Psi)^{1-\theta}}{1-\theta} + \beta EV(s') \right\} \quad (19)$$

subject to the conditions (3) to (10), where the state of the economy is denoted by  $s = (k, h, M, z, u, v)$  and a prime (') indicates the next-period values. A competitive equilibrium consists of a set of policy functions  $c(s)$ ,  $x(s)$ ,  $l(s)$ ,  $n(s)$ ,  $f(s)$ ,  $s_G(s)$ ,  $s_H(s)$ ,  $q(s)$ ,  $d(s)$ ,  $k'(s)$ ,  $h'(s)$ ,  $M'(s)$ , pricing functions  $P(s)$ ,  $w(s)$ ,  $r(s)$ ,  $R_Q(s)$ ,  $P_Q(s)$  and a value function  $V(s)$ , such that:

- (i) the consumer maximize utility, given the pricing functions and the policy functions, so that  $V(s)$  solves the functional equation (19);
- (ii) the bank firm maximizes profit similarly in equation (12) subject to the technology of equation (11)
- (iii) the goods producer maximizes profit similarly, with the resulting functions for  $w$  and  $r$  being given by equations (16) and (17);
- (iv) the goods, money and credit markets clear, in equations (11), (15) and (18).

### 3.6 Balanced-Growth Path Equilibrium

As derived from the equilibrium above, a partial set of equilibrium conditions along the balanced-growth path (BGP) are given here to describe the

deterministic balanced-growth path equilibrium, and how inflation affects it. The balanced-growth rate is denoted by  $g$ , and dropping time subscripts on stationary variables the BGP conditions are

$$\left(\frac{P_{Qt}}{P_t}\right) = R; \quad (20)$$

$$1 + R = (1 + \pi)(1 + r - \delta_k); \quad (21)$$

$$\frac{x}{\alpha c_t} = \frac{1 + \tilde{R}}{wh_t}; \quad (22)$$

$$\tilde{R} = \left(1 - \frac{q_t}{d_t}\right) R + \left(\frac{q_t}{d_t}\right) \gamma R; \quad (23)$$

$$\frac{q_t}{d_t} = 1 - (A_F e^{v_t})^{\frac{1}{1-\gamma}} \left(\frac{\gamma R}{w}\right)^{\frac{\gamma}{1-\gamma}}; \quad (24)$$

$$r_H \equiv \varepsilon A_H \left(\frac{s_H k_t}{l_H h_t}\right)^{(1-\varepsilon)} (1 - x); \quad (25)$$

$$(1 + g)^\theta = \frac{1 + r_H - \delta_H}{1 + \rho} = \frac{1 + r - \delta_K}{1 + \rho}. \quad (26)$$

The relative price of credit is its marginal cost and by equation (20) this is equal to the nominal interest rate. At the optimum, the nominal interest  $R$  of equation (21) equals zero and no credit is used. But as inflation rises, the agent substitutes from goods towards leisure while equalizing the margin of the ratio of the shadow price of goods to leisure,  $x/(\alpha c_t) = [1 + \tilde{R}] / (wh_t)$ , in equation (22). Here  $\tilde{R}$ , as given in equation (23), is the average exchange cost per unit of output; this equals the average cost of using cash,  $R$ , weighted by  $1 - \frac{q_t}{d_t}$  and the average cost of using credit,  $\gamma R$ , weighted by  $\frac{q_t}{d_t}$ . That  $\gamma R_t$  is an average cost can be verified by dividing the total cost of credit production by the total output of credit production. And this total exchange cost determines how much substitution there is from money to credit, and from goods to leisure. The solution for consumption-normalized money demand,  $1 - \frac{q_t}{c_t}$ , is derived from equation (9), (10) and (24); from here it is clear that the consumption velocity of money, denoted by  $v_t \equiv \frac{c_t}{\frac{M_t}{P_t}}$ , rises at an increasing rate as the nominal interest rate rises (see Gillman and Kejak, 2005).

Inflation-induced substitution towards leisure causes a fall in the human capital return of  $r_H \equiv \varepsilon A_H (s_H k_t / l_H h_t)^{(1-\varepsilon)} (1 - x)$ , given in equation (25). The marginal product of physical capital  $r$ , in equation (17), also falls, while

the real wage  $w$  in equation (16) rises. This causes a Tobin (1965)-type substitution from labor to capital across both goods and human capital investment sectors in response to the higher real wage to real interest rate ratio; the Tobin (1965) like rise in  $s_H k_t / l_H h_t$  mitigates but does not reverse the fall in the return to human capital  $r_H$  caused by the increase in leisure. The growth rate, in equation (26), falls as  $R$  rises since both  $r_H$  and  $r$  fall. But as the inflation rate continues to rise, the credit substitution channel allows the growth rate to decline at a decreasing rate, as increasingly more credit and less leisure are used as the substitute for the inflation-taxed good (Gillman and Kejak 2005).

## 4 Model Simulation

### 4.1 Calibration

Table 1 presents the parameters for the calibration which are chosen in order to match the Table 2 target values of certain variables; the targets are the average annual values from US time series for 1919-2004. These values reflect issues raised by Gomme and Rupert (2007), in their study of the two sector real business cycle model, in that our human capital sector is a second sector with some comparison to the household sector in Gomme and Rupert.

The capital share in the goods sector is set at  $1 - \alpha = 0.36$  as in Jones, Manuelli, and Siu (2005), the annual discount factor is set at  $\beta = 0.96$ , and log-utility is assumed so that  $\theta = 1$ . The US average annual output growth rate  $g$  is set at 2.4% as in the data. The baseline investment to output ratio target value is  $i/y = 0.26$ . For comparison this is 0.13 in Gomme and Ruppert for postwar market structures, equipment and software. But also including consumer durables in Gomme and Ruppert adds 0.10, and housing adds another 0.056, for a postwar total of 0.29. Our education sector will include some of this investment, causing a rate less than 0.29. However, there are alternative ways to measure  $i/y$  as discussed in Gomme and Ruppert. The 0.26 value implies that the annual depreciation rate of capital is  $\delta_K = 0.031$ . In turn this gives the goods sector capital to effective labor ratio and the real interest rate net of depreciation of  $r - \delta_K = 0.067$ .

The rate of depreciation of human capital is set at  $\delta_H = 0.025$  as in Jones et al. (2005) and Jorgenson and Fraumeni (1989). The allocation of time is similar to Gomme and Rupert (2007), with the working time set at  $l = 0.24$

and leisure at  $x = 0.55$ . Time in human capital investment is set at  $n = 0.2$ . Given  $n$ ,  $g$ , and  $\delta_H$  and equation (7) implies the capital to effective labor ratio in the human capital sector and so the value of the capital share in the education sector, which is  $\varepsilon = 0.83$ . The chosen values imply  $A_H = 0.21$ , with the weight on the leisure in the utility function given by  $\psi = 1.84$ .

In the banking sector we set the value of the inverse of the consumption velocity of money,  $m/c$ , equal to the average annual value for the period 1919-2004, which is 0.38. The average annual inflation rate,  $\pi$ , over the same period is 2.6% which implies that the annual money growth,  $\sigma$ , is equal to 5%. Using an approximate cost of an exchange credit card (American Express) at \$100, and the per capita annual consumption expenditure,  $c = \$15780$ , both at 2006 prices, the share of the labor in the banking sector is  $\gamma = 100/[R(1 - [m/c])c] = 0.11$  (for further details see the calibration in Benk et al., 2008).

Table 1 also includes the parameters characterizing the shock processes of equation (1); these are chosen through an iterative process by which the assumed shock parameters converge with the actual shock parameters that are in turn estimated from the constructed shock processes described in Appendix A3. In particular, estimated parameters are inputted back into the model, shocks are re-constructed and parameters re-estimated until convergence is achieved in the parameter structure.

## 4.2 Effects of Shocks on Output Growth and Inflation

In order to solve the model, we log-linearize the equilibrium conditions of the model around its deterministic steady state, with variables that grow along the balanced-path normalized to stationary variables by dividing them by the human capital stock  $h_t$ . The impulse responses of the shocks are given in the Appendix in Figures 13-18. The initial impact of the shocks in the first period involves no change in the capital stocks, so that starting from the BGP equilibrium the changes in levels go in the same direction as the changes in growth rates. And the percentage changes of non-state variables like consumption are equal to the changes of the related normalized values. Indicating the percentage deviation from the balanced growth path by  $\widehat{\cdot}$ , then for example this means that  $\widehat{c}_t = (\widehat{c_t/h_t})$ .

A positive money growth shock,  $u_t > 0$ , causes the inflation rate and nominal interest rate to deviate upwards;  $\widehat{\pi}_t > 0$ ,  $\widehat{R}_t > 0$ . Consumption

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|                            |       |   |
|----------------------------|-------|---|
| Preferences                |       |   |
| $\theta$                   | 1     | Relative risk aversion parameter          |
| $\psi$                     | 1.84  | Leisure weight                            |
| $\beta$                    | 0.96  | Discount factor                           |
| Goods Production           |       |   |
| $\alpha$                   | 0.64  | Labor share in goods production           |
| $\delta_K$                 | 0.031 | Depreciation rate of goods sector         |
| $A_G$                      | 1     | Goods productivity parameter              |
| Human Capital Production   |       |   |
| $\varepsilon$              | 0.83  | Labor share in human capital production   |
| $\delta_H$                 | 0.025 | Depreciation rate of human capital sector |
| $A_H$                      | 0.21  | Human capital productivity parameter      |
| Banking Sector             |       |   |
| $\gamma$                   | 0.11  | Labor share in credit production          |
| $A_F$                      | 1.1   | Banking productivity parameter            |
| Government                 |       |   |
| $\sigma$                   | 0.05  | Money growth rate                         |
| Shocks processes           |       |   |
| Autocorrelation parameters |       |   |
| $\varphi_z$                | 0.84  | Production productivity                   |
| $\varphi_u$                | 0.74  | Money growth rate                         |
| $\varphi_v$                | 0.73  | Banking productivity                      |
| Variances                  |       |   |
| $\sigma_z$                 | 0.77  | Production productivity                   |
| $\sigma_u$                 | 0.50  | Money growth rate                         |
| $\sigma_v$                 | 1.16  | Banking productivity                      |

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Table 1: Parameters of Calibration

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|       |        |   |
|-------|--------|---|
| $g$   | 0.024  | Avg. annual output growth rate          |
| $\pi$ | 0.026  | Avg. annual inflation rate              |
| $l$   | 0.248  | Labor used in goods sector              |
| $n$   | 0.20   | Labor used in human capital sector      |
| $f$   | 0.0018 | Labor used in banking sector            |
| $i/y$ | 0.26   | Investment-output ratio in goods sector |
| $m/c$ | 0.38   | Share of money transactions             |

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Table 2: Target Values of Calibration

declines on impact of the shock, so that  $\widehat{c}_t < 0$ , because of the increased shadow price  $\widehat{R}_t > 0$ . Investment drops some;  $\widehat{i}_t < 0$ . And so output drops as both consumption and investment decrease;  $\widehat{y}_t < 0$ . Thus the growth rate of output declines from a money shock. Another perspective of the output decrease is that the return to physical capital falls: by log-linearizing (17),  $\widehat{r}_t \approx -\beta \left( \widehat{s}_{Gt} - \widehat{l}_t + (\widehat{k}_t/\widehat{h}_t) \right)$ , or since the capital stock is constant at impact,  $\widehat{r}_t \approx -\beta \left( \widehat{s}_{Gt} - \widehat{l}_t \right)$ . It results that  $\widehat{r}_t < 0$ , so that  $|\widehat{l}_t| > |\widehat{s}_{Gt}|$ . Since the share of labor and capital in goods production both decrease it follows that  $\widehat{y}_t \approx (1 - \beta) \widehat{s}_{Gt} + \beta \widehat{l}_t < 0$ . So output also falls by that route. The related effect on the output of human capital and its return is revealed from the change in the real wage: by log-linearizing (16),  $\widehat{w}_t \approx (1 - \beta) \left( \widehat{s}_{Gt} - \widehat{l}_t \right) > 0$ . This implies that the shares in human capital output go up, so that  $\widehat{s}_{Ht} > 0$  and  $\widehat{l}_{Ht} > 0$  and output in human capital increases as does its output growth rate:  $\widehat{g}_{Ht} > 0$ . And because  $\widehat{w}_t - \widehat{R}_t \left( \widetilde{R}/(1 + \widetilde{R}) \right) < 0$ , the consumption shadow price increases relative to the leisure shadow price, inducing substitution from consumption towards leisure, so that  $\widehat{x}_t > 0$ . This leisure increase causes a lower return to human capital (see equation 25) and a consequent lower growth rate of consumption, denoted by  $\widehat{g}_{ct} < 0$ .

A positive credit shock,  $v_t > 0$ , on the contrary leads to a decreased cost of exchange, and works in reverse as compared to a monetary shock. A goods productivity shock,  $z_t > 0$ , directly increases the interest rate,  $\widehat{r}_t > 0$ , and the wage rate,  $\widehat{w}_t > 0$ . Since the return to physical capital is larger than the return to human capital, resources move into the goods sector so that  $\widehat{s}_{Gt} > 0$ ,  $\widehat{l}_t > 0$  and  $\widehat{y}_t > 0$ . Increased consumption and real money demand cause a decrease in the inflation rate;  $\widehat{\pi}_t < 0$ . So a positive goods productivity shock causes an increase in output growth and a decrease in the inflation rate.

The effect of shocks on inflation can also be seen from by log-linearizing equations (9) and (18):

$$\widehat{\pi}_t = \widehat{g}_{Vt} - \widehat{g}_{ct} + u_t \quad (27)$$

where  $u_t$  is monetary supply growth rate shock and  $g_{Vt}$  is the growth rate of consumption velocity  $V_t$  defined as  $V_t = P_t c_t / M_{t+1}$ . A positive money shock directly causes inflation to deviate upwards;  $\widehat{\pi}_t > 0$ . And since this shock causes velocity to rise and consumption growth to fall ( $\widehat{g}_{Vt} > 0$  and  $\widehat{g}_{ct} < 0$ ), these other factors both go in the same direction so as to further amplify the inflation rate increase. If the shock effects on  $\widehat{g}_{ct}$  and  $\widehat{g}_{Vt}$  are small,

then  $\widehat{\pi}_t \simeq u_t$ . However, the other shocks can be important, such as the shift up on velocity when credit was deregulated in the early 1980s; this would have raised inflation sharply above the level of the money supply growth rate, which is broadly consistent with the "missing money" at that time. A positive credit shock causes both velocity and consumption growth to rise ( $\widehat{g}_{vt} > 0$  and  $\widehat{g}_{ct} > 0$ ), resulting in opposing effects. A positive shock to goods productivity causes consumption velocity to be somewhat affected (see Benk et al., 2008) while making  $\widehat{g}_{ct} > 0$  so that inflation decreases;  $\widehat{\pi}_t < 0$ .

### 4.3 Simulation Results

Table 3 presents US data stylized facts and simulations of the model, in terms of moments of a set of variables for the period 1919-2004; Tables 4 and 5 present the same for the 1919-1954 and 1955-2004 subperiods. The data series have been detrended using the Christiano and Fitzgerald (2003) asymmetric frequency filter with a band of 2-86 years (where 86 is the sample size). And the covariance matrix is separately computed for each of the two subperiods and for the whole period.

Results are divided into the real side, or RBC, and the more monetary side, or Monetary. On the real side, consumption, investment and output growth volatilities relative to output volatility, along with output and output growth correlations are simulated rather well in the full sample and both subsamples. However, simulated investment has an output correlation above the data in 1919-1954; simulated consumption volatility is low relative to data in the second volatility cycle of 1955-2004; and simulated consumption correlation with output growth is too high. Employment, defined as labor hours in the goods and banking sector or  $l_t + f_t$ , has a simulated relative volatility that is right on the data.

Monetary results show that simulated velocity volatilities are close to data; and there is at most a 0.17 difference between simulated velocity correlation with output and output growth in the three samples. The simulated real money (normalized by human capital) gets the relative volatility very close to the data, with output and output growth rate correlations close in the full sample but less close in the 1955-2004 period.

While the inflation correlation with output is not well captured, in contrast the correct signs of the inflation correlation with output growth are well captured. And even though the sign of the inflation correlation with output

| Simulation Results       | RELATIVE   |      | OUTPUT      |      | OUTPUT GROWTH RATE |      |
|--------------------------|------------|------|-------------|------|--------------------|------|
| Data: 1919-2004          | VOLATILITY |      | CORRELATION |      | CORRELATION        |      |
|                          | Simulated  | Data | Simulated   | Data | Simulated          | Data |
| <b>RBC</b>               |            |      |             |      |                    |      |
| Consumption              | 0.51       | 0.64 | 0.71        | 0.56 | 0.30               | 0.01 |
| Investment               | 2.97       | 4.09 | 0.94        | 0.53 | 0.31               | 0.17 |
| Output Growth Rate       | 0.29       | 0.48 | 0.35        | 0.20 | 1.00               | 1.00 |
| Employment               | 0.73       | 0.75 | 0.84        | 0.91 | 0.26               | 0.30 |
| <b>Monetary</b>          |            |      |             |      |                    |      |
| Income Velocity of Money | 1.21       | 1.39 | -0.03       | 0.06 | -0.05              | 0.01 |
| Normalized Real Money    | 1.60       | 1.61 | 0.65        | 0.47 | 0.26               | 0.14 |
| Inflation Rate           | 0.81       | 0.44 | -0.44       | 0.40 | 0.10               | 0.32 |

Note: See Appendix for data sources. All data series represent the cyclical component of the data filtered with the Christiano and Fitzgerald (2003) asymmetric frequency filter with a band of 2-86 years (86=sample size). Series are in logs except those that represent rates. Relative volatility is measured as the ratio of standard deviation of the series to the standard deviation of GDP

Table 3: US Business Cycle Facts, 1919-2004, and Simulations

growth changes across prewar and postwar subperiods, the model captures this. And this can be seen as support for the model's central feature of the inflation tax effect on output growth. Figure 5 shows the inflation rate and the GDP growth rate over the whole sample: both the positive correlation between these variables in the first half of the sample, before 1955, and the negative correlation between these variables apparent in after 1955, is captured in the 1919-1954 and the 1955-2004 simulation results.

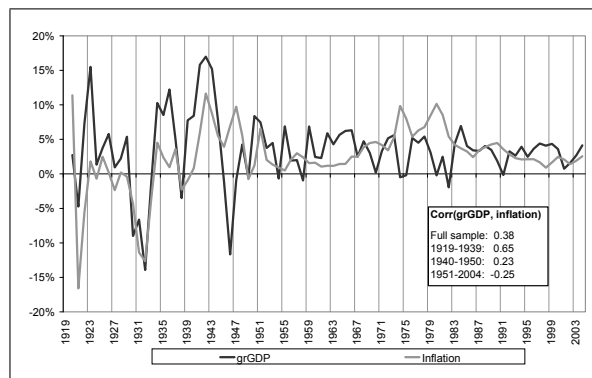


Figure 5: GDP Growth Rate and the Inflation Rate, 1919-2004

| Simulation Results       | RELATIVE   |      | OUTPUT      |       | OUTPUT GROWTH RATE |      |
|--------------------------|------------|------|-------------|-------|--------------------|------|
| Data: 1919-1954          | VOLATILITY |      | CORRELATION |       | CORRELATION        |      |
|                          | Simulated  | Data | Simulated   | Data  | Simulated          | Data |
| RBC                      |            |      |             |       |                    |      |
| Consumption              | 0.47       | 0.61 | 0.66        | 0.52  | 0.29               | 0.01 |
| Investment               | 3.11       | 4.05 | 0.95        | 0.49  | 0.22               | 0.19 |
| Output Growth Rate       | 0.32       | 0.48 | 0.28        | 0.23  | 1.00               | 1.00 |
| Employment               | 0.76       | 0.75 | 0.87        | 0.95  | 0.17               | 0.33 |
| Monetary                 |            |      |             |       |                    |      |
| Income Velocity of Money | 0.96       | 1.22 | 0.10        | -0.03 | -0.07              | 0.05 |
| Normalized Real Money    | 1.32       | 1.54 | 0.69        | 0.57  | 0.27               | 0.13 |
| Inflation Rate           | 0.73       | 0.42 | -0.48       | 0.38  | 0.16               | 0.45 |

Note: See Appendix for data sources. All data series represent the cyclical component of the data filtered with the Christiano and Fitzgerald (2003) asymmetric frequency filter with a band of 2-86 years (86=sample size). Series are in logs except those that represent rates. Relative volatility is measured as the ratio of standard deviation of the series to the standard deviation of GDP

Table 4: US Business Cycle Facts, 1919-1954, and Simulations

| Simulation Results       | RELATIVE   |      | OUTPUT      |       | OUTPUT GROWTH RATE |       |
|--------------------------|------------|------|-------------|-------|--------------------|-------|
| Data: 1955-2004          | VOLATILITY |      | CORRELATION |       | CORRELATION        |       |
|                          | Simulated  | Data | Simulated   | Data  | Simulated          | Data  |
| RBC                      |            |      |             |       |                    |       |
| Consumption              | 0.49       | 0.94 | 0.70        | 0.85  | 0.30               | 0.03  |
| Investment               | 3.01       | 3.87 | 0.95        | 0.86  | 0.22               | 0.14  |
| Output Growth Rate       | 0.31       | 0.59 | 0.28        | 0.04  | 1.00               | 1.00  |
| Employment               | 0.76       | 0.72 | 0.86        | 0.41  | 0.18               | 0.06  |
| Monetary                 |            |      |             |       |                    |       |
| Income Velocity of Money | 2.86       | 2.55 | 0.24        | 0.40  | 0.05               | -0.12 |
| Normalized Real Money    | 2.79       | 2.28 | 0.11        | -0.03 | 0.06               | 0.17  |
| Inflation Rate           | 1.31       | 0.59 | -0.01       | 0.39  | -0.08              | -0.50 |

Note: See Appendix for data sources. All data series represent the cyclical component of the data filtered with the Christiano and Fitzgerald (2003) asymmetric frequency filter with a band of 2-86 years (86=sample size). Series are in logs except those that represent rates. Relative volatility is measured as the ratio of standard deviation of the series to the standard deviation of GDP

Table 5: US Business Cycle Facts, 1955-2004, and Simulations

## 5 Shocks and Volatilities

The money shocks,  $u_t$ , and the productivity shocks in credit and goods production,  $v_t$  and  $z_t$ , are recovered by a least squares procedure using the equilibrium solution of the model and data series for six of the unknown variables of the model. This process is described in Appendix A.1, as in Benk et al. (2005, 2008). The actual constructed shocks both under endogenous and exogenous growth are found below. One difference from previous work is that we use a band pass filter that takes out only the 86 year trend from the data, a minimalist filter desirable for leaving in the longer run features, along with business cycle and short run features (see Section 5.1 for the filter).

### 5.1 Band Pass Filters

Figure 6 shows the band pass filters of the three shocks at the different frequencies across the whole sample period. A result is that the short run fluctuations of all three shocks (the righthand panels) are much more apparent in the first volatility cycle period than in the second. Fluctuations at the business cycle frequency (middle panels) are larger during the depression and WWII. The long run fluctuations (lefthand panels) are more severe for the productivity shock during the first volatility cycle period, but rather equal across both cycle periods for the money shock. This last result is apparently due to the large inflation build-up during the 1960s and 1970s that rivaled the deflation of the depression in terms of the amplitude of the fluctuation. In sum, all three frequencies indicate non-trivial and plausible aspects of the shocks.

### 5.2 Variance Decompositions

A variance decomposition of output growth and inflation is presented for both endogenous and exogenous growth versions of the model.<sup>4</sup> Tables 6 and 7 show how much of the total variance in the data is explained within each sub-period by each of the model's shocks: the productivity (PR), money (M) and credit (CR) shocks. Variance is further decomposed by frequencies, across the various subperiods. The short-run (SR) frequency band corresponds to cycles of 2-3 years, the business cycle (BC) frequency band to cycles of 3-8

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<sup>4</sup>The parameters for the exogenous growth model are the same as for the endogenous growth model parameters of Table 1, except for the lack of human capital parameters.

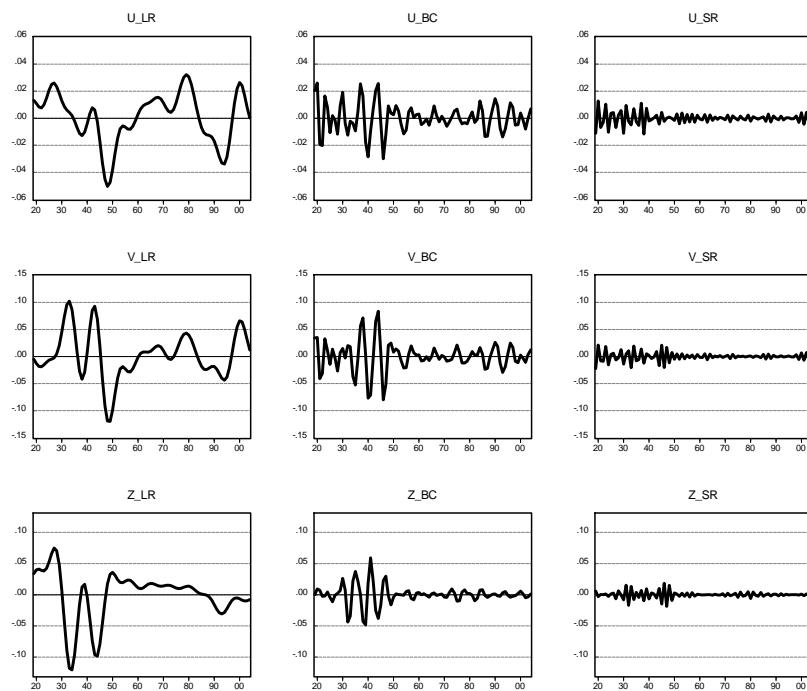


Figure 6: The decomposition of money (u), credit (v) and productivity (z) shocks into their long run (LR), business cycle (BC) and short run (SR) components

years, and the long-run (LR) band to cycles of 8 years and longer; the spectral density of the series is normalized by the series variance, and then its integral is computed over the corresponding frequency band. This gives a 9 element, three-by-three, submatrix within the two tables for each subperiod. The fourth and eighth columns are marked `FREQ` and these show the total variance found within each frequency by the respective endogenous and exogenous growth models; the sum of the columns in contrast is the amount of variation within each frequency of the data that is explained by the model.

The results are reported for the entire 1919-2004 period, and for 1919-1935, 1936-1954, 1955-1982, and 1983-2004, corresponding approximately to the rise and fall of the two volatility cycles. Note that in Figure 4, inflation volatility peaks in the first volatility cycle around 1935, but the GDP volatility continues to have another double peak during WWII and only then recedes. So this 1935 date is based more on the inflation peak and other dates could be used. For the second volatility cycle, Figure 4 shows that GDP volatility troughs in 1954 (when the Korean War was over, with an armistice signed on July 27, 1953), but GDP growth and inflation troughed in 1963 (on August 7, 1964 the US approved the use of military force in the Vietnam War, without declaring war, through the Gulf of Tonkin Resolution); so the 1954 dividing point might alternatively be substituted by 1963.

For GDP growth volatility, Table 6 shows that consistently more than double the total simulated variation takes place in the long run than in the SR and BC frequencies, for the endogenous growth versus the exogenous growth models, as seen by comparing the `FREQ` columns. But how do the models perform in terms of explaining the data's volatility? For the entire period of 1919-2004, the last three rows show that the endogenous growth model explains only a total of 22% (the sum of the 9 three-by-three elements) of the total variation versus 46% for exogenous growth. But looking at the subperiods gives a different story. Overall, the endogenous growth model explains 65% of the Great Depression subperiod, 44%, 47% and 49% for the other subperiods, versus 64%, 48%, 59% and 26% for the exogenous growth model. Even though the average subperiod explained volatility is comparable at 51% for the endogenous growth and 49% for the exogenous growth models, the Great Moderation subperiod is the standout difference, with the endogenous growth model explaining almost double the volatility.

More particularly, productivity shocks explain between 7 and 21% of the

|                  |    | Endogenous |     |     |      | Exogenous |     |     |      |
|------------------|----|------------|-----|-----|------|-----------|-----|-----|------|
|                  |    | PR         | M   | CR  | FREQ | PR        | M   | CR  | FREQ |
| <b>1919-1935</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 7%         | 7%  | 7%  | 25%  | 13%       | 11% | 3%  | 37%  |
|                  | BC | 9%         | 4%  | 5%  | 38%  | 16%       | 1%  | 7%  | 45%  |
|                  | LR | 2%         | 18% | 6%  | 37%  | 3%        | 3%  | 7%  | 18%  |
| <b>1936-1954</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 0%         | 7%  | 7%  | 28%  | 1%        | 15% | 16% | 38%  |
|                  | BC | 3%         | 6%  | 6%  | 41%  | 4%        | 18% | 2%  | 48%  |
|                  | LR | 4%         | 7%  | 4%  | 31%  | 2%        | 0%  | 0%  | 14%  |
| <b>1955-1982</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 8%         | 2%  | 2%  | 18%  | 18%       | 4%  | 2%  | 36%  |
|                  | BC | 12%        | 1%  | 1%  | 33%  | 19%       | 1%  | 12% | 44%  |
|                  | LR | 1%         | 11% | 9%  | 49%  | 1%        | 0%  | 2%  | 20%  |
| <b>1983-2004</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 3%         | 3%  | 3%  | 15%  | 7%        | 0%  | 0%  | 35%  |
|                  | BC | 4%         | 10% | 11% | 30%  | 9%        | 0%  | 0%  | 44%  |
|                  | LR | 5%         | 5%  | 5%  | 55%  | 1%        | 8%  | 1%  | 21%  |
| <b>1919-2004</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 2%         | 4%  | 4%  | 19%  | 6%        | 14% | 4%  | 34%  |
|                  | BC | 5%         | 2%  | 3%  | 34%  | 8%        | 7%  | 4%  | 46%  |
|                  | LR | 0%         | 1%  | 1%  | 47%  | 0%        | 0%  | 3%  | 20%  |

Table 6: Decomposition of Variance of GDP growth by Frequency, 1919-2004

variation across the four subperiods, with more explained during the volatility upswings than the downswings. The average for productivity shocks is 16% of the variation each subperiod, with the average for money and credit shocks being 20% and 16%. Money shocks explain 29% of the variation during the Great Depression subperiod, with much of this in the LR spectrum. However credit shocks explain relatively the most during the Great Moderation subperiod (19% of the variation).

For inflation, Table 7 shows the endogenous growth and exogenous growth give many similar results, with an across-subperiod average of 72% of the data's variation explained in endogenous growth and 80% in exogenous growth, and with a substantial amount of this in the LR spectrum. The total explained variation by subperiod is 98%, 88%, 54% and 50% of the volatility for endogenous growth, and 87%, 90%, 58% and 84% for exogenous growth; in the entire period sample this is 70% versus 61% for endogenous and exogenous growth. The average explained variance by shock in the endogenous growth model, for PR, M and CR, is 19%, 25% and 27% and in exogenous growth, 20%, 32% and 16%. With endogenous growth, money and credit shocks explain most of the inflation variation during the Great Depression subperiod, while in the Great Moderation the goods productivity PR shock explains the most total variation, with a equal split in the total contribution of the M and CR shocks.

The effects of the shocks on output and inflation can also be graphi-

|                  |    | Endogenous |     |     |      | Exogenous |     |     |      |
|------------------|----|------------|-----|-----|------|-----------|-----|-----|------|
|                  |    | PR         | M   | CR  | FREQ | PR        | M   | CR  | FREQ |
| <b>1919-1935</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 1%         | 16% | 21% | 37%  | 2%        | 22% | 9%  | 36%  |
|                  | BC | 4%         | 15% | 23% | 45%  | 7%        | 26% | 1%  | 45%  |
|                  | LR | 5%         | 11% | 3%  | 18%  | 9%        | 1%  | 10% | 19%  |
| <b>1936-1954</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 4%         | 14% | 15% | 33%  | 5%        | 21% | 4%  | 33%  |
|                  | BC | 18%        | 4%  | 5%  | 43%  | 15%       | 5%  | 11% | 39%  |
|                  | LR | 24%        | 2%  | 2%  | 23%  | 28%       | 0%  | 1%  | 28%  |
| <b>1955-1982</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 0%         | 9%  | 8%  | 32%  | 0%        | 2%  | 1%  | 28%  |
|                  | BC | 1%         | 8%  | 8%  | 40%  | 1%        | 21% | 10% | 40%  |
|                  | LR | 0%         | 9%  | 11% | 27%  | 1%        | 8%  | 14% | 33%  |
| <b>1983-2004</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 10%        | 5%  | 6%  | 32%  | 7%        | 3%  | 2%  | 25%  |
|                  | BC | 0%         | 4%  | 4%  | 41%  | 0%        | 8%  | 5%  | 35%  |
|                  | LR | 12%        | 5%  | 4%  | 27%  | 7%        | 13% | 39% | 39%  |
| <b>1919-2004</b> |    |            |     |     |      |           |     |     |      |
|                  | SR | 1%         | 15% | 19% | 34%  | 1%        | 20% | 6%  | 33%  |
|                  | BC | 7%         | 10% | 13% | 43%  | 7%        | 14% | 4%  | 38%  |
|                  | LR | 0%         | 3%  | 3%  | 23%  | 0%        | 8%  | 1%  | 29%  |

Table 7: Decomposition of Variance of Inflation by Frequency, 1919-2004

cally illustrated using the regression estimation methodology of Benk et al. (2005). Figure 7 shows that the productivity shock caused the depression era drop in output, as expected, and that the money shock also contributed to the depressionary output drop. But even more prolonged was the negative effect of the money shock on GDP during the Great Inflation period of the 1970s, during which time the productivity shock had only positive effects. The credit shock helped GDP to increase during the 1933 banking reorganization and the start of federal deposit insurance; and there is a positive CR effect on GDP during the financial deregulation in the early 1980s. The money shock effect on inflation is in evidence during the 1930s deflation, and the 1970s and 1980s inflation. The credit shock lowered inflation during the depression and the late 1970s and early 1980s.

## 6 Discussion

A comparison of the model's results can be made to the empirical literature on the Great Moderation, such as the succinct summary and extension by Giannone, Lenza, and Reichlin (2008). They find that the larger the VAR model, the more of the Great Moderation that is explained by a change in the structure of the shock process ("good policy"), and the less by a change in the variation of the shock process ("good luck"). Giannone et al. also

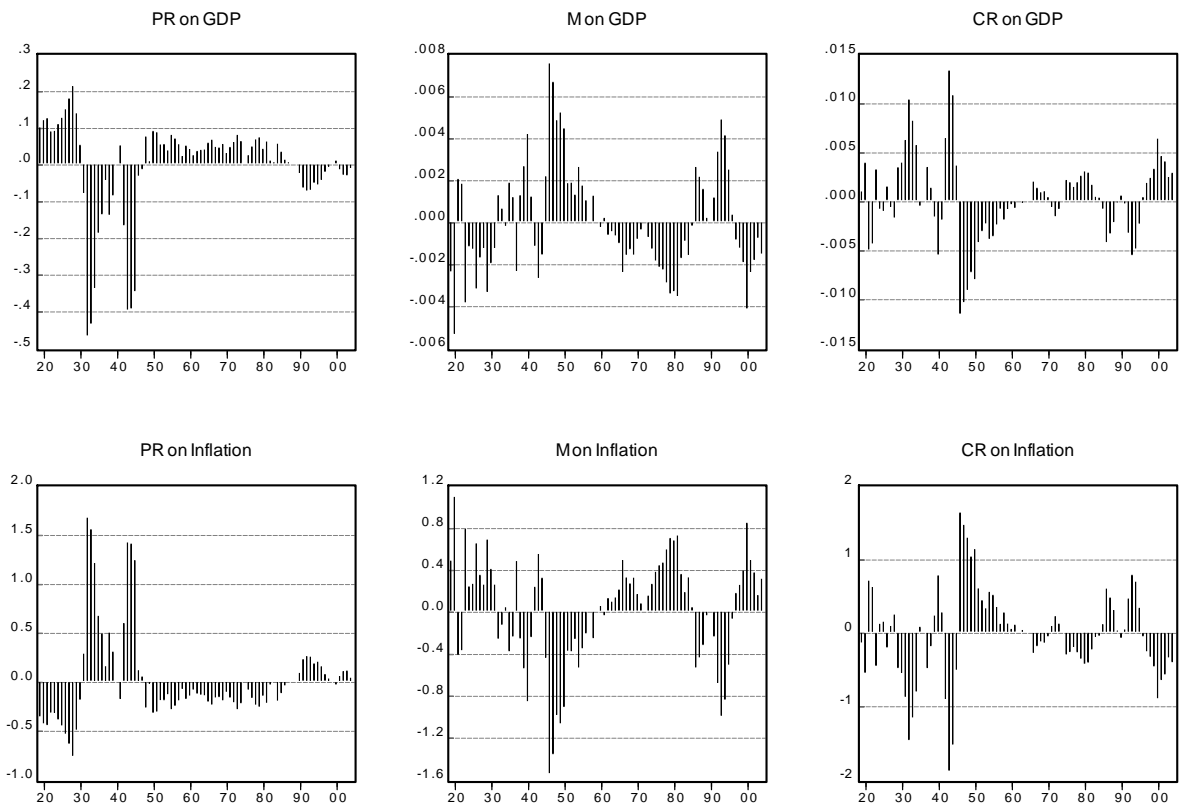


Figure 7: Effect of shocks on GDP and Inflation

|                  |    | StdDev       | Correlation |       |       |
|------------------|----|--------------|-------------|-------|-------|
|                  |    |              | PR          | M     | CR    |
| <b>1919-1935</b> |    |              |             |       |       |
|                  | PR | <b>0.074</b> | 1.00        | 0.50  | -0.72 |
|                  | M  | <b>0.020</b> | 0.50        | 1.00  | 0.18  |
|                  | CR | <b>0.053</b> | -0.72       | 0.18  | 1.00  |
| <b>1936-1954</b> |    |              |             |       |       |
|                  | PR | <b>0.053</b> | 1.00        | -0.54 | -0.82 |
|                  | M  | <b>0.026</b> | -0.54       | 1.00  | 0.92  |
|                  | CR | <b>0.085</b> | -0.82       | 0.92  | 1.00  |
| <b>1955-1982</b> |    |              |             |       |       |
|                  | PR | <b>0.007</b> | 1.00        | 0.00  | -0.23 |
|                  | M  | <b>0.013</b> | 0.00        | 1.00  | 0.97  |
|                  | CR | <b>0.021</b> | -0.23       | 0.97  | 1.00  |
| <b>1983-2004</b> |    |              |             |       |       |
|                  | PR | <b>0.011</b> | 1.00        | 0.54  | 0.30  |
|                  | M  | <b>0.021</b> | 0.54        | 1.00  | 0.96  |
|                  | CR | <b>0.040</b> | 0.30        | 0.96  | 1.00  |
| <b>1919-2004</b> |    |              |             |       |       |
|                  | PR | <b>0.043</b> | 1.00        | 0.17  | -0.51 |
|                  | M  | <b>0.023</b> | 0.17        | 1.00  | 0.75  |
|                  | CR | <b>0.053</b> | -0.51       | 0.75  | 1.00  |

Table 8: Correlation Matrix of Shocks Across Subperiods, and Whole Period

emphasize that missing information biases estimates of the shock variance, an omitted variable bias. They suggest that covariance between GDP and other variables like inflation can increase predictability, and that estimates of the shocks must take into account such multivariate information.

Our equilibrium involves a large number of equations, and the shocks of our model are derived from these equilibrium conditions and time series variables. The covariance is estimated from the shocks, while taking into account the way in which, for example, money supply growth affects output by causing inflation to rise and output growth to decrease. Table 8 presents the covariance structure of the shocks across the different subperiods and for the whole period.

The covariance matrices across subperiod give several results:

1. Standard Deviation (SD): The SD of the goods productivity shock ( $z$ ) is much greater in the first than the second volatility cycle, as expected (a change in "luck"). The SD of the money shock ( $u$ ) is historically similar across subperiods, except that it is lower during the Great Inflation subperiod. This suggests the contribution of the monetary shocks was more in terms of "policy"; or looking at the Great Moderation compared to the previous subperiod, the "luck" was even a bit worse. The SD of the credit shock ( $v$ ) is less stable but again lowest during the Great Inflation subperiod.

2. Covariances: The credit shock is negatively correlated with the goods productivity shock in the first three subperiods, but positively correlated in the Great Moderation. And it is highly correlated with the money shock in the last three subperiods, but negatively correlated in the Great Depression subperiod.

One interpretation is that in the Great Depression, credit was constrained by the collapse of the banking sector, leading to the negative correlation of credit shocks with goods productivity shocks. Restrictions on credit may have similarly produced a like negative correlation between CR and PR in the next two subperiods. In the Great Moderation subperiod, credit was liberalized with financial deregulation, leading to a positive correlation of CR with PR. This credit deregulation would have allowed the economy to be more insulated from the inflation tax, which in turn allowed credit to take up an historically different, positive, role of helping output growth in part by insulating the economy from money shocks.

Figure 8 presents the volatility of the constructed credit shock in comparison to that of GDP. The volatilities moved together everywhere except especially in 1927-1938 and 1983-2004 (as well as in WWII). When credit was repressed by banking failure in the Great Depression, GDP volatility went way up while credit shock volatility did not rise as much. And when deregulation began in 1981, credit volatility rose while GDP volatility fell. This suggests that the high velocity volatility and M1 money volatility after 1983 (Figure 4), along with the high credit volatility in Figure 8, reflect the financial deregulation effect and explain how velocity and M1 volatility could rise even while inflation volatility fell.

These money and credit shocks are almost like the structural shocks that Chari, Kehoe and McGrattan (2008) define as being invariant to policy changes, being interpretable in a plausible fashion and even in terms of good shocks versus bad shock with the possibility of trying to offset the bad shocks. They write that a consensus on the need for such structural shocks within the dynamic macro models is emerging, with the focus on a goods productivity, or "efficiency" shock, and a labor wedge shock. Our shocks include this same goods productivity shock, and the monetary and credit shocks both affect primarily the goods to leisure, or "labor", margin (through the shadow exchange cost of goods  $\tilde{R}$  in equation 23).

Our shocks do reflect policy however, as the money shock is based on government action, either directly through the money supply, or perhaps it

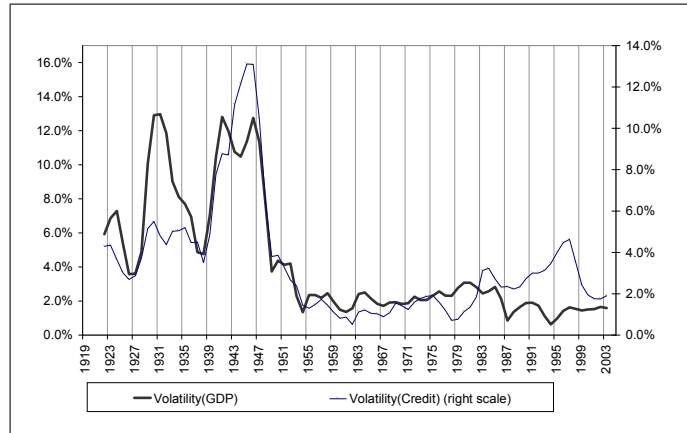


Figure 8: Volatilities of GDP and the Credit Shock, 1919-2004

can be viewed as indirectly through inflation tax finance of deficits (especially during wartime). And credit shocks are linked potentially to changes in banking laws. This interpretation, seen also in Benk et al. 2005, is related to that of Jermann and Quadrini (2006) and is not inconsistent with Perri and Quadrini's (2008) finding that financial integration decreases business cycle volatility. In terms of good and bad shocks: the good shocks to credit are those enhancing credit productivity, as during financial deregulation, and the bad shocks are those restricting credit, such as the bank collapses of the Great Depression. These bad shocks can be offset or minimized, such as through efficient forms of banking insurance, and the good ones can be enhanced such as through liberalization of markets combined with good regulation aimed towards full information revelation including proper accounting. The money shocks, on the other hand, contribute directly to inflation volatility which is linked closely to GDP volatility, thereby suggesting one possible conclusion that their volatility should be minimized.

## 7 Conclusion

The endogenous growth model explains 49% of the volatility of GDP, and 50% of the volatility of inflation, during the Great Moderation period, through a combination of money supply, credit productivity and goods productivity shocks. And with these shocks, the paper explains a particular puzzle through the role of the credit shock: the divergence of higher velocity and

money aggregate volatilities post 1983 from lower GDP and inflation volatilities post 1983. The model's constructed credit shock also rises in volatility during the Great Moderation subperiod, while being relatively low during the Great Depression subperiod. We interpret the post 1983 increased credit volatility as reflecting the unleashing of credit through long run financial innovation during deregulation that created an inflation escape valve. This allowed monetary aggregate volatility to be manifested through higher credit volatility rather than turning into higher inflation and GDP volatility as in previous time, in particular during the Great Depression period. Credit liberalization appears to have diminished some of the inflation tax fluctuations that high money supply volatility can otherwise entail. With GDP volatility coinciding with the inflation volatility rather than the credit volatility, this helped lead to the lower GDP and inflation volatilities during the Great Moderation. Then, looking forward, this model predicts both greater inflation and output volatility at times when credit is constrained and the money supply is shocked upwards, such as during the recent credit crisis, with a subsequently greater inflation and output volatility.

One extension of the model that we are studying is to include investment in the exchange constraint, as in Stockman (1981), as this makes the inflation tax fall on a fraction of investment as well as consumption, which may be more realistic. This creates a negative effect of inflation on investment even while leaving the Tobin effect operative as manifested through an inflation-induced rise in the capital to effective labor across sectors. Another extension is to include the intermediation of intertemporal savings and investment through the banking system. Credit productivity shocks could then affect the share of loans going through to the goods producer. This might lead to a greater use of government bonds and a lesser supply of savings during the crisis, as may be consistent with evidence. And it could be a micro-founded banking component that is useful in extending the RBC model to explain banking crises, although perhaps a credit constraint as in Kocherlakota (2000) in addition may be necessary.

## References

- ASSENMACHER-WESCHE, K., AND S. GERLACH (2008): "Interpreting Euro Area Inflation at High and Low Frequencies," *European Economic Review*, 52(6), 964–986.

- BENK, S., M. GILLMAN, AND M. KEJAK (2005): “Credit Shocks in the Financial Deregulatory Era: Not the Usual Suspects,” *Review of Economic Dynamics*, 8(3), 668–687.
- (2008): “Money Velocity in an Endogenous Growth Business Cycle with Credit Shocks,” *Journal of Money, Credit, and Banking*, 40(6), 1281–1293.
- CHARI, V. V., P. J. KEHOE, AND E. R. MCGRATTAN (2008): “New Keynesian Models: Not Yet Useful for Policy Analysis,” NBER Working Papers 14313, National Bureau of Economic Research.
- CHRISTIANO, L. J., AND T. J. FITZGERALD (2003): “The Band Pass Filter,” *International Economic Review*, 44(2), 435–465.
- CLARK, J. A. (1984): “Estimation of Economies of Scale in Banking Using a Generalized Functional Form,” *Journal of Money, Credit, and Banking*, 16(1), 53–68.
- COOLEY, T. F., AND G. D. HANSEN (1989): “The Inflation Tax in a Real Business Cycle Model,” *American Economic Review*, 79(4), 733–48.
- CROWDER, W. J. (1998): “The Long-Run Link Between Money Growth and Inflation,” *Economic Inquiry*, 36(2), 229–43.
- FEENSTRA, R. C. (1986): “Functional Equivalence Between Liquidity Costs and the Utility of Money,” *Journal of Monetary Economics*, 17(2), 271–291.
- FOUNTAS, S., AND M. KARANASOS (2007): “Inflation, Output Growth, and Nominal and Real Uncertainty: Empirical Evidence for the G7,” *Journal of International Money and Finance*, 26, 229–250.
- FOUNTAS, S., M. KARANASOS, AND J. KIM (2006): “Inflation Uncertainty, Output Growth Uncertainty and Macroeconomic Performance,” *Oxford Bulletin of Economics and Statistics*, 68(3), 319–343.
- FRIEDMAN, M., AND A. J. SCHWARTZ (1963): *A Monetary History of the United States, 1867-1960*. Princeton University Press, Princeton, NJ.

- GIANNONE, D., M. LENZA, AND L. REICHLIN (2008): “Explaining The Great Moderation: It Is Not The Shocks,” *Journal of the European Economic Association*, 6(2-3), 621–633.
- GILLMAN, M., M. N. HARRIS, AND L. MATYAS (2004): “Inflation and Growth: Explaining a Negative Effect,” *Empirical Economics*, 29(1), 149–167.
- GILLMAN, M., AND M. KEJAK (2005): “Inflation and Balanced-Path Growth with Alternative Payment Mechanisms,” *Economic Journal*, 115(500), 247–270.
- GOMME, P. (1993): “Money and Growth: Revisited,” *Journal of Monetary Economics*, 32, 51–77.
- GOMME, P., AND P. RUPERT (2007): “Theory, Measurement, and Calibration of Macroeconomic Models,” *Journal of Monetary Economics*, 54(2), 460–497.
- HANCOCK, D. (1985): “The Financial Firm: Production with Monetary and Nonmonetary Goods,” *Journal of Political Economy*, 93(5), 859–880.
- INGRAM, B., N. KOCHERLAKOTA, AND N. SAVIN (1994): “Explaining Business Cycles: A Multiple-Shock Approach,” *Journal of Monetary Economics*, 34, 415–428.
- JERMANN, U. J., AND V. QUADRINI (2006): “Financial Innovations and Macroeconomic Volatility,” NBER Working Papers 12308, NBER.
- JONES, L. E., R. E. MANUELLI, AND H. E. SIU (2005): “Fluctuations in Convex Models of Endogenous Growth II: Business Cycle Properties,” *Review of Economic Dynamics*, 8(4), 805–828.
- JORGENSON, D. W., AND B. M. FRAUMENI (1989): “The Accumulation of Human and Non-Human Capital, 1948-1984,” in *The Measurement of Savings, Investment and Wealth*, ed. by R. E. Lipsey, and H. S. Tice, chap. 6, pp. 272–282. The University of Chicago Press, Chicago.
- JORGENSON, D. W., AND K. J. STIROH (2000): “Raising the Speed Limit: U.S. Economic Growth in the Information Age,” OECD Economics Department Working Papers 261, OECD.

- KEHOE, T. J., AND E. C. PRESCOTT (2002): “Great Depressions of the Twentieth Century,” *Review of Economic Dynamics*, 5(1), 1–18.
- KING, R. G., AND S. REBELO (1990): “Public Policy and Economic Growth: Developing Neoclassical Implications,” *Journal of Political Economy*, 98(5), S126–50.
- KOCHERLAKOTA, N. (2000): “Creating Business Cycles Through Credit Constraints,” *Federal Reserve Bank of Minneapolis Quarterly Review*, 24(2), 2–10.
- KUZNETS, S. (1941): *National Income and its Composition, 1919-1938*. National Bureau of Economic Research, New York.
- LEVY, D., AND H. DEZHBAKHSH (2003): “On the Typical Spectral Shape of an Economic Variable,” *Applied Economics Letters*, 10(7), 417–423.
- LUCAS, JR., R. E. (1988): “On the Mechanics of Economic Development,” *Journal of Monetary Economics*, 22, 3–42.
- MATTHEWS, K., AND J. THOMPSON (2008): *The Economics of Banking*. John Wiley and Sons, Ltd, West Sussex, second edn.
- MIRON, J. A., AND C. D. ROMER (1990): “A New Monthly Index of Industrial Production, 1884-1940,” *Journal of Economic History*, 50, 321–337.
- MULLER, U. K., AND M. W. WATSON (2008): “Testing Models of Low-Frequency Variability,” *Econometrica*, 76(5), 979–1016.
- PERRI, F., AND V. QUADRINI (2008): “Understanding the International Great Moderation,” Society of Economic Dynamics Conference Paper Session 61, MIT, Boston.
- STOCKMAN, A. (1981): “Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy,” *Journal of Monetary Economics*, 8(3), 387–393.
- TOBIN, J. (1965): “Money and Economic Growth,” *Econometrica*, 33(4/2), 671–684.

## A Appendix.

### A.1 Data Sources

Data used in the paper has been constructed on annual frequency, for the 1919 - 2004 time period. The main data sources were the Bureau of Economic Analysis (BEA) and the IMF International Financial Statistics (IFS). Series have been extended backwards until 1919 based on the series published in Kuznets (1941), Friedman and Schwartz (1963) (F&S) and the online *NBER Macroeconomic Database* (<http://www.nber.org/databases/macroeconomic/contents/>) (NBER).<sup>5</sup>

The data series are as follows:

*Gross Domestic Product (BEA, Kuznets).*

*Consumer Price Index (BEA, F&S).*

*Price Index for Gross Domestic Product (BEA, Kuznets).*

*Personal Consumption expenditures (BEA, Kuznets).*

*Gross private domestic investment (BEA, Kuznets).*

*Wage and salary accruals (BEA, Kuznets).*

*Wage and salary accruals, Finance, insurance, and real estate (BEA, Kuznets).*

*Full-time equivalent employees (BEA, Kuznets).*

*Full-time equivalent employees, Finance, insurance, and real estate (BEA, Kuznets).*

*M0 (IFS, NBER).*

*M1 (IFS, NBER).*

*M2 (IFS, NBER).*

*Treasury Bill rate (IFS, NBER).*

### A.2 Variance Decomposition

The decomposition of the variance of the GDP growth and velocity by shocks is based on the principle described in Ingram, Kocherlakota, and Savin (1994), and has been done as follows: Let  $z$ ,  $v$  and  $u$  be the three, possibly

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<sup>5</sup>Note that Romer's revised historical data for GDP was alternatively used. Miron and Romer (1990) reports Industrial Production rather than GDP, for the period up to 1939. This was chained to the GDP data for 1940 and after. Use of this alternative GDP series results in more volatility in the level and in the growth rate of output. But the spectral decomposition results on volatility were not qualitatively affected. Therefore these alternative results are not reported.

correlated shocks. Let's assume the ordering  $z-v-u$ , that is, the movements in  $z$  are responsible for any comovements between  $z$  and  $v$  or  $z$  and  $u$ , and that movements in  $v$  are responsible for any comovements between  $v$  and  $u$ . We can formalize this notion by defining  $v_t^e$  to be the residuals in a regression of  $v_t$  on the vector  $(z_t, \dots, z_{t-s})$  and  $u_t^e$  to be the residuals in a regression of  $u_t$  on the vector  $(z_t, \dots, z_{t-s}, v_t, \dots, v_{t-s})$ . Thus we interpret  $v_t^e$  as capturing the movements of  $v$  that are not associated with current, future, or past movements in  $z$ .

Given this particular ordering, consider the decomposition of the variance of GDP growth ( $\dot{\Delta}y_t$ ) into the components due to the various shocks that is obtained by running the regression:

$$\dot{\Delta}y_t = \underbrace{\sum_{s=0}^S \beta_{z,s} z_{t-s}}_{\dot{\Delta}y_t^z} + \underbrace{\sum_{s=0}^S \beta_{v,s} v_{t-s}^e}_{\dot{\Delta}y_t^v} + \underbrace{\sum_{s=0}^S \beta_{u,s} u_{t-s}^e}_{\dot{\Delta}y_t^u} + \varepsilon_t \quad (28)$$

Then the fraction of the variance of  $\dot{\Delta}y_t$  explained by each shock is given by:  $P^z = \frac{Var(\dot{\Delta}y_t^z)}{Var(\dot{\Delta}y_t)}$ ,  $P^v = \frac{Var(\dot{\Delta}y_t^v)}{Var(\dot{\Delta}y_t)}$ ,  $P^u = \frac{Var(\dot{\Delta}y_t^u)}{Var(\dot{\Delta}y_t)}$ . A similar regression to that of (28) is run on velocity and the same shocks to determine its variance decomposition.

Unless the shocks  $z$ ,  $v$  and  $u$  are orthogonal to each other, the results are sensitive to the ordering adopted. We considered all six possible orderings of the shocks. Results presented are the average for the two cases when the goods productivity shock is ordered first.

The proportion of variance of a series due to SR, BC and LR components can be obtained as in Levy and Dezhbakhsh (2003): it amounts to estimating the spectral density of the series, normalizing it by the series variance, and then computing its integral over the corresponding frequency band. If we denote by  $f(\omega)$  the spectral density of the series and by  $\sigma^2$  its variance, then the fraction of variance due to each frequency component is given by  $H^{SR} = \int_{2\pi/3}^{2\pi/2} f(\omega)/\sigma^2 d\omega$ ,  $H^{BC} = \int_{2\pi/8}^{2\pi/3} f(\omega)/\sigma^2 d\omega$ ,  $H^{LR} = \int_{2\pi/\infty}^{2\pi/8} f(\omega)/\sigma^2 d\omega$ . The frequency bands are determined by the mapping  $\omega = 2\pi/p$ , where  $p$  measures the cycle length (2, 3 or 8 years).

We are using an alternative, equivalent measure for the fractions of variance (suggested also by Levy and Dezhbakhsh (2003)): this consists of passing the series through a band-pass filter, estimating the variance of the filtered series and relating it to the variance of the original series. We employ the

Christiano and Fitzgerald (2003) asymmetric band-pass filter with the aforementioned 2 – 3, 3 – 8 and > 8 year bands. This procedure is applied to the simulated series of output growth and velocity, where simulations have been run by feeding back the estimated variance-covariance structure of the shocks into the model. The variance-covariance matrices have been estimated separately for each of the subperiods, this way we obtained simulated series and decompositions that differ by subperiods.

To assess the fraction of variance explained by each shock in turn at each frequency, we decompose each of the frequency component further, by shocks. The variance decomposition procedure is similar to that described in equation (28). The difference consists in pre-filtering the target series and the shock series to extract the adequate frequency component. According to this, the Christiano and Fitzgerald (2003) asymmetric band-pass filter with the 2-3, 3-8 and >8 year bands is applied to the output growth and velocity series, as well as to the productivity, money and credit shock series.

### A.3 Construction of Shocks

Assume that  $\bar{\xi}$  denotes the steady state value of variable  $\xi$ , and  $\hat{\xi}$  denotes its percentage deviation from the steady state ( $\hat{\xi} = \log(\xi) - \log(\bar{\xi})$ ). With  $\tilde{k} \equiv k/h$ , and any variable with a tilde above indicated the ratio of that variable to  $h$ , and using the solution of the model from section 2, the log-deviations of the model variables can be written as linear functions of the state  $s = (\tilde{k}, z, u, v)$ . By stacking the equations, the solution can be written in matrix form as follows:

$$X_t = A \begin{bmatrix} \tilde{k}_t \end{bmatrix} + B \begin{bmatrix} z_t & u_t & v_t \end{bmatrix}', \quad (29)$$

where  $X = \begin{bmatrix} \hat{c} & \hat{x} & \hat{l} & \hat{n} & \hat{f} & \hat{s}_G & \hat{a} & \hat{\pi} \end{bmatrix}'$ , and  $\tilde{c} \equiv c/h$ . From (29), one can construct the solution of any variable of the model, by forming the appropriate linear combination of the appropriate rows of (29), the linear combinations being given by the linearized versions of equations (3)-(8).

Given the model solution (29) (that is, knowing the value of matrices A and B), the series of shocks  $\begin{bmatrix} z_t & u_t & v_t \end{bmatrix}$  can be constructed by using data on  $X_t$  and  $\tilde{k}_t$  and "solving" the system of linear equations (29). It can be easily seen, that in order to identify the three series of shocks, we need data on at least three variables from  $X_t$ . In a three-variable case the shocks represents

the solution of a system of three linear equation. If more that three variables are used, then the shocks are "overidentified" as we have more equations than unknowns. In such a case we apply a least-square procedure as we illustrate below.

In the procedure of constructing the shocks, we employ the variables on which we were able to find reliable data. We construct stationary variables  $c/y$ ,  $i/y$ ,  $\pi$  and  $m/y$ , and on which we use data to construct the shocks. We also use data on labor hour in banking sector  $f$ . and on the wage rate in banking - the latter series being used as a proxy for the marginal product of labor in banking ( $mplb$ ). The data series on  $\widehat{k}$  is constructed by using for  $k$  the capital accumulation equation (5), data on investment to compute  $\widehat{i}_t$  and the initial condition  $\widehat{k}_{-1} = 0$ . For human capital, because of a lack of a series going back to 1919, we use a smooth trend, as data in Jorgenson and Stiroh (2000) for 1959-1998 indicates.

Having the data series on  $\widehat{k}$ ,  $\widehat{c/y}$ ,  $\widehat{\pi}$ ,  $\widehat{i/y}$ ,  $\widehat{m/y}$ ,  $\widehat{f}$  and  $\widehat{mplb}$ , we set up a system of linear equations:

$$XX_t = AA \begin{bmatrix} \widehat{k}_t \end{bmatrix} + BB \begin{bmatrix} z_t & u_t & v_t \end{bmatrix}', \quad (30)$$

where  $XX = \begin{bmatrix} \widehat{c/y} & \widehat{i/y} & \widehat{\pi} & \widehat{m/y} & \widehat{f} & \widehat{mplb} \end{bmatrix}'$  and the rows of the matrices  $AA$  and  $BB$  result from the linear combinations of the corresponding rows of matrices  $A$  and  $B$ , the appropriate linear combinations being given by the linear equations that define the variables from  $XX$  as functions of the variables from  $X$ . The marginal product of labor in banking, is derived from equation (11), while the definition of the other terms of the matrix  $XX$  is straightforward.

The least square "estimates" for the shock series are computed as follows:

$$est \begin{bmatrix} z_t & u_t & v_t \end{bmatrix}'_t = (BB'BB)^{-1}BB'(XX_t - AA \begin{bmatrix} \widehat{k}_t \end{bmatrix}).$$

In this approach we used six variables to construct the economy's three shocks. To test for the robustness of the process of shock construction, we repeated the computation by using combinations of six variables taken five at a time, six taken five at a time and six taken four at a time, allowing for twenty-one more possible ways to construct the shocks. The results show that all combinations that include  $\pi$ ,  $m/y$ , either  $c/y$  or  $i/y$ , and either  $f$  or  $mplb$  generate nearly the same shock series, while other combinations show

randomness and lack of conformity. Thus, we found that the results are robust as long as the variables are included that correspond to the model's three sectors in which the three shocks occur.

## A.4 Shock Profiles

The next three figures show each of the computed shocks. Figure 9 shows the money shock, with a rise in the money supply during the 1920s, up until 1931 and then a large drop until 1939. After a WWII bounce, it then rises up with a long surge in the 1960s and 1970s, after which it falls again. There are some differences between the exogenous and endogenous growth models, such as during WWII, and in the late 1990s.

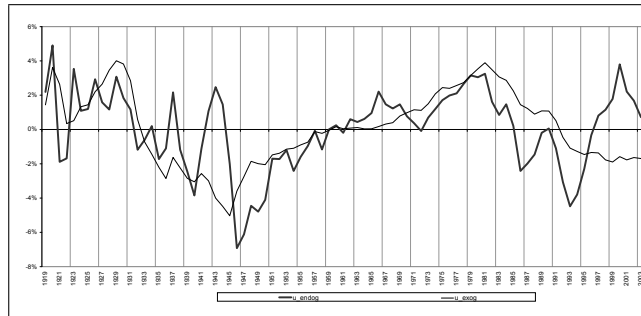


Figure 9: Money Shocks, Endogenous and Exogenous Growth Models, 1919-2004

Figure 10 shows a strong negative effect of the goods sector productivity shock during the Great Depression, consistent with total factor productivity stories of the Great Depression (Kehoe and Prescott 2002). The shock is little changed from the exogenous growth version of the model.

Figure 11 shows the credit shock. During the Depression, the baseline endogenous growth shows a positive effect of the credit shock in the early part until 1932, when banking could partially provide a means of exchange instead of money; this was reversed then by the subsequent banking collapse. And some positive effects are apparent during the 1970s and 1980s, when interest ceilings were controverted by new non-bank banks and when deregulation began.

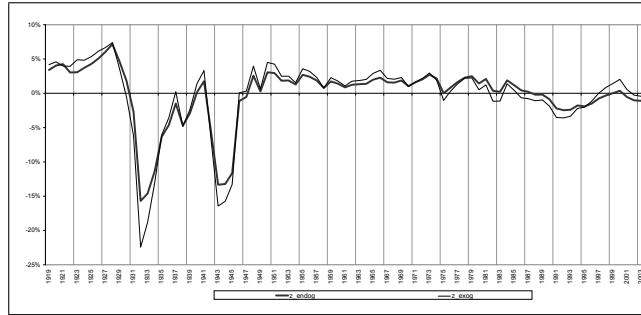


Figure 10: Productivity Shocks, Endogenous and Exogenous Growth Models, 1919-2004

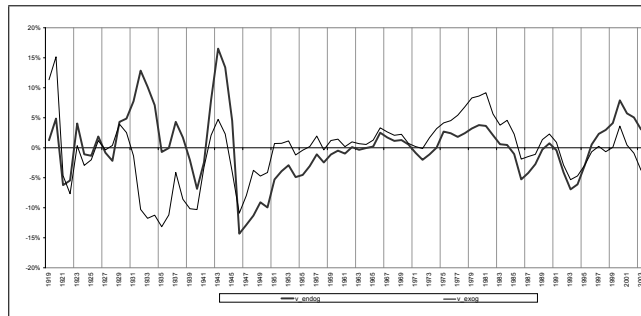


Figure 11: Credit Shocks, Endogenous and Exogenous Growth Models, 1919-2004

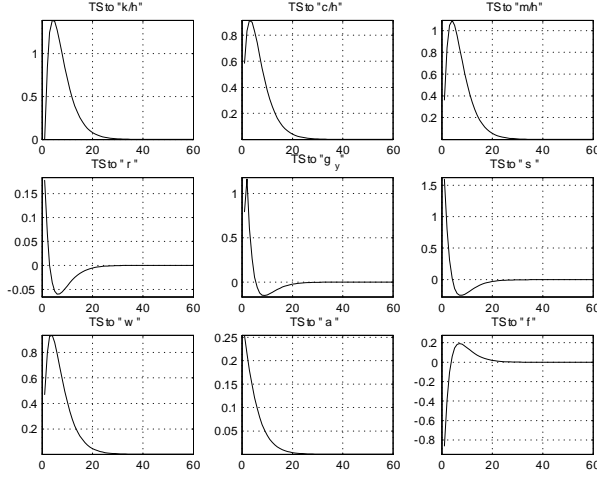


Figure 12: Impulse responses to the technological shock - Part 1

## A.5 Impulse Responses

The impulse responses show how the shocks affect the economy in the short run, in that the shocks eventually die out. Here we report the simulated impulse responses of the baseline model's variables to the three shocks on goods sector productivity (TS), the money supply growth rate (MS) and the credit sector productivity (CS), with two panels for each of the three shocks.

The first set of goods productivity impulses show that the output growth rate ( $g_y$ ), the real interest rate ( $r$ ), the real wage ( $w$ ), normalized consumption ( $c/h$ ) and money demand ( $m/h$ ), the money to consumption ratio ( $a$ ) and the capital to effective labor in the goods sector ( $s$ ) all initially rise, while bank labor ( $f$ ) falls, and the physical capital to human capital ratio ( $k/h$ ) gradually rises after the first period.

The second set shows that normalized output and investment, labor in goods production, the income velocity of money ( $vel$ ) and leisure all rise initially, while the growth of human capital ( $g_h$ ) the labor in human capital production, inflation and the nominal interest rate all fall initially.

Money shocks cause an initial drop in the capital to human capital ratio, normalized consumption and money demand, the real interest rate, the output growth rate, the capital to effective labor ratio in goods production, and the money to consumption ratio, while causing an increase in the real wage and labor time spent in the bank sector.

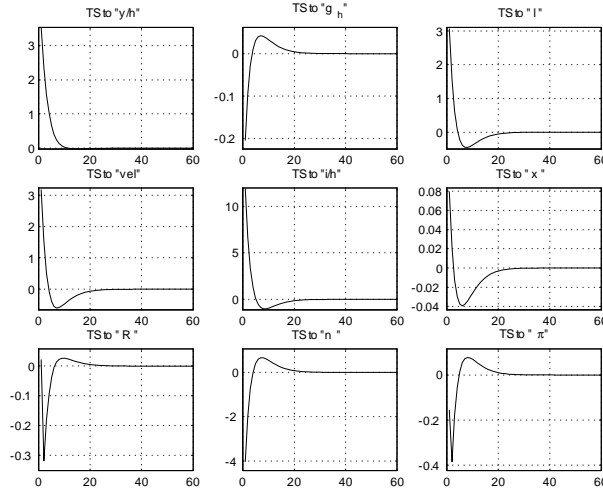


Figure 13: Impulse responses to the technological shock - Part 2

The decrease in the real interest rate is like a liquidity effect, even while the nominal interest rate rises because of expected inflation. But rather than the real interest rate decrease being due to more capital entering the capital markets, as with a liquidity effect of a money supply increase, here the dynamics are that the physical capital investment decreases and output growth decreases, even as the growth rate of human capital increases.

It also causes a fall in normalized output, labor in goods production, and normalized investment ( $i/h$ ), while raising the growth rate of human capital, the income velocity of money, leisure, the nominal interest rate and the inflation rate, and time in human capital investment.

A credit shock causes almost the exact opposite to a money shock.

In sum, the goods sector and bank sector productivity shocks increase output growth and decrease inflation, while the money shock has the opposite effect on these variables.

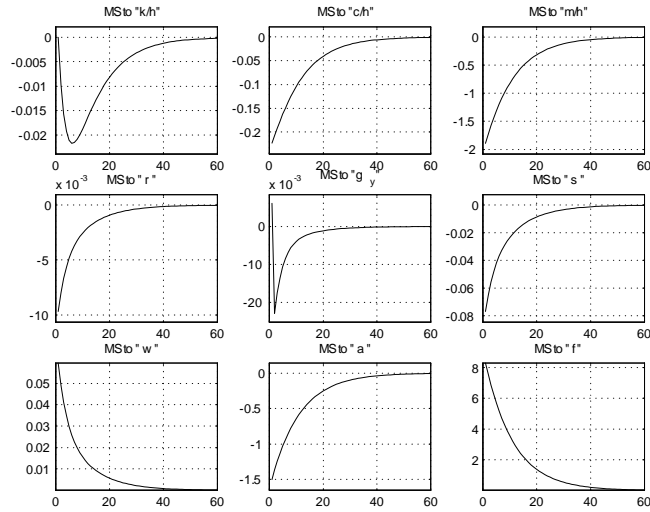


Figure 14: Impulse responses to the monetary growth shock - Part 1

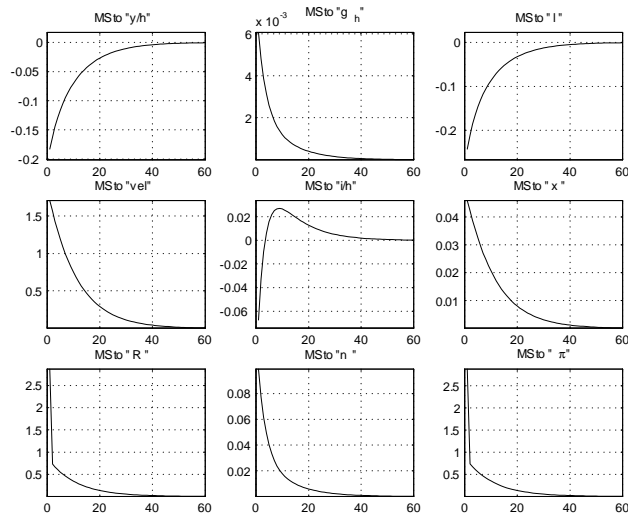


Figure 15: Impulse responses to the monetary growth shock - Part 2

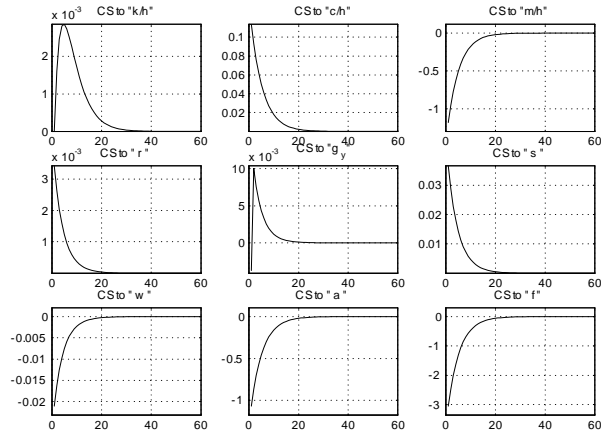


Figure 16: Impulse responses to the credit sector productivity shock - Part 1

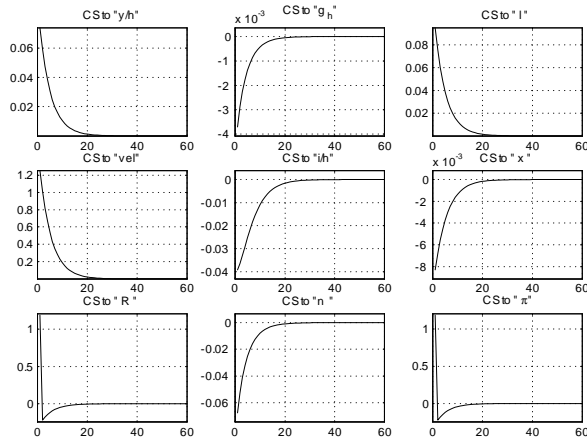


Figure 17: Impulse responses to the credit sector productivity shock - Part 2