INTERNATIONAL TRADE, GROWTH, AND PER CAPITA INCOME
Preliminary Version

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Abstract

I construct a dynamic, two-country model of international trade and endogenous economic growth where agents have nonhomothetic preferences. I study the role that per capita income plays in jointly determining economic growth and international trade. The model exhibits multiple steady states. I find that in a steady state where all varieties are traded inequality has no effect on the world growth rate and the volume of trade. In a steady state where only part of all varieties are traded inequality has a positive effect on world growth and a negative effect on the volume of trade. (JEL D30, F10, F12, O30)

What role does per capita income play in jointly determining growth and trade? The model presented aims to shed light on this question by incorporating nonhomothetic preferences in a simple dynamic, two country model of trade and growth.

The influence of per capita income on international trade has been discussed since Linder (1961). The Linder hypothesis states that the more similar the demand structures of countries, the more intensive they will trade among each other. Linder argues that the more similar the demand patterns of countries, the larger will be their overlap of production and consumption patterns. He makes the point that this implies a more similar commodities composition of trade and a greater volume of trade. He considers per capita income to be the most important determinant of the demand structure. Thus, the Linder hypothesis essentially predicts international trade to be more intensive between countries the more equal their per capita incomes are. There is also empirical evidence that per capita income affects international trade. Hunter and Markusen (1988) reject the hypothesis of homotheticity and suggest that differences in demand play an important role in explaining trade. Hunter (1991) estimates that nonhomothetic preferences may account for more than one quarter of inter-industry

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trade. Bernasconi (2009) finds evidence for the Linder hypothesis over the extensive margin of trade.

There is a vast literature that studies the channels through which international trade influences economic growth (for a survey see Grossman and Helpman (1994)). Grossman and Helpman (1991a) identify several features of the global economy which they regard as important for understanding trade and growth. For example, comparative advantage leading to specialization in the creation of knowledge, large scale of the world economy, or spillovers of knowledge in trade partners (knowledge transfer). However, Grossman and Helpman (1990a) also mention in their conclusion: 'We need to learn much more about the mechanisms by which knowledge and technology diffuse across international borders ...'. Grossman and Helpman (1990) postulate perfect knowledge spillovers. They argue that each country has access to the knowledge stock of the world, the sum of all blueprints worldwide. Grossman and Helpman (1990) also consider the cases if only a fraction of the global knowledge stock is accessible, or, if there is a time lag in the diffusion of knowledge. There are models of North-South trade (Grossman and Helpman (1991), Dinopoulos and Segerstrom (2003), and Matsuyama (2000), among others) which assume that the developed North innovates differentiated goods whereas the South imitates those differentiated goods.

The bulk of the literature on trade and growth neglects or plain ignores demand side issues by assuming homothetic preferences. This implies that per capita income has no role to play in determining the demand structure. However, the effect of per capita income on the demand structure can only be studied in a model where agents have nonhomothetic preferences. There is a narrow strand of literature that is concerned with nonhomothetic preferences in trade and/or growth models. However, these models are either static trade models (Foellmi et al. (2008)) or are not models of endogenous growth in the sense of Romer (1990) where new differentiated goods are invented (Spilimbergo (2000)). The model closest to the one developed in this paper is Hanslin (2005). She develops a North-South model of endogenous growth and nonhomothetic preferences. Technological progress happens in the North through investment in research and development that creates new blueprints. The South does not create new blueprints but imitates goods that have been invented in the North. Households have nonhomothetic preferences of the zero-one form (see Matsuyama (2000)). All households in the North are equal whereas in the South there are two groups of households, namely rich and poor. It is assumed that the households in the North are wealthier than the rich in the South. There is free trade between countries. The analysis is restricted to the steady-state. Hanslin finds that an increase in inequality has a positive effect on the growth rate and the fraction of goods that are imitated. The fundamental difference of my model to Hanslin (2005) is the different structure of the North-South model. Furthermore, trade in her model is free across borders. Thus, her model is not able to analyze the interaction of nonhomothetic preferences and trade liberalization.

In section 1 I describe the framework of the model in detail. Section 2 elaborates the balanced growth path properties. The model exhibits multiple steady states which are discussed in sections 2.1 and 2.2. Section 3 concludes.

1 The Model

I construct a dynamic, two-country model of costly international trade and endogenous growth. The two countries are labeled $i = A, B$. Each country is populated by identical households. However, I allow households to be potentially different with respect to their endowment of efficiency units of labor across countries. The model follows Foellmi et al. (2008), Hanslin (2005), and Foellmi and Zweimüller (2006).

1.1 The Demand Side

Households in this world are all identical with respect to their preferences. Furthermore, households within a country are not only equal in what concerns their preferences but also their endowment with efficiency units of labor. However, I allow for potential heterogeneity on the country level, in particular, households in country A can have a larger endowment of efficiency units of labor than households in country B. There are L^i households in country $i = A, B$ each endowed with l_i efficiency units of labor.

Households in country i have preferences of the following form

$$U_i(0) = \int_0^{\infty} \frac{1}{1-\sigma} u_i(t)^{1-\sigma} e^{-\rho t} dt \tag{1}$$

where $1/\sigma$ is the intertemporal elasticity of substitution (σ represents the (constant) relative risk aversion), ρ is the subjective discount rate, and $u_i(t)$ denotes the instantaneous utility function given by

$$u_i(t) = \int_0^{N_i(t)} c_i(j,t) dj.$$

Consumption is a zero-one decision modeled by the indicator function $c_i(j,t)$ that takes the value 1 if good j is consumed in quantity 1 by a household in country i at time t, and zero otherwise. Zero-one preferences imply that households' consumption choice is restricted to the extensive margin. In contrast, with constant elasticity of substitution preferences households can only choose consumption along the intensive margin. $N_i(t)$ is the measure of differentiated goods purchased by a household in country i. Since preferences are additive separable, the household faces essentially a two-stage budgeting problem. In a first stage the household optimally allocates her expenditure, $E_i(t)$, across differentiated goods at a given point in time t. The household's optimization problem in the first stage can be formulated as

$$
\begin{aligned}
&\max_{c_i(j,t)} \int_0^{N_i(t)} c_i(j,t)dj \\
&\text{s.t.} \\
&\int_0^{N_i(t)} p^i(j,t)c_i(j,t)dj = E_i(t).
\end{aligned}
$$

Here, $p^i(j,t)$ denotes the price of variety j at time t charged in country i. The first-order (Kuhn-Tucker) conditions of the problem are given by

$$
c_i(j,t) = \begin{cases} 1, & p^i(j,t) \leq 1/\lambda_i(t) \\ 0, & p^i(j,t) > 1/\lambda_i(t) \end{cases} \tag{2}
$$

where $\lambda_i(t)$ is the Lagrange multiplier (here the marginal utility of income of a household from i at time t) associated with the optimization problem of the first stage. Substituting the first-order condition back into the budget constraint yields the following demand equation

$$
N_i(t) = \frac{E_i(t)}{p^i(t)}.
$$

Note that I have used the fact from the first-order conditions that in equilibrium, prices charged in country i will be symmetric. It follows that the indirect instantaneous utility function, $v_i(t)$, of a household from country i can be written as

$$
v_i(t) = N_i(t) = \frac{E_i(t)}{p^i(t)}.
$$

In the second-step a household needs to allocate her expenditure optimally across periods given the optimal choice of consumption within each period. The household's optimization problem in the second stage is

$$
\begin{aligned}
&\max_{\{E_i(t)\}_{t=0}^{\infty}} \int_0^{\infty} \frac{1}{1-\sigma} \left(\frac{E_i(t)}{p^i(t)}\right)^{1-\sigma} e^{-\rho t}dt \\
&\text{s.t.} \\
&\int_0^{\infty} E_i(t)e^{-R^i(t)}dt = \int_0^{\infty} w^i(t)l_i e^{-R^i(t)}dt + V_i(0)
\end{aligned}
$$

where $w^i(t)$ denotes the market-clearing wage in country i at date t, $V_i(0)$ is the initial wealth endowment of a household from country i, and l_i the stationary efficiency units of labor endowment of a household in country i. Since the labor market is competitive in country i the wage rate $w^i(t)$ is the same for each household and taken as given. $R^i(t) = \int_0^t r^i(s)ds$ denotes the cumulative interest rate in country i. The solution to this optimization problem is characterized by

$$
\left(\frac{E_i(t)}{p^i(t)}\right)^{-\sigma} \frac{1}{p^i(t)} e^{-\rho t} = \mu_i e^{-R^i(t)}
$$

where μ_i is the (time-independent) Lagrange multiplier associated with the second stage optimization problem. I rewrite the first-order condition as

$$N_i(t)^{-\sigma}\frac{1}{p^i(t)}e^{-\rho t} = \mu_i e^{-R^i(t)}, \tag{3}$$

taking logs and differentiating with respect to time yields

$$\frac{\dot{N}_i(t)}{N_i(t)} = \frac{r^i(t) - \rho - \frac{\dot{p}^i(t)}{p^i(t)}}{\sigma}. \tag{4}$$

Note that $\frac{\partial R^i(t)}{\partial t} = \frac{\partial}{\partial t}\int_0^t r^i(s)ds = r^i(t)$. Equation (4) is the Euler equation telling me how the optimal consumption path of a household in country i evolves over time.

1.2 The Supply Side

I abstract from heterogeneity in the supply side. All firms in the world are identical with respect to their technology. The technology they employ requires labor as its only input factor. Technology is characterized as follows. A firm has to create a design or blueprint which grants the firm access to a linear production technology. The use of the designs is then protected by infinitely lived patents that are fully enforced. Hence, firms are in a monopoly position and can charge prices above their marginal costs. The creation of a new design requires $F^i(t) = F/N(t)$ units of labor. Here, $N(t)$ denotes the measure of known designs at time t in the entire world. Let the stock of designs at time t in country i be $N^i(t)$. The stock of designs in the world is then the sum of the stocks of designs of all countries in the world, in particular, $N(t) = N^A(t) + N^B(t)$. Production of 1 unit of final output requires $b^i(t) = b/N(t)$ units of labor. This technology implies perfect knowledge spillovers. Labor is not mobile across borders. The labor market is competitive and $w^i(t)$ denotes the market-clearing wage rate in country i at time t. Thus, in country i the cost of creating a new design is $w^i(t)F^i(t)$, and the cost of producing 1 unit of output is $w^i(t)b^i(t)$. I am free to choose a numeraire (one in each period t). I set the marginal cost of production in country A equal to one, so that $w^A(t)b^A(t) = 1$ for all t. This implies that the wage rate in country A, $w^A(t) = N(t)/b$, grows at the same rate as the stock of designs in the world, and that the innovation cost in country A remains constant over time, $w^A(t)F^A(t) = F/b$.

2 Balanced Growth Path

2.1 Full Trade Equilibrium

I consider a situation where the inequality between country A and country B is sufficiently low, so that all households in the world consume all differentiated goods available in equilibrium. I assume that households in country A have a higher labor endowment than households in country B, in particular $l_A > l_B$. Trade across borders is costly. The transport costs, denoted by τ, are assumed to take the form of iceberg costs, i.e. to make sure that one unit of the good reaches its destination $\tau \geq 1$ units have to be shipped.

2.1.1 Prices and Interest Rate

I conjecture an equilibrium where all households in the world consume all differentiated goods available in the world. How can such a situation be an equilibrium? First, in equilibrium, firms will charge prices in country i that equal the willingness to pay of households in country i as can be seen from the first-order conditions (2). This implies that prices in country i are identical for all j since within country i all households have the same willingness to pay (remember I abstract from within country inequality). I assume that households in country A are richer than households in country B, which will indeed turn out to be the case in equilibrium since households in country A have a higher labor endowment, and in equilibrium the wage rates will be equalized across countries. Furthermore, discounted profits of monopolistic firms owned by the households (each household in country i holds an equal share in the stocks of a monopolist) will be equal to the cost of innovation in equilibrium. I assume that households in country i hold equal shares only in monopolistic firms from country i, i.e. there is a perfect home bias in the composition of a household's portfolio. This implies that prices in country A will be higher in equilibrium than in country B since the willingness to pay of households in country A is higher than in country B.

There is the following caveat. In principle a monopolist can perfectly price discriminate between households from country A and B. However, the price setting behavior of a monopolist is restricted if I allow for parallel imports. In that case, the threat of parallel imports disciplines the price setting behavior of the monopolist in the sense that if the difference between the willingness to pay of households in country A and B is high, arbitrage possibilities emerge. In particular, the price setting restriction becomes binding if the willingness to pay of households in the rich country exceeds the willingness to pay of the households in the poor country plus transport costs. In that case, an arbitrageur could travel to the poor country, purchase some good j at a low price, ship it back to the rich country at costs τ, and sell it there at a price epsilon lower than the price at which the producer of good j sells it. If the price wedge exceeds the transport costs the arbitrageur would make a profit.

In the following analysis I assume that the inequality between country A and B is sufficiently low such that the price setting restriction for monopolists is not binding, i.e. the willingness to pay of households in country A does not exceed the willingness to pay of households in country B plus transport costs. Thus monopolists charge prices in country A and B that are equal to the willingness to pay. This implies that households in country A and B exhaust their budget constraints in equilibrium. In other words, monopolists can perfectly price discriminate. There is no incentive to deviate for a monopolist and sell only to households in country A. Thus the proposed equilibrium is indeed a (Nash) equilibrium.

First, consider the operating profits at time t of a monopolist j in country A, given by

$$\pi^A(j,t) = \left[p^A(j,t) - 1\right] L^A + \left[p^B(j,t) - \tau\right] L^B \quad \forall j, t.$$

Operating profits at time t of a monopolist j in country B are

$$\pi^B(j,t) = \left[p^A(j,t) - \tau b \frac{w^B(t)}{N(t)}\right] L^A + \left[p^B(j,t) - b \frac{w^B(t)}{N(t)}\right] L^B \quad \forall j, t.$$

From the first-order conditions follows that one single (symmetric) price will prevail in equilibrium, namely $p^A(j,t) = p^A(t)$ in country A, and $p^B(j,t) = p^B(t)$ in country B. Thus $\pi^A(j,t) = \pi^A(t)$ and $\pi^B(j,t) = \pi^B(t)$, for all j. I assume that the number of households is equal in both countries, $L^A = L^B \equiv L$. I conjecture a balanced growth path on which profit flows are constant over time. This implies that prices on the balanced growth path have to be stationary, $\dot{p}^i(j,t)/p^i(j,t) = 0$. It further implies that the wage rate in country B has to grow at a rate equal to the growth rate of the stock of designs in the world. I define the growth rate of the world stock of designs as $g^W(t) \equiv \dot{N}(t)/N(t)$. On the balanced growth path the stock of designs in the world grows at a constant rate equal to $g^W(t) = g^W$. Using the Euler equation (4) and imposing the conditions on the balanced growth path, I get

$$g^W = \frac{r^A - \rho}{\sigma},$$

and

$$g^W = \frac{r^B - \rho}{\sigma}.$$

Combining the two expressions yields $r^A = r^B \equiv r$, i.e. the (instantaneous) interest rate has to be constant and equal across countries. I write the Euler equation in the following form

$$g^W = \frac{r - \rho}{\sigma}. \tag{5}$$

The Euler equation (5) applies to both countries. Hence, on the balanced growth path considered the evolution of consumption of households in country A and B is identical. Note that for an equilibrium with positive sustained growth the (market) interest rate must exceed the subjective discount rate, $r > \rho$.

2.1.2 Resource Constraints

The resource constraint in country i is composed of two parts. The first part determines how much resources are allocated to the research and development (R&D) sector whereas the second part determines how much resources are allocated to the production sector of the economy. The resources required in the R&D sector in country A, respectively country B, are determined by

$$\begin{aligned} F^A(t)\dot{N}^A(t) = \; & F\tfrac{\dot{N}^A(t)}{N(t)} = \frac{1}{1+n(t)} F g^A(t) \\ F^B(t)\dot{N}^B(t) = \; & F\tfrac{\dot{N}^B(t)}{N(t)} = \frac{n(t)}{1+n(t)} F g^B(t) \end{aligned}$$

where $g^A(t) \equiv \dot{N}^A(t)/N^A(t)$, $g^B(t) \equiv \dot{N}^B(t)/N^B(t)$ and $n(t) \equiv N^B(t)/N^A(t)$. In the production sector the resources required in country A, respectively B, are determined by

$$\begin{aligned}
\int_0^{N^A(t)} b^A(t)\left[c_A^A(j,t)L + \tau c_B^A(j,t)L\right] dj &= \frac{bL(1+\tau)}{1+n(t)} \\
\int_0^{N^B(t)} b^B(t)\left[c_B^B(j,t)L + \tau c_A^B(j,t)L\right] dj &= \frac{n(t)bL(1+\tau)}{1+n(t)}
\end{aligned}$$

where the superscript on $c(j,t)$ denotes where the differentiated good j is produced, and the subscript denotes where j is consumed. A competitive labor market ensures that demand equals supply in the economy at every point in time. It follows that the resource constraint of the economy in country A, respectively B, is given by

$$\begin{aligned}
l_A L &= \frac{1}{1+n(t)}\left[F g^A(t) + bL(1+\tau)\right] \\
l_B L &= \frac{n(t)}{1+n(t)}\left[F g^B(t) + bL(1+\tau)\right].
\end{aligned}$$

Imposing the conjectured balanced growth path requires that $g^A(t) = g^A$, $g^B(t) = g^B$, and $n(t) = n = N^B/N^A$, which implies that $g^A = g^B = g^W$ (which is consistent with the Euler equations). Thus the resource constraints on the balanced growth path can be written as

$$l_A L = \frac{1}{1+n}\left[F g^W + bL(1+\tau)\right] \tag{6}$$

$$l_B L = \frac{n}{1+n}\left[F g^W + bL(1+\tau)\right]. \tag{7}$$

2.1.3 Zero-Profit Conditions

There is free entry into the R&D sector that leads to zero-profits in equilibrium in the sense that the discounted value of future profits, $V(t)$, is equal to the cost of innovation, F/b. $V^i(t)$, with $i = A, B$, is given by

$$V^i(t) = \int_t^{\infty} \pi^i(\tau) e^{-R^i(\tau)} d\tau$$

where $R^i(\tau) = \int_t^{\tau} r^i(s)ds$. Differentiating $V^i(t)$ with respect to time t yields

$$\dot{V}^i(t) = r^i(t)V^i(t) - \pi^i(t).$$

The zero-profit condition for country i demands that

$$V^i(t) = \frac{F}{b}.$$

Differentiating $V^i(t)$ with respect to time t gives $\dot{V}^i(t) = 0$. Thus the zero-profit condition for country i can be written as

$$\frac{\pi^i(t)}{r^i(t)} = \frac{F}{b}.$$

In particular for country A, respectively B, the zero-profit condition is given by

$$\begin{aligned}
\frac{\left[p^A(t)-1\right]L+\left[p^B(t)-\tau\right]L}{r^A(t)} &= \frac{F}{b} \\
\frac{\left[p^A(t)-\tau b\frac{w^B(t)}{N(t)}\right]L+\left[p^B(t)-b\frac{w^B(t)}{N(t)}\right]L}{r^B(t)} &= \frac{F}{b}.
\end{aligned}$$

Imposing the conjectured balanced growth path on which profits, prices, and the interest rate are constant (note that the conjectured balanced growth path is consistent with the zero-profit conditions) yields

$$\frac{\left[p^A-1\right]L+\left[p^B-\tau\right]L}{r} = \frac{F}{b} \tag{8}$$

$$\frac{\left[p^A-\tau b\frac{w^B(t)}{N(t)}\right]L+\left[p^B-b\frac{w^B(t)}{N(t)}\right]L}{r} = \frac{F}{b}. \tag{9}$$

2.1.4 Balance of Payments

In general, the balance of payments requires that the present discounted value of imports of country i be equal to the present discounted value of exports of country i. On the balanced growth path the balance of payments has to hold at every point in time t. Thus on the balanced growth path the balance of payments for country A can be written as

$$\underbrace{p^A\tau N^B L}_{\text{Value of Imports}} = \underbrace{p^B\tau N^A L}_{\text{Value of Exports}}$$

$$n = \frac{p^B}{p^A}. \tag{10}$$

2.1.5 Budget Constraints

One can show that by Walras' Law the budget constraints are implied by the resource constraints, the zero-profit conditions, and the balance of payments. For completeness I state the intertemporal budget constraints in appendix A.

2.1.6 General Equilibrium

In this section I will discuss the solution of the full trade equilibrium on the balanced growth path. The system of equations and the corresponding solutions are given in appendix B.

Growth Rate First, I will discuss the growth rate of the economy which is given by

$$g^W = \frac{L}{F}\left[(l_A + l_B) - b\,(1+\tau)\right]. \tag{11}$$

Note that $g^W > 0$ if and only if $(l_A + l_B) > b(1+\tau)$.

The growth rate is independent of preference parameters like the subjective discount rate of households ρ, and the intertemporal elasticity of substitution $1/\sigma$. To understand why the growth rate is independent of preference parameters consider a decrease in the subjective discount rate ρ which translates into households being more patient. As in standard endogenous growth models a decrease in ρ leads to a decrease in the interest rate r. Differentiating equation (32) with respect to ρ yields $\partial g^W/\partial\rho = 1$. This increases the present discounted value of future profits of monopolistic firms (the value of the firms) and thus induces more firms to enter the market. This effect increases the growth rate. However, in this model it is crucial to notice that markups [1] of monopolistic firms are endogenous. If households become more patient the willingness to pay for differentiated goods today relative to tomorrow falls. Thus prices of varieties, markups and hence the present discounted value of profits fall, reducing the incentive to innovate for firms. This effect reduces the number of monopolistic firms entering the market thereby lowering the growth rate. On the balanced growth path these two effects exactly offset each other such that all households still consume all varieties. It is straightforward to prove this claim. To see how prices change with ρ I differentiate equations (33) and (34) with respect to ρ

$$\begin{aligned}
\frac{\partial p^A}{\partial\rho} &= \frac{l_A}{l_A + l_B}\frac{F}{bL} \\
\frac{\partial p^B}{\partial\rho} &= \frac{l_B}{l_A + l_B}\frac{F}{bL}.
\end{aligned}$$

I have established that prices (markups) fall as the subjective discount rate ρ decreases, i.e. households become more patient. Next, I rewrite the zero-profit condition (25) as

$$\frac{Fr(\rho)}{bL} = p^A(\rho) + p^B(\rho) - (1+\tau)$$

where $r(\rho)$, $p^A(\rho)$, and $p^B(\rho)$ denote functions of ρ. Differentiating the zero-profit condition with respect to ρ yields

$$\begin{aligned}
\frac{F}{bL}\frac{\partial r}{\partial\rho} &= \frac{\partial p^A}{\partial\rho} + \frac{\partial p^B}{\partial\rho} \\
\frac{F}{bL}1 &= \frac{F}{bL}\left[\frac{l_A + l_B}{l_A + l_B}\right].
\end{aligned}$$

The zero-profit condition does not change. Thus, I have shown that the two effects exactly offset each other.

[1] Markups are defined as the ratio of price to marginal costs. Since marginal costs are equal to one in equilibrium, markups are equal to prices.

A different way to understand why the growth rate is independent of preference parameters is to look at the resource constraint. If all households consume all differentiated goods in the economy the resources for producing these goods are pinned down since in the case of zero-one preferences there is no intensive margin that depends on the preference parameters like in the constant elasticity of substitution case. Thus the remaining resources are allocated to research and development and determine the growth rate.

I note that

$$\frac{\partial g^W}{\partial \tau} = -\frac{Lb}{F} < 0.$$

A marginal increase in transport costs τ lowers the growth rate in the economy. There are two opposing effects at work here. First, an increase in transport costs lowers the interest rate which increases the present discounted value of firms. This induces more firms to enter the market thereby increasing the growth rate of the economy. Second, an increase in transport costs lowers instantaneous profits of monopolistic firms. This decreases the present discounted value of firms inducing less firms to enter the market. The second effect outweighs the first effect.

Note that

$$\frac{\partial g^W}{\partial l_i} = \frac{L}{F} > 0$$

with $i = A, B$. The growth rate is increasing in the endowment of efficiency units of labor in both countries, l_A and l_B. A higher endowment of efficiency units of labor translates into a higher income and thus a higher willingness to pay. Hence firms can charge higher prices, markups and therefore profits are higher, increasing the incentive to innovate. Note that the degree of inequality on this growth path has no bearing on the growth rate. A mean preserving spread does not alter the growth rate. Intuitively, if both countries consume all varieties that are produced in the world, inequality does not matter. However, inequality can not rise without bound for this equilibrium to be sustainable. If inequality reaches a certain level (yet to be determined) the economy will transition to a balanced growth path where only part of all varieties are traded.

The population size L has a positive effect on the growth rate. Due to increasing returns a larger market means that monopolistic firms can produce at lower average costs (scale effect). This increases profits and thus the incentive to innovate.

An increase in the amount of labor needed to create a new blueprint, F, or to produce one unit of final output, b, decreases the present discounted value of profits and thus the incentive to innovate. This results in a lower growth rate.

Ratio of Varieties Produced in Country B to Country A The measure of differentiated goods produced in country B relative to the measure of differentiated goods produced in country A, $n = N^B/N^A$, is given by

$$n = \frac{l_B}{l_A} < 1$$

since $l_A > l_B$ by assumption. I study the effect of a mean preserving spread on n as follows. I can write the total amount of efficiency units of labor in the world, denoted by L^W, as

$$\begin{aligned} L^W &= (l_A + l_B)L \\ l_B &= \frac{L^W}{L} - l_A. \end{aligned}$$

Increasing l_A decreases l_B leaving the total amount of efficiency units of labor in the world L^W constant, i.e. increasing l_A is a mean preserving spread. Substituting the expression for l_B into the expression for n yields $n = \frac{L^W}{Ll_A} - 1$. The effect of a mean preserving spread can be analyzed by differentiating n with respect to l_A

$$\frac{\partial n}{\partial l_A} = -\frac{L^W}{Ll_A^2} < 0.$$

Executing a mean preserving spread by increasing l_A lowers the ratio of differentiated goods produced in country B to A. Intuitively, increasing the number of efficiency units of households in country A by taking the equivalent amount of efficiency units away from households in country B, increases the measure of differentiated goods produced in country A, N^A, and decreases the measure of goods produced in country B, N^B. Thus, the ratio N^B/N^A falls. However, the total measure of differentiated goods produced in the world, $N = N^A + N^B$, does not change since N^A increases by the same mass as N^B decreases. Note that increasing l_A leaving l_B fixed, increases N^A and thus N. Since all households consume all differentiated goods, the volume of trade increases. Furthermore, transport costs do not affect the volume of trade in this equilibrium.

2.1.7 Discussion

In sum, I have found a balanced growth path where all differentiated goods produced in the world are traded. On this balanced growth path inequality in per capita income (i.e. a mean preserving spread) across countries has no effect on the growth rate of the world economy (and thereby on the growth rate of the individual countries). Furthermore, income inequality does not affect the volume of trade. A mean preserving spread simply shifts production from one country to the other in a one-to-one manner since both countries use the same technology. Thus, since households in both countries consume all differentiated goods produced in the world, the volume of trade does not change. However, inequality in per capita income across the world cannot rise without bound for a full trade equilibrium to be sustainable.

2.2 Partial Trade Equilibrium

I now turn to the same set-up as before but where the inequality in per capita incomes between country A and B is sufficiently high, so that the price setting restriction of monopolists becomes binding. Monopolists can no longer set prices in country A equal to the willingness to pay of households in A but instead are forced to set prices equal to the willingness to pay of households in B plus transport costs. In such a situation only part of all differentiated goods produced in country A will be shipped to B. Some producers in country A will sell to households from A exclusively, and thus can set a price equal to the willingness of households in A. They might miss out on a larger market but instead they can charge a higher price. Of course, in equilibrium these two opposing effects will be balanced such that producers are indifferent as to serve the world market or the market in A exclusively.

2.2.1 Prices and Interest Rate

I conjecture a partial trade equilibrium that is characterized as follows. Households from country A consume all differentiated goods produced in A and B, whereas households from B can only afford a subset of differentiated goods produced in A but consume all differentiated goods produced in B. The subset of differentiated goods produced in country A that is not traded will be priced according to the willingness to pay of households in A. Differentiated goods that are produced in country A and traded will have a price in A equal to the willingness to pay of households in B plus transport costs. All differentiated goods produced in country B will be imported by A at a price equal to the willingness to pay of households in B plus transport costs. All differentiated goods sold in country B have the same price which is equal to the willingness to pay of households in B.

First, I consider operating profits at time t of a monopolist j in country A, that sells exclusively to households in A

$$\pi_A^A(j,t) = \left[p_{NT}^A(j,t) - 1\right] L \quad \forall j, t.$$

Operating profits at time t of a monopolist j in country A, selling to households in A and B are equal to

$$\pi_{tot}^A(j,t) = \left[p^A(j,t) - 1\right] L + \left[p^B(j,t) - \tau\right] L \quad \forall j, t.$$

A monopolist j in country B sells both to households in A and B and makes the following operating profits at time t

$$\pi_{tot}^B(j,t) = \left[p^A(j,t) - \tau b \frac{w^B(t)}{N(t)}\right] L + \left[p^B(j,t) - b \frac{w^B(t)}{N(t)}\right] L \quad \forall j, t.$$

Note that the first-order conditions imply that in equilibrium prices are symmetric and given by $p_{NT}^A(t) = 1/\lambda_A(t)$, $p^A(t) = \tau/\lambda_B(t)$, and $p^B(t) = 1/\lambda_B(t)$. Again, I conjecture a balanced growth path on which profit flows are constant over time. This implies that prices have to be constant as well, $\dot{p}^A_{NT}(t)/p^A_{NT}(t) = 0$, $\dot{p}^A(t)/p^A(t) = 0$, and $\dot{p}^B(t)/p^B(t) = 0$. It further implies that $w^B(t)/N(t)$ has to be constant, which means that wages in country B grow at the same rate as the stock of designs in the world.

The Euler equation in country A is given by

$$\frac{\dot{N}_A(t)}{N_A(t)} = \frac{r^A(t) - \rho - \frac{\dot{p}^B(t)}{p^B(t)}}{\sigma}$$

where $N_A(t)$ denotes the number of differentiated goods the representative household in country A consumes at time t. Using the fact that $N_A(t) = N(t)$ and imposing the balanced growth path results in

$$g^W = \frac{r^A - \rho}{\sigma}. \tag{12}$$

The Euler equation on the balanced growth path in country B is given by

$$\frac{\dot{N}_B(t)}{N_B(t)} = \frac{r^B(t) - \rho}{\sigma}$$

where where $N_B(t)$ denotes the number of differentiated goods the representative household in country B consumes at time t. $N_B(t)$ consists of all differentiated goods produced in country B, $N^B(t)$, and the subset of differentiated goods produced in A that is traded, $N^A_T(t)$. Differentiating $N_B(t) = N^A_T(t) + N^B(t)$ with respect to time t and rearranging terms yields

$$\frac{\dot{N}_B(t)}{N_B(t)} = \frac{\dot{N}^A_T(t)}{N^A_T(t)}\frac{N^A_T(t)}{N_B(t)} + \frac{\dot{N}^B(t)}{N^B(t)}\frac{N^B(t)}{N_B(t)} \tag{13}$$

I further conjecture[2] that on the balanced growth path the measure of differentiated goods produced in country A that is traded, N^A_T, and the measure of differentiated goods produced in country B, N^B, grow at the same rate as the world stock of designs, N, namely, g^W. Imposing the conjectured balanced growth path in equation (13) implies $\dot{N}_B/N_B = g^W$. Thus, the Euler equation on the balanced growth path for country B becomes

$$g^W = \frac{r^B - \rho}{\sigma}.$$

Combining the Euler equations for country A and B implies $r^A = r^B = r$, the interest rate is constant and equal across countries. I write the Euler equation as

$$g^W = \frac{r - \rho}{\sigma}. \tag{14}$$

[2]This conjecture implies a balanced growth path along which the allocation of resources is constant over time.

The Euler equation (14) applies to both countries. Hence consumption in country A and B grow at the same rate, namely at the rate the world stock of designs grows. For $g^W > 0$, the (market) interest rate r has to exceed the subjective discount rate ρ.

2.2.2 Resource Constraints

The resource constraints are derived in an analogue way to the previous section. First, the resources required in the R&D sector in country A are determined by

$$F^A(t)\dot{N}^A(t) = F\frac{\dot{N}^A(t)}{N^A(t)}\frac{N^A(t)}{N(t)} = Fg^A(t)\frac{1}{1+n(t)}$$

whereas in country B required resources for R&D are given by

$$F^B(t)\dot{N}^B(t) = F\frac{\dot{N}^B(t)}{N^B(t)}\frac{N^B(t)}{N(t)} = Fg^B(t)\frac{n(t)}{1+n(t)}.$$

The production sector in country A requires the following resources

$$\int_0^{N^A(t)} b^A(t)c_A^A(j,t)Ldj + \int_0^{N_T^A(t)} b^A(t)\tau c_B^A(j,t)Ldj = \frac{bL}{1+n(t)} + bL\tau\frac{N_T^A(t)}{N(t)}.$$

In country B the production sector demands the following resources

$$\int_0^{N^B(t)} b^B(t)\left[c_B^B(j,t)L + \tau c_A^B(j,t)L\right]dj = \frac{n(t)bL(1+\tau)}{1+n(t)}.$$

A competitive labor market ensures that demand equals supply in the economy at every point in time. Thus, the resource constraint in country A is given by

$$l_A L = \frac{Fg^A(t)}{1+n(t)} + \frac{bL}{1+n(t)} + bL\tau\frac{N_T^A(t)}{N(t)}$$

whereas in country B the resource constraint of the economy is given by

$$l_B L = \frac{n(t)}{1+n(t)}\left[Fg^B(t) + bL(1+\tau)\right].$$

On the conjectured balanced growth path along which the allocation of resources across sectors remains constant over time the resource constraints imply that $g^A = g^B = g^W$, and that N_T^A must grow at the same rate as N. On the balanced growth path the resource constraint of country A becomes

$$l_A L = \frac{Fg^W}{1+n} + \frac{bL}{1+n} + bL\tau\frac{N_T^A}{N} \tag{15}$$

The resource constraint on the balanced growth path for country B is given by

$$l_B L = \frac{n}{1+n}\left[Fg^W + bL(1+\tau)\right]. \tag{16}$$

2.2.3 Zero-Profit Conditions

Free entry into the R&D sector implies that monopolistic firms will make zero profits in equilibrium. The present discounted value of future profits, $V(t)$, has to be equal to the cost of innovation, F/b. The zero-profit condition is derived in the same way as in section 2.1.3, and takes the following form

$$\frac{\pi(t)}{r(t)} = \frac{F}{b}.$$

Imposing the conjectured balanced growth path the zero-profit conditions for monopolistic firms located in country A exclusively selling to households in country A, for monopolistic firms located in country A selling to households in country A and B, and for monopolistic firms located in country B selling to households in country A and B, respectively, are given by

$$\begin{aligned}
\frac{\left[p_{NT}^{A} - 1\right] L}{r} &= \frac{F}{b} && (17)\\
\frac{\left[\tau p^{B} - 1\right] L + \left[p^{B} - \tau\right] L}{r} &= \frac{F}{b} && (18)\\
\frac{\left[\tau p^{B} - \tau b \frac{w^{B}(t)}{N(t)}\right] L + \left[p^{B} - b \frac{w^{B}(t)}{N(t)}\right] L}{r} &= \frac{F}{b}. && (19)
\end{aligned}$$

2.2.4 Balance of Payments

The balance of payments requires that the present discounted value of imports of country i equal the present discounted value of exports of country i. On the balanced growth path the balance of payments has to hold at every point in time t. Thus on the conjectured balanced growth path the balance of payments for country A can be written as

$$\begin{aligned}
\underbrace{\tau p^{B} \tau N^{B} L}_{\text{Value of Imports}} &= \underbrace{p^{B} \tau N_{T}^{A} L}_{\text{Value of Exports}} \\
\tau N^{B} &= N_{T}^{A}. && (20)
\end{aligned}$$

2.2.5 Budget Constraints

Again, one can show that by Walras' Law the budget constraints are implied by the resource constraints, the zero-profit conditions, and the balance of payments. For completeness I state the intertemporal budget constraints in appendix C.

2.2.6 General Equilibrium

In this section I will discuss the solution of the partial trade equilibrium on the balanced growth path. The system of equations and the corresponding solutions are given in appendix D.

Growth Rate Again, I will first discuss the growth rate of the economy which is given by

$$g^W = \frac{L}{2F}\left[(l_A + l_B) - b(2+\tau) + \sqrt{(l_A - l_B + b\tau)^2 + 4(l_A - b\tau^2)l_B}\right] \qquad (21)$$

Note that $g^W > 0$ if and only if $(l_A+l_B)+\sqrt{(l_A - l_B + b\tau)^2 + 4(l_A - b\tau^2)l_B} > b(2+\tau)$.

Again, the world growth rate is independent of preference parameters like the subjective discount rate ρ, and the intertemporal elasticity of substitution $1/\sigma$. I will not discuss the intuition again here since it is identical to the one given in section 2.1.6 (as is the proof).

Note that

$$\frac{\partial g^W}{\partial \tau} \gtrless 0 \Leftrightarrow 2\tau l_B - l_A \gtrless \frac{b\tau^2}{1+2\tau}$$

The effect of transport costs τ on the world growth rate is ambiguous. However, the case where $2\tau l_B < l_A$ is more likely since we are in an equilibrium where inequality is rather high. Thus, it is likely that an increase in transport costs lowers the world growth rate. The intuition is the same as in section 2.1.6.

Further note that

$$\begin{aligned} \frac{\partial g^W}{\partial l_A} &> 0 \\ \frac{\partial g^W}{\partial l_B} &< 0. \end{aligned}$$

Thus, increasing the efficiency units of labor of households in country A, l_A, ceteris paribus increases the world growth rate whereas an increase in the efficiency units of labor of households in country B, l_B, ceteris paribus decreases the world growth rate. This already hints that higher inequality in per capita incomes might have a growth enhancing effect. However, increasing l_A or decreasing l_B does not only increase inequality but also increases, respectively decreases, the aggregate amount of efficiency units in the world. In order to confirm my hunch I perform a mean preserving spread identical to the one in section 2.1.6. I use $l_A = L^W/L - l_B$ to write the world growth rate as

$$g^W = \frac{L}{2F}\left[\frac{L^W}{L} - b(1+\tau) + \sqrt{\left(\frac{L^W}{L} - 2l_B + b\tau\right)^2 + 4\left(\frac{L^W}{L} - l_B - b\tau^2\right) l_B}\right].$$

Differentiating the expression for the world growth rate with respect to l_B yields $\partial g^W/\partial l_B < 0$. Thus, a mean preserving spread which decreases inequality by increasing l_B (redistributing efficiency units of labor from country A to B while keeping the aggregate amount of efficiency units in the world constant) decreases the world growth rate. The intuition behind this result is the same as in Foellmi and Zweimüller (2006). Inequality in per capita income works through a market size effect and a price effect on the world growth rate. The price effect works as follows. If inequality in per capita income is high monopolistic firms entering the market with a new differentiated product can charge high prices. Due to discounting this increases the present discounted value of profits, and thus the value of monopolistic firms. The market size effect means that with higher inequality in per capita incomes there is a small mass market (a larger market means that monopolistic firms can produce at lower average costs due to increasing returns to scale) which decreases the incentive to innovate. In sum, the price effect dominates the market size effect, so that higher inequality in per capita incomes increases the world growth rate[3].

As in the previous section 2.1.6 the population size L has a positive effect on the world growth rate as before due to increasing returns to scale. An increase in the amount of labor needed to create a new blueprint, F, or to produce one unit of final output, b, decreases the world growth rate.

Ratio of Varieties Produced in Country B to Country A The measure of differentiated goods produced in country B relative to the measure of differentiated goods produced in country A, $n = N^B/N^A$, is given by

$$n = \frac{-(l_A - l_B + b\tau)^2 + \sqrt{(l_A - l_B + b\tau)^2 + 4(l_A - b\tau^2)l_B}}{2(l_A - b\tau^2)}$$

where $l_A > b\tau^2$. One can show that a mean preserving spread yields the following condition

$$\frac{\partial n}{\partial l_B} \gtrless 0 \Leftrightarrow l_A + l_B \gtrless 2 + b\tau(1 + 2\tau).$$

The effect of a mean preserving spread on the ratio of varieties produced in country B to A is ambiguous.

2.2.7 Discussion

Summarizing the results I note that an increase in inequality unambiguously increases the world growth rate. The positive price effects of higher inequality outweigh the negative market size effects. The effect of inequality on the volume of trade is negative. Increasing the inequality reduces the number of varieties produced in country A that are traded thereby lowering the volume of trade.

3 Conclusion

I studied a simple dynamic, two-country model of international trade and endogenous growth where agents have nonhomothetic preferences. I find that the model exhibits

[3] For a detailed discussion on inequality and growth see Foellmi and Zweimüller (2006).

multiple steady states. In a steady state where all differentiated goods worldwide are traded inequality has no effect on the growth rate of the world economy and the volume of trade. In a steady state where only part of all varieties are traded I find that inequality has a positive effect on the world growth rate and a negative effect on the volume of trade.

Future research will be directed towards a welfare analysis, and an investigation of the dynamic properties of the model discussed above.

A Appendix Full Trade Equilibrium: Budget Constraints

The intertemporal budget constraints of the representative households in country A and B, respectively, are given by

$$\begin{aligned}
& \int_0^\infty \int_0^{N^A(t)} p_A^A(j,t)c_A^A(j,t)dje^{-R^A(t)}dt + \int_0^\infty \int_0^{N^B(t)} p_A^B(j,t)c_A^B(j,t)dje^{-R^A(t)}dt \\
= & \int_0^\infty w^A(t)l_A e^{-R^A(t)}dt + V_A(0) \\
& \int_0^\infty \int_0^{N^B(t)} p_B^B(j,t)c_B^B(j,t)dje^{-R^B(t)}dt + \int_0^\infty \int_0^{N^A(t)} p_B^A(j,t)c_B^A(j,t)dje^{-R^B(t)}dt \\
= & \int_0^\infty w^B(t)l_B e^{-R^B(t)}dt + V_B(0)
\end{aligned}$$

where $V_i(0)$ is the initial wealth endowment of a household in $i = A, B$. The initial wealth endowment is equal to the value of the household's equity holdings. The value of shares at time zero is equal to the market value of monopolistic firms at time zero. All households hold equal shares in monopolistic firms. The market value of monopolistic firms in country i is equal to the discounted value of their future profits at time zero. Imposing the conjectured balanced growth path the budget constraints become

$$\begin{aligned}
p^A &= \frac{l_A}{b} + \frac{V_A(0)(r-g^W)}{N(0)} \\
p^B &= \frac{l_B}{b} + \frac{V_B(0)(r-g^W)}{N(0)}.
\end{aligned}$$

The zero-profit conditions imply that the market value of monopolistic firms equals the cost of creating a new design, F/b. Hence the intertemporal budget constraints can be written as

$$\begin{aligned}
p^A &= \frac{l_A}{b} + \frac{F}{bL}\frac{(r-g^W)}{1+n(0)} \\
p^B &= \frac{l_B}{b} + \frac{F}{bL}\frac{n(0)(r-g^W)}{1+n(0)}.
\end{aligned}$$

B Appendix Full Trade Equilibrium: System of Equations and Solutions

The system of equations is given by

$$\begin{aligned}
p^A &= \frac{1}{\lambda_A} && (22)\\
p^B &= \frac{1}{\lambda_B} && (23)\\
r &= \rho + \sigma g^W && (24)\\
\frac{F}{b} &= \frac{\left[p^A - 1\right] L + \left[p^B - \tau\right] L}{r} && (25)\\
\frac{F}{b} &= \frac{\left[p^A - \tau b \frac{w^B(t)}{N(t)}\right] L + \left[p^B - b \frac{w^B(t)}{N(t)}\right] L}{r} && (26)\\
l_A L &= \frac{1}{1+n}\left[F g^W + bL(1+\tau)\right] && (27)\\
l_B L &= \frac{n}{1+n}\left[F g^W + bL(1+\tau)\right] && (28)\\
n &= \frac{p^B}{p^A}. && (29)
\end{aligned}$$

Equations (22)-(29) form a system of eight equations in the eight unknowns λ_A, λ_B, r, $p^A, p^B, g^W, w^B(t)/N(t), n$. The corresponding solutions are derived as

$$\begin{aligned}
\lambda_A &= \left[\sigma \frac{l_A}{b} + \frac{l_A}{l_A + l_B}\left[\frac{F\rho}{bL} - (\sigma - 1)(1+\tau)\right]\right]^{-1} && (30)\\
\lambda_B &= \left[\sigma \frac{l_B}{b} + \frac{l_B}{l_A + l_B}\left[\frac{F\rho}{bL} - (\sigma - 1)(1+\tau)\right]\right]^{-1} && (31)\\
r &= \rho + \sigma \frac{L}{F}[(l_A + l_B) - b(1+\tau)] && (32)\\
p^A &= \sigma \frac{l_A}{b} + \frac{l_A}{l_A + l_B}\left[\frac{F\rho}{bL} - (\sigma - 1)(1+\tau)\right] && (33)\\
p^B &= \sigma \frac{l_B}{b} + \frac{l_B}{l_A + l_B}\left[\frac{F\rho}{bL} - (\sigma - 1)(1+\tau)\right] && (34)\\
g^W &= \frac{L}{F}[(l_A + l_B) - b(1+\tau)] && (35)\\
\frac{w^B(t)}{N(t)} &= 1 && (36)\\
n &= \frac{l_B}{l_A}. && (37)
\end{aligned}$$

C Appendix Partial Trade Equilibrium: Budget Constraints

The intertemporal budget constraints of the representative household in country A and B, respectively, are given by

$$
\begin{aligned}
& \int_0^\infty \int_0^{N_T^A(t)} \tau p^B(j,t) c_A^{A,T}(j,t) dj e^{-R^A(t)} dt + \int_0^\infty \int_0^{N_{NT}^A(t)} p_{NT}^A(j,t) c_A^{A,NT}(j,t) dj e^{-R^A(t)} \\
+ \quad & \int_0^\infty \int_0^{N^B(t)} \tau p^B(j,t) c_A^B(j,t) dj e^{-R^A(t)} dt = \int_0^\infty w^A(t) l_A e^{-R^A(t)} dt + V_A(0) \\
& \int_0^\infty \int_0^{N^B(t)} p^B(j,t) c_B^B(j,t) dj e^{-R^B(t)} dt + \int_0^\infty \int_0^{N_T^A(t)} p^B(j,t) c_B^{A,T}(j,t) dj e^{-R^B(t)} dt \\
= \quad & \int_0^\infty w^B(t) l_B e^{-R^B(t)} dt + V_B(0).
\end{aligned}
$$

Imposing the conjectured balanced growth path the budget constraints can be written as follows

$$
\begin{aligned}
\tau p^B \left[N_T^A(0) + N^B(0)\right] + p_{NT}^A N_{NT}^A(0) &= \frac{l_A}{b} N(0) + V_A(0)\left(r - g^W\right) \\
p^B \left[N_T^A(0) + N^B(0)\right] &= \frac{l_B}{b} N(0) + V_B(0)\left(r - g^W\right).
\end{aligned}
$$

Using the zero-profit conditions to determine $V_A(0)$ and $V_B(0)$ the budget constraints can be rewritten as

$$
\begin{aligned}
\tau p^B \left[N_T^A(0) + N^B(0)\right] + p_{NT}^A N_{NT}^A(0) &= \frac{l_A}{b} N(0) + \frac{F N^A(0)}{bL}\left(r - g^W\right) \\
p^B \left[N_T^A(0) + N^B(0)\right] &= \frac{l_B}{b} N(0) + \frac{F N^B(0)}{bL}\left(r - g^W\right).
\end{aligned}
$$

D Appendix Partial Trade Equilibrium: System of Equations and Solutions

The system of equations is given by

$$
\begin{aligned}
p_{NT}^{A} &= \frac{1}{\lambda_A} && (38)\\
p^{B} &= \frac{1}{\lambda_B} && (39)\\
r &= \rho + \sigma g^{W} && (40)\\
\frac{F}{b} &= \frac{\left[p_{NT}^{A} - 1\right] L}{r} && (41)\\
\frac{F}{b} &= \frac{\left[\tau p^{B} - 1\right] L + \left[p^{B} - \tau\right] L}{r} && (42)\\
\frac{F}{b} &= \frac{\left[\tau p^{B} - \tau b \frac{w^{B}(t)}{N(t)}\right] L + \left[p^{B} - b \frac{w^{B}(t)}{N(t)}\right] L}{r} && (43)\\
l_A L &= \frac{F g^{W}}{1+n} + \frac{bL}{1+n} + bL\tau \frac{N_T^A}{N} && (44)\\
l_B L &= \frac{n}{1+n}\left[F g^{W} + bL(1+\tau)\right] && (45)\\
\tau &= \frac{N_T^A}{N^B}. && (46)
\end{aligned}
$$

Equations (38)-(46) form a system of nine equations in the nine unknowns λ_A, λ_B, r, $p_{NT}^{A}, p^{B}, g^{W}, w^{B}(t)/N(t), n, N_T^A/N^B$. The corresponding solutions below are derived for the case[4] of $l_A > b\tau^2$. The solutions are given by

$$\lambda_A = \left(p_{NT}^A\right)^{-1} \tag{47}$$

$$\lambda_B = \left(p^B\right)^{-1} \tag{48}$$

$$r = \rho + \frac{\sigma L}{2F}\left[(l_A + l_B) - b(2+\tau) + \sqrt{(l_A - l_B + b\tau)^2 + 4(l_A - b\tau^2)l_B}\right] \tag{49}$$

$$p_{NT}^A = \frac{bL + F\rho}{bL} + \frac{\sigma}{2b}\left[(l_A + l_B) - b(2+\tau) + \sqrt{(l_A - l_B + b\tau)^2 + 4(l_A - b\tau^2)l_B}\right] \tag{50}$$

$$p^B = \frac{bL(1+\tau) + F\rho}{(1+\tau)bL} + \frac{\sigma}{2b(1+\tau)}\left[(l_A + l_B) - b(2+\tau) + \sqrt{(l_A - l_B + b\tau)^2 + 4(l_A - b\tau^2)l_B}\right] \tag{51}$$

$$g^W = \frac{L}{2F}\left[(l_A + l_B) - b(2+\tau) + \sqrt{(l_A - l_B + b\tau)^2 + 4(l_A - b\tau^2)l_B}\right] \tag{52}$$

$$\frac{w^B(t)}{N(t)} = 1 \tag{53}$$

$$n = \frac{-(l_A - l_B + b\tau)^2 + \sqrt{(l_A - l_B + b\tau)^2 + 4(l_A - b\tau^2)l_B}}{2(l_A - b\tau^2)} \tag{54}$$

$$\frac{N_T^A}{N^B} = \tau. \tag{55}$$

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[4] There is a second set of solutions for the case of $l_A < b\tau^2$. So far I have not been able to determine the circumstances under which one or both cases emerge. I suspect that the relevant case is the one discussed in the text since the condition $l_A > b\tau^2$ is reminiscent of a condition determining if country A is trading with country B at all. However, there might be multiple balanced growth paths in this partial trade equilibrium.

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