Lowest Unique Bid Auctions over the Internet: Ability, Lottery or Scam?

Andrea Gallice [a,b,*]

[a]BRICK, Collegio Carlo Alberto. Via Real Collegio 30, 10024 Moncalieri, IT

[b]University of Siena (DEPFID). P.za San Francesco 7, 53100 Siena, IT

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Abstract

A lowest unique bid auction allocates a good to the agent who submits the lowest bid that is not matched by any other bid. This peculiar auction format is getting increasingly popular over the internet. We show that when bidders are rational such a selling mechanism can lead to positive profits only if there is a large mismatch between the auctioneer's and bidders' valuation. On the contrary, the mechanism becomes highly lucrative if bidders are myopic. In this second case, we analyze the key role played by the existence of some private signals that the seller sends to the bidders. Data about actual auctions confirm the profitability of the mechanism and the limited rationality of the bidders.

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* *E-mail:* andrea.gallice@brick.carloalberto.org. *Tel.:* +39 0116705287. *Fax:* +39 0116705088.

1 Introduction

A new wave of websites is intriguing consumers over the internet. These websites sell goods of considerable value (electronic equipment, watches, holidays and even cars and houses) through quite a peculiar auction mechanism: the winner is the bidder who submits the lowest *unique* offer, i.e., the lowest offer that is not matched by any other bid. Such a mechanism is commonly called lowest unique bid auction (LUBA) and leads to impressively low selling prices: one of these websites reports that an iPod (value 200 Euros) has been sold for 0.25 Euros, a Sony Playstation 3 (400 €) for 0.81 € and a new Volkswagen Beetle Cabriolet (32,000 €) for 32.83 €. These are not exceptions: as a rule of thumb, goods are usually sold for a price that is in the order of 0.1-0.2% of the market value.

Websites offering LUBAs first appeared in Scandinavian countries in early 2006. Since then, they rapidly developed in many other European countries (France, Germany, Holland, Italy, Spain, UK).[1] Word of mouth is fast and this auction format is gaining increasing media attention. Some people say LUBAs are a game of strategy, some say they are just a lottery, some suspect they are a plain scam. In this paper we contribute to this debate by studying this selling mechanism from a game theoretic point of view. But before moving to the formal analysis let us introduce the functioning of a LUBA in more detail. As a first step, agents must register to one of these websites and transfer an amount of money of their choice to a personal deposit.[2] Users can then browse through the items on sale and submit as many bids as they want on the items they are interested in. Bids are expressed in cents and are

[1] Here is a far from exhaustive list of some of these websites: bidster.com (Scandinavian Countries); youbid.fr, bidcrown.fr, mamabid.com (France); auktionclick.com, betsmart.eu (Germany); bidster.com (Holland); bidplaza.it, bidpremium.it, lowbid.it (Italy); pujamasbajo.com, subin.es, menudapuja.com (Spain); bidplaza.uk, bidlowest2win.com, bidster.com (UK).

[2] The registration is for free but users must provide quite detailed personal data (name, address, e-mail, phone number).

private information. Every time that a user places a bid a fixed amount of money (typically 2 Euros) is deducted from his deposit. The auctioneer justifies this cost as a price for a (compulsory) "packet of information" that he sends to the bidder. In fact, as soon as a bid is submitted, the user gets one of the three following messages: 1) Your bid is currently the unique lowest bid; 2) Your bid is unique but is not the lowest; 3) Your bid is not unique. During the bidding period, which usually lasts for a few days, users can at any time log in to their account in order to check the current status of their bids, add new ones or refill the deposit. Once that the auction closes, the object is sold to the bidder who submitted the lowest unique bid. For instance, if agents A and B offer 1 cent, C offers 2 cents, A and D offer 3 cents and E offers 6 cents, the object is sold to C for a price of 2 cents.

This allocation mechanism is therefore considerably different with respect to traditional auction formats.[3] In particular, it is the requirement about the uniqueness of the winning bid that represents a novelty. On one hand, this requirement undermines key objectives that lie at the core of standard auction theory, like for instance the efficiency of the final allocation. On the other hand, it adds some new strategic elements. Indeed, from a strategic point of view, a LUBA is more similar to other well known games rather than to a standard auction. For instance, given that agents want to outguess the rivals, the game has something in common with a Guessing Game (Nagel, 1995). There is an important difference though. In the Guessing Game the pattern of best responses follows a unique direction. This does not happen in a lowest unique bid auction. In fact, a player that expects all the opponents to bid 1 cent maximizes his payoff by bidding 2 cents. But if the player expects all the opponents to bid 2 cents then he should switch back and bid 1 cent. Therefore, the game is not dominance solvable. On the other hand, some other

[3] Detailed reviews about the rich auction theory literature can be found in Klemperer (1999) or Krishna (2002).

features of the game (possibility of multiple bidding, fixed cost for each bid and instantaneous knowledge of the bids' status) makes it similar to a War of Attrition (Maynard Smith, 1974). But certainly the closest relative of the lowest unique bid auction is the Dollar Auction Game (Shubik, 1971). This is a public auction in which the prize (say, one dollar) is won by the highest bidder but both he and the second highest bidder must pay their bids. When participants are not fully rational this game can lead to some paradoxical results that highly reward the auctioneer. We will see that something analogous can easily happen in the case of lowest unique bid auctions.

Apart from these classical contributions, there are also some very recent papers that explicitly study various versions of unique bid auctions. Houba *et al.* (2008) and Rapoport *et al.* (2009) analyze the pure and mixed equilibria of a LUBA in which bidders submit a unique bid, there is a non-negative bidding fee and the winner pays his bid. Both papers find that in the symmetric mixed equilibrium bidders randomize with decreasing probabilities over a support that comprises the lowest possible bid and is made of consecutive numbers.[4] Östling *et al.* (2008) obtain a similar result for what they call a LUPI (Lowest Unique Positive Integer) game in which players can again submit a single bid but there are no bidding fees and the winner does not have to pay his bid. The peculiarity of this study is that the number of participants is unknown and is assumed to follow a Poisson distribution. Finally, Eichberger and Vinogradov (2008) analyze a LUBA (that they call LUPA, i.e., Least Unmatched Price Auction) where bidders can submit multiple costly bids and the winner must pay his winning bid. Given that no information about other bidders' behavior is released during the auction, they model the game as a simultaneous

[4] Rapoport *et al.* (2009) also analyze HUBAs, i.e., unique bid auctions in which the winner is the bidder who submits the highest unmatched offer. Such a mechanism is also studied by Raviv and Virag (2007).

game. For some special ranges of the parameters, they show the existence of a unique Nash equilibrium in which agents mix over bidding strings that comprise the minimum allowed bid and are made of consecutive numbers. In addition to the theoretical analysis the papers by Houba *et al.* (2008) and by Rapoport *et al.* (2009) propose some algorithms for computing the symmetric mixed strategy equilibrium. The papers by Östling *et al.* (2008) and Eichberger and Vinogradov (2008) have instead an empirical part which is based on field and/or experimental data. Theoretical predictions find some empirical evidence at the aggregate level but a much lower one at the individual level.

With respect to this ongoing literature our paper differs in a number of ways. The main novelty is the analysis of the role played by the signals that the bidders receive about the status of their submitted bids. We study how these signals influence the bidding strategies and we show them to be a key element of the mechanism, especially for what concerns out of equilibrium play. Second, we explicitly model bidders' decision about how much to invest in the auction (i.e., how many bids to submit). We frame the problem as a rent-seeking game and we study how the optimal level of investment is influenced by the parameters of the game. Finally, by modelling LUBAs as a sequential game that captures the actions of both the bidders and the auctioneer, we focus on the study of the profitability of the mechanism. We show that if bidders are rational then the expected profits of the auctioneer can be positive only if his valuation of the good is (much) lower than the retail price. This would imply that websites offering LUBAs should not proliferate the way they do. We then adopt a more behavioral approach and show how a LUBA can become highly profitable when bidders lack the necessary commitment to stick to equilibrium strategies. The profitability of this selling mechanism and the limited rationality of the bidders find an empirical confirmation in the analysis of a dataset that collects information about actual LUBAs.

The remaining of the paper is organized as follows: Section 2 formalizes the strategic situation and characterizes its equilibria under the assumption of perfect rationality of the players. Section 3 investigates what happens when some of the bidders are boundedly rational. Section 4 examines a dataset which collects detailed information about 100 LUBAs and then performs an out-of-sample exercise using 10 additional auctions. Section 5 concludes.

2 The game and its equilibria

We introduce and analyze a sequential game that captures the key features of a lowest unique bid auction. The game spans over $T+2$ periods with $t \in \{-1, 0, 1, ..., T\}$ and has $(N+1)$ risk-neutral players: an auctioneer (a) and $N \geq 2$ potential buyers. At period $t = -1$ the auctioneer, whose outside option is $u_a = 0$, can decide to auction a certain good. We indicate with V_a the value of the good for the auctioneer and with V the homogeneous valuation of any potential buyer $i \in N$. We assume that $V_a \leq V$.[5] If a opens the auction he credibly commits to sell the good to the buyer who offers the lowest positive bid that is not matched by any other bid. The N buyers must then solve two distinct and subsequent problems. In the first one, which takes place at $t = 0$ and which we label the "investment decision", each potential buyer must decide how much he is willing to invest in bids. In fact, bidders must pay to the seller a fixed amount $c \in [1, V-1]$ for every bid they submit. In the second problem, which we label the "bidding phase", each bidder must decide where and when to place his bids. The bidding phase starts at $t = 1$ (the opening of the auction) and ends at $t = T$ (the closing of the auction) where T is common

[5] V can be interpreted as the retail price of the good. The assumption $V_a \leq V$ captures the fact that the auctioneer may pay the good less than its retail price because of quantity discount or marketing reasons.

knowledge. At any period $t \in \{1, ..., T\}$ each player i plays $x_i^t \in \{\phi\} \cup \{1, ..., \infty\}$.[6] Action $x_i^t = \phi$ indicates that agent i does not bid at period t. Action $x_i^t \neq \phi$ indicates that agent i submits at time t the bid $x_i^t \in \{1, ..., \infty\}$. As soon as a bid $x_i^t \neq \phi$ has been placed, player i is charged c and receives from the auctioneer a truthful and private signal $\sigma^t\left(x_i^t\right) \in \{W, M, L\}$. It is common knowledge that the signals mean the following:

- $\sigma^t\left(x_i^t\right) = W$ indicates that x_i^t is currently the Winning bid (i.e., at time t x_i^t is the lowest unique bid).
- $\sigma^t\left(x_i^t\right) = M$ indicates that x_i^t Might be the winning bid (i.e., at time t x_i^t is unique but it is not the lowest).
- $\sigma^t\left(x_i^t\right) = L$ indicates that x_i^t is a Losing bid (i.e., x_i^t is not unique).[7]

The status of some bids can thus change over the course of the auction. In particular, the signal $\sigma^t\left(x_i^t\right) = W$ can be updated by $\sigma^s\left(x_i^t\right) = M$ (a bidder j places at time $s \in \{t+1, ..., T\}$ the bid $x_j^s < x_i^t$ and $\sigma^s\left(x_i^s\right) = W$) or $\sigma^s\left(x_i^t\right) = L$ (a bidder j bids $x_j^s = x_i^t$). For similar reasons the signal $\sigma^t\left(x_i^t\right) = M$ can be updated by $\sigma^s\left(x_i^t\right) = W$ or $\sigma^s\left(x_i^t\right) = L$. At the opposite, the signal $\sigma^t\left(x_i^t\right) = L$ cannot be updated as the status of a bid that is not unique cannot change any more. Each bidder can check the current status of his bids at any time and at no cost.

In order to formalize players' payoffs we let η_i^t be the number of bids submitted by agent i up to period t such that η_i^T is the number of bids submitted by i over the course of the entire auction $\left(\text{i.e., the cardinality of the set } \left\{x_i^t \mid x_i^t \neq \phi\right\}_{t=1}^T\right)$. Moreover we use the notation $\hat{x}_i^t$ to indicate the winning bid. Stressing that all the

[6] If at the end of the auction a unique offer does not exist, then the good is sold to the bidder that submitted first the lowest offer. This same tie breaking rule is the one used in reality.

[7] Notice that a bidder that receives the signal $\sigma^t\left(x_i^t\right) = L$ does not know if x_i^t is higher or lower than the current winning bid.

monetary values $\left(V_a, V, c \text{ and } \left\{x_i^t\right\}_{t=1}^T \text{ for all } i\right)$ are expressed in the same unit (say Euro cents), payoffs take the following form:

$$u_a = \begin{cases} \sum_{i \in N} \eta_i^T c + \hat{x}_i^t - V_a & \text{if } a \text{ opens the auction} \\ 0 & \text{otherwise} \end{cases}$$

$$u_i = \begin{cases} V - \eta_i^T c - \hat{x}_i^t & \text{if } \exists \hat{x}_i^t \in \left\{x_i^t\right\}_{t=1}^T \text{ s.t. } \sigma^T\left(\hat{x}_i^t\right) = W \\ -\eta_i^T c & \text{otherwise} \end{cases} \quad \text{for } i \in N$$

Notice that the payoffs of the bidders comprise their outside option of not participating to the auction as $u_i = 0$ when $\eta_i^T = 0$. We solve the game by backwards induction. Therefore, we first focus on the subgame that takes place among the bidders (this subgame comprises the investment decision and the bidding phase) and then we will move to the decision of the auctioneer if to open or not the auction.

2.1 The investment decision

Bidders accumulate sunk costs at rate $c > 0$ for every bid they submit. A rational bidder i should then first commit to a certain amount of money $\omega_i \in [0, V]$ that he is willing to invest in the auction. The bidder must in fact trade off the probability of winning and make positive profits with the risk of not winning and lose his investment. With the help of two simplifying assumptions, we model such a choice as a symmetric rent-seeking game. The first assumption is that each bidder believes that the probability of winning the auction is increasing in ω_i and decreasing in ω_j for any $j \neq i$. Notice that this is not necessarily true in a lowest unique bid auction where these relations hold only in their weak formulation. Second, at this stage of the game, agents do not consider that they will also have to pay their winning bid $\hat{x}_i^t$ in case they win. We already mentioned in the introduction that $\hat{x}_i^t$ is usually extremely small with respect to V and thus unlikely to really affect the investment decision. Starting with the work of Tullock (1980), various forms of rent-seeking games have been carefully analyzed in the economic literature (see among others Baye et al., 1994 and Kooreman and Schoonbeek, 1997). The following proposition states and proves the well-known solution for the symmetric version of the game where each agent's probability of winning increases proportionally to his investment (i.e., ω_i is raised to the power of 1 for every i).

Proposition 1 *The investment decision of the symmetric lowest unique bid auction with N bidders has solution $\omega = \frac{N-1}{N^2}V$.*

Proof. Each agent i solves $\max_{\omega_i} E(u_i) = \left(\frac{\omega_i}{\omega_i+\sum_{j\neq i}\omega_j}\right)V - \omega_i$. First order condition is given by $\left(\frac{\sum_{j\neq i}\omega_j}{\left(\omega_i+\sum_{j\neq i}\omega_j\right)^2}\right)V - 1 = 0$ and expected utility is concave in ω_i given that $\frac{\partial^2 E(u_i)}{\partial \omega_i^2} = \left(-\frac{\left(\sum_{j\neq i}\omega_j\right)\left(2\left(\omega_i+\sum_{j\neq i}\omega_j\right)\right)}{\left(\omega_i+\sum_{j\neq i}\omega_j\right)^4}\right)V < 0$. Imposing symmetry ($\omega_i = \omega_j = \omega$) the FOC can be expressed as $\left(\frac{(N-1)\omega}{(N\omega)^2}\right)V - 1 = 0$ which is satisfied by $\omega = \frac{N-1}{N^2}V$.
■

In line with what intuition suggests, ω is increasing in V and decreasing in N. Moreover ω uniquely determines the number of bids that each agent is willing to submit. In fact, introducing the "floor" operator $\lfloor\cdot\rfloor$ such that $\lfloor z\rfloor$ maps the real number z into the integer n with $n \leq z < n+1$, we can state the following lemma.

Lemma 1 *Each agent submits up to $\eta^{\max}$ bids with $\eta^{\max} = \left\lfloor\frac{\omega}{c}\right\rfloor = \left\lfloor\frac{N-1}{N^2 c}V\right\rfloor$.*

Proof. Each bid costs c, such that, given ω, $\frac{\omega}{c}$ is the number of bids an agent would submit if bids were perfectly divisible. But the number of bids must be an integer such that $\eta^{\max} = \left\lfloor\frac{\omega}{c}\right\rfloor$. This is the maximum number of bids an agent is willing to submit: bidders' payoff is decreasing in η_i^T such that, conditional on winning the auction, an agent is strictly better off if $\eta_i^T < \eta^{\max}$. ■

The integer $\eta^{\max}$ is a weakly increasing function of ω such that $\eta^{\max}$ is weakly increasing in V and weakly decreasing in N. Moreover $\eta^{\max}$ is weakly decreasing in c.

2.2 The bidding phase

According to Lemma 1 each bidder has up to $\eta^{\max} = \left\lfloor \frac{N-1}{N^2 c} V \right\rfloor$ bids to submit in the auction. It follows that $\eta^{\max} = 0$ if $V < \frac{N^2 c}{N-1}$. If this is the case then no bidder submits any bid and $u_i = u_a = 0$.[8] Things are more interesting when $\eta^{\max} \geq 1$ as each bidder must then choose when and where to place his bids. We first solve for the timing dimension of the game and we assume $T >> \eta^{\max}$ such that agents have enough periods to use all their available bids if so they wish. This assumption is not particularly restrictive given that in real LUBAs the bidding period usually lasts for a few days while the time needed to submit a bid amounts to a few seconds.

Lemma 2 *For any $t \geq 1$ and any possible $x_i^t \neq \phi$, x_i^t weakly dominates x_i^{t+k} with $k \in \{1, ..., T-t\}$.*

Proof. This comes directly from the tie breaking rule that states that if a lowest unique bid does not exist the winner is the one who first submitted the lowest bid. Therefore bidders have an incentive to submit their bids as soon as possible. ■

To study the bidding behavior of the participants we differentiate between two cases: $\eta^{\max} = 1$ and $\eta^{\max} > 1$. The second situation is obviously more complex as bidders must condition their behavior on their former bids and on the associated signals. Still, the two cases share some common features. First, in both

[8] Despite a totally different modelling strategy this result is in line with Houba *et al.* (2008) and Rapoport *et al.* (2009) that also show that entry in the auction does not occur if N or c are too high or V is too low.

situations an equilibrium surely exists. In fact, the number of players is finite and so is their strategy space once that strictly dominated actions are eliminated, i.e., $x_i^t \in \{1, ..., V - c\}$ rather than $x_i^t \in \{1, ..., \infty\}$. Indeed, it is easy to notice that equilibria actually abound. In particular there exist a large number of asymmetric equilibria in pure strategies.[9] Still, given the symmetry and the anonymity of the bidders, we restrict our attention to symmetric equilibria. Symmetric equilibria in pure strategies cannot exist: bidders want to outguess the rivals such that for any $N > 2$ a profitable deviation surely exists from any symmetric pure strategy profile. Therefore, in line with the current literature on LUBAs, we conjecture that the symmetric equilibrium is in mixed strategies. In such a mixed strategy each player chooses where to place his initial bid following a common discrete probability distribution $p = (p(1), ..., p(V - c))$ with $p(x) \geq 0$ for any $x \in \{1, ..., V - c\}$ and $\sum_x p(x) = 1$. In the case of $\eta^{\max} > 1$, bidders submit their additional bids updating the initial distribution according to their previous bids and the signals they receive from the auctioneer.

2.2.1 The case with $\eta^{\max} = 1$

The case with $\eta^{\max} = 1$ is analogous to a LUBA in which the rules of the auction specify that each player can submit a single bid. This situation has been carefully investigated by Houba *et al.* (2008) and Rapoport *et al.* (2009). In line with their findings, the following proposition describes some features of the equilibrium distribution.

Proposition 2 *In the symmetric equilibrium of the LUBA with* $\eta^{\max} = 1$*, each bidder chooses where to place* x_i^1 *according to the distribution* p *such that:*

[9] For example, if $N = 3$ and $\eta^{\max} = 1$ the profiles $\{x_i^1 = 1, x_j^1 = 1, x_k^1 = 2\}$ and $\{x_i^1 = 1, x_j^1 = 2, x_k^1 = 3\}$ are Nash equilibria as there are no (strictly) profitable deviations.

- *p has support $S(p) = \{1, ..., K\}$ with $K \leq V - c$.*

- *$p(x)$ is strictly decreasing in x.*

Proof. Assume that there exists an equilibrium in which $p(k) = 0$ for some $k \in \{1, ..., K\}$ but $p(\lambda) > 0$ for $\lambda > k$. Then pure strategy $x_i = \lambda$ would be strictly dominated by strategy $x_i = k$. As a consequence, λ should not be played in the mixed equilibrium, a negation of the initial assumption. Therefore, the support of the distribution comprises 1 and has no gaps. Moreover $K \leq V - c$ because any strategy $x_i > V - c$ is strictly dominated as it would lead to a negative payoff even in case of winning. As for the second point imagine that the support of $p(x)$ is $\{1, 2\}$ and that in equilibrium all the $j \neq i$ bidders mix uniformly over 1 and 2. Then $E\left(u_i|x_i^1 = 1\right) > E\left(u_i|x_i^1 = 2\right)$. In fact, if $p(1) = p(2)$, then both 1 and 2 are equally likely to result into a unique bid such that it is better to bid on the lowest of the two. Moreover, in case of winning, lower is the price that the bidder should pay. But $E\left(u_i|x_i^1 = 1\right) > E\left(u_i|x_i^1 = 2\right)$ contradicts the fact that both 1 and 2 are in the support of $p(x)$. For this to be the case $E\left(u_i|x_i = 1\right) = E\left(u_i|x_i = 2\right)$ must hold which requires $p(1) > p(2)$ in order to balance the advantages of bidding on 1. A similar argument can be applied to any two consecutive numbers in the support such that $p(x)$ is strictly decreasing in $x \in \{1, ..., K\}$. ■

In the symmetric equilibrium all the bidders mix according to p. It follows that every player is equally likely to win. Signals do not matter in this context: each bidder receives the signal $\sigma^1(x_i^1) = \{W, M, L\}$ but he does not have any additional bid to submit. Therefore $x_i^t = \phi$ for any i and any $t \in \{2, ..., T\}$ and $\sigma^T(x_i^1) = \sigma^1(x_i^1)$ for any i.

2.2.2 The case with $\eta^{\max} > 1$

If $\eta^{\max} > 1$ bidders have the option to submit multiple bids. Because of Lemma 2, every bidder submits his first bid at $t = 1$. In choosing where to place x_i^1, we assume that bidders use the same probability distribution that characterizes the equilibrium when $\eta^{\max} = 1$. We now indicate this distribution with p^1 where the superscript indicates that this is the distribution from which agents draw their first bid. Bidders then receive the signal $\sigma^1(x_i^1)$ and can decide if to submit additional bids. Who will do so? The following two lemmas answer this question.

Lemma 3 *At any $t \geq 1$, there exist at least $N-1$ bidders for which $\nexists x_i^r \in \{x_i^r\}_{r=1}^t$ such that $\sigma^t(x_i^r) = W$.*

Proof. In any period, and for any possible distribution of actual bids, either a lowest unique bid exists or it does not. It follows that at any $t \geq 1$ at least $N-1$ bidders do not hold the current winning bid. ■

Lemma 4 *At any $t \geq 1$, $x_i^t \neq \phi$ if and only if $\nexists x_i^r \in \{x_i^r\}_{r=1}^{t-1}$ for which $\sigma^{t-1}(x_i^r) = W$ and $\eta_i^{t-1} < \eta^{\max}$. Otherwise $x_i^t = \phi$.*

Proof. The fact that every player who does not hold the current winning bid will keep on submitting bids until he reaches $\eta^{\max}$ directly derives from the investment decision analyzed in Proposition 1 and from Lemma 2. ■

In other words all the players who do not hold the current winning bid submit additional bids until they reach $\eta^{\max}$. After that they stop bidding no matter the status of their submitted bids. Notice that subsequent bids are clearly not independent. Not only a rational player will not submit the same bid more than once but he will also condition his bidding strategy on the signals associated to his standing bids. The following proposition describes how a rational player updates the probability distribution from which he draws x_i^t. Notice the subscript i attached to this distribution. In fact, while $p_i^1 = p^1$, subsequent distributions may differ across bidders given that these depend on bidders' specific bidding histories and on the associated signals.

Proposition 3 *For any $t > 1$ a bidder i for which $x_i^t \neq \phi$ chooses x_i^t according to p_i^t where p_i^t is such that:*

- $S(p_i^t) = \left\{ \left\{ 1, ..., \min \left\{ \left\{ x_i^r - 1 | \sigma^{t-1} \left(x_i^r \right) = M \right\}_{r=1}^{t-1} \cup \{K\} \right\} \right\} \setminus \left\{ x_i^r | \sigma^{t-1} \left(x_i^r \right) = L \right\}_{r=1}^{t-1} \right\}$

- $p_i^t(x)$ is strictly decreasing in x for $x \in S(p_i^t)$

- p_i^t is derived from p_i^{t-1} according to Bayes's rule.

Proof. The fact that a player must exclude from the support the bids that he already submitted is obvious. Similarly, the upper bound of the support must be updated with the predecessor of the smallest bid whose associated signal is $\sigma^{t-1} \left(x_i^r \right) = M$. In fact such a signal implies that the current winning bid lies somewhere between 1 and $x_i^r - 1$. The bidder must bid in this interval in order to either match the current winning bid and hope to transform x_i^r in the winning bid or to find a new lowest and unique bid. The proof that $p_i^t(x)$ is strictly decreasing over $S(p_i^t)$ mimics the proof of Proposition 2. The fact that players update the probability distribution according to Bayes's rule directly derives from the assumption of rational behavior.

■

2.3 The auctioneer's decision

Having analyzed buyers' bidding behavior, we can move backwards and study the auctioneer's decision if to open the auction or not. This decision obviously depends on auctioneer's expected profits. We show that, in front of rational bidders, these expected profits are bounded below. It follows that the auctioneer certainly opens the auction whenever this lower bound is positive.

Proposition 4 *For any* $V_a < ((N-1)\eta^{\max} + 1)\,c + 1$ *auctioneer's profits are surely positive.*

Proof. Auctioneer's profits are given by $u_a = \sum_{i \in N} \eta_i c + \hat{x}_i^t - V_a$. Because of Lemma 4, in equilibrium the $N-1$ losing bidders submit $\eta^{\max}$ bids while the winning bidder submits at least one bid. Moreover the lowest possible winning bid is 1 cent. It follows that u_a is bounded below by $u_a^{\min} = ((N-1)\eta^{\max} + 1)\,c + 1 - V_a$ which is strictly positive for any $V_a < ((N-1)\eta^{\max} + 1)\,c + 1$. ■

An example can clarify the situation.

Example 1 *Assume a lowest unique bid auction for an item for which* $V = 10,000$*,* $c = 200$ *and* $N = 10$*. These parameters imply* $\eta^{\max} = \left\lfloor \frac{N-1}{N^2 c} V \right\rfloor = 4$*. Auctioneer's profits are bounded below by* $u_a^{\min} = ((36+1) * 200) + 1 - V_a$ *such that* $u_a^{\min} > 0$ *for any* $V_a < 7,401$*. It follows that the auctioneer certainly opens the auctions if he can get the good at less than* 74% *of its retail price.*

It is interesting to compare auctioneer's profits with the profits that would arise in a situation in which signals were not available. If agents do not get any feedback about the status of their bids then, in line with Proposition 1, all the N bidders would submit their $\eta^{\max}$ available bids.[10] Payoff for the seller would then be bounded below by $u_a^{\min} = N\eta^{\max} c + 1 - V_a$. It follows that expected profits with signals

[10] Bidders would choose x_i^t according to p_i^t where p_i^t has support $S(p_i^t) = \left\{\{1, ..., K\} \setminus \{x_i^r\}_{r=1}^{t-1}\right\}$ and is derived from $p^1(x)$ according to Bayes's rule.

cannot be larger than expected profits without signals. This consideration leads to question why the auctioneer sends the signals. Two are the possible answers: either the auctioneer adopts a sub-optimal behavior or the bidders do not behave as equilibrium analysis indicates. Given that the first option seems unlikely, we now turn to analyze the second possibility.

3 The game with (some) boundedly rational bidders

The previous section showed that a lowest unique bid auction can be profitable for the seller even when bidders are rational and play according to the equilibrium strategies. Still a necessary condition for ensuring positive profits is the existence of a (possibly large) mismatch between the retail price of the good (V) and the auctioneer's valuation (V_a). This finding, while interesting, hardly rationalizes what we observe in reality, namely the continuous opening of websites that offer LUBAs. On the contrary, this trend suggests that the business is much more profitable than what equilibrium analysis indicates. In this section we relax the assumption of perfect rationality of the players and we show how LUBAs can become highly profitable when at least some bidders lack the necessary commitment to stick to equilibrium strategies.[11] In particular we show how a kind of war-of-attrition mechanism is triggered and amplified by the existence of the signals.

Assume that $\eta^{\max} \geq 1$ and that the auction proceeds according to the equilibrium analysis. Lemmas 3 and 4 state that the auction reaches the period $t = t^*$ in which at least $N - 1 \geq 1$ i bidders have used all their available bids ($\eta_i^{t^*} = \eta^{\max}$) and none of them holds the winning bid. Therefore $u_i^{t^*} = -\eta^{\max} c$. A rational bidder who committed to $\eta_i^T \leq \eta^{\max}$ (see Proposition 1) accepts this loss. In other words, he

[11] A similar approach is adopted by Malmendier and Szeidl (2008) that show how the presence of a minority of overbidding behavioral agents disproportionally inflates profits in the case of standard auctions.

plays $x_i^t = \phi$ for any $t \in \{t^* + 1, ..., T\}$ such that $u_i^T = -\eta^{\max} c$. On the contrary, a boundedly rational bidder may be tempted to keep on bidding hoping to win the auction and turn the sunk costs into a positive payoff. We start by better defining what we mean by boundedly rational behavior in the context of a LUBA.

Definition 1 *A bidder i for which, at time t, $\eta_i^t \geq \eta^{\max}$ and $\sigma^t(x_i^r) \neq W$ for any $x_i^r \in \{x_i^r\}_{r=1}^t$ is boundedly rational if:*
- he is myopic in the sense that he only cares about u_i^{t+1}.
- he holds a probability weighting function $w_i^t(q_i^t) > q_i^t \geq 0$ where $q_i^t = q_i^t(x_i^{t+1})$ is the probability that an additional bid $x_i^{t+1} \neq \phi$ will lead to the signal $\sigma^{t+1}(\hat{x}_i^r) = W$ for a $\hat{x}_i^r \in \{x_i^r\}_{r=1}^{t+1}$.
- he lacks the commitment to stop at $\eta_i^T = \eta^{\max}$.

Out of the many possible distributions of actual bids at any time t there are certainly cases in which an additional bid of bidder i can result as the lowest unique bid. For example, if all bidders bid 1 at $t = 1$ then $x_i^2 = 2$ will receive $\sigma^2(x_i^2) = W$. Therefore the event of winning the auction with an additional bid has an objective probability $q_i^t \geq 0$. For realistic values of N and $\eta^{\max}$, this probability, when positive, is certainly small. In line with prospect theory (Kanheman and Tversky, 1979) and the empirical evidence about probability weighting functions (Prelec, 1998), a boundedly rational player overestimates this probability, i.e., he holds the beliefs $w_i^t(q_i^t) > q_i^t$. Many are the well-known behavioral biases that can shape such a subjective probability assessment: loss-aversion, over-confidence, over-optimism, wishful thinking. The bidder will submit an additional bid if his beliefs are biased enough to justify this choice and if he cannot commit to $\eta_i^T = \eta^{\max}$.

Proposition 5 *A boundedly rational bidder for which* $\sigma^t(x_i^r) \neq W$ *for any* $x_i^r \in \{x_i^r\}_{r=1}^t$ *and* $\eta_i^t \geq \eta^{\max}$ *plays* $x_i^{t+1} \neq \phi$ *if* $w_i^t(q_i^t) > \frac{c}{V-\hat{x}_i^r} \simeq \frac{c}{V}$. *Moreover if this condition holds at time* t *then it also holds at time* $t+k$ *such that* $x_i^{t+k} \neq \phi$ *for any* $k \in \{1,...,T-t\}$ *whenever* $\sigma^{t+k-1}(x_i^r) \neq W$ *for all* $x_i^r \in \{x_i^r\}_{r=1}^{t+k-1}$.

Proof. By Definition 1 a boundedly rational bidder is not committed to $\eta_i^t < \eta^{\max}$ and therefore he submits an additional bid if $w_i^t(q_i^t)(V - (\eta_i^t+1)c - \hat{x}_i^r) + (1 - w_i^t(q_i^t))(-(\eta_i^t+1)c) > -\eta_i^t c$. Solving for $w_i^t(q_i^t)$ we get $w_i^t(q_i^t) > \frac{c}{V-\hat{x}_i^r} \simeq \frac{c}{V}$ given that $\hat{x}_i^r$ is negligible. The lower bound for the subjective probability assessment is not a function of η_i^t such that it remains constant at every period. This means that if the constraint is satisfied at time t it will be satisfied at any period $t+k$ with $k \in \{1,...,T-t\}$ given that $w_i^t(q_i^t)$ is non-decreasing in η_i^t. ■

Example 2 *Consider the situation described in Example 1 with* $V = 10,000$, $c = 200$, $N = 10$ *and* $\eta^{\max} = 4$. *Assume that there are at least* $2 \leq J \leq 10$ *boundedly rational players (Definition 1). At least* $J-1$ *of them will reach at period* t_j^* *the situation* $\eta_j^{t_j^*} = 4$ *and* $\sigma^{t_j^*}(x_j^r) \neq W$ *for any* $x_j^r \in \left\{x_j^r\right\}_{r=1}^{t_j^*}$. *According to Proposition 5 each one of these* j *bidders keeps on submitting an additional bid whenever they do not hold the winning bid and* $w_j^{t_j^*}\left(q_j^{t_j^*}\right) > \frac{200}{10,000-\hat{x}_i^r} \simeq 0.02$.

Example 2 shows that the presence of two boundedly rational bidders is enough to trigger a costly vicious circle. An upper bound to this process is given either by T (the closing of the auction) or by bidders' budget constraints. Whenever these limits are not binding, this sort of war of attrition can continue even when the costs associated with the number of bids exceed the value of the auctioned good. To see this, imagine that the auction has reached the stage t^* in which bidder A leads the auction such that $u_A^{t^*} = V - \eta_A^{t^*}c - \hat{x}_A^r > 0$ with $r \in \{1,...,t^*\}$and $u_B^{t^*} = -\left(\eta_A^{t^*} - 1\right)c$.

Still one more bid of B can potentially lead to $u_B^{t^*+1} = V - (\eta_A^{t^*} + 1)c - \hat{x}_B^r < 0$ with $r \in \{1, ..., t^* + 1\}$. Agent B compares $u_B^{t^*}$ and $u_B^{t^*+1}$. Both values are negative. Nevertheless $V - (\eta_A^{t^*} + 1)c - \hat{x}_B^r > -\left(\eta_A^{t^*} - 1\right) c$ such that, for an appropriate probability weighting function $w_B^{t^*}\left(q_B^{t^*}\right) > 0$, agent B still prefers to outbid A in the hope of diminishing his own loss. Now assume that indeed by bidding $x_B^{t^*+1}$ agent B conquers the winning bid at $t^* + 1$. Bidder A would then find himself in the situation in which B was at period t^*. Therefore, the same logic applies and the mechanism perpetuates itself.

This feature of lowest unique bid auctions is very reminiscent of the Dollar Auction Game (Shubik, 1971). The Dollar Auction Game is a public ascending auction where N bidders compete for a dollar. The auction is won by the agent who submits the highest bid but both him and the second highest bidder must pay their bids. Also in this case, the auction is unprofitable for the seller if agents are rational. But if multiple entry occurs, this starts off a bidding war between the two leading bidders such that the winner may end up paying the dollar more than what it is worth. In both games, the bidding escalation is detrimental for the bidders but is obviously beneficial for the auctioneer. In fact, as Morgan and Krishna (1997) show, war of attritions yield revenues that are superior to standard auction mechanisms.

Going back to the analysis of LUBAs, notice that the assumption about the presence of some boundedly rational bidders is not sufficient by itself to trigger the bidding escalation. It is in fact the combination of boundedly rational behavior and of the existence of the signals that accomplish this task. To appreciate the fundamental role that signals play, consider how different the situation would be if agents were not receiving any kind of feedback about the status of their bids. In such a case, each player would hold the legitimate hope to win the auction with their $\eta^{\max}$ bids such that the incentives to submit additional bids would be much weaker. And when at the closure of the auction the winner is declared, it would be too late for the losers to submit additional bids. In other words, in terms of ambiguity, a LUBA without signals would resemble a traditional lottery. On the other hand, signals make the game more similar to a "scratch and win" lottery as they immediately solve the ambiguity about having no chances to win. This clearly encourages overbidding given that an agent that faces potential losses is tempted to submit additional bids in order to catch up. Quoting Malmendier and Szeidl (2008) "if agents are subject to bidding fever, sellers may instigate this bias using salient messages informing the buyer that he has been outbid". Indeed, the entire signaling mechanism seems to have been designed with the goal of stimulating emotional responses that may lead to an irrational escalation of commitment. Given that the auctioneer aims to maximize the number of received bids, this obviously comes as no surprise.

4 Empirical analysis

In this section we analyze a dataset that collects information about 100 lowest unique bid auctions that took place in the period February 6th, 2008 - April 6th, 2008. These auctions have been organized by the website bidplaza.it, the leader of the Italian market with more than 1,000,000 contacts per month.[12] The rules implemented by this auctioneer are exactly the ones explained in the introduction. In particular the cost associated with each bid is set at 2 Euros. For each auction we know the good sold, its market value, the winning bid and, most importantly, the complete list of submitted bids. Overall, our dataset collects 100,940 bids.[13]

[12] Bidplaza vs Bidster....

[13] The dataset, which we manually assembled by retrieving the appropriate information from the website bidplaza.it (section "closed auctions"), is available upon request. This is the list of goods to which our data refer. The notation k (V, n) indicates that good k whose retail price is V has been offered in n different auctions (therefore $\sum_k n = 100$): Sony Playstation 3 $(400, 10)$, Sony

On the other hand, we do not have information about the number of bidders and about how many and which bids each bidder submitted. Nevertheless, the data allow to clearly distinguish some interesting patterns as well as to discriminate between rational versus irrational bidding behavior.

First of all, the auctions attracted many participants and many bids. On average, each auction received 1,009 bids (min.: 119, max.: 2,917, st. dev.: 635). Not surprisingly, there is a positive relationship between the market value of the auctioned good and the number of received bids (Pearson's $r = 0.645$). As a consequence of the high number of bids, the auctioneer made positive profits in every single auction. A cautious estimate shows that profits per auction total on average to the 441% of the market value of the good (min.: 19%, max.: 1,082%, st. dev.: 237%).[14] In absolute terms, revenues amount to 151,410 Euros and costs to 27,490 Euros. The fact that the auctioneer made positive profits on every single auction allows us to immediately reject the hypothesis of rational behavior. In fact, if bidders are rational and $V_a = V$, auctioneer's profits can be positive only if the winning bid is very large.[15] This is not what we observe in the data where winning bids amount on average to just 0.23% of V.

As for the number of bidders, it is possible to establish a lower bound. In fact,

Playstation Portable Slim & Lite $(190, 10)$, Digital Camera IXUS 860IS $(350, 9)$, iPod Shuffle 1 GB $(80, 7)$, iPod Nano 8 GB $(200, 9)$, iPod Touch 16 GB $(400, 8)$, Bose Companion 3 multimedia speaker system $(295, 10)$, Samsung CE 1070TS microwawe oven $(240, 9)$, Nintendo Wii $(250, 10)$, Philips Digital PhotoFrame Wood 10FF2CWO $(250, 4)$, TomTom One V3 Portable GPS Navigation System $(200, 9)$, XBOX 360 Elite $(450, 5)$.

[14] Despite knowing the market value of the goods, the number of bids received and the unitary cost of 2 Euros per bid, profits cannot be computed with certainty. In fact, this website offers a welcome bonus such that a user's first deposit of money is doubled. Therefore, some of the bids are virtually for free. We adopt the conservative approach of assuming that only 75% of the bids generated actual revenues for the seller. Moreover, in the estimation of profits we assume that $V_a = V$ despite the alternative assumption $V_a < V$ seems more likely because of quantity discounts and/or marketing reasons.

[15] Profits are bounded above by $u_a^{\max} = N\eta^{\max}c + \hat{x}_i^t - V_a$ with $\hat{x}_i^t \in \{1, ..., V - \eta^{\max}\}$. Noticing that $\eta^{\max} \leq \frac{N-1}{N^2 c}V$ we have that $u_a^{\max} \leq \frac{N-1}{N}V + \hat{x}_i^t - V_a$. If $V_a = V$, $u_a^{\max}$ is certainly negative for any $\hat{x}_i^t < \frac{1}{N}V$.

by assuming that no agent submitted more than once the same bid in the same auction, the minimum number of bidders can be inferred by the frequency of the most frequent bid. Therefore, we can say that on average each auction attracted a minimum of 15.3 bidders (min.: 5, max.: 38, st. dev.: 6.45). This value is surely extremely conservative as it would imply that every bidder submitted 66 bids, and thus invested 122 Euros, in every auction. With a more credible guess of 10 bids per individual per auction, the average number of participants increases to 101.

Some features of bidders' behavior also noticeably stand out. Focusing on the 97,225 bids that picked numbers belonging to the set $\{1, ..., 500\}$ (96.3% of total bids, the remaining ones are mainly outliers), what emerges is that the majority of them picked odd numbers. More precisely, 54,230 bids (55.8%) are odd while only 42,995 (44.2%) are even. A normally approximated binomial test shows that this difference is significant at the 1% level. In line with this tendency, 9 out of the 10 most frequent bids are odd.[16] The preference for odd numbers has an intuitive explanation. In a lowest unique bid auction, players want to submit bids that no one else chooses. Therefore, agents tend to submit bids that they perceive to be original: odd numbers (a part from those whose trailing digit is 5) and, even better, prime numbers. But the aggregate result of this individual strategy is quite paradoxical as agents end up converging on non-focal numbers. A similar behavior emerges also in the auctions studied by Östling *et al.* (2008) and is analogous to the one first described in Crawford and Iriberri (2007) for what concerns Hide and Seek games. Such a bidding behavior confirms the hypothesis of boundedly rational bidders. In particular, agents erroneously think to be smarter than the opponents and only perform a limited number of steps of reasoning. Indeed, the data show

[16] The complete top ten list, with aggregate frequency in brackets, is the following: 1 (1,287), 11 (1,039), 17 (936), 3 (841), 13 (822), 111 (813), 23 (798), 7 (777), 2 (766), 27 (741). As a matter of comparison, round numbers like 10, 20 and 100 attracted respectively 506, 498 and 471 bids.

that submitting an odd bid is suboptimal as the large majority of winning bids are even numbers (68 vs. 32, with the difference being significant at 1% level). Notice moreover that our data do not allow to control for the level of experience of the players. The bias in submitting odd bids would probably be even more pronounced if only agents that play the game for the first few times were considered.

A part from the general indication that even numbers are more likely to win, data eyeballing seems to suggest that winning bids follow quite a random pattern, even for what concerns homogeneous goods.[17] Still, we investigate if there are some specific variables that may systematically influence the actual realizations of the winning bids. In particular, given our sample of 100 auctions k, we implement the following OLS regression:

$$WB_k = \beta_0 + \beta_1 (V)_k + \beta_2 (WBprevious)_k + \beta_3 \left(\frac{WBprevious}{Vprevious}\right)_k + \beta_4 (AverageWBprevious5)_k + \beta_5 (WBlastequal)_k + \epsilon_k$$

where the dependent variable WB_k is the winning bid in auction k. The independent variables are the following: $(V)_k$ is the retail price of the auctioned good; $(WBprevious)_k$ is the winning bid of the previous auction; $\left(\frac{WBprevious}{Vprevious}\right)_k$ is the ratio between the winning bid and the retail price of the good in the previous auction; $(AverageWBprevious5)_k$ is the average of the winning bids in the previous five auctions; $(WBlastequal)_k$ is the winning bid of the last auction that offered the same good.

Not all these informations are equally accessible to the bidders. The web page where users submit their bids clearly displays the retail price of the auctioned good. On the same page, a column on the right reports the list of the last five auctions with the objects sold, their retail price and the winning bids. Therefore, a bidder imme-

[17] For instance, in the 10 auctions that offered a Sony Playstation 3, the winning bids (€ cents) are the following: 60, 262, 180, 140, 81, 337, 70, 148, 55, 176.

diately knows $(V)_k$ and $(WBprevious)_k$ while he can easily compute $\left(\frac{WBprevious}{Vprevious}\right)_k$ and $(AverageWBprevious5)_k$. As for $(WBlastequal)_k$, the bidder must click to a different page and browse through the list of closed auctions (on average the same good is auctioned every 14 auctions, i.e., 5-6 days). We do not consider the number of received bids as a possible regressor because this information is not available to the bidders during the bidding period.

	1	2	3	4	5	6
$Intercept$	8.080 (18.214)	86.193* (7.6038)	82.266* (12.074)	95.107* (16.300)	79.377* (10.812)	0.309 (24.158)
V	0.0029* (0.0006)					0.0034* (0.0007)
$WBprevious$		0.009 (0.009)				0.012 (0.008)
$\frac{WBprevious}{Vprevious}$			1,873.853 (2,956.352)			2,888.816 (2,713.244)
$AverageWBprevious5$				−0.038 (0.087)		−0.041 (0.080)
$WBlastequal$					0.105 (0.097)	−0.118 (0.100)
$R-squared$	0.183	0.011	0.004	0.002	0.012	0.220
$Observations$	100	100	100	100	100	100

Table 1: OLS regressions for winning bid. St. err. in brackets. * Significant at 5%.

Table 1 reports the results of OLS regressions. Columns 1 to 5 show the coefficients of the five regressors when these are considered individually. Only the retail price of the auctioned good is characterized by a significant (positive) coefficient. Column 6 refers to a regression that includes all the covariates. The retail price of the auctioned good confirms to be the only significant regressor. In any case, all the specifications have little explanatory power.

4.1 An out-of-sample exercise

The regressions that appear in Table 1 may be of some interest for an external observer who wants to study the pattern of the winning bids but they would not help much an individual who wants to actually participate in a lowest unique bid auction. In fact, even assuming that the model had a good predictive power, a bidder that uses it would be at best able to match the existing winning bid. This would simply invalidate both bids as they would not be unique. The bidder needs something more. As a matter of fact, and considering that usually the winning bid is lower than the lowest number that did not receive any bid (in our sample this happens 91 times out of 100), the bidder's goal is twofold. On one hand, he must match and eliminate the otherwise winning bid. On the other hand, he must place the new winning offer. A more elaborate bidding strategy is then to combine the model that appears in the last column of Table 1 (call it Model 1) with a model (Model 2) that may help in finding the lowest empty number.

In order to test this strategy we first need to estimate Model 2. Therefore, we regress the lowest empty number (LEN) on the same regressors that we used in the previous section. The regression equation is the following:

$$LEN_k = \beta_0+\beta_1 (V)_k+\beta_2 (WBprevious)_k+\beta_3 \left(\frac{WBprevious}{Vprevious}\right)_k+\beta_4(AverageWBprevious5)_k+ \beta_5(WBlastequal)_k + \epsilon_k$$

and the results are reported in Table 2. Also in this case the only significant regressor is the value of the auctioned good.

	$Intercept$	V	$\frac{WB}{previous}$	$\frac{WBprevious}{Vprevious}$	$\frac{AverageWB}{previous5}$	$\frac{WB}{lastequal}$
	5.631 (30.582)	0.0042* (0.0009)	0.0007 (0.0102)	6,165.428 (3,434.737)	−0.099 (0.101)	0.056 (0.127)
R-squared	0.264					
Obs.	100					

Table 2: OLS regressions for Lowest Empty No. St. err. in brackets. * Significant at 5%.

We now perform an out-of-sample exercise that applies the estimated models to 10 additional auctions. These auctions followed the original 100 auctions and offered the same goods. Given that it would be too much to pretend the models to be able to exactly identify the two variables we are interested in (the current winning bid and the lowest empty number) we adopt a much more generous approach. More precisely, we imagine that the models are used by a bidder who is willing to invest in bids (remember that each bid costs 2 €) up to half of the value of the auctioned good. This is admittedly quite a strong assumption given that such a level of investment exposes the bidder to the risk of incurring considerable losses. On the other hand, it obviously increases the possibility of the models to succeed, i.e., it provides a non demanding test of their usefulness.

As a starting point, let us better specify the bidding strategy that we want to test. Call $\hat{b}_1$ the predicted value of Model 1 (the estimate for the current winning bid) and $\hat{b}_2$ the predicted value of Model 2 (the estimate for the lowest empty number). Our imaginary bidder starts by submitting $x^1 = \hat{b}_2$ in the hope of finding an empty number, possibly the lowest one. The bidder then optimally reacts to the signals he receives from the auctioneer. If $\sigma(x^1) = L$ then x^1 is not unique and the bidder submits additional bids that follow the pattern $x^2 = \hat{b}_2 - 1$, $x^3 = \hat{b}_2 + 1$, $x^4 = \hat{b}_2 - 2$, $x^5 = \hat{b}_2 + 2$ ... This process ends when either the agent runs out of bids or he submits the bid x^{t^*} that receives the signal $\sigma(x^{t^*}) \neq L$. In particular, if $\sigma(x^{t^*}) = W$, the bidder wins the auction.[18] If $\sigma(x^{t^*}) = M$ the bidder found an empty number but this is not the lowest one. The agent must then try to match the current lowest unique bid in order to eliminate it. Therefore, he will bid $x^{t^*+1} = \hat{b}_1$, $x^{t^*+2} = \hat{b}_1 - 1$, $x^{t^*+3} = \hat{b}_1 + 1$, $x^{t^*+4} = \hat{b}_1 - 2$, $x^{t^*+5} = \hat{b}_1 - 2$ ...[19] Again this procedure can end in various ways: 1) the agent runs out of bids; 2) the signal about x^{t^*} is updated from $\sigma(x^{t^*}) = M$ to $\sigma'(x^{t^*}) = W$; 3) a different bid receives the signal $\sigma(x^{t^*+k}) = W$.

The logic of the test is then very simple: we check how many times this bidding strategy would have been successful and which would have been the overall payoff of a bidder that adopted it. The following example refers to the first of the 10 additional auctions and helps in clarifying the situation.

Example 3 (Auction for a 1GB iPod shuffle) *The object is worth 80 Euros such that the agent is willing to submit up to 20 bids. The predicted value for the current winning bid (Model 1) is $\hat{b}_1 = 26$. The predicted value for the lowest empty number (Model 2) is $\hat{b}_2 = 34$. The agent submits $x^1 = 34$ and gets $\sigma(x^1) = M$ given that* 34 *is an empty number (but not the lowest one, see Figure 1.a). The agent then bids $x^2 = 26$. This happens to be exactly the current winning bid as all the values between* 1 *and* 21 *received multiple bids. Still, the agent simply receives the signal $\sigma(x^2) = L$ because* 26 *is now covered by two bids (see Figure 1.b). Notice that, after this second offer, the current winning bid would be* 32*. The agent keeps following the suggested strategy and submits $x^3 = 25$ (and $\sigma(x^3) = L$), $x^4 = 27$ ($\sigma(x^4) = L$), $x^5 = 24$ ($\sigma(x^5) = L$) and $x^6 = 28$ ($\sigma(x^6) = W$).*

[18] We assume that the rivals do not bid anymore. This is a very restrictive assumption that again strongly facilitates the success of the bidding strategy.

[19] In the case in which $\sigma(x^{t^*+k}) = M$, the bidder modifies his strategy to $x^{t^*+k+1} = x^{t^*+k} - 1$, $x^{t^*+k+2} = x^{t^*+k} - 2$ ...

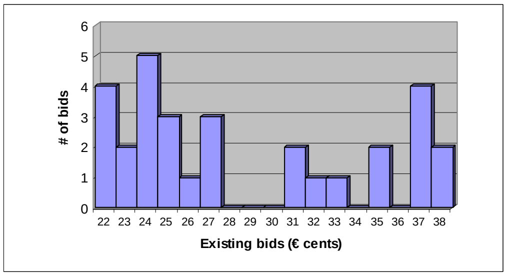

Fig. 1.a: the actual bids in the auction.

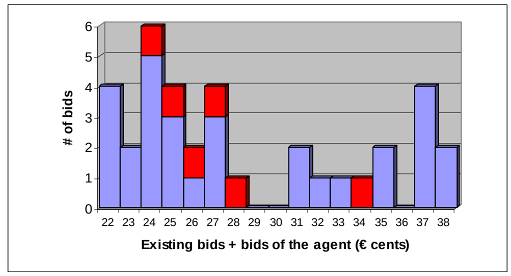

Fig. 1.b: the suggested bidding strategy.

The bidder wins the auction with his bid of 28 *cents and realizes positive profits of* $80 - (6 * 2) - 0.28 = 67.72$ *Euros.*

Despite this positive example, the overall performance of the strategy falls short. Out of the 10 auctions, the bidder would have won 3 and lost 7 and he would have obtained a negative total payoff (-377.63 €). The bidder would have won one more auction (i.e., 4 wins and 6 losses) and realized positive payoff (181.95 €) by adopting an even more sophisticated strategy: follow the indications of the previous strategy but, in addition, submit only even bids. Still this result must be judged with caution. First, such a strategy is very risky: for instance, without the last auction, the accrued payoff would have stayed negative (-115.5 €). Second, these simulations have been done under very favorable assumptions: a high level of investment (the bidder was willing to potentially waste $1,206$ €) and no further reactions from the rivals.

To sum up the content of this empirical section, we can say that data about actual auctions convey three main results:

- Contrary to equilibrium prediction, lowest unique bid auctions are (highly) profitable for the sellers.
- Bidders are boundedly rational.

- In spite of some (weak) regularities and of the existence of the signals, to place the winning bid remains a complicated issue. Luck certainly plays a major role.

5 Discussion

The paper introduced and analyzed a peculiar selling mechanism that is getting increasingly popular over the internet: lowest unique bid auctions (LUBAs) that allocate valuable goods to the agent who submits the lowest bid that is not matched by any other bid. We showed that if bidders are rational a LUBA can be profitable for the seller only if his valuation of the good is much lower that the one of the potential buyers. But we also showed why in reality this auction format is so successful: boundedly rational bidders may lack the necessary commitment to stick to equilibrium strategies and thus they may get locked in a costly war of attrition that highly rewards the auctioneer. In particular, we highlighted how such a mechanism is driven by the existence of the signals the auctioneer sends about the current status of players' bids. It is therefore ironic to notice how websites that organize LUBAs overstress, surely a bit in bad faith, the alleged positive role of these signals.[20] While it is clear why they do so (they have to justify the fixed cost associated with each bid and they want to distinguish themselves with respect to pure lotteries and gambling), the paper showed that signals are at best a double edged weapon. Similarly, in presenting the functioning of the auction mechanism, the auctioneers tend to overstate the role of bidders' ability and to minimize the luck component.[21] Our empirical analysis showed instead that winning bids follow

[20] For instance, one of these websites claims that "Relying on these signals, using different strategies and different levels of investment, to win the auction becomes a matter of a complex use of various abilities".

[21] Another website declares: "The investment, the signals and the bidding stategies make the auction void of any element of luck and based exclusively on the bidder's ability".

quite a random pattern and that luck matters a lot.

Lowest unique bid auctions also suffer from other potential problems that should suggest prudence. For instance, they share the technological hitches that characterize on-line auctions: problems of connectivity, delays or congestion, possibly due to last minute bidding or "sniping" (see for instance Roth and Ockenfels, 2002, for the case of eBay and Amazon auctions). Collusive behaviors are also an important issue. While collusion among bidders seems unlikely due to the secrecy of agents' identities and to problems of coordination, collusion between the auctioneer and a single bidder looks much more easily implementable. Bids are private information but the auctioneer get to know them in real time. In theory, he could then indicate to a third party where to place a winning bid seconds before the auction closes. Obviously this would turn the auction into a scam. We do not think that lowest unique bid auctions are scams: the mechanism is too profitable for risking to ruin it with such a trick. And indeed, to speak the truth, these websites put quite some effort in trying to build and maintain a reputation for being a trustable and transparent outlet.

To sum up, lowest unique bid auctions are a very smart selling mechanism. On one hand, by giving the possibility to win goods of considerable value for very little money, they share the appeal of lotteries. On the other hand, they give bidders the illusion to be in control of what they do and they convey the idea that winning is just a matter of being smarter than the others. The combination of these two factors makes the business successful and this in turn explains continuous entry in the industry. Entry will surely stimulate competition and lead to better conditions for the players: lower participation fees, higher initial bonuses, lower number of opponents. Nevertheless, the basic mechanism underlying the auction format will remain the same such that the analysis of this paper will continue to be valid. We conclude by stressing once more the similarities that lowest unique bid auctions have with other well known games like the war of attrition and the Dollar Auction Game. It is obviously not a coincidence that these games are used as archetypes for describing situations where irrational behavior leads to an inefficient waste of resources.

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