Conjectural Cost Variations in a Differentiated Good Oligopoly*

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Abstract

In the homogenous good case, the relationship between market structure and efficiency was studied extensively. Assuming a standard quadratic utility with quantity competition, this paper carries on the analysis in a differentiated good context. It can be shown that there is a positive relationship between market heterogeneity and efficiency, too. In contrast to the homogenous good case, consumer surplus as well as producer surplus increases with the dispersion of marginal costs.

JEL classification: L13, L11, L4

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1 Introduction

This paper analyzes the relationship between efficiency and market heterogeneity in a differentiated good oligopoly. Market heterogeneity is caused by differently efficient firms. The pivotal question is whether society is better off in case of a more heterogeneous market structure or not. Assuming standard quadratic utility according to Dixit, social surplus is the measure for Pareto-optimality since preferences are quasi-linear. The impact of a conjectural marginal cost variation on consumer surplus as well as producer surplus and therefore social surplus is analyzed. An arbitrarily marginal cost variation is decomposed into an average component and a heterogeneity component. The former increases or decreases all marginal costs to the same degree. The latter increases or decreases the dispersion of marginal costs and lets average marginal costs unchanged.

In the homogenous good case there is a positive relationship between market heterogeneity and efficiency. Consumer surplus solely depends on aggregated output which in turn only depends on average marginal costs. Total cost of production, however, decreases with the dispersion of marginal costs. Since total revenue (equal to aggregated expenditure) is constant, producer surplus increases with the dispersion of marginal costs. Hence, there is a positive relationship between market heterogeneity (given by the distribution of marginal costs) and efficiency in the homogenous good case. In case of differentiated goods consumer surplus not only depends on aggregated output but also on its distribution. The goods are not perfectly substitutable and marginal utility of each good diminishes. Therefore, consumers prefer the differentiated goods in equal quantity. Hence, gross utility decreases with the diversity of the goods if aggregated output is constant. Since the willingness to pay for each good does not only depend on aggregated quantity but also on its distribution, aggregated expenditures (equal to total revenue) varies in case of a mean preserving cost variation. In contrast to the homogenous good case total revenue (equal to total expenditures) is not constant in case of a mean preserving cost variation. Gross utility, aggregated expenditures, total revenue and total cost of production changes. Hence, the relationship between market heterogeneity and consumer surplus as well as producer surplus and therefore social surplus is ambiguous. Furthermore, there may be additional inefficiencies due to firms exercising their market power since goods are no longer perfect substitutes. One would expect that at least consumers should be worse off in more heterogeneous market structures.

It can be shown, however, that the exact opposite is true. Diminishing total expenditures outweigh declining gross utility. Consequently, consumer surplus increases with the dispersion of marginal costs and vice versa. Declining total costs of production overcompensate sales collapse. Thus, producer surplus increases with the dispersion of marginal costs, too. Since consumers and producers are better off in case of a mean preserving conjectural cost variation there remains a positive relationship between market heterogeneity and efficiency as in the homogenous good case.

In the context of homogenous goods there is a huge amount of literature analyzing the relationship between market structure and producer surplus as well as consumer surplus (thus welfare). Dixit and Stern (1982) analyze a homogenous good oligopoly with iso-elastic demand. They show that equilibrium prices depend on average marginal costs and decrease with the number of firms and elasticity of demand. Industry profits are increasing with the Herfindahl-Hirschman Index. Market concentration (hence industry profits) increases in case of a cost reduction of a single firm if the respective firm is more efficient than the average firm. Consumers benefit from this cost reduction. Dixit and Stern allow for different reaction functions including the Cournot case. Farrell and Shapiro (1990) consider a homogenous Cournot oligopoly and analyze the relationship between market concentration and welfare. They show that even a (conjectural) reduction of the output of a single firm increases welfare if the market concentration measured by the Herfindahl-Hirschman Index increases sufficiently. This is due to a shift in production from less efficient to more efficient firms. Kimmel (1992) analyzes the impact of an increase of all marginal costs on equilibrium profits and the market price in context of homogenous goods. While consumers are always worse off, the equilibrium profit of a firm increases if inverse demand is sufficiently concave (convex) and respective market share is sufficiently small (big). Salant and Shaffer (1999) use the results from Bergstrom and Varian (1985) and show that aggregate cost of production strictly decreases with the variance of marginal costs. Since gross revenue is invariant, industry profits increase while consumer surplus remains unchanged. Van Long and Soubeyran (2001) show that aggregated profits are an increasing function of the dispersion of marginal costs if average marginal costs are constant. Since aggregate output and consumer surplus remains unchanged, social welfare increases with the dispersion of marginal costs too. Furthermore there is a stringent (inverse) relationship between the market concentration measured by the Herfindahl-Hirschman index and the distribution of marginal costs. Février and Linnemer (2004) analyze the impact of an arbitrary marginal cost variation on consumer surplus, producer surplus and welfare as well as on market concentration in a homogenous Cournot oligopoly in an extensive manner. They replicate the results of the aforementioned papers and allow for a simultaneous change of all marginal costs. The effect of an arbitrary cost variation on the variables of interest is decomposed into an average impact and a heterogeneity impact.

Lahiri and Ono (1988) show that a reduction of the marginal costs of a single firm may reduce welfare if respective firm is relatively inefficient. They also show that closing down a sufficiently inefficient firm increases social surplus. Zhao (2001) continues the analysis of Lahiri and Ono (1988) and derives threshold values for marginal cost and respective market shares such that a cost reduction reduces welfare. Smythe and Zhao (2006) refine the analysis of Zhao (2001) and allow for nonlinear demand and nonlinear costs as well as technological spill-over. Wang and Zhao (2007) extend the analysis of Lahiri and Ono (1988) and Zhao (2001) in a differentiated good context. Assuming a utility originated by Shubik (1980) they derive conditions under which marginal cost reductions reduce welfare in Cournot and Bertrand competition.

Even though most of the goods are not perfectly substitutable, there are only a few studies analyzing the relationship between efficiency and market heterogeneity in a differentiated good context. Assuming Dixit-utility, Singh and Vives (1984) compare equilibrium prices under Bertrand and Cournot competition in a differentiated good duopoly. They show that consumer surplus and social surplus are higher under Bertrand competition whereas producer surplus is higher under Cournot (Bertrand) competition if the goods are substitutes (complements). Häckner (2000) continues the analysis of Singh and Vives (1984) and shows that duopoly results do not hold generally in the oligopoly case. Koh (2008) assumes a Dixit-utility and analyzes a symmetric oligopoly with fixed cost under Bertrand and Cournot competition. He shows that profits are always lower under Bertrand competition and derives conditions depending on the fixed cost under which there is excessive entry. Zanchettin (2006) investigates an asymmetric differentiated good duopoly allowing for quality and cost asymmetries. Depending on the degree of substitutability he derives conditions under which (industry) profits are higher under Cournot compared to Bertrand competition. Symeonidis (2003) analyzes the impact of quality heterogeneity on consumer surplus and producer surplus thus on social welfare in a vertically differentiated good context. Assuming a Dixit-utility he finds that consumer surplus as well as producer surplus and therefore social welfare increase with the quality heterogeneity if the average quality is unvaried. The market heterogeneity is caused only by quality differences since firms are assumed to have identical cost functions.

The aim of the paper is to analyze the relationship between efficiency and market structure in a differentiated good oligopoly in an extensive manner. Firms are assumed to compete in quantities and have constant return to scale without fixed cost. The impact of an arbitrary marginal cost variation is decomposed into an average and a heterogeneity impact. While the former influences all firms in equal manner, the latter is a mean preserving cost variation. Furthermore the effect of a cost variation on social surplus is decomposed into its components consumer surplus and producer surplus. The results are contrasted to the homogeneous good case.

This paper is organized as follows: the following section describes the framework of the model. Section 3 presents the central results. Section 4 finally concludes.

2 The model

Consider an oligopoly consisting of $n \geq 2$ firms competing in quantities. Each firm produces one differentiated good Q_i with $i = 1, \dots, n$. Abstracting from fixed cost, each firm incurs constant marginal cost c_i. Let q_i denote the quantity produced by firm $i = 1, \dots, n$. The quasi-linear preferences of the representative household are described by a quadratic utility according to Dixit (1979). Firm $i = 1, \dots, n$ faces the following inverse demand:

$$p_i = 1 - q_i - \nu Q_{-i} \tag{2.1}$$

$Q_{-i} := \sum_{j \neq i} q_j$ denotes aggregated output of the competitors of firm $i = 1, \dots, n$ and ν denotes the parameter of substitution. In case of $\nu > 0$ goods are substitutes and in case of $\nu < 0$ goods are complements. For $\nu = 0$ the goods are independent. To secure that utility is concave the parameter of substitution is assumed to be $\nu \in \left(-\frac{1}{n-1}, 1\right)$. For further insight see appendix A. Each firm maximizes its profit choosing an optimal quantity. Let Q^* denote aggregated output in equilibrium. Summing up all first order conditions given by $1 - 2q_i^* - \nu Q_{-i}^* - c_i = 0$ and solving for Q^* yields:

$$Q^* = \frac{n(1-\overline{c})}{2+\nu(n-1)} \tag{2.2}$$

Let $\overline{c} := \frac{1}{n}\sum_{i=1}^{n} c_i$ denote average efficiency which is assumed not to exceed 1. Comparable to the homogenous good case, aggregated output depends just on the average of marginal costs and not on its distribution. Industry output Q^* is unchanged in case of a mean preserving cost variation. Since goods are differentiated, however, the (heterogeneity) impact of a mean preserving cost variation on consumer surplus is different to the homogenous good case. I will come back to this point later. In contrast to aggregated output the derivation of equilibrium output q_i^* is little more tricky. The derivation is delegated to the appendix.

Lemma 2.1 (Equilibrium output) *Equilibrium output of firm $i = 1, \ldots, n$ is given as follows:*

$$q_i^* = \frac{(2-\nu) - [2+\nu(n-2)]c_i + \nu \sum_{j \neq i} c_j}{(2-\nu)[2+\nu(n-1)]}$$

Intuitively equilibrium output is in reverse proportion to its marginal costs and increases with the sum of competitors marginal costs irrespective its distribution. As shown in the appendix, corresponding equilibrium price p_i^* is given by $p_i^* = q_i^* + c_i$. Comparable to the homogenous good case, equilibrium profit $\Pi_i^* := (p_i^* - c_i)q_i^*$ equals its squared quantity.

$$\Pi_i^* = (q_i^*)^2 \tag{2.3}$$

Since entry or exit is not subject of investigation I assume $p_i^* - c_i = q_i^* > 0$ for $i = 1, \ldots, n$. Solving $q_i^* > 0$ for c_i yields the expression is the following assumption:

Assumption 2.1 (Oligopoly of n firms) *To ensure an oligopoly consisting of n firms, I assume $q_i^* > 0$ for $i = 1, \ldots, n$ which is equivalent to the following inequality:*

$$c_i < \frac{2-\nu}{2+\nu(n-2)} + \frac{\nu}{2+\nu(n-2)} \sum_{j \neq i} c_j$$

Note that in case of substitutes assumption 2.1 requires marginal costs not to exceed 1 (equal to the maximum willingness to pay). In case of complements, however, marginal cost may exceed 1 if rivals are sufficiently efficient.

In case of complements the willingness to pay for a good increases with the consumption of rivals' output which in turn is in reverse proportion to respective marginal costs.

3 Results

In the following, the central results concerning producer surplus, consumer surplus and social surplus are presented. In the terminology of Février and Linnemer (2004) the impact of an arbitrary conjectural marginal cost variation on the aforementioned variables is decomposed into an average and a heterogeneity impact. In contrast to Février and Linnemer these impacts are not analyzed simultaneously and I analyze no subgroup shocks. Analytically, the average impact and the heterogeneity impact are given by directional derivatives. The average effect reduces (increases) marginal cost of all firms to the same degree while the variance is constant. The heterogeneity effect, however, comprises the reduction of the marginal cost of a single firm. In return the marginal cost of another firm increases to the same degree. The heterogeneity component increases or decreases the variance of marginal costs while average efficiency is unchanged.

Definition 3.1 (Average and heterogeneity impact) *Let* AIF *denote the average impact and* HIF *the heterogeneity impact on F. In this study F is given by producer surplus* PS, *consumer surplus* CS *and social surplus* W. *The total derivative of F is given by $dF = \sum_{k=1}^{n} \frac{\partial F}{\partial c_k} dc_k$. The average impact is characterized by $dc_1 = \ldots = dc_n = dc$. Without loss of generality the heterogeneity impact is given by a conjectural variation of c_k and c_l with $k < l$ and $dc_k = -dc_l > 0$.* AIF *and* HIF *are given as follows:*

$$\text{AIF} := \sum_{i=1}^{n} \frac{\partial F}{\partial c_i} \qquad \text{HIF} := \frac{\partial F}{\partial c_k} - \frac{\partial F}{\partial c_l}$$

Note that the 'directions' $dc_1 = \ldots = dc_n$ and $dc_k = -dc_l$ just equal the Eigenvectors of the matrix of coefficients characterizing the Cournot-Nash equilibrium given by (B.2).

3.1 Producer surplus

In the following, the relationship between producer surplus and market structure is analyzed. Producer surplus $\mathrm{PS}^* := \sum_i \Pi_i^*(q_i^*, Q_{-i}^*)$ is just the sum of all equilibrium profits.

Proposition 3.1 (Average Impact) *The average impact on equilibrium profit of firm $i = 1, \ldots, n$ and producer surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation.*

Proof: Due to linearity, the average impact on producer surplus is just the sum of the average impact on Π_i^*. $\mathrm{AIPS}^* = \sum_j \mathrm{AIPS}_j^*$ with $\mathrm{AIPS}_j^* = \sum_i \partial_i \left(q_j^*\right)^2$ since $\Pi_i^* = \left(q_i^*\right)^2$. It holds:

$$\mathrm{AIPS}_j^* = \sum_i \partial_i \left(q_j^*\right)^2 = 2q_j^* \sum_i \partial_i q_j^*$$

$$= 2q_j^* \left(\frac{-[2+\nu(n-2)] + \nu(n-1)}{(2+\nu)[2+\nu(n-1)]} \right) \quad (3.1)$$

$$= \frac{-2q_j^*}{2+\nu(n-1)} < 0 \quad (3.2)$$

The average impact on producer surplus is just the sum of all AIPS_j^*.

$$\mathrm{AIPS}^* = \sum_j \frac{-2q_j^*}{2+\nu(n-1)} = \frac{-2Q^*}{2+\nu(n-1)} < 0 \quad (3.3)$$

All firms are worse off in case of a cost variation making all firms less efficient and vice versa. □

The average impact on equilibrium profit has two opposite components. On the one hand making all competitors more efficient has a negative effect on the equilibrium profit, since all substitutes of the product are getting cheaper and, therefore, more attractive. This effect is given by $\nu(n-1)$ in (3.1). On the other, hand each firm benefits by a reduction of its marginal cost. This effect is given by $-[2+\nu(n-2)]$ in (3.1). The latter effect, however, outweighs the former effect. The profit of each firm increases in case of a cost variation decreasing all marginal costs and vice versa.

This result coincides with the homogenous good case since producer surplus decreases if all firms are negatively affected unless market concentration is sufficiently high and inverse demand is sufficiently concave. Since inverse demand is linear in this model, firms are always worse off increasing all marginal costs. In the homogenous good case a firm benefits by an increase of all marginal costs if its market share is sufficiently big and inverse demand sufficiently concave. This is due to a shift in production from the inefficient to the efficient firms. Compare Seade (1985), Kimmel (1992) or Février and Linnemer (2004).

In the following, the dispersion of marginal costs is varied while keeping average efficiency constant. The results concerning the heterogeneity impact on equilibrium profit and producer surplus are summarized in the following proposition.

Proposition 3.2 (Heterogeneity Impact) *Producer surplus increases with the dispersion of marginal costs and vice versa.*

Proof: According to (2.3) equilibrium profit is given by $\Pi_i^* = (q_i^*)^2$. The heterogeneity impact $\mathrm{HIQ}^* := \partial_k q_i^* - \partial_l q_i^*$ on equilibrium output q_i^* is given as follows:

$$\mathrm{HIQ}_i^* = \begin{cases} \frac{-1}{2-\nu}, & \text{for } i = k, \\ \frac{1}{2-\nu}, & \text{for } i = l, \\ 0, & \text{else.} \end{cases} \tag{3.4}$$

Intuitively equilibrium output of the firm which is positively (negatively) affected by the cost variation increases (decreases). The heterogeneity impact on the equilibrium profit of the unaffected firms $i \neq k, l$ is zero. Therefore, the heterogeneity impact on producer surplus is composed of the heterogeneity impacts on Π_k^* and Π_l^*.

$$\begin{aligned} \mathrm{HIPS}^* &= \mathrm{HIPS}_k^* + \mathrm{HIPS}_l^* \\ &= \left[\partial_k (q_k^*)^2 - \partial_l (q_k^*)^2\right] + \left[\partial_k (q_l^*)^2 - \partial_l (q_l^*)^2\right] \qquad (3.5) \\ &= 2q_k^* \,\mathrm{HIQ}_k^* + 2q_l^* \,\mathrm{HIQ}_l^* \\ &\overset{(3.4)}{=} 2\,\mathrm{HIQ}_k^* (q_k^* - q_l^*) \qquad (3.6) \end{aligned}$$

Equilibrium quantity is in reverse proportion to efficiency.

$$
\begin{aligned}
q_k^* - q_l^* &= \frac{-[2+\nu(n-2)](c_k - c_l) + \nu(c_l - c_k)}{(2+\nu)[2+\nu(n-1)]} \\
&= \frac{-1}{2-\nu}(c_k - c_l)
\end{aligned}
\tag{3.7}
$$

Inserting (3.7) in (3.6) yields:

$$
\text{HIPS}^* = \frac{2}{(2-\nu)^2}(c_k - c_l) \tag{3.8}
$$

Producer surplus increases in case of a cost variation increasing the dispersion of marginal costs and vice versa. □

Intuitively, the firm which is advantaged by the cost variation profits and the disadvantaged firm looses. Reducing the marginal cost of a firm increases its equilibrium output as well as its price-cost margin since $p_i^* - c_i = q_i^*$. The heterogeneity effect on the more efficient firm outweighs the effect on the less efficient one. Therefore, producer surplus increases with the dispersion of marginal costs.

The heterogeneity impact on equilibrium profit and producer surplus coincides with the homogenous good case. Compare Bergstrom and Varian (1985) or Février and Linnemer (2004). This result, however, is not self-evident. In contrast to the homogenous good case, the heterogeneity impact on total revenue is not constant but falls with the diversity of marginal costs. It can be shown, however, that the effect on total costs overcompensates the effect on total revenue. I will get back to this later.

Furthermore, the results coincide with those of Symeonidis (2003). Assuming Dixit-utility he analyzes a vertically differentiated good oligopoly. He finds that industry profits under Cournot competition increase with the dispersion of quality levels if average quality is constant.

The heterogeneity impact on producer surplus can be explained by another point of view: in the following, the heterogeneity effect on its components, total revenue and total cost, is analyzed. In contrast to the homogenous good case, gross revenue decreases with the dispersion of marginal costs.

Lemma 3.1 (Total revenue versus total cost) *Both total revenue and total cost decreases with the disparity of marginal costs. The heterogeneity impact* *on total cost, however, outweighs the heterogeneity impact on total revenue. Hence, producer surplus increases with the disparity of marginal costs.*

In the homogenous good case producer surplus increases with the dispersion of marginal cost, since gross revenue (equal to total expenditure) is unchanged and total cost decrease with the disparity of marginal cost. Compare Salant and Shaffer (1999) for instance. Thus, in the homogeneous good case as well as in the differentiated good context producer surplus increases with market heterogeneity.

3.2 Consumer surplus

Are consumers better off in a more heterogeneous market structure characterized by some big and several small firms? Does a more homogeneous market structure solely consisting of equipollent firms involve more favorable conditions? Consumer surplus caused by the consumption of the goods q_i^* with $i = 1, \ldots, n$ is defined as follows: $\mathrm{CS}^* := U(m - \sum_{i=1}^n p_i^* q_i^*, q_1^*, \ldots, q_n^*) - U(m, 0, \ldots, 0)$. The consumption of the numeraire good q_0 is given by $q_0^* = m - \sum_{i=1}^n p_i^* q_i^*$. Let m denote the income of the representative household which is assumed to be exogenous. In the following the average effect on consumer surplus is analyzed.

Proposition 3.3 (Average Impact) *Consumer surplus decreases with average marginal costs and vice versa.*

A reduction of all marginal costs increases all equilibrium quantities and, therefore, consumers are unambiguously better off. This result again coincides with the homogenous good case. Compare Février and Linnemer (2004) for instance. In case of homogenous goods consumer surplus increases with industry output which again is negatively correlated with average efficiency.

In the following, the relationship between the dispersion of marginal costs and consumer surplus is analyzed. Are there inefficiencies due to firms exercising their market power in highly concentrated markets? Since goods are not perfectly substitutable, firms have more market power to enforce higher price-cost margins. As shown above, the price-cost margin increases with efficiency. Compare (3.4) and (2.3). Since marginal utility decreases, consumers prefer the goods in equal quantity if aggregated output is constant. Indeed, gross utility decreases with the dispersion of marginal costs. Therefore, the results concerning the heterogeneity impact on consumer surplus are surprising.

Proposition 3.4 (Heterogeneity Impact) *Consumer surplus increases with the dispersion of marginal costs.*

In case of differentiated goods a more heterogeneous market structure is favorable not only for producers but also for consumers. Although price-cost margin increases with efficiency and variance of equilibrium output increases, consumers are better off in case of heterogeneous market structures. In the limit case of perfect substitutes (i.e. $\nu \to 1$) the result coincides with classical homogenous good models. Consumer surplus solely depends on industry output which again depends on average efficiency. Compare Février and Linnemer (2004), for instance.

This result also corresponds with the insight of Symeonidis (2003). Assuming a Dixit-utility he finds that in a vertically differentiated good oligopoly producer surplus as well as consumer surplus increase with the variance of the quality levels if average quality is constant.

The heterogeneity impact on consumer surplus can be explained by decomposing the effect on its components: gross utility and total expenditure. Since households' expenditures just equal gross revenue, the results concerning firms revenue given by (D.6) can be employed for this analysis. It remains to analyze the heterogeneity impact on gross utility.

Lemma 3.2 (Total expenditure versus gross utility) *Total expenditures as well as gross utility decrease with the disparity of marginal costs. The heterogeneity impact on total expenditure, however, outweighs the effect on gross utility. Therefore, consumer surplus increases with the dispersion of marginal cost.*

This result is essentially different to the homogenous good case since gross utility as well as total expenditures decrease with market heterogeneity. Ultimately, consumers are better off in more heterogeneous market structures. In the following the heterogeneity effect on consumer surplus is analyzed by another point of view. Consumer surplus is just the sum of the net benefits of each single commodity. Let CS_i denote the net utility caused by the consumption of good $i = 1, \ldots, n$:

$$\mathrm{CS}_i := q_i - \frac{1}{2}q_i^2 - \frac{\nu}{2}q_i Q_{-i} - p_i q_i$$

The term $q_i - \frac{1}{2}q_i^2$ reflects the direct utility caused by the consumption of commodity q_i^*. The term $\frac{\nu}{2}q_i Q_{-i}$ describes the additional utility (or disutility) caused by simultaneous consumption of the other commodities. Associated expenditures are given by $p_i q_i$. It is easy to prove that consumer surplus CS is just aggregated net utility of all n goods.

Obviously, the net utility of the non-affected goods is unchanged in case of mean preserving cost variation since aggregated concurrence output is unchanged and according to (3.4) the heterogeneity impact on non-affected quantities and equilibrium prices is zero. Due to linearity, the heterogeneity impact on consumer surplus is the sum of heterogeneity impacts on the affected goods.

Lemma 3.3 (Net utility of a single commodity) *The net utility of a single commodity is in reverse proportion to its marginal costs. The absolute value of the heterogeneity effect is proportional to efficiency. The effect on the more efficient firm outweighs the effect on the less efficient one. Therefore, consumer surplus increases with the dispersion of marginal costs.*

Proof: Consumer surplus can be expressed as follows:

$$\begin{aligned} \mathrm{CS}^* =& q_k - \frac{1}{2}q_k^2 - \frac{\nu}{2} q_k Q_{-k} - p_k q_k && (3.9)\\ &+ q_l - \frac{1}{2}q_l^2 - \frac{\nu}{2} q_l Q_{-l} - p_l q_l && (3.10)\\ &+ \sum_{j\neq k,l} \left(q_j - \frac{1}{2}q_j^2 - \frac{\nu}{2} q_j Q_{-j} - p_j q_j \right) \end{aligned}$$

According to (3.4) the impact on equilibrium quantity and price of the unaffected goods is zero. Since aggregated output solely depends on average efficiency (cf. (2.2)) the heterogeneity impact on aggregated concurrence output is zero. Hence the effect on the net utility of the unaffected goods $j \neq k, l$ is zero. Therefore the heterogeneity impact on consumer surplus is just the sum of HICS_k and HICS_l.

$$\begin{aligned} \mathrm{HICS}_k^* = &\partial_k q_k^* \left(1 - q_k^* - \frac{\nu}{2} Q_{-k}^*\right) - \frac{\nu}{2} q_k^* \partial_k Q_{-k}^* - \partial_k p_k^* q_k^* - p_k^* \partial_k q_k^* \\ &- \partial_l q_k^* \left(1 - q_k^* - \frac{\nu}{2} Q_{-k}^*\right) + \frac{\nu}{2} q_k^* \partial_l Q_{-k}^* + \partial_l p_k^* q_k^* + p_k^* \partial_l q_k^* \end{aligned}$$

Note that the equilibrium price is just given by $p_i^* = 1 - q_i^* - \nu Q_{-i}^*$ for $i = 1, \ldots, n$. Furthermore $p_i^* = q_i^* + c_i$ for $i = 1, \ldots, n$. Hence HICS_k is given as follows:

$$
\begin{aligned}
\mathrm{HICS}_k = & -\frac{\nu}{2} q_k^* \partial_k Q_{-k}^* + \partial_k q_k^* \frac{\nu}{2} Q_{-k}^* - \partial_k p_k^* q_k^* \\
& + \frac{\nu}{2} q_k^* \partial_l Q_{-k}^* - \partial_l q_k^* \frac{\nu}{2} Q_{-k}^* + \partial_l p_k^* q_k^*
\end{aligned}
$$

The heterogeneity impact on equilibrium output q_i^* is denoted by HIQ_i^*. Furthermore the heterogeneity impact $\mathrm{HIQ}_{-k} := \partial_k Q_{-k}^* - \partial_l Q_{-k}^*$ on aggregated concurrence output is given by $\mathrm{HIQ}_{-k} = \mathrm{HIQ}_l^* = -\mathrm{HIQ}_k^*$. Since equilibrium price is given by $p_i^* = q_i^* + c_i$ the heterogeneity impact on p_i^* is given by $\mathrm{HIP}_i^* = \mathrm{HIQ}^* + \mathrm{HIC}_i$. Let HIC_i denote the 'heterogeneity impact' on the marginal cost of firm $i = 1, \ldots, n$ with $\mathrm{HIC}_k = 1$, $\mathrm{HIC}_l = -1$ and $\mathrm{HIC}_i = 0$ for $i \neq k, l$.

$$
\begin{aligned}
\mathrm{HICS}_k &= \frac{\nu}{2} Q_{-k}^* \,\mathrm{HIQ}_k^* + \frac{\nu}{2} q_k^* \,\mathrm{HIQ}_k^* - q_k^* \,\mathrm{HIQ}_k^* - q_k^* \\
&= \frac{\nu}{2} Q^* \,\mathrm{HIQ}_k^* - \left(\frac{1-\nu}{2-\nu} \right) q_k^* < 0
\end{aligned}
$$

Since $\mathrm{HIQ}_k^* < 0$ the heterogeneity impact on CS_k is negative irrespective of the distribution of marginal costs or the degree of substitutability ν. Similarly CS_l can be derived which is given by $\mathrm{CS}_l = -\frac{\nu}{2} Q^* \,\mathrm{HIQ}_k^* + \left(\frac{1-\nu}{2-\nu}\right) q_l^* > 0$. Summing up CS_k and CS_l yield the heterogeneity impact on consumer surplus given by (F.1). □

Hence, consumers haven´t worry about heterogeneous market structures. Net utility of a commodity is in reverse proportion to its marginal costs. Therefore, a mean preserving cost variation increasing the disparity of marginal costs makes consumers better off. In the homogeneous good case, however, consumers have no preferences about the distribution of marginal cost as long as average efficiency is constant.

3.3 Social surplus

In the following the relationship between market structure and efficiency is analyzed. It can be shown that a heterogeneous market structure is not a hostile environment for society. It provides a more efficient market outcome compared to more homogenous market structures. Social surplus is an increasing function of the dispersion of marginal costs, if average marginal costs are constant. Since preferences are quasi-linear, social surplus is the measure for Pareto-optimality.

$$W := U\left(m - \sum_{i=1}^{n} c_i q_i, q_1, \ldots, q_n\right) - U(m, 0, \ldots, 0)$$

The consumption of the numeraire-good q_0 is given by $q_0 = m - \sum_i c_i q_i$. Naturally, social surplus abstracts from the distribution of total surplus on consumers and producers. Social surplus is just the sum of producer surplus and consumer surplus. Therefore, the average impact on social surplus is the sum of the average impacts on both components.

Corollary 3.1 (Average Impact) *The average impact on social surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation.*

Decreasing all marginal cost makes society unambiguously better off and vice versa. In the homogeneous good case social surplus increases due to a cost variation making all firms less efficient if inverse demand is sufficiently concave and market concentration is sufficiently high. In this case there is a shift in production from inefficient firms to efficient firms. This phenomena cannot occur since demand is linear in this model.

Since consumer surplus as well as producer surplus increases with the dispersion of marginal costs, the following result is no longer surprising.

Corollary 3.2 (Heterogeneity Impact) *Social surplus increases with the disparity of marginal costs if average marginal costs is constant.*

Thus, society benefits from a mean preserving cost variation increasing the market heterogeneity irrespective the distribution of marginal costs or parameter of substitution. A more heterogeneous market structure is beneficial for both consumers as well as producers and therefore society. This result is well known in the homogeneous good case and can be brought forward into the differentiated good context. In the homogeneous good case consumers are indifferent between market structures with same average efficiency. In case of differentiated goods, however, society is better off since producer surplus as well as consumer surplus increases with market heterogeneity. This result also coincides with related research in vertically differentiated good models (cf. Symeonidis (2003)). Consumer surplus as well as producer surplus increases with the dispersion of quality levels if average quality is constant. Therefore, market heterogeneity either in terms of quality differences or in terms of differently efficient firms provides favorable conditions for efficient market outcomes.

4 Conclusion

This paper analyzes the impact of a marginal cost variation on consumer surplus, producer surplus and social surplus in a differentiated good context. The effect of an arbitrary cost variation is decomposed into an average and a heterogeneity component. It can be shown that there is a positive relationship between the dispersion of marginal costs and efficiency. In contrast to the homogenous good case consumer surplus as well as producer surplus increases with the dispersion of marginal costs. On the one hand these results coincide with the homogenous good case. On the other hand the results are similar to related research analyzing vertically differentiated good oligopolies. Consumer surplus as well as producer surplus increases with the dispersion of quality levels if average quality is constant. Therefore, heterogeneous market structures provide favorable conditions for consumers as well as producers.

A Utility

The quadratic utility according to Dixit (1979) is given as follows:

$$U(q_0, q_1, \ldots, q_n) = q_0 + \sum_i q_i - \frac{1}{2}\boldsymbol{q}^T H \boldsymbol{q}$$

Let q_0 denote the numeraire good and the matrix of substitution H is given as follows:

$$H = \begin{pmatrix} 1 & \nu & \cdots & \nu \\ \nu & 1 & \cdots & \nu \\ \vdots & \vdots & \ddots & \vdots \\ \nu & \nu & \cdots & 1 \end{pmatrix}$$

The corresponding Hessian $\nabla^2 U = -H$ is real and symmetric and can be decomposed by $P^{-1}DP = -H$. Let D denote the matrix containing the Eigenvalues and let P denote the matrix consisting of the Eigenvalues of the Hessian H. The correctness can be proved by calculating $-HP = PD$. Compare Jänich (2002), p. 219.

$$D = \begin{pmatrix} -1+\nu & 0 & \cdots & 0 & 0 \\ 0 & -1+\nu & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & -1+\nu & 0 \\ 0 & 0 & \cdots & 0 & [-1-\nu(n-1)] \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & 1 \\ -1 & 1 & \cdots & 0 & 0 & 1 \\ 0 & -1 & \cdots & 0 & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & -1 & 1 & 1 \\ 0 & 0 & \dots & 0 & -1 & 1 \end{pmatrix}$$

Utility is concave if the corresponding Hessian is negative definit. This requires negative Eigenvalues. Compare Königsberger (1993), p.74. Hence: $-1+\nu < 0 \Leftrightarrow \nu < 1$ and $-1-\nu(n-1) < 0 \Leftrightarrow \nu > -\frac{1}{n-1}$. Thus, I assume: $\nu \in \left(-\frac{1}{n-1}, 1\right)$. Utility can also be expressed as follows:

$$U(q_0, q_1, q_2, \ldots, q_n) = \sum_i q_i - \frac{1}{2}\sum_i (q_i)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i q_j \tag{A.1}$$

B Proof of lemma 2.1

Competing in quantities, firm $i = 1, \ldots, n$ maximizes its profit $\Pi_i = p(q_i + Q_{-i})q_i - c_i q_i$ choosing an optimal q_i. Inverse demand $p(q_i + Q_{-i}) = 1 - q_i - \nu Q_{-i}$ is given by (2.1). The first order condition of firm $i = 1, \ldots, n$ is given by $1 - 2q_i - \nu Q_{-i} - c_i = 0$. In matrix form all first order conditions can be expressed as follows:

$$\begin{pmatrix} 2 & \nu & \cdots & \nu \\ \nu & 2 & \cdots & \nu \\ \vdots & \vdots & \ddots & \vdots \\ \nu & \nu & \cdots & 2 \end{pmatrix} \boldsymbol{q} = \begin{pmatrix} 1-c_1 \\ 1-c_2 \\ 1-c_3 \\ \vdots \\ 1-c_n \end{pmatrix}$$

Let A denote the matrix of coefficients. $\boldsymbol{c}^T = (1-c_1, \ldots, 1-c_n)$ is the vector of constants. A is real and symmetric and can be decomposed by $A = PDP^{-1}$. Hence $A\mathbf{q} = \mathbf{c}$ can be expressed by $PDP^{-1}\boldsymbol{q} = \boldsymbol{c}$. Let P denote the matrix of Eigenvectors. The diagonal matrix D contains the corresponding Eigenvalues. It is easy to proof that $\lambda_1 = 2 - \nu$ is an $n-1$ fold Eigenvalue of A and $\lambda_2 = 2 + \nu(n-1)$ is the n-th Eigenvalue. The diagonal matrix D is given as follows:

$$D = \begin{pmatrix} 2-\nu & 0 & \cdots & 0 & 0 \\ 0 & 2-\nu & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 2-\nu & 0 \\ 0 & 0 & \cdots & 0 & [2+\nu(n-1)] \end{pmatrix} \tag{B.1}$$

The matrix P containing the corresponding Eigenvectors $\boldsymbol{v}_i$ with $i = 1, \ldots, n$ is given as follows:

$$P = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & 1 \\ -1 & 1 & \cdots & 0 & 0 & 1 \\ 0 & -1 & \cdots & 0 & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & -1 & 1 & 1 \\ 0 & 0 & \ldots & 0 & -1 & 1 \end{pmatrix} \tag{B.2}$$

Prove the accuracy of (B.1) and (B.2) by calculating $AP = PD$. The Cournot-Nash equilibrium q_i^* for $i = 1, \ldots, n$ is determined by solving $PDP^{-1}\boldsymbol{q}^* = \boldsymbol{c}$ in two steps. Firstly $PD\boldsymbol{z}^* = \boldsymbol{c}$ is solved for $\boldsymbol{z}^* := P^{-1}\boldsymbol{q}$. Then the solution of $\boldsymbol{q}^*$ can be derived by calculating $\boldsymbol{q}^* = P\boldsymbol{z}^*$. The optimal $\boldsymbol{z}^*$ must solve the following system of linear equations $PD\boldsymbol{z}^* = \boldsymbol{c}$:

$$\begin{pmatrix} 2-\nu & 0 & \cdots & 0 & 0 & [2+\nu(n-1)] \\ -(2-\nu) & 2-\nu & \cdots & 0 & 0 & [2+\nu(n-1)] \\ 0 & -(2-\nu) & \cdots & 0 & 0 & [2+\nu(n-1)] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -(2-\nu) & 2-\nu & [2+\nu(n-1)] \\ 0 & 0 & \cdots & 0 & -(2-\nu) & [2+\nu(n-1)] \end{pmatrix} \boldsymbol{z}^* = \begin{pmatrix} 1-c_1 \\ 1-c_2 \\ \vdots \\ 1-c_n \end{pmatrix}$$

Summing up the first and the second row yields the new second row. The new second row is added to the third row which again yields the new third row and so on. The resulting row echelon form is given as follows:

$$\begin{pmatrix} 2-\nu & 0 & \cdots & 0 & 0 & [2+\nu(n-1)] \\ 0 & 2-\nu & \cdots & 0 & 0 & 2[2+\nu(n-1)] \\ 0 & 0 & \cdots & 0 & 0 & 3[2+\nu(n-1)] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 2-\nu & (n-1)[2+\nu(n-1)] \\ 0 & 0 & \cdots & 0 & 0 & n[2+\nu(n-1)] \end{pmatrix} \boldsymbol{z}^* = \begin{pmatrix} 1-c_1 \\ 2-c_1-c_2 \\ \vdots \\ (n-1)-\sum_{i=1}^{n-1} c_i \\ n-\sum_{i=1}^{n} c_i \end{pmatrix}$$

Solving the last row for z_n^* yields:

$$z_n^* = \frac{n-\sum_{i=1}^{n} c_i}{n[2+\nu(n-1)]} \tag{B.3}$$

Inserting z_n^* given by (B.3) in the row before last which is given as follows

$$(2-\nu)z_{n-1}^* + (n-1)[2+\nu(n-1)]z_n^* = (n-1) - \sum_{i=1}^{n-1} c_i$$

yield the solution for z_{n-1}^* which is given as follows:

$$
\begin{aligned}
z_{n-1}^* &= \frac{1}{2-\nu}\left((n-1) - \sum_{i=1}^{n-1} c_i - (n-1)[2+\nu(n-1)]z_n^*\right) \\
&\overset{(B.3)}{=} \frac{1}{2-\nu}\left((n-1) - \sum_{i=1}^{n-1} c_i - (n-1)[2+\nu(n-1)]\left(\frac{n-\sum_{i=1}^{n} c_i}{n[2+\nu(n-1)]}\right)\right) \\
&= \frac{1}{2-\nu}\left((n-1) - \sum_{i=1}^{n-1} c_i - (n-1) + \frac{n-1}{n}\sum_{i=1}^{n} c_i\right) \\
&= \frac{1}{2-\nu}\left(\frac{n-1}{n}\sum_{i=1}^{n} c_i - \sum_{i=1}^{n-1} c_i\right) \qquad \text{(B.4)}
\end{aligned}
$$

Equilibrium quantities q_i^* are given by $\boldsymbol{q}^* = P\boldsymbol{z}^*$. Therefore, the solution for q_n^* is given by $q_n^* = -z_{n-1}^* + z_n^*$ with z_{n-1}^* and z_n^* given by (B.3) and (B.4) respective. Hence:

$$
\begin{aligned}
q_n^* &= \frac{-1}{2-\nu}\left(\frac{n-1}{n}\sum_{i=1}^{n} c_i - \sum_{i=1}^{n-1} c_i\right) + \frac{n-\sum_{i=1}^{n} c_i}{n[2+(n-1)\nu]} \\
&= \frac{(2-\nu) - \frac{2-\nu}{n}\sum_{i=1}^{n} c_i + [2+(n-1)\nu]\left(\sum_{i=1}^{n-1} c_i - \frac{n-1}{n}\sum_{i=1}^{n} c_i\right)}{(2-\nu)[2+(n-1)\nu]} \\
&= \frac{(2-\nu) - \frac{2-\nu}{n}\sum_{i=1}^{n} c_i + 2\sum_{i=1}^{n-1} c_i + (n-1)\nu\sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]} \\
&\quad + \frac{-2\frac{n-1}{n}\sum_{i=1}^{n} c_i - \frac{(n-1)^2}{n}\nu\sum_{j=1}^{n} c_j}{(2-\nu)[2+(n-1)\nu]}
\end{aligned}
$$

Rearranging the terms by collecting the coefficients of c_n and c_i for $i \neq n$ yields:

$$
\begin{aligned}
q_n^* = & \frac{(2-\nu) + \left[-\frac{2-\nu}{n} - 2\frac{n-1}{n} - \frac{n-1}{n}(n-1)\nu\right] c_n}{(2-\nu)[2+(n-1)\nu]} \\
& + \frac{\left[-\frac{2-\nu}{n} + 2 + (n-1)\nu - 2\frac{n-1}{n} - \frac{(n-1)^2}{n}\nu\right] \sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]} \\
= & \frac{(2-\nu) + \left[-2 + \nu\left(\frac{1}{n} - \frac{(n-1)^2}{n}\right)\right] c_n + \left[\nu\left(\frac{1}{n} + (n-1) - \frac{(n-1)^2}{2}\right)\right] \sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]} \\
= & \frac{(2-\nu) - [2+\nu(n-2)]c_n + \nu \sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]}
\end{aligned}
$$

Equilibrium output q_i^* for $i = 1, \ldots, n-1$ can be derived analogously. q_i^* for $n = 1, \ldots, n$ is given as follows:

$$
q_i^* = \frac{(2-\nu) - [2+\nu(n-2)]c_i + \nu \sum_{j \neq i} c_i}{(2-\nu)[2+\nu(n-1)]} \quad \square \tag{B.5}
$$

C Proof of equation 2.3

In the following I show that equilibrium profit Π_i^* is just its squared quantity. Equilibrium price can be obtained by inserting equilibrium quantities given by (B.5) in the inverse demand. It holds $p_i^* - c_i = 1 - q_i^* - \nu Q_{-i}^* - c_i$. Aggregated concurrence output $Q_{-i}^* = \sum_{j \neq i} q_j^*$ is given as follows:

$$
Q_{-i}^* = \frac{(n-1)(2-\nu) - [2+\nu(n-2)]\sum_{j\neq i} c_j + \nu(n-1)c_i + \nu(n-2)\sum_{j\neq i} c_j}{(2-\nu)[2+\nu(n-1)]}
$$

It remains to show that $p_i^* - c_i = q_i^*$:

$$
\begin{aligned}
p_i^* - c_i =& - q_i^* + \frac{(2-\nu)[2+\nu(n-1)] - \nu(n-1)(2-\nu)}{(2-\nu)[2+\nu(n-1)]} \\
&+ \frac{-\nu^2(n-1) - (2-\nu)[2+\nu(n-1)]}{(2-\nu)[2+\nu(n-1)]} c_i \\
&+ \frac{\nu[2+\nu(n-2)] - \nu^2(n-2)]}{(2-\nu)[2+\nu(n-1)]} \sum_{j \neq i} c_j \\
=& - q_i^* + \frac{2(2-\nu) + [-4 + (2-n)2\nu]c_i + 2\nu \sum_{j \neq i} c_j}{(2-\nu)[2+\nu(n-1)]} \\
=& - q_i^* + 2q_i^* \\
=& q_i^* \quad \square
\end{aligned}
$$

D Proof of lemma 3.1

The heterogeneity impact $\text{HIR}^* := \partial_k R^* - \partial_l R^*$ on total revenue $R^* := \sum_i p_i^* q_i^*$ is just the sum of the heterogeneity impacts on each firms revenue.

$$
\begin{aligned}
\text{HIR}^* :=& \, \partial_k R^* - \partial_l R^* \\
=& \, \partial_k \sum_i R_i^* - \partial_l^* \sum_i R_i^* \\
=& \sum_i \Big(\partial_k R_i^* - \partial_l R_i^* \Big) \\
=& \sum_i \text{HIR}_i^*
\end{aligned}
$$

According to (3.4) the heterogeneity impact on the output of the unaffected firms is zero. Since $p_i^* = q_i^* + c_i$ the heterogeneity impact on the equilibrium price of the unaffected firms is zero. Hence the heterogeneity impact on total revenue is given as follows:

$$
\text{HIR}^* = \text{HIR}_k^* + \text{HIR}_l^* \tag{D.1}
$$

whereas the heterogeneity impact on revenue HIR_i^* is given as follows:

$$\begin{aligned}
\mathrm{HIR}_i^* &:= \partial_k R_i^* - \partial_l R_i^* \\
&= \partial_k (p_i^* q_i^*) - \partial_l (p_i^* q_i^*) \\
&= \partial_k p_i^* q_i^* + p_i^* \partial_k q_i^* - (\partial_l p_i^* q_i^* + p_i^* \partial_l q_i^*) \\
&= (\partial_k p_i^* - \partial_l p_i^*) q_i^* + (\partial_k q_i^* - \partial_l q_i^*) p_i^* \\
&= \mathrm{HIP}_i^* \, q_i^* + \mathrm{HIQ}_i^* \, p_i^* \qquad \text{(D.2)}
\end{aligned}$$

whereas HIP_i^* denotes the heterogeneity impact on the equilibrium price of firm i. Since the equilibrium price $p_i^* \overset{(2.3)}{=} q_i^* + c_i$ the heterogeneity impact on market price p_i^* is given as follows:

$$\begin{aligned}
\mathrm{HIP}_i^* &= \partial_k (q_i^* + c_i) - \partial_l (q_i^* + c_i) \\
&= \mathrm{HIQ}_i^* + \mathrm{HIC}_i \qquad \text{(D.3)}
\end{aligned}$$

Let $\mathrm{HIC}_i := \partial_k c_i - \partial_l c_i$ denote the 'heterogeneity impact' on the marginal cost of firm $i = 1, \ldots, n$ with

$$\mathrm{HIC}_i = \begin{cases} 1, \ \text{for } i = k, \\ -1, \ \text{for } i = l, \\ 0, \ \text{else.} \end{cases} \qquad \text{(D.4)}$$

Hence the heterogeneity impact on revenue is given as follows:

$$\begin{aligned}
\mathrm{HIR}_i^* &\overset{(D.2)}{=} \mathrm{HIP}_i^* \, q_i^* + \mathrm{HIQ}_i^* \, p_i^* \\
&\overset{(D.3)}{=} (\mathrm{HIQ}_i^* + \mathrm{HIC}_i) q_i^* + \mathrm{HIQ}_i^* \, p_i^* \\
&= \mathrm{HIQ}_i^* (q_i^* + p_i^*) + \mathrm{HIC}_i \, q_i^* \\
&\overset{(2.3)}{=} \mathrm{HIQ}_i^* (2q_i^* + c_i) + \mathrm{HIC}_i \, q_i^*
\end{aligned}$$

Since the heterogeneity impact on the quantity and the marginal cost of the unaffected firms $j \neq k, l$ is zero and $\mathrm{HIQ}_k = -\mathrm{HIQ}_l = \frac{-1}{2-\nu}$, the heterogeneity impact on revenue is given as follows:

$$\mathrm{HIR}_i^* = \begin{cases} \frac{-1}{2-\nu}(2q_k^* + c_k) + q_k^*, \ \text{for } i = k, \\ \frac{1}{2-\nu}(2q_l^* + c_l) - q_l^*, \ \text{for } i = l, \\ 0, \ \text{else.} \end{cases}$$

Since $2\,\mathrm{HIQ}_k + 1 = \frac{-\nu}{2-\nu}$ the heterogeneity impact on the revenue of firm i is given as follows:

$$\mathrm{HIR}_i^* = \begin{cases} \left(\frac{-\nu}{2-\nu}\right) q_k^* + \frac{-1}{2-\nu} c_k, & \text{for } i = k, \\ \left(\frac{\nu}{2-\nu}\right) q_l^* - \frac{-1}{2-\nu} c_l, & \text{for } i = l, \\ = 0, & \text{else.} \end{cases} \tag{D.5}$$

Hence in case of substitutes (i.e. $\nu \geq 0$) the heterogeneity impact on revenue k is negative and the heterogeneity impact on revenue l is positive. Note that in case of complements this is not true in general. Hence the heterogeneity impact on total revenue is given as follows:

$$\begin{aligned}
\mathrm{HIR}^* &\overset{(D.1)}{=} \mathrm{HIR}_k^* + \mathrm{HIR}_l^* \\
&\overset{(D.5)}{=} \left[\left(\frac{-\nu}{2-\nu}\right) q_k^* + \frac{-1}{2-\nu} c_k\right] + \left[\left(\frac{\nu}{2-\nu}\right) q_l^* - \frac{-1}{2-\nu} c_l\right] \\
&= \left(\frac{-\nu}{2-\nu}\right)(q_k^* - q_l^*) + \frac{-1}{2-\nu}(c_k - c_l) \\
&\overset{(3.7)}{=} \frac{-\nu}{2-\nu}\left[\frac{-(c_k - c_l)}{(2-\nu)}\right] + \left(\frac{-1}{2-\nu}\right)(c_k - c_l) \\
&= -2\frac{(1-\nu)}{(2-\nu)^2}(c_k - c_l) \begin{cases} < 0, & \text{for } c_k > c_l, \\ = 0, & \text{for } c_k = c_l, \\ > 0, & \text{for } c_k < c_l. \end{cases}
\end{aligned} \tag{D.6}$$

Hence total revenue diminishes (increases) if the more (less) efficient firm is getting more efficient. This result is true in case of substitutes and complements even though the heterogeneity impact on revenue R_k must not be negative in case of complements (cf. (D.5)). In the following the heterogeneity impact on total costs $C^* := \sum_i c_i q_i^*$ is investigated.

$$
\begin{aligned}
\partial_k C^* - \partial_l C^* &= \partial_k \sum_i c_i q_i^* - \partial_l \sum_i c_i q_i^* \\
&= \sum_i \Big(\partial_k (c_i q_i^*) - \partial_l (c_i q_i^*) \Big) \\
&= \sum_i \Big(\partial_k c_i q_i^* + c_i \partial_k q_i^* - \partial_l c_i q_i^* - c_i \partial_l q_i^* \Big) \\
&= \sum_i \Big(\mathrm{HIC}_i \, q_i^* + \mathrm{HIQ}_i^* \, c_i \Big) \\
&\overset{(D.4)}{=} q_k^* - q_l^* + \mathrm{HIQ}_k^* \, c_k + \mathrm{HIQ}_l^* \, c_l \\
&\overset{(3.7)}{=} \frac{-2(c_k - c_l)}{(2-\nu)} \begin{cases} > 0, & \text{for } c_k < c_l, \\ = 0, & \text{for } c_k = c_l, \\ < 0, & \text{for } c_k > c_l. \end{cases}
\end{aligned}
\tag{D.7}
$$

Hence the heterogeneity impact on total costs is negative (positive) if the more (less) efficient firm is getting more efficient. Obviously the heterogeneity impact on total revenue outweighs the heterogeneity impact on total costs for $c_k > c_l$:

$$
\begin{aligned}
\partial_k C^* - \partial_l C^* \overset{(D.7)}{=} \frac{-2}{2-\nu}(c_k - c_l) &< \frac{-2(1-\nu)}{(2-\nu)^2}(c_k - c_l) \overset{(D.6)}{=} \partial_k R^* - \partial_l R^* \\
\Leftrightarrow 1 &> \frac{1-\nu}{2-\nu}
\end{aligned}
$$

Hence for $c_k > c_l$ the diminishing total costs outweigh the diminishing revenue and vice versa. Thus the heterogeneity impact on producer surplus is positive (negative) if the more (less) efficient firm is getting more efficient. Note that the heterogeneity impact on producer surplus is just the difference between the heterogeneity impact on revenue and total costs. Hence subtracting (D.7) from (D.6) yields (3.8). □

E Proof of proposition 3.3

According to (A.1) the Dixit-utility is given as follows:

$$U(q_1^*, q_2^*, \dots, q_n^*) = \sum_i q_i^* - \frac{1}{2}\sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} q_i^* q_j^*$$

The average impact $\text{AICS}^* := \sum_{i=1}^n \partial_i \, \text{CS}^*$ on consumer surplus in equilibrium is given as follows:

$$\begin{aligned}
\text{AICS}^* &= \sum_k \partial_k \left(\sum_i q_i^* - \frac{1}{2}\sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} q_i^* q_j^* - \sum_i p_i^* q_i^* \right) \\
&= \sum_k \Bigg\{ \sum_i \partial_k q_i^* - \sum_i q_i^* \partial_k q_i^* - \nu \left(\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} (\partial_k q_i^* q_j^* + q_i^* \partial_k q_j^*) \right) \\
&\qquad - \sum_i (\partial_k p_i^* q_i^* + p_i^* \partial_k q_i^*) \Bigg\}
\end{aligned}$$

Since market price p_i^* is given by $p_i^* = q_i^* + c_i$ the average impact is given as follows:

$$\begin{aligned}
\text{AICS}^* = \sum_k \Bigg\{ &\sum_i \partial_k q_i^* - \sum_i \partial_k q_i^* q_i^* - \nu \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} q_j^* \partial_k q_i^* - \nu \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} q_i^* \partial_k q_j^* \\
&- \sum_i q_i^* \partial_k (q_i^* + c_i) - \sum_i (q_i^* + c_i) \partial_k q_i^* \Bigg\}
\end{aligned}$$

Rearranging the terms deftly allows to factor out $p_i^* = 1 - q_i^* - \nu Q_{-i}^*$.

$$\begin{aligned}
\text{AICS}^* &= \sum_k \left\{ \sum_i \partial_k q_i^* \left(1 - q_i^* - \nu Q_{-i}^* - c_i\right) - 2\sum_i q_i^* \partial_k q_i^* - q_k^* \right\} \\
&= \sum_k \left\{ \sum_i \partial_k q_i^* \left(p_i^* - c_i\right) - 2\sum_i q_i^* \partial_k q_i^* - q_k^* \right\}
\end{aligned}$$

Since $p_i^* - c_i = q_i^*$ the average impact is given as follows:

$$\begin{aligned}\text{AICS}^* &= \sum_k \left\{ \sum_i q_i^* \partial_k q_i^* - 2 \sum_i q_i^* \partial_k q_i^* - q_k^* \right\} \\ &= \sum_k \left\{ - \sum_i q_i^* \partial_k q_i^* - q_k^* \right\} \qquad \text{(E.1)} \\ &= - \sum_i q_i^* \sum_k \partial_k q_i^* - Q^* \end{aligned}$$

Note that the average impact $\text{AIQ}_i^* := \sum_k \partial_k q_i^*$ on the equilibrium output of firm $i = 1, \ldots, n$ is given by $\text{AIQ}_i^* = \frac{-1}{2+\nu(n-1)}$.

$$\begin{aligned}\text{AICS}^* &= - \sum_i q_i^* \,\text{AIQ}_i - Q^* \\ &= - \text{AIQ}_i \, Q^* - Q^* \\ &= -(\text{AIQ}_i + 1) Q^* \\ &= - \frac{1 + \nu(n-1)}{2 + \nu(n-1)} Q^* \qquad \text{(E.2)} \end{aligned}$$

Since $-\frac{1+\nu(n-1)}{2+\nu(n-1)} \leq 0$ for $\nu \in \left(-\frac{1}{n-1}, 1\right)$ and $Q^* > 0$ the average impact on consumer surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation. □

F Proof of proposition 3.4

In the following the heterogeneity impact on consumer surplus $\text{HICS}^* := \partial_k \text{CS}^* - \partial_l \text{CS}^*$ is derived. The partial derivatives $\partial_k \text{CS}^*$ and $\partial_l \text{CS}^*$ are given as follows:

$$\begin{aligned}\partial_k \text{CS}^* &= \partial_k \left(\sum_i q_i^* - \frac{1}{2} \sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* q_j^* - \sum_i p_i^* q_i^* \right) \\ &\overset{(E.1)}{=} - \sum_i q_i^* \partial_k q_i^* - q_k^* \\ \partial_l \text{CS}^* &\overset{(E.1)}{=} - \sum_i q_i^* \partial_l q_i^* - q_l^* \end{aligned}$$

Hence the heterogeneity impact on consumer surplus $\mathrm{HICS}^* := \partial_k \mathrm{CS} - \partial_l \mathrm{CS}$ is given as follows:

$$\begin{aligned}
\mathrm{HICS}^* &= -\sum_i q_i^* \partial_k q_i^* - q_k^* - \left(-\sum_i q_i^* \partial_l q_i^* - q_l^*\right) \\
&= -\sum_i q_i^* \left(\partial_k q_i^* - \partial_l q_i^*\right) - (q_k^* - q_l^*) \\
&= -\sum_i q_i^* \,\mathrm{HIQ}_i - (q_k^* - q_l^*)
\end{aligned}$$

According to (3.4) the heterogeneity impact on the equilibrium output of the unaffected firms is zero.

$$\begin{aligned}
\mathrm{HICS}^* &= -q_k^* \,\mathrm{HIQ}_k - q_l^* \,\mathrm{HIQ}_l - (q_k^* - q_l^*) \\
&= -(q_k^* - q_l^*) \,\mathrm{HIQ}_k - (q_k^* - q_l^*) \\
&= -(\mathrm{HIQ}_k + 1)(q_k^* - q_l^*) \\
&= -\left(\frac{1-\nu}{2-\nu}\right)(q_k^* - q_l^*) \\
&\overset{(3.7)}{=} \frac{1-\nu}{(2-\nu)^2}(c_k - c_l)
\end{aligned} \tag{F.1}$$

Thus the heterogeneity impact on consumer surplus is positive (negative) if the more inefficient (efficient) firm is getting more efficient. □

G Proof of lemma 3.2

Note that households‘ expenditures just equal to firms‘ total revenue which was analyzed already in appendix D. Hence the heterogeneity impact on households expenditures is given by (D.6). Thus it remains analyzing the heterogeneity impact on consumers utility $U(q_0^*, q_1^*, \ldots, q_n^*)$ given by $\partial_k U^* - \partial_l U^*$.

$$
\begin{aligned}
\partial_k U^* &= \partial_k \left(\sum_i q_i^* - \frac{1}{2} \sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* q_j^* \right) \\
&= \sum_i \partial_k q_i^* - \sum_i \partial_k q_i^* q_i^* - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_j^* \partial_k q_i^* - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* \partial_k q_j^* \\
&= \sum_i \partial_k q_i^* \left(1 - q_i^* - \nu Q_{-i}^*\right) \\
&\overset{(2.1)}{=} \sum_i \partial_k q_i^* p_i^*
\end{aligned}
$$

Hence the heterogeneity impact on consumers utility is given as follows:

$$
\begin{aligned}
\partial_k U^* - \partial_l U^* &= \sum_i \left(\partial_k q_i^* - \partial_l q_i^*\right) p_i^* \\
&= \sum_i \mathrm{HIQ}_i^* \, p_i^*
\end{aligned}
$$

According to (3.4) the heterogeneity impact on the output of the unaffected firms is zero. Since $\mathrm{HIQ}_k = -\mathrm{HIQ}_l$ it holds:

$$
\begin{aligned}
\partial_k U^* - \partial_l U^* &= \mathrm{HIQ}_k \, p_k^* + \mathrm{HIQ}_l \, p_l^* \\
&= \mathrm{HIQ}_k (p_k^* - p_l^*) \\
&\overset{(2.3)}{=} \mathrm{HIQ}_k [q_k^* + c_k - (q_l^* + c_l)] \\
&= \mathrm{HIQ}_k (q_k^* - q_l^*) + \mathrm{HIQ}_k (c_k - c_l) \\
&\overset{(3.7)}{=} \frac{-1}{2-\nu} \left[\frac{-1}{2-\nu} (c_k - c_l) \right] + \frac{-1}{2-\nu} (c_k - c_l) \\
&= -\frac{(1-\nu)}{(2-\nu)^2} (c_k - c_l) \qquad \text{(G.1)}
\end{aligned}
$$

Hence the heterogeneity impact on consumer surplus is negative (positive) if the more (less) efficient firm is positively affected by the cost variation. It is easy to check that the heterogeneity impact on consumer expenditures outweighs the heterogeneity impact on consumer utility.

$$
\partial_k U^* - \partial_l U^* \overset{(G.1)}{=} -\frac{(1-\nu)}{(2-\nu)^2} (c_k - c_l) \overset{c_k > c_l}{>} -2 \frac{(1-\nu)}{(2-\nu)^2} (c_k - c_l) \overset{(D.6)}{=} \partial_k R^* - \partial_l R^*
$$

Since the heterogeneity impact on consumers expenditures outweighs the heterogeneity impact on consumers utility the heterogeneity impact on consumer surplus is positive (negative) if the more (less) efficient firm is getting more efficient. □

References

[1] Amir, R. and J. Jin (2001): Cournot and Bertrand equilibria compared: substitutability, complementarity and concavity, *International Journal of Industrial Organization*, 19, 303-317

[2] Bergstrom, T.C. and H.R. Varian (1985a): Two Remarks on Cournot Equilibria, *Economics Letters*, 19, 5-8

[3] Bester, H. and E. Petrakis (1993): The incentives for cost reduction in a differentiated industry, *International Journal of Industrial Organization*, 11, 519-534

[4] Dixit, A (1979): A Model of Duopoly Sugggesting a Theory of Entry Barriers, *Journal of Economics*, 10, 20-32

[5] Dixit, A. and N. Stern (1982): Oligopoly and Welfare: A unified Presentation and Application to Trade and Development, *European Economic Review*, 19, 123-143

[6] Farrell, J. and C. Shapiro (1990): Horizontal Mergers: an Equilibrium Analysis, *American Economic Review*, 80, 107-126

[7] Février, P. and L. Linnemer (2004): Idiosyncratic Shocks in an Asymmetric Cournot Oligopoly, *International Journal of Industrial Organization*, 22, 835-848

[8] Häckner, J. (2000): A Note on Price and Quantity Competition in Differentiated Oligopolies, *Journal of Economic Theory*, 93, 233-239

[9] Hsu J. and H. Wang (2005): On Welfare under Cournot and Bertrand Competition in Differentiated Oligopolies, *Review of Industrial Organization*, 27, 185-191

[10] Jänich, K. (2002): Lineare Algebra, 9. Auflage, Springer Verlag, Berlin

[11] Kimmel, S. (1992): Effects of Cost Changes on Oligopolists´ Profits, *Journal of Industrial Economics*, 40, 441-449

[12] Koh, W. (2008): Market competition, social welfare in an entry-constrained differentiated-good oligopoly, *Economics Letters*, 100, 229-233

[13] Königsberger, K. (1993): Analysis 2, Springer Verlag, Berlin

[14] Lahiri, S. and Y. Ono (1988): Helping Minor Firms Reduces Welfare, *The Economic Journal*, 98, 1199-1202

[15] Salant, S.W. and G. Shaffer (1999): Unequal Treatment of Identical Agents in Cournot Equilibrium, *Americican Economic Review*, 89, 585-604

[16] Seade, J. (1985): Profitable Cost Increases and the Shifting of Taxation: Equilibrium Response of Markets in Oligopoly, *The Warwick Economic Research Paper Series*, 260

[17] Shubik, M. (1980): Market Structure and Behavior, Havard University Press, Cambridge

[18] Singh, N. and X. Vives (1984): Price and quantity competition in a differentiated duopoly, *Rand Journal of Economics*, 15, 546-554

[19] Smythe, D.J. and J. Zhao (2006): The Complete Welfare Effects of Cost Reductions in a Cournot Oligopoly, *Journal of Economics*, 87, 181-193

[20] Symeonidis, G. (2003): Quality heterogeneity and welfare, *Economics Letters*, 78 (1), 1-7

[21] van Long, N. and A. Soubeyran (2001): Cost manipulation games in oligopoly, with cost of manipulating, *International Economic Review*, 42 (2), 505-533

[22] Wang, H. and J. Zhao (2007): Welfare reductions from small cost reductions in differentiated oligopoly, *International Journal of Industrial Organization*, 25, 173-185

[23] Zanchettin, P. (2006): Differentiated Duopoly with Asymmetric Costs, *Journal of Economics and Management Strategy*, 15, 999-1015

[24] Zhao, J (2001): A Characterization for the Negative Welfare Effects of Cost Reduction in Cournot Oligopoly, *International Journal of Industrial Organization*, 19, 455-469