Product and Process Innovation in a Growth Model of Firm Selection*

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Abstract

Recent empirical evidence based on firm level data emphasizes firm heterogeneity in innovation activities and the different effects of process and product innovations on productivity and productivity growth. To match this evidence, my paper develops an endogenous growth model with two sources of firm heterogeneity: productivity and quality of the variety produced. Both productivity and quality evolve endogenously through firms' innovation choices. Growth is driven by selection among incumbent firms and sustained by innovation and by entrants who imitate the best incumbents. The model disentangles the different effects of selection and innovation as well as product and process innovation on growth. Compared to single attribute models of firm heterogeneity, my model provides a more complete characterization of firms' innovation choices and optimal pricing. Additionally, it explains the partition of firms among different innovation strategies and the effect on the price distribution.

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1 Introduction

Globalization and the rise of new technologies have challenged firms' abilities in developing innovation strategies to face increasing market competition. Innovation has become a fundamental source of economic growth[1]. The literature has widely analyzed the relation between innovation and growth, however few attention has been given to the relation between firm heterogeneity and innovation activities, and even less to the relation between firm heterogeneity and different innovation strategies as well as their impact on firms' competitiveness and productivity growth. Understanding the determinants of firms' innovation strategies and their impact on growth is therefore crucial and can also contribute to enhance the effectiveness of policies aimed at fostering growth.

This paper propose a new framework to analyze the different effects of process and product innovation on firm dynamics, competitiveness and growth. For this purpose, an endogenous growth model with two sources of firm heterogeneity, productivity and quality, is developed. Besides the model is consistent with recent empirical evidence on firm dynamics, it stresses the importance of reallocation of resources not only among less efficient firms to more efficient ones, but also among different innovation strategies. Moreover, it disentangles the effect of cost reduction and quality improvement innovations on growth. Additionally, it partially endogenizes the evolution of firm states, which is purely exogenous in the firm dynamics literature.

Existing literature distinguishes only between two types of innovations: horizontal innovation which expands the number of varieties in the market, and vertical innovation which increases the quality or reduces the production cost of existing goods. In these models quality improvement (product innovation) or cost reduction (process innovation) are seen as interchangeable and yield the same prediction. Contrastingly, recent empirical evidence at the firm level emphasizes that firms perceive in a very different way quality improvement or cost reduction innovations. Firms not only have different incentives to invest either in quality improvement or in cost reduction or even in both innovations simultaneously but also their impact on firms' pricing strategies, productivity and TFP growth is different. Firms innovate on their existing products, aiming at increasing product differentiation and hence prices, thereby exploiting consumers willingness to pay for a higher quality good. Instead process innovation reduces the unit cost of production or

[1] For instance, Huergo and Jaumandreu (2007) estimate that the contribution of firms that perform innovations explains between 45% and 85% of productivity growth in the industry with intermediate or high innovation activity. Moreover, Bartelsman and Doms (2000) report evidence of a self-reinforcing mechanism between productivity and innovation. Profitable firms have a higher propensity to innovate and innovation is positively related with productivity and productivity growth.

increases the firms production efficiency. This leads to higher firm productivity, lower prices and a larger scale of production. Process and product innovations are often complementary: product innovation allows new product designs but this new designs become profitable only when they are affordable. Standard models with only one source of innovation cannot capture this price effect. In addition, the heterogenous firm literature is usually based only on one factor of heterogeneity: either productivity or instead the ability of producing quality. These models share the feature that a single attribute monotonically predicts firms' revenue, competitiveness and innovation. This property then implies a threshold firm size above which all firms innovate and below none do.

However, empirical evidence[2] at the firm level emphasizes the existence of a more complex link between firm size and innovation. Fritsch and Mescede (2001) testing Cohen and Klepper (1996) hypothesis find that the investment in process and product innovation is indeed positively correlated with firm size but less than proportional. Three main aspects are highlighted by the empirical studies. A first point is the *heterogeneity* in innovation strategies which appears clearly from table 1.

Table 1: Heterogeneity in Innovation Strategies

Country	**Share of Innovative Firms**			
	No Innovation	**Process**	**Product**	**Both Process and Product**
Spain	55.4%	12.2%	12.4%	20%
Italy	20%	28%	12%	40%
Germany	41%	10.2%	21%	27.4%
Great Britain	60.5%	11%	14.2%	14.3%
Netherlands	36.6%	5.8%	18.8%	42.7%

Jamandreu (2003) in a sample of Spanish firms in the manufacturing sectors finds that half of the firms never innovate, 30% do either process or product innovation and 20% of the firms do both types of innovations. Huergo and Jaumandreu's (2004) paper on process innovation, highlights that 42% of a Spanish panel with 2300 never innovate and only 15% of firms innovate every year. Similar statistics are provided by Parisi et

[2]The empirical literature reported focuses on European data sets which distinguish between process and product innovation. The same is not possible with American data where innovation is measured as patents and therefore the two dimensions are more difficult to analyze. However, for a concise summary Klette and Kortum (2004) report a list of stylized facts concerning firm R&D, innovation and productivity.

al. (2006) on the MCC Italian data set on medium and large firms for innovative sectors: 20% of firms never innovate, 40% do both process and product innovation, 28% do only process and 12% only product. Similar data are also available for Germany, Great Britain and Nederlands. A second point is the *asymmetry* of the innovation strategies: Huergo and Jamandreu (2004) estimate that process innovation increases productivity by 14% and product innovation by 4% over three years period and that process innovation contributes for about 77% of the yearly growth rate of aggregate productivity. Additionally, product innovation can account for about 23% only total productivity growth. As expected, innovating firms are characterized by a productivity distribution that stochastically dominates the productivity distribution for non innovators. But in the case of product innovation the distribution becomes more skewed to the right. The third point is the *interaction* between the innovation strategies. Process innovation is more frequent than product innovation, the probability of introducing a product innovation is higher for firms that also introduce a process innovation in the same period. However, process innovation does not necessarily imply product innovation.

When talking about firm dynamics I cannot abstract from entry and exit which play an important role in explaining the reallocation of resources and therefore growth. Foster, Haltiwanger and Krizan (2001) find that in the U.S. Census Manufactures, more than a quarter of the increase in aggregate productivity between 1997 and 1978 was due to entry and exit. In addition, exit is associated with a lower level of preexit innovations and entrants present a high probability of innovation. However, this does not imply that entrants are always the major source of innovation, although entry stimulates incumbents to introduce new products and new processes to face the increasing competition.

This paper takes the inconsistency between the existing theoretical literature and the empirical evidence as a starting point and develops a general equilibrium heterogenous firms model with endogenous evolution through innovation of productivity and quality of the variety produced. The set up draws the industry structure from Hopenhayn (1992) and the competitive structure from Meliz (2003), using monopolistic competition instead of perfect competition. Firms produce differentiated goods and are heterogeneous in their productivity and in the quality of the variety produced. The evolution of both productivity and quality is given by a stochastic component and by an endogenous deterministic component derived by the optimal investment decision taken by the firm. Growth arises and is sustained endogenously by the interaction between selection & imitation and innovation. Each period, non profitable incumbents exit the industry, implying that the average productivity of the remaining firms increases. Entrants enter in the market as in Gabler and Licandro (2005): they try to imitate the incumbents that are close to the technological frontier but they do not success fully but on average entrants are more productive than exiting firms increasing the average productivity of the industry. This is reinforced by innovation: successful firms innovate increasing the selection in the industry and as a result also growth. The model is able to generate a consistent partition of firms among the different innovation strategies and in addition shows that firms with low productivity but high quality can not only survive in the industry but also innovate specializing first in process innovation to increase the market share and then in both types of innovations. The double effect of innovation on prices is also captured generating a more accurate price distribution. If the price of a firm does not change this does not mean that nothing has happened, as the one factor heterogenous firm models would predict, instead it can be that the firm performs both process and product innovation and their effects offset, but the position of the firm in the distribution has improved. Calibrating the model generates moments that closely match their empirical counterpart providing a suitable framework for further work aimed at inspecting the effects of innovation policies on growth and welfare. Finally, the model is able to generate concave policy functions for the innovation decisions consistently with evidence.

To summarize the contribution of the paper is to propose a more nuanced characterization of the determinants of firm innovations and productivity growth providing a growth model consistent with facts on firm dynamics and on recent empirical evidence.

1.1 Related literature

[TO ADD]

2 The model

This section develops a general equilibrium model in discrete time, indexed by t, and the horizon is infinite. Firstly, I analyze the representative consumer problem and then from section 2.2 the focus moves to characterize the decision taken by the firms and the industry dynamic generated by two factors of firm heterogeneity. The production problem of each monopolistic firm is defines as well as the different innovation choices that each firm can individually taken to improve either one or both the states of its technology, i.e. firm productivity and product quality. Firm dynamics is discussed in sections 2.2.4 and 2.2.5 in which firm exit and entry are respectively defined. After that the cross-sectional firm distribution is characterized and the stationary equilibrium is defined in section 2.4.

2.1 Consumer problem

The representative consumer maximizes his utility choosing consumption and supplying inelastically labor at the wage rate w. Its lifetime utility is assumed to take the following form

$$U = \sum_{t=0}^{\infty} \beta^t ln(U_t) \tag{1}$$

where $\beta < 1$ is the discount factor. In every period consumer faces the problem of maximizing its current consumption across a continuum of differentiated products indexed by $i \in I$ where I is a measure of the available varieties in the economy. Specifically, the preferences are represented by an augmented Dixit-Stiglitz utility function with constant elasticity of substitution between any two goods $\sigma = 1/(1-\alpha) > 1$ with $\alpha \in (0,1)$. Hence, the utility function at time t is:

$$U_t = \left(\int_{i \in I} (q_t(i)x_t(i))^{\alpha} di \right)^{\frac{1}{\alpha}}. \tag{2}$$

where $x(i)$ is the quantity of variety $i \in I$ and $q(i)$ is the quality of variety $i \in I$. This utility function is augmented to account for quality variation across products and quality acts as an utility shifter: for a given price consumer prefers products with high quality rather than products with low quality.

The representative consumer maximizes his utility subject to the budget constraint $E_t = \int_{i \in I} p_t(i)x_t(i)di$ where E_t is total expenditure at time t and $p_t(i)$ is the price of variety $i \in I$ at time t. Solving the intra-temporal consumer problem yields the demand for each variety $i \in I$

$$x_t(i) = \left(\frac{P_t q_t^{\alpha}(i)}{p_t(i)} \right)^{\frac{1}{1-\alpha}} X_t = \left(\frac{P_t^{\alpha} q_t^{\alpha}(i)}{p_t(i)} \right)^{\frac{1}{1-\alpha}} E_t \tag{3}$$

with P_t define by

$$P_t = \left(\int_{i \in I} \left(\frac{p_t(i)}{q_t(i)} \right)^{\frac{\alpha}{\alpha-1}} \mathrm{d}i \right)^{\frac{\alpha-1}{\alpha}} \quad \text{and} \quad X_t = U_t. \tag{4}$$

P_t is the price quality index at time t of all the bundle of varieties consumed and X_t is the aggregate set of varieties consumed. Since α is lower than 1, the price index is positively related to products prices and inversely related to product quality. This implies that P can be seen as a measure of market competition: the higher the price index, the higher the prices charged by the individual firms and hence the lower the competition in the market. Using the budget constraint, the aggregate expenditure can be rewritten as

$$E_t = \int_{i \in I} \left(\frac{q_t(i)P_t}{p_t(i)} \right)^{\frac{\alpha}{1-\alpha}} \mathrm{d}i E_t. \tag{5}$$

Each good has a positive demand and the consumer devotes a higher share of expenditure to goods with higher quality price ratio.

Finally, the optimal inter-temporal allocation of consumption yields the standard Euler equation:

$$\frac{X_{t+1}}{X_t} = \beta(1 + r_t). \tag{6}$$

where r_t is the return on asset holding.

2.2 Firms

This section outlines a dynamic two factors heterogeneous firm model. The first source of heterogeneity is *productivity*, $a(i) \in \mathbb{R}_{++}$, which increases the marginal productivity of labor, as in the seminal paper of Hopenhayn (1992), and the second source is *quality* of the firm's variety, $q(i) \in \mathbb{R}_{++} \backslash (0, 1)$, which decreases the marginal productivity of labor. In this respect, a higher quality variety has a higher variable cost as in Verhoogen (2008). Firms distribute over productivity and quality. Let define $\widetilde{\mu}(a, q) = \mu(a, q)I$ the measure of firm with state (a, q) at time t, where I is the number of firms in the industry and $\mu(a, q)$ is a density function. I assume that each firm produces only one variety so that the index i identifies both the firm and the corresponding variety produced by that firms and I represents both the set of varieties and the mass of incumbent firms active in the industry. Let's define the set of all productivities with A, the set of all qualities with Q and the state space $A \times Q$ with Ω.

In every period active firms pay a fixed operational cost and then they learn their technology level, (a, q). Firm productivity and quality evolve over time affected by different innovation strategies carried out in the previous periods and by a stochastic process. More precisely, every period both productivity and quality are hit by idiosyncratic shocks and increased endogenously by past $R\&D$ investments. Based on the resulting technology, firms optimize their profits choosing the monopolistic price of their variety and the innovation strategy. Before undertaking any innovation, firms have to pay a fixed cost and then they can decide how much to invest in improving next period technology. Then firms choose employment, the representative consumer supplies labor and the real wage adjusts to clear the labor market. Finally, firms decide whether to stay or to exit the industry at the end of the period. To summarize, incumbent firms face three decisions: an intra-temporal decision about pricing and production, and two inter-temporal decisions concerning whether to innovate and whether to stay or to exit the industry. Besides active firms, potential entrants decide whether to enter or not in the industry comparing the expected value of entry with a fixed entry cost. Upon entry, new firms pay a sunk entry cost and then they draw a technology, (a, q), from a distribution of entrants technologies which is linked to the incumbent distribution via an imitation mechanism.

2.2.1 Production decision

After paying a fixed operational cost c_f, expressed in terms of labor, active firms receive their new technology level (a, q). Firms produce and price their own products under the assumption of monopolistic competition: each firm is small relative to the size of the market and hence unable to affect its competitors' prices and aggregate variables. The production decision is particularly simple since it involves only an intra-temporal dimension of profit maximization given consumer demand, (4), and firm technology. Close to Hallak and Sivadasan (2008), the production function is assumed to be linear in labor, n, which is the unique input, increasing in firm productivity, a, and decreasing in firm's product quality, q. That is, $x_t(i) = a_t(i)q_t(i)^{-\eta}n_t(i)$ with $\eta \in (0, 1)$. The parameter η introduces asymmetry between firm efficiency and product quality and measures the difficulties in producing a higher quality variety: the higher η the more difficult and costly is to produce a high quality product. This particular functional form is justified by the empirical evidence at the base of this paper. In particular, it generates a price distribution consistent with the estimates of Smolny (1998) and complementarity between process and product innovation.

Precisely, the intra-temporal maximization problem, faced by each firm, is formulated as

$$\pi_t(a(i), q(i)) = \max_{p(i)} p_t(i)x_t(i) - w_t n_t(i) \quad \text{s.t.} \quad x_t(i) = \left(\frac{P_t q_t^{\alpha}(i)}{p_t(i)}\right)^{\frac{1}{1-\alpha}} X_t \qquad (7)$$
$$x_t(i) = a_t(i)q_t(i)^{-\eta}n(i)$$

where w_t is the wage rate at time t common to all firms. The first order condition with respect to price yields the optimal pricing rule:

$$p_t(a(i), q(i)) = \frac{w_t q_t^{\eta}(i)}{\alpha a_t(i)}. \qquad (8)$$

$1/\alpha$ is the constant mark-up associated with the CES demand function. In contrast to the standard models with a single factor of firm heterogeneity, firms' prices depend on both firms productivity and quality of their products. Consistently with both the theoretical predictions and the empirical estimates, the price schedule is increasing in the quality of the variety produced by the firms and decreasing in firms productivity. As in Melitz (2003) the nominal wage is normalized to one. Using the monopolistic price to solve for the optimal demand for each variety yields:

$$x_t(a(i), q(i)) = \left(\frac{\alpha a_t(i) P_t^\alpha}{q_t(i)^{\eta-\alpha}}\right)^{\frac{1}{1-\alpha}} E_t. \tag{9}$$

Firm output is an increasing function of both the aggregates (P, E) and of the productivity level of firms. A high price index is associated with low competition and hence each firm can enjoy a high market quota relatively to an economy in which the price index is low. In contrast to the prediction of quality based model with a single factor of heterogeneity, quality is not monotonically related to the optimal quantity demanded. For a given level of productivity, the relation between product quality and output is ambiguous and depends on the relation between α, coming from the consumer demand and linked to the elasticity of substitution, and η, coming from firm production function and measuring the difficulty to produce a high quality variety. If $\eta > \alpha$ then firm output is decreasing in the product quality: high quality varieties are characterized by a relatively lower market share. In this case, the positive effect of quality on consumer utility is completely offset by the related high market price. The opposite is true when $\alpha > \eta$, though this last scenario appears to be counterfactual.

The optimal labor demand is given by

$$n_t(a(i), q(i)) = \left(a_t(i) q_t(i)^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}} \left(\alpha P_t^\alpha\right)^{\frac{1}{1-\alpha}} E_t. \tag{10}$$

Labor input is an increasing function of both productivity and quality of the variety produced by the firm. Consequently, firms with more advanced technology demand more labor input. Finally, the net per period profit of firm i is given by

$$\pi_t(a(i), q(i)) = \left(a_t(i) q_t(i)^{1-\eta} \alpha\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) P_t^{\frac{\alpha}{1-\alpha}} E_t - c_f. \tag{11}$$

Although product quality has an ambiguous effect on the optimal output of firms, profits are increasing in both the states, productivity and quality. This provides incentives for firms in improving endogenously their position in the technology distribution via firms' innovation policies. In this respect, the model predicts that a change in productivity impacts more firm's profit than a change in quality.

The different effects of firm productivity and quality on the monopolistic price, on the output and on the profits provide a suitable framework in which to study the interplay among different innovation choices taken by the firms and their effects on firms competitiveness. An innovation in product, aimed at increasing product quality, results in a higher market price for the given variety and, for appropriate parameters, in a contraction of the market quota. This then determines an incentive to invest also in process innovation and hence increase firm efficiency. This in turn leads to a lower market price and to an unambiguous larger market share. Innovation is then introduced in the next section.

2.2.2 Innovation decision

As mentioned before, firms are heterogeneous in two dimensions, productivity and quality of the variety produced. Firms receive idiosyncratic shocks on both states, i.e. firms' log productivity and log quality follow a random walk. This is a way of capturing the role of firm specific characteristics and the persistency of firm productivity and quality which is established in the empirical literature. For instance, we can capture factors as absorption techniques, managerial ability, gain and losses due to the change in the labor composition and so on. Besides the exogenous random walks, firms can endogenously affect the evolution of their states through private innovation activities. I define two different types of innovation: *process innovation* and *product innovation*. Process innovation refers to the decision of firms to invest labor aiming at increasing firm productivity, while product innovation refers to the decision of firms to invest labor aiming at increasing the quality of the variety produced.

Accordingly to the theoretical growth literature, the benefits derived by firms' innovation investments are proportional to the amount of resources spent. In particular, I assume that innovation introduces an endogenous drift in the random walk processes which reflects the amount of variable labor that firms optimally invest in R&D. In addition to this variable cost of innovation firms have to pay also a fixed cost of innovation, c_a and c_r for process and product innovation, respectively. This is a way of capturing the costs necessary to set up an R&D department, to conduct market analysis and technically it allows to generate the partition of firms among different innovation strategies. In other words, depending on the firms specific technology state, some firms decide optimally to innovate either in process or in product or in both types of innovation. The innovative firms pay the fixed costs to access the innovation facilities and then they hire the optimal amount of labor to be assigned to either product or process R&D. The variable labor hired then translates in a proportional drift in the evolution of either productivity or quality or both states: today investment in process or product innovation results in tomorrow higher firm productivity and/or product quality. This also implies that the innovation choice becomes history dependent. In either forms innovation comes, it represent a first sources of endogenous growth since it shifts the binomial firm distribution to the right.

Specifically, let's assume that log productivity evolves according to

$$\log a_{t+1} = \begin{cases} \log a_t + \varepsilon^a_{t+1} & \text{when } \; z_t = 0 \\ \log a_t + \lambda^a \log z_t(a,q) + \varepsilon^{az}_{t+1} & \text{otherwise} \end{cases} . \tag{12}$$

Shocks are firm specific and distributed as $\varepsilon^a_{t+1} \sim N(0, \sigma^2_a)$, $\varepsilon^{az}_{t+1} \sim N(0, \sigma^2_{az})$ where σ^2_a is the variance of the random walk when innovation does not occur and σ^2_{az} is the variance of the process when innovation takes place. The mean of the shocks is normalized to zero to eliminate exogenous sources of growth. In fact, abstracting from innovation and firm selection, in expectation firms do not grow. $z_t(a,q) > 0$ is the labor that a firm with states (a,q) decide optimally to invest in process innovation. $\lambda^a > 0$ is a parameter that, together with the log form of the innovation drift, scales the effects of innovation. The log functional form chosen for the innovation drift is important because together with firm selection assures a bounded growth and hence the existence of a stationary distribution. Similarly log quality evolves

$$\log q_{t+1} = \begin{cases} \log q_t + \varepsilon^q_{t+1} & \text{when } \; l_t = 0 \\ \log q_t + \lambda^q \log l_t(a,q) + \varepsilon^{ql}_{t+1} & \text{otherwise} \end{cases} . \tag{13}$$

Again $\varepsilon^q_{t+1} \sim N(0, \sigma^2_q)$, $\varepsilon^{ql}_{t+1} \sim N(0, \sigma^2_{ql})$ where σ^2_q and σ^2_{ql} are the two variances without and with innovation. $l_t(a,q)$ is the variable labor devoted to product innovation and $\lambda^q > 0$ is the related scale parameter.

The random component ε is independent both across firms and over time. Moreover, I assume that the two processes, productivity and quality, are independent. This simplification does not affect qualitatively the model predictions but has the advantage to narrow the set of parameters to calibrate since it is possible to ignore the covariances of the two processes. Let's define the density function of the productivity process a_{t+1} conditional on a_t as $f(a_{t+1}|a_t)$ when there is no process innovation and as $f(a_{t+1}|a_t, z_t)$ when there is process innovation. The corresponding conditional distribution functions are indicated with capital letters, $F(a_{t+1}|a_t)$ and $F(a_{t+1}|a, z_t)$. Similarly, the density functions of q_{t+1} conditional on q_t are $p(q_{t+1}|q_t)$ when there is not product innovation and $p(q_{t+1}|q_t, l_t)$ when there is product innovation. Again in capital letters are indicated the conditional distribution functions, $P(q_{t+1}|q_t)$ and $P(q_{t+1}|q_t, l_t)$.

The transition of the two state variables depends on the firms innovation decisions and the idiosyncratic shocks. Considering jointly the two transition functions, let define $\Phi : \Omega \to \Omega$ as the joint transition function, which moves firms' quality and productivity states. The corresponding transition probability function is defined as $\phi : \Omega \times \Omega \to [0,1]$, which gives the probability of going from state (a, q) to state $(a\prime, q\prime)$. The transition probability takes different forms depending on the innovation decisions and on the exit decision defined below. If the two processes are independent then $\phi(\cdot) = f(\cdot)p(\cdot)$.

2.2.3 Firm value function

Incumbent firms face a dynamic optimization problem of maximizing their expected value. If we abstract from the innovation decision this is a particularly simple problem since it is a sequence of static optimizations. With the innovation scheme, today investment in innovation affects the transition probabilities and thus the value of tomorrow technology. This generates a dynamic interplay between firm technology, productivity and quality, and the innovative position taken by the firm. The following value function summarizes this interplay:

$$v(a, q) = \max\{v^P(a, q), v^A(a, q), v^{AQ}(a, q), v^Q(a, q)\}. \tag{14}$$

The $\max$ operator indicates that in each period firms face different discrete choices which depend on the current level of firm productivity and product quality. Defining with prime the next period variables

$$v^P(a, q) = \max_p \left\{ \pi(a, q) + \frac{1}{1+r} \max \left\{ \int_\Omega v(a', q')\phi(a', q'|a, q)\mathrm{d}a'\mathrm{d}q', 0 \right\} \right\} \tag{15}$$

is the Belman equation when no innovation investments occurred and firm takes only the static decision about pricing and production. The profit function includes the fixed operational cost and the inner max operator indicates the option to exit the market illustrated in detail in the next section. In this case the transition probabilities are given by $F(a'|a)$ for the evolution of the productivity state and $P(q'|q)$ for the evolution of the quality state, jointly they read $\phi(a', q'|a, q)$. Next,

$$v^A(a, q) = \max_{p,z} \left\{ \pi(a, q) - z(a, q) - c_a + \frac{1}{1+r} \max \left\{ \int_\Omega v(a', q')\phi(a', q'|a, q, z)\mathrm{d}a'\mathrm{d}q', 0 \right\} \right\}, \tag{16}$$

is the firm value when firm produces and innovates in process aiming at increasing next period productivity. This leads to consider the fixed cost and variable cost related to process innovation, c_a and $z(a, q)$, respectively. A firm that optimally decides to invest to improve its efficiency anticipate that today investment, $z(a, q)$ affects next period value through the transition function $\phi(a', q'|a, q, z)$. Analogously, the value function when, besides production, both process and product innovation occur reads

$$v^{AQ}(a,q) = \max_{p,z,l}\Bigg\{\pi(a,q) - (z(a,q)+l(a,q)) - c_a - c_r + \tag{17}$$
$$+ \frac{1}{1+r}\max\Big\{\int_{\Omega} v(a',q')\phi(a',q'|a,q,z,l)\mathrm{d}a'\mathrm{d}q', 0\Big\}\Bigg\}.$$

This time the fixed cost are given by the sum of $c_a + c_r$ and the variable costs by $z(a,q) + l(a,q)$. Finally,

$$v^{Q}(a,q) = \max_{p,l}\Bigg\{\pi(a,q) - l(a,q) - c_r + \frac{1}{1+r}\max\Big\{\int_{\Omega} v(a',q')\phi(a',q'|a,q,l)\mathrm{d}a'\mathrm{d}q', 0\Big\}\Bigg\}. \tag{18}$$

is the value function when firms optimally specialize only in product innovation.

These value functions characterize a partition of firms among the different decisions (only produce or produce and innovate and in the latter case if innovate in process, or in product or in both process and product at the same time) which depends on the relation between the technological state of each firm and the fixed costs. In fact, given the specific position of a firm inside the binomial distribution of technology, the fixed costs of innovation generate different firms decisions consistently with equation (14). Two sources of firm heterogeneity implies that the thresholds, characterizing the border among the different innovation strategies, are given by infinite combinations of (a,q) couples. For this reason, it becomes convenient to express the reservation values in terms of productivity as a function of quality[3], $a(q)$ and to obtain *cut off functions* rather than cut off values as in one factor heterogeneous firm models. For given $q \in Q$ it is possible to define the following cut off functions:

- $a_A(q) = \inf\{a(q) \in A \quad \text{s.t.} \quad v^A(a(q),q) = v(a(q),q)\}$. Inferior limit of $a(q)$ such that firms optimally choose process innovation. All firms with $a(q) \geq a_A(q)$ undertake process innovation.

- $a_{AQ}(q) = \inf\{a(q) \in A \quad \text{s.t.} \quad v^{AQ}(a(q),q) = v(a(q),q)\}$. Inferior limit of $a(q)$ such that firms choose optimally to innovate both in productivity and in quality of their variety.

- $a_Q(q) = \inf\{a(q) \in A \quad \text{s.t.} \quad v^Q(a(q),q) = v(a(q),q)\}$. Inferior limit of $a(q)$ such that firms choose optimally to specialize in product innovation only.

[3]It is equivalent to express product quality as a function of productivity, $q(a)$. Using a specific formulation for the cut-off function does not affect the implications of the model.

The cut off functions are decreasing in q, highlighting a non monotonic relation between the innovation strategies and firms productivity. In contrast with one factor heterogeneous firm models, also less productive firms (firms with a lower level of a) but characterized by a product with high quality innovate. Notice that firm profits $\pi(a, q)$ are increasing in both productivity and quality generating the incentives to innovate which are slowed down by the log form in which the innovation drift is modeled. Abstracting from the fixed costs of innovation which introduces a discontinuity in the value function, the more advanced the firm technology, the higher the innovation investment but the lower the benefit due to the diminishing returns of innovation.

2.2.4 The exit decision

Firms exit the industry after a bad technological draw such that the expected value of continuing is lower than the exit value which has been normalized to zero, that is $v(a\prime, q\prime) < 0$. Notice that exit is triggered by the assumption of fixed operation costs, c_f, paid by active firms each period. Without fixed operational costs, a firm hit by a series of bad shocks instead of exiting the market could temporary shut down its production and just wait for better periods when positive shocks hit its technology and then start again producing. Anyway, since firm value is increasing in both states, productivity and quality of the variety, the exit reservation value is decreasing in both of them. Again I can define a cut off function $a_x(q)$ such that

$$E[v(a\prime(q), q\prime)|(a_x(q), q)] = 0. \tag{19}$$

For each quality level, there is a maximum productivity level such that below this maximum firm value is negative and therefore firms find optimally to exit the industry. Interestingly, the cut off function $a_x(q)$ is decreasing in quality: for given productivity firms with a high quality product can survive longer in the market when hit by a bad productivity shock.

Firms innovation decisions, exit and the law of motion of (a, q) define the transition function $\Phi_{xI} : A \backslash A_x \times Q \rightarrow (A_p \cup A_A \cup A_Q \cup A_{AQ} \cup A_x) \times Q$ where for given q the support of productivity is partitioned into $A_x = \{a(q) \in A : a(q) < a_x(q)\}$ (exit support), $A_P = \{a(q) \in A : v(a(q), q) = v^P(a(q), q)\}$ (support in which firms specialize only in production), $A_A = \{a(q) \in A : v(a(q), q) = v^A(a(q), q)\}$ (support in which firms specialize in process innovation), $A_Q = \{a(q) \in A : v(a(q), q) = v^Q(a(q), q)\}$ (support in which firms specialize in product innovation) and $A_{AQ} = \{a(q) \in A : v(a(q), q) = v^{AQ}(a(q), q)\}$ (support in which firms specialize in process and product innovation). These partitions differ across different elements of Q. The corresponding transition probability of going from state $(a, q) \in (A_p \cup A_A \cup A_Q \cup A_{AQ}) \times Q$ to $(a\prime, q\prime) \in (A_p \cup A_A \cup A_Q \cup A_{AQ} \cup A_x) \times Q$ is given by a function $\phi_{xI}(\cdot)$.

2.2.5 Firms entry

Every period there is a mass of potential entrants in the industry which are a priori identical. To enter firms have to pay a sunk entry cost c_e expressed in terms of labor. This cost can be interpreted as an irreversible investment into setting up the production facilities. After paying the initial cost, firms draw their initial productivity level, a, and their initial product quality, q, from a common bivariate density function $\gamma(a, q)$. The associated distribution is denoted by $\Gamma(a, q)$ and has support in $\mathbb{R}_+ \times \mathbb{R}_+$. Define $\overline{\gamma}_e$ the mean of the joint distribution and σ^2_{ea} and σ^2_{eq} the variances of the entrants productivity and quality processes (the covariance is zero given the current assumption of independency between the evolution of the two states). As standard in the literature on firm dynamics, in equilibrium the free entry condition holds: potential entrants enter until the expected value of entry is equal to the entry cost.

$$v^e(a, q) = \int_{\Omega_e} v(a, q) d\Gamma(a, q) = c_e, \tag{20}$$

M_t is the mass of firms that enter in the industry at time t. At the stationary equilibrium also a stability condition needs to be satisfied: the mass of new entrants exactly replaces the mass of unsuccessful incumbents who are hit by a bad shock and exit the market: $M = \int_{a_x(q)} \int_Q \mathrm{I}\mu(a, q)$.

The average technology of surviving incumbent firms grow due to randomness and innovation. This implies that the demand of labor grows over time at a positive rate and, given a fixed exogenous supply, the wage rate rises. Hence, if the joint distribution of entrants productivity and product quality, $\gamma(a, q)$ were completely exogenous and constant, then the expected value of entry would be driven to zero and no firms would eventually enter the market. To avoid this scenario the entrants technology is linked to incumbent firms technology through an imitation mechanisms related to Luttmer (2007) and used also in Gabler and Licandro (2007). Entrants imitate incumbent firms: the mean of the entrant distribution is a constant fraction $\psi_e \in (0, 1)$ of the mean of the joint distribution of incumbents defined as $\overline{\mu}$. That is, $\overline{\gamma}_e = \psi_e \overline{\mu}$. For given quality, and consistently with empirical evidence, entrants are on average less productive than incumbents. However, as the distribution of incumbents shifts rightward due to growth so does the distribution of entrants due to imitation. Imitation is then needed to guarantee a positive measure of new firms in every period and hence, together with selection, to assure the existence of a stationary distribution. This knowledge spillover, that goes from incumbent firms to entrants, is the only externality present in the model[4] and combined with firm selection and together with innovation generates endogenous growth.

2.3 Cross sectional distribution and aggregates

All firms' choices and the processes for the idiosyncratic shocks yield the low of motion of firms distribution across productivities and qualities, $\mu(a, q)$

$$\begin{aligned}\mu'(a', q') = &\int_{A_P}\int_Q \mu(a, q)\phi(a', q'|a, q)\mathrm{d}q\mathrm{d}a + \\ &\int_{A_A}\int_Q \mu(a, q)\phi(a', q'|a, q, z)\mathrm{d}q\mathrm{d}a + \int_{A_{AQ}}\int_Q \mu(a, q)\phi(a', q'|a, q, z, l)\mathrm{d}q\mathrm{d}a \\ &+ \int_{A_Q}\int_Q \mu(a, q)\phi(a', q'|a, q, l)\mathrm{d}q\mathrm{d}a + \int_A\int_Q \frac{M}{\mathrm{I}}\gamma(a, q)\mathrm{d}q\mathrm{d}a\end{aligned} \tag{21}$$

Tomorrow density is given by the contribution of all surviving firms (the domain of the integral is restricted to surviving firms only) and of entrants. The contribution of entrants is represented by the last term of (21). The first integral represents the share of surviving firms that only produce and do not invest neither in process nor in product innovation, the second integral shows the contribution of the firms that successfully produce and also invest in process innovation. The third one instead represents the firms that produce and do both types of innovation and finally the forth one highlights the share of producers that specialize in product innovation only.

Since the industry is populated by a continuum of firms and only independent idiosyncratic shocks occurs the aggregate distribution evolves deterministically. As a consequence, thought the identity of any firms i associated to a couple (a, q) is not determined their aggregate measure is deterministic. For the same reason also the other aggregate variables evolve deterministically.

To summarize the information about the average firm productivity and product quality, a weighted mean of firm technology can be introduced. That is,

$$\overline{\mu} = \left(\int_{a_x(q)}\int_Q \left(aq^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}}\mu(a, q)\mathrm{d}q\mathrm{d}a\right)^{\frac{1-\alpha}{\alpha}}. \tag{22}$$

Differently from Melizt (2003), this weighted mean not only depends on two states, productivity and quality of the firm variety, but also the weights reflect the relative quality

[4]Eeckhout and Jovanovic (2002) used a wider mechanisms of knowledge spillover in which all firms and not only entering firms, can imperfectly imitate the whole population of firms.

adjusted outoput shares of firms with different technology levels rather than the simple output shares. Moreover, the weighted mean can be also seen as the aggregate technology incorporating all the information contained in $\mu(a,q)$. In fact, it has the property that the aggregate variables can be expressed as a function of only $\overline{\mu}$ disregarding the technology distribution $\mu(a,q)$. Hence, the price index, the aggregate consumption and the aggregate profits can be rewritten as

$$P = \left(\int_{a_x(q)} \int_Q \left(\frac{p(a,q)}{q(a,q)} \right)^{\frac{\alpha}{\alpha-1}} \mathrm{I}\mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{\alpha-1}{\alpha}} = \mathrm{I}^{\frac{\alpha-1}{\alpha}} p(\overline{\mu}), \tag{23}$$

$$X = \left(\int_{a_x(q)} \int_Q \left(qx(a,q) \right)^{\alpha} \mathrm{I}\mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{1}{\alpha}} = I^{\frac{1}{\alpha}} x(\overline{\mu}). \tag{24}$$

$$\Pi = \left(\int_{a_x(q)} \int_Q \pi(a,q)\mu(a,q)\mathrm{d}q\mathrm{d}a \right) = I\pi(\overline{\mu}) \tag{25}$$

2.4 Equilibrium definition

In equilibrium the representative consumer maximizes its utility, firms maximize their discounted expected profit and markets clear. The stationary equilibrium of this economy is a sequences of prices $\{p_t\}_{t=0}^{\infty}$, $\{P_t\}_{t=0}^{\infty}$, real numbers $\{I_t\}_{t=0}^{\infty}$ $\{M_t\}_{t=0}^{\infty}$, $\{X_t\}_{t=0}^{\infty}$ functions $n(a,q;\mu)$, $z(a,q;\mu)$, $l(a,q;\mu)$ $v(a,q;\mu)$, cut off functions $a_x(q)$, $a_A(q)$, $a_{AQ}(q)$ and $a_Q(q)$ and a sequence of probability density function $\{\mu_t\}_{t=0}^{\infty}$ such that:

- the representative consumer chooses asset holding and consumption optimally so that to satisfy the Euler Equation (6),
- all active firms maximize their profits choosing a price that satisfies (8) and employment & innovation policies that satisfy $n(a,q;\mu)$, $z(a,q;\mu)$ and $l(a,q;\mu)$ yielding the value function $v(a,q)$ as specified by equation (14) and its components,
- innovation is optimal such that the cut off functions $a_A(q)$, $a_{AQ}(q)$ and $a_Q(q)$ satisfy the previous conditions,
- exit is optimal such that $a_x(q)$ is given by equation (19) and firms exit if $a(q) < a_x(q)$,
- entry is optimal: firms enter until equation (20) and the aggregate stability condition are satisfied,

- the number of active firms I adjusts till the labor market clears: the aggregate demand of labor is equal to the exogenous labor supply

$$\int_A \int_Q \big(n(a,q) + l(a,q) + z(a,q)\big) \mathrm{I}\mu(a,q)\mathrm{d}q\mathrm{d}a + I \int_{A_A} \int_Q \mu(a,q) c_a \mathrm{d}q\mathrm{d}a$$
$$+ I \int_{A_Q} \int_Q \mu(a,q) c_r \mathrm{d}q\mathrm{d}a + I \int_{A_{AQ}} \int_Q \mu(a,q)(c_a + c_r)\mathrm{d}q\mathrm{d}a + I c_f + M c_e = N^s,$$

- the stationary distribution of firms evolves accordingly to (21) given μ_0, I, M and the cut off values.

In equilibrium a_x, a_A, a_{AQ} a_Q, I and M are such that the sequence of firms distribution is consistent with the law of motion generated by the entry and exit rules.

Hopenhayn (1992)'s paper proves the existence of equilibrium for similar economies. All the elements of the model have been assembled and the next step is to characterize its balance growth path, to discuss the determinants of growth and to provide an algorithm to solve the model.

3 Endogenous Growth

This section defines the balanced growth path of the economy and explains the interplay between the two mechanisms in action that generate endogenous growth.

3.1 Balanced growth path

In general, on the balanced growth path output, consumption, real wage, prices and the aggregate technology grow at a constant rate, the binomial distribution of productivity and quality shifts to the right by constant steps, its shape is time invariant and the interest rate, the aggregate expenditure, the aggregate profit, the profit and the labor demand distributions, the number of firms, the firm turnover rate and the other characteristics of the firm distribution are constant.

Define g as the growth rate of the mean of the joint distribution of productivity and quality, $\overline{\mu}$. It is given by a combination of the growth rate of the productivity state, denoted by g_a, and of the growth rate of the product quality state, indicated by g_q. Intuitively, growth arises because every period the log of the joint aggregate technology shifts to the right by a factor g, meaning that the average productivity and the average product quality present in the industry grow. Higher technology adopted by incumbent firms rises labor demand which in turn drives up the real wage, $w^r = 1/P$, given a fixed supply of labor.

The effect on the real wage is a consequence of the constant decline of the price index. From the first order conditions output and aggregate output grow and this results in a constant expenditure.

In details, let's define $G_A = \frac{a_{t+1}}{a_t} = 1 + g_a$ and $G_Q = \frac{q_{t+1}}{q_t} = 1 + g_q$ the growth factors of firm productivity and product quality with g_a and g_q the respective growth rates. From the labor market clearing condition, given the assumption of a constant labor supply, N_s, also the number of incumbent firms, I, and the number of entrants, M, has to be constant as well as the share of labor allocated to production and innovation[5]. As standard in models of monopolistic competition, aggregate expenditure, E, has to be equal to the aggregate labor income, N_s, given the wage normalization. This in turn implies that on the balanced growth path E is constant too and hence also Π has to be constant. The profit distribution, equation (11), clearly shows that $\pi(a, q)$ has to be constant because of constant fixed operational costs. Given a constant expenditure, profits are constant only if $aq^{1-\eta}P$ is constant. For positive growth rate of the technology, the previous condition holds if the price index growth factor is inversely related to the the average technology growth factor, i.e. $G_P = (G_A G_Q^{1-\eta})^{-1}$. in other words as the industry grows and the average technology advances the price index diminishes. The equation for the price index shows this directly once the price distribution (8) is plugged in

$$P = \left(\int_{a_x(q)} \int_Q \left(\frac{1}{\alpha a q^{1-\eta}} \right)^{\frac{\alpha}{\alpha-1}} \mathrm{I}\mu(a, q)\mathrm{d}a\mathrm{d}q \right)^{\frac{\alpha-1}{\alpha}}. \tag{26}$$

With the same reasoning also the distribution of manufacturing labor, equation (10), is time invariant, which together with the labor market clearing condition implies that also the distributions of the labor hired for the innovation activities, $z(a, q)$ and $l(a, q)$ is constant. From the consumer problem $E = PX$, which hold only if the aggregate consumption X grows at a constant factor $(G_A G_Q^{1-\eta})$. This results in a constant interest rate as shown by the Euler equation $r = (1 + g)\beta - 1$. The price distribution $p(a, q)$ decreases at a factor equal to $\frac{G_Q^{\eta}}{G_a}$ which is lower than than the growth rate of the price index. This is a consequences of the fact that the price index is adjusted to consider the growth in the product quality. Finally, the quantity distribution $x(a, q)$ has a growth factor of $\frac{G_A}{G_Q^{\eta}}$.

A balanced growth path equilibrium exists if a there are a g_a and a g_q consistent with the stationary equilibrium. To find the growth rates consistent to the equilibrium and to characterize the equilibrium itself and the stationary firm distribution it is necessary to transform the model such that all the variables are constant along the balanced growth path

[5]If there was population growth then the number of varieties, and the number of entrant firms would grow at the same rate as population grow.

and they are denoted with a " $\sim$ ". Hence, all growing variables need to be divided by the corresponding growth factor, i.e. $\widetilde{z} = z/G_z^t$, and the stochastic processes in productivity and quality need to be detrended by the respective growth rates, i.e. $\log \widetilde{a}_t = \log a_t - g_a t$ and $\log \widetilde{q}_t = \log q_t - g_q t$. In fact, in expected terms both average firm productivity and average product quality increase and thus in expectation in every period each firm falls back relative to the distribution. This transformation affects also the transition functions and hence log productivity and log quality, in the stationarized economy, evolve according to

$$\log \widetilde{a}_{t+1} = \begin{cases} \log \widetilde{a}_t - g_a + \varepsilon^a_{t+1} \\ \log \widetilde{a}_t - g_a + \lambda^a \log \widetilde{z}_t + \varepsilon^{az}_{t+1} \end{cases} \tag{27}$$

$$\log \widetilde{q}_{t+1} = \begin{cases} \log \widetilde{q}_t - g_q + \varepsilon^q_{t+1} \\ \log \widetilde{q}_t - g_q + \lambda^q \log \widetilde{l}_t + \varepsilon^{ql}_{t+1}. \end{cases} \tag{28}$$

For positive growth rates firm productivity and quality follows a random walk with negative drifts. This negative drift determines a finite expected lifetime for any level of technology and hence the existence of a stationary distribution in the detrended economy is guaranteed. The reason will become clear in the next section.

3.2 Growth rate determinants

Two mechanisms drive growth: *selection & imitation* and *innovation*. The first mechanism results from the assumption of a random walk for productivity and quality together with entrants' imitation. Abstracting from the endogenous drift introduced by firms innovations, the random walk process, for a given set of firms, is characterized by a constant expectation and by a variance that grows over time. However, firms at the bottom of the distribution exit the industry truncating the joint distribution from below and allowing the distribution to grow only to the right through higher level of productivity and quality. The selection of firms with low productivity and low quality and the consequent reallocation of resources from these firms to more efficient ones increases the average productivity and quality of the surviving set of firms. However, as time goes by the distribution of incumbent firms shrinks as exit is an absorbing state and firms keep exiting the industry. So selection alone is not enough to sustain growth and also imitation is needed. In equilibrium the mass of entrants has to be equal to the mass of firms exiting the market. However entrants are on average more productive than exiting firms otherwise they would not find optimal to enter in the market. The combination of selection and imitation results into a firm distribution that moves upwards every period. Selection is important to emphasize the role of reallocation of resources in the growth process. Growth could still be generated without selection assuming that entrants distribution shifts every period exogenously by g. However in this way growth would just result from entry and exit which is at odds with the data.

Innovation with its endogenous drift reinforces growth. For a given set of innovative firms, not only the variance grows over time but also the expectation. The expectation of firm technology depends on the initial state and on the sequence of resources invested in innovation implying that the average technology shift upwards due to the endogenous drift. Again an important role is played by the reallocation of workers from exiting and non innovative firms to innovators. However, innovation has decreasing returns through the log form in which the innovation drift is modeled. For this reason the reallocation effect from non innovators to innovators is controlled by the selection effect and the result is that growth is reinforced but still bounded and a stationary distribution exists. The reason is that in the detrended economy (and hence thanks to the negative drifts in equations (27) and (28)) the average technology of the incumbent firms improves at a rate that it in not too high relative relative to the rate at which the technology available to entrants firms improves. Technically, a stationary distribution exists because firm life time is finite for any combination of (a, q). This is assured by the combination of decreasing return on innovation, which implies that the innovation drift is decreasing in the technological state, and by the downward drift in the random walk. Any successful firm which performs innovation will not be innovator forever but eventually it will exit the market, leading to a finite expectation and to a finite variance of the distribution.

When innovation occurs the productivity and quality processes have also a higher variances of the stochastic component. This increases the probability of a bad shock hitting the innovative firms and the dispersion of the innovator distribution against the distribution of non innovators and exiting firms. On the one hand, selection results in a higher average technology for innovators because relatively bad firms fall among the pool of non innovators causing that only relatively high productive and high quality firms keep innovating. On the other hand, the pool of non innovators becomes larger implying a higher weight to the distribution of non innovators which has a lower average technology. The final effect of higher variances of the innovation random walks on the mean of the joint distribution is ambiguous. The negative effect is then reinforced by the fact that the value function is convex in both states. Thus, a higher variance implies that the continuation values for the innovation strategies are higher relaxing the general cut off function between innovate or not to innovate. This reduces selection and therefore growth. However, calibrating the model to both German and Spanish data shows that the positive effect of innovation always outweighs the negative one.

4 Numerical Analysis

After describing the algorithm used to solve for the stationary equilibrium, this section explains the calibration of the parameters of the model to match the German manufacturing sector. The calibrated model is then used to quantify the impact of process and product innovation on economic growth as well as the impact of the reallocation of resources due to entry and exit of firms.

4.1 Algorithm

The algorithm, used to solve the model in the stationary equilibrium, is given by the following steps. The state space $A \times Q$ is discretized. The grid chosen is of 20 points for each state[6] yielding 400 technology combinations, (a, q). Firms value function is computed through value function iteration. The unknown variables are the growth rates g_a and g_q which combines in the growth rate of the aggregate technology g and the aggregate expenditure and price index summarized by $k = P^{\frac{\alpha}{1-\alpha}} E$. For given g and k compute the stationary profit $\widetilde{\pi}(a, q; g, k)$ and then the firm value function $\widetilde{v}(a, q; g, k)$. While iterating the value function, the optimal policies for the investment in process and product innovation, $\widetilde{z}(a, q; g, k)$ and $\widetilde{l}(a, q; g, k)$, are computed and the random walk processes, that govern the transition of firm productivity and product quality, are approximated using the method explained by Tauchen (1987). This step is time consuming since firms problem has to be solved via first order conditions for each single single couple of states, (a, q), till convergency is reached. Once the value function is approximated the algorithm computes the cut-off functions $a_x(q; g, k)$,$a_A(q; g, k)$, $a_Q(a; g, k)$ and $a_{AQ}(q; g, k)$. Then the transition matrix Φ_{xI} is computed. This is the final transition matrix which takes into account the exit and the innovation decisions. After guessing an initial distribution for entrant firms and normalizing its initial joint mean to zero, the expected value of entry is computed. Since $\widetilde{v}^e(a, q; g, k)$ is decreasing in k, the free entry condition is used to pin down

[6] The choice of 20 grid points for each state is due to the fact that the algorithm is computationally heavy given the presence of two states and the endogenization of the dynamic choice of the innovation investment. On the one hand, increasing the grid size would improve the precision of the calibration but would not affect qualitatively the results. On the other hand, the technology combination (a, q) available to firms would increase quadratically in the grid size and the code would eventually become unfeasible. Hence, given that the results are not qualitatively affected by the grid size, a quality and productivity grid of 20 points is a reasonable restriction.

the equilibrium value of k resulting from the first iteration of the algorithm. Using the equilibrium k then compute the firm value, the cut-off function and the transition matrices for given initial g. The binomial firm distribution is then determined using the formula for the ergodic distribution $\widetilde{\mu} = (I - T_{xI})^{-1}G$ as proved by Hopenhayn (1992). The algorithm is close using the condition on the mean of the entrant distribution, $\overline{\gamma}_e = \psi_e \overline{\mu}$ and pinning down the equilibrium growth rate, g, that satisfies this equation. All these steps are repeated until all conditions are jointly satisfied and convergence is reached.

4.2 Calibration

Fifteen parameters, linked to both firm dynamics characteristics and firms specific innovation behavior, need to be chosen. Since all of them interact with each other to determine the stationary equilibrium only two of them are parametrized, twelve are jointly calibrated to match the German manufacturing sector[7] and a last one is individually calibrated to match the condition on the entrants mean. The two parameters fixed are the discount factor β, set equal to 0.95, and the parameter α associated to the elasticity of substitution among varieties. α is set equal to 0.73, so that the price mark-up charged by the monopolistic firm is of 36% over the marginal cost in line with Ghironi and Melitz (2003). A so high mark-up could be seen at odds with the macro literature that delivers a standard mark-up of around 20% over the marginal/average cost. In my model, a higher mark-up is justified by the presence of the fixed costs. In fact, given the free entry condition firms on average break even which implies that on average firms price at the average cost leading to a reasonable high mark-ups over the average cost. The other twelve parameters are calibrated using a genetic algorithm as described by Dorsey and Mayer (1995). The main idea is to jointly calibrate the parameters in order to minimize the mean relative squared deviation of twelve model moments with respect to the corresponding moments in the data. Since the problem is highly non-linear, the minimization can be characterized by many local minima and the genetic algorithm used has the nice feature to increase the probability of choosing the global minima.

The twelve parameters calibrated are: the ratio among the fixed costs, c_e/c_f, c_a/c_f and c_r/c_f, the quality parameter η, the four variances of the incumbent random walks σ_a, σ_{az},

[7]The German economy has been empirically widely studied in both the dimensions studied in this paper: the new dimension related to firm innovation behavior and the traditional dimension related to firm dynamics. Hence, from the German data it is possible to obtain enough information to calibrate successfully the model. Similar studies [ADD REF] are available also for other European countries, for instance Spain. In an extension of the model, not yet included in the present paper, the results of the model calibrated to German data are compared to the one obtained by calibrating the model to match the Spanish economy.

σ_q and σ_{ql}, the two variances of the entrant random walks σ_{ea} and σ_{eq} and finally the two parameters that scale the innovation drifts into the stochastic processes λ_a and λ_q. These parameters jointly determine the shape, the truncation functions of the stationary distribution of firms and the partition of firms among the different innovation strategies. They are calibrated using as targets static and dynamic empirical moments that are informative and related to the main objective of the paper. Given the novelty of the the European Surveys at the firm level, the empirical moments used in the calibration are not taken from the same data source but results from a combination of information extracted by both OECD and national surveys. It is useful to distinguish between two main targets subsets. A first set of targets gives the information about the innovation behavior of firms and help to set the fixed cost of process and product innovation, η, λ_a, λ_q and the variances when innovation occurs. These are the share of firms performing process innovation, 10.2%, product innovation 21% both process and product innovation $27,4\%$ and the share of process R&D expenditure over sales equal to 0.059 as reported by Smolny (2003) studying a panel of German manufacturing firms. Fritsch and Meschede (2001) also report that 61% of the $R\&D$ expenditure of German firms is devoted to product innovation. The second set of moments gives information about the variances of the productivity shocks and on the entrants distribution. These are the survival rate of firms after four and seven years upon entry, firms yearly turnover rate, the job turnover rate derived by firm entering and exiting the market, the fraction of firms below average productivity, the productivity spread and the relative size of seven year old entrants relative to new entrants. Accordingly to Bartelsman, Scarpetta and Schivardi (2003), the two and seven year survival rates for German firms in the manufacturing sector are estimated to be 53% and 45%, respectively. In other words, for a given cohort of entrants, 20% of firms exit after two years of operation and more than half after only seven years. Those numbers are aligned to the one reported by other developed countries as UK and Spain (Garcia and Puente, (2006)). They report also a yearly firm turnover rate of 12% and a job turnover rate due to entry and exit equal to 3%. Job turnover rate is computed as the sum of job creation and job destruction due to entry and exiting of firms in a year and divided by total employment in the same year. Moreover, Bartelsman, Haltiwanger and Scarpetta (2004) estimate that the fraction of German firms below average productivity is equal to 83%, highlighting a right skewed firm size distribution, and that the size of seven years old firms relative to new entrants is 1.4 for all Germany and 1.1 for West Germany. This means that in West Germany entering firms are bigger with respect to the national level which, in the current model, would translate in a easier imitation mechanism for Western German firms. The last informative moment is the productivity spread between the 85^{th} and 15^{th} percentile which is estimated to be between 3 and 4.

The remaining parameter to calibrate is ψ_e which relates the joint mean of the entrants distribution with the joint mean of the incumbents. Given the importance of this parameter in determining the growth rate of the economy it is set individually to match its empirical counterpart. That is, ψ_e is chosen such that the average size of entrants is 43% of the size of incumbent firms as estimated by Bartelsman, Scarpetta and Schivardi (2003). Finally, the weight assigned to process and product innovation in contributing to the aggregate growth rate g is of 76% and 24%, respectively as estimated by Smonly (2003).

Table 2 shows the adopted parameters resulting from the calibration. The fixed costs are expressed in terms of the average output produced by the industry. It may be argued that the fixed operational cost, c_f is low compared to the standard values calibrated in the literature on firm dynamic. However in those model the fixed operational cost represents all the costs firms pay every period not only as administrative cost but also as the costs incurred to survive. In my model the latter costs are model separately as the fixed cost necessary to innovate. The parameter η associated with the difficulty to copy is equal to 0.76 and hence slightly higher than α. This implies that the higher the quality of a variety the lower the its production as equation (9) shows. F

Table 3 shows the calibration targets and the corresponding statistics generated by the model. Despite the large set of parameters, the moments of the model seem to match well all the targets and in particular the statistics related to the innovation choices of firms which are the main objective of this paper. Moreover, the model generates a consumption growth rate of 3.7% which is given by the combination of the growth rate of productivity equal to 2.9% and the growth rate of product quality equal to 1%. Finally, the model predicts that only 13% of the growth rate of consumption/technology is due to the process of reallocation of resources of entering and exiting firms. This highlight as economic growth is sustained by the growth of incumbents firms and the consequent reallocation of resources among successful firms who innovate. The next section will insect this last aspect.

4.3 The role of innovation

For a better understanding of the interaction between two attributes of firm heterogeneity and the different innovation strategies, figure 1 displays the cutoff functions and the equilibrium partition among the innovation choices. These partitions results from the interplay between the fixed costs and firms heterogeneity. Thought the graph does not give any information about firm density in each point, it illustrates the combinations of

Table 2: Calibration

Parameter	Value	Description
Parametrization		
α	0.73	Elasticity of substitution
β	0.95	Discount factor
Calibrated Parameters		
c_e	207.56%	Entry cost, % of average firm output
c_f	1.02%	Fixed cost, % of average firm output
c_a	3.08%	Process innovation cost, % of average firm output
c_r	10.8%	Product innovation cost, % of average firm output
η	0.76	Quality parameter
σ_a	0.10	Variance of productivity shock
σ_{az}	0.19	Variance of productivity shock with innovation
σ_q	0.18	Variance of quality shock
σ_{ql}	0.16	Variance of quality shock with innovation
σ_{ea}	0.81	Variance of log productivity distribution of entrants
σ_{eq}	0.78	Variance of log quality distribution of entrants
λ_a	0.1	Scale coefficient for process innovation
λ_q	0.08	scale coefficient for product innovation

productivity and quality for which the different choices faced by firms are optimal. Having in mind that firm distribution is right skewed meaning that the largest firm density is concentrate in the left bottom corner, we can analyze how firm behavior changes as productivity and quality change.

The first area on the left represents the firms with productivity lower than $a_x(q)$ which optimally exit the market. The exit cut off function is the border between the exit region and the region where firms remain active but do only production. As mentioned before, due to the trade off between quality and productivity this cut off function is decreasing in quality. In the second region firms are sufficiently profitable to be active but not enough to be innovators, $v(a, q) = v^P(a, q)$. Thanks to the quality feature also low productive firms can survive in the market when the quality of the variety they produce is high. The optimal

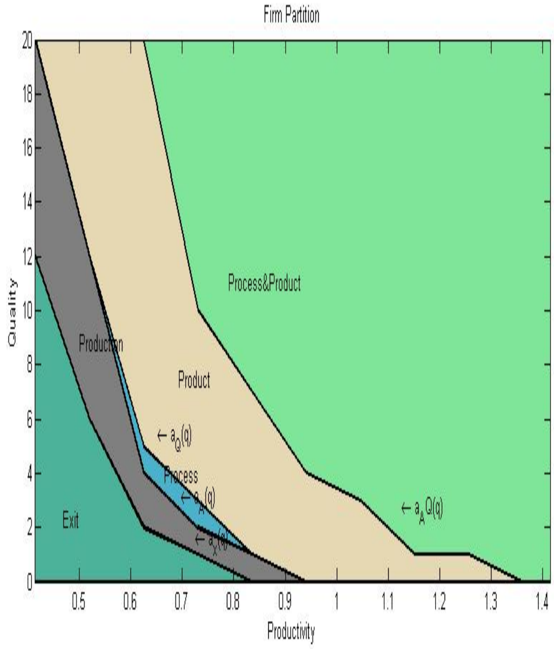

Figure 1: Firm Partition

price charged by these firms will be relatively high affecting negatively the market share. This negative effect is completely offset by the positive shift in the demand due to the high quality offered. As quality decreases the positive shift in the demand side becomes less strong implying that productivity has to rise to remain profitable. Prices charged by these firms with low quality and high productivity will be relatively low allowing the firms to survive in the industry. The blue small region that represents roughly 10% of the firms gives the area in which firms decide optimally to pay the fixed innovation cost and to perform process innovation. In this area the marginal return of process innovation are higher than the return of product innovation. The innovation incentive is in fact direct to lower the price of a high quality good leading to a higher market share. The third region represents the firms that optimally perform product innovation, i.e. about 20% of all firms. These are firms with relatively high productivity such that the marginal return of process innovation is lower than the one of product innovation or in contrast they are already characterized by products of high quality. The bigger area is represented by about 20% of firms that optimally perform both types of innovation.

Figure 2 plots the price distribution highlighting the relation previously mentioned. High productive firms can charge low prices and prices grow with quality of the variety produced. Firms move in the price distribution due to the idiosyncratic shocks and to innovation. Process and product innovation affect prices in the opposite direction imply-

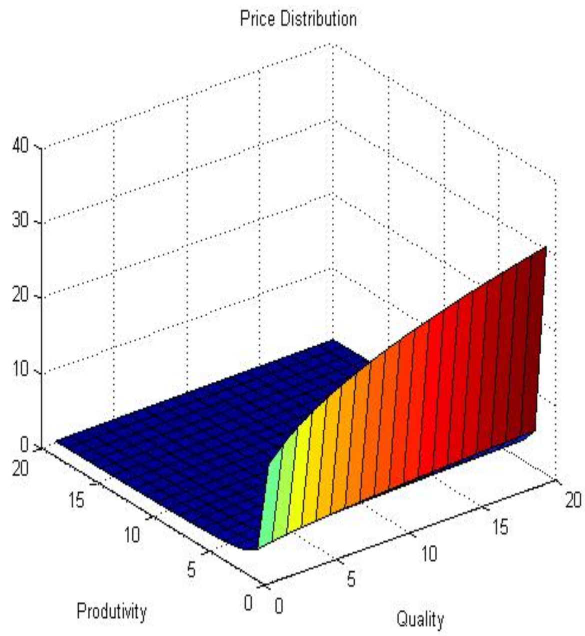

Figure 2: Price distribution

ing that the price may stay the same although firm improve its position in the technology distribution.

Incumbent firms and entrants binomial distribution is plotted in Figure 3 while the marginal distributions over productivity and quality are plotted in figure 4. Consistently with well established empirical evidence, firm distribution is right skewed. Notice that due to the imitation process characterizing entry, incumbents distribution is centered on higher levels of productivity and quality with respect to the entrant distribution. Additionally, incumbents distribution is also less spread than entrant distribution as a consequence of a lower calibrated variance for the former distribution. The marginal distributions of firms both over productivity and quality show once again the long right tail of the firm distribution. In accordance with empirical studies the distribution over quality is more skewed than the one over productivities.

The last table presents the results of an exercise that allows to gain some insights about the innovation strategies and how innovation affects firm dynamics and growth. It compares the growth rates, distributions means and economic aggregates obtained by simulating the model assuming first that no innovation occurs (benchmark) and then introducing innovation. In the benchmark case the innovation channel is shut down and the only engine of growth is selection & imitation. The role played by innovation is evident: the growth rate in the full model is almost double with respect to the benchmark case. We

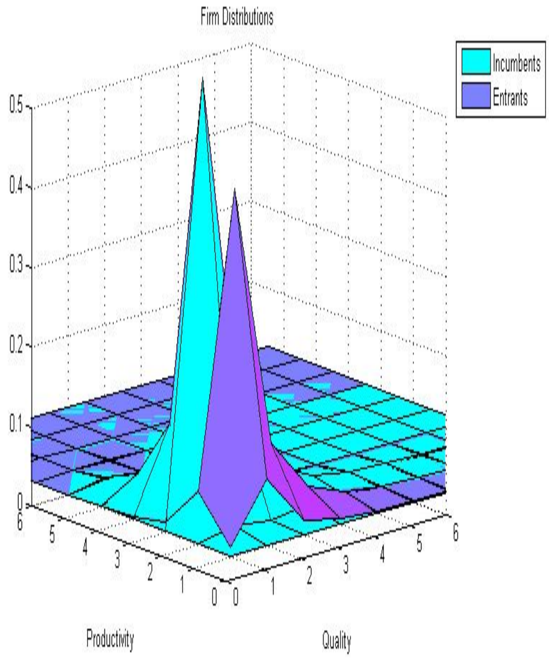

Figure 3: Firm distribution

can then conclude that innovation contributes for about 65% to the observed growth rate, while the mechanism of selection and imitation of exiting and entering firms accounts for 13% and the pure reallocation of resources between unsuccessful incumbents to more dynamic ones for a residual 22%. Additionally, the industry characterized by innovation activities has a higher average productivity, a higher average quality of the products produced and in general a higher joint mean of the technology distribution. Also the turnover is higher, implying a lower number of incumbents and a higher number of entering and exiting firms. Thus, innovation increases competition, indicated by a lower price index, and eventually increases selection and reallocation of resources too. This has an implication on welfare per worker, $W = 1/P$ which rises in presence of innovation. Finally, the innovative industry has a higher aggregate expenditure and a higher aggregate output although the number of firms is lower indicating a higher concentration of the market.

5 Conclusions

This paper proposes an endogenous growth model with heterogeneous firms where firms differ in two dimensions: productivity and quality of the variety produced. Both dimensions are subject to idiosyncratic shocks but firms can affect endogenously their evolution through process, product or both types of innovations. Growth arises due to

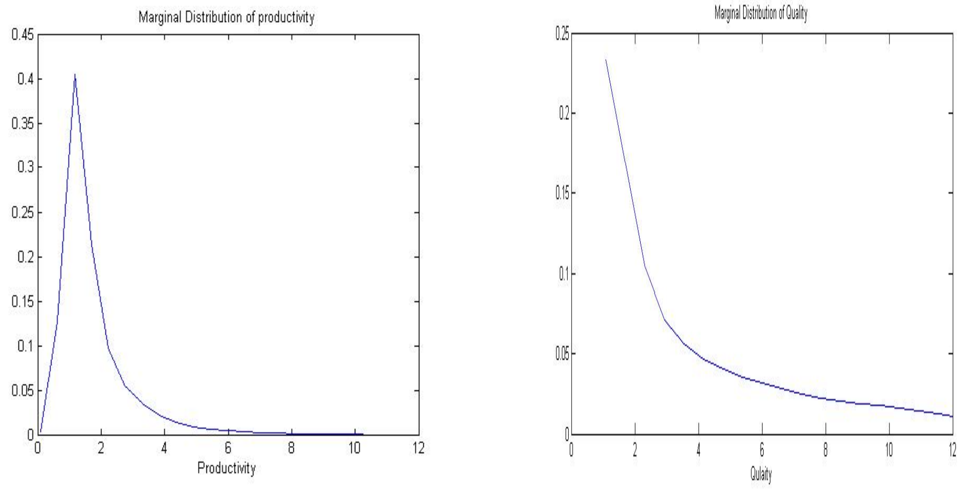

Figure 4: Marginal distributions over productivity and quality

selection & imitation and innovation. Selection eliminates the bad firms from the market, thereby increasing the average technology of incumbents. Entrants imitating the best firms are on average more productive than exiting firms. The result is that the firm distribution shifts upwards generating growth. This mechanism is amplified by innovation not only because it increases directly the average technology of firms but also because it increases selection.

The model provides a more complete description of firm's innovation choices and leads to interesting implications. Firm's size is not the sole determinant of innovation and the choice of innovation changes depending on both productivity and quality. A small firm can also choose optimally to innovate when its product is of high quality. In this case a firm finds optimally to innovate in both dimensions: process and product. On the other hand, a large firm with a lower quality good prefers to specialize in product innovation. The interaction between productivity and quality results in an innovation behavior that is non monotonically related to firm size. In addition, the effect of innovation on firms' prices is richer than in the one factor heterogenous firms model. Process innovation increases firm efficiency, thus the price charged by the firm is reduced. Contrastingly, product innovation increases the marginal cost of production and shifts the demand for the variety outwards, allowing firms to charge higher prices.

Calibrating the model to match the German manufacturing sector allows to closely match newly empirical moments related to the innovation dimension of firms and more standard moments associated to the distribution of firms. The contribution of firm innovation on economic growth is evident: 65% of the growth rate is explained by the innovation activities of firms.

This nuanced and empirically consistent characterization of firms' innovation behavior has opened a interesting room for policy implication aimed at fostering both economic growth and welfare. My further work will focus on a combined measure to incentivate both process and product innovation to implement the first best. Moreover, introducing international trade will allow to study the interaction among different innovation strategies and export behavior and their effect of economics growth.

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Table 3: Model Statistics and Empirical Targets

Targets	**Data**	**Model**
Share process innovation	10.2%	12.97%
Share product innovation	21%	23.37%
Share both process and product	27.4%	22.40%
Product $R\&D$/total $R\&D$	0.612	0.72
Process $R\&D$/total sales	0.059	0.055
4 year survival rate	0.53	0.90
7 year survival rate	0.45	0.90
Firm turnover rate	0.12	0.09
Firm below average productivity	0.83	0.81
Job turnover due to entry and exit	0.03	0.02
Relative size after 7	1.4	1.56
Productivity spread	[3, 4]	2.8
Entrant size/incumbent size	0.43	0.43
Growth Rate decomposition		
Economy average growth rate	$g = 0.037$	
Process innovation average growth rate	$g_A = 0.029$	
Product average growth rate	$g_Q = 0.010$	
Share of growth due to entry and exit	0.13	

Table 4: An Exercise

	Benchmark	**Innovation**
Growth rate	0.024	0.037
Average productivity	2.16	2.34
Average quality	1.55	1.97
Joint mean	4.12	5.16
Turnover rate	0.018	0.098
Price index	0.0058	0.0050
Aggregate expenditure	2. 17e+7	2.20e+7
Aggregate output	3.7e+9	4.4e+9