Should central banks care about investment?*

Markus Hörmann†, *University of Dortmund and Ruhr Graduate School in Economics*

Abstract

We ask how including endogenous capital formation into a New-Keynesian model affects optimal monetary policy. We find that the response of Ramsey optimal policy to a persistent cost-pushing shock is unconventional: In response to the shock, the central bank persistently reduces the nominal interest rate below its steady state. We find that this is due to a decrease in the natural interest rate and does not reflect a desire to choose a systematically different point on the policy frontier. However, the central bank's tradeoff is affected in the sense that inflation stabilization can become more costly: When analyzing optimal simple rules, we find that these can imply welfare losses which substantially exceed those of Ramsey optimal policy. The reason is that active interest rate policy magnifies output fluctuations by destabilizing the capital stock. When introducing adjustment cost, our results return to standard: First, Ramsey optimal policy increases the interest rate as a response to a cost-pushing shock. Second, active policy becomes more successful in mimicking the allocation a Ramsey planner would choose.

1 Introduction and motivation

This paper seeks to contribute to the literature on optimal monetary policy in New-Keynesian models by analysing how endogenous capital accumulation affects the tradeoff faced by monetary policy. A paradigm in the literature on optimal monetary policy in New-Keynesian models is the prescription of active interest rate policy: Schmitt-Grohé and Uribe (2007) and argue that simple policy rules, where the central bank raises the interest rate in response to inflation overproportionally, resolve the central bank's tradeoff in a near-optimal way. The intuition behind this policy it that an increasing real rate in face of inflationary pressure will curb economic activity and thereby dampen inflation. Schmitt-Grohé and Uribe (2007) further find that a policy of complete inflation stabilization yields

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†Email: Markus.Hoermann@tu-dortmund.de, Phone: +492317555403 Fax: +492317553069

a level of welfare virtually identical to that implied by optimal policy. Woodford (2004), in his survey of the literature, finds that "it is not a bad first approximation to say that the goal of monetary policy should be price stability". While we do not wish to contest that claim, our goal is to analyse how the inclusion of endogenous capital formation affects the central bank's policy tradeoff. In particular, we present the Ramsey optimal policy for a model with endogenous capital formation and compare it to optimal policy in the standard model. We further ask to what extent simple rules can mimic the allocation achieved by Ramsey optimal policy when capital accumulation is endogenous.

In the canonical New-Keynesian model, as presented for instance in Clarida, Galí, and Gertler (1999), the capital stock is assumed to be either absent or fixed, so that the central bank has an impact on aggregate activity exclusively due to its influence on agents' consumption-saving decision. This implies that, in absence of capital, the interest rate set by monetary policy has no impact on the intertemporal allocation of resources: The central bank can reduce aggregate demand by increasing the interest rate without having an impact on future allocations. When allowing endogenous capital accumulation, an increase in the real interest rate implies a reduction of investment which affects future marginal cost and production. This so-called investment channel is empirically important, in particular in the euro area: Angeloni, Kashyap, Mojon, and Terlizzese (2003) finds that output changes are predominantly driven by investment fluctuations, as opposed to fluctuations in consumption.

We present three main results in this paper. First, we find that optimal policy reacts to a cost-push shock by reducing the nominal and real interest rates for a prolonged time period. However, this does not reflect a systematically changed tradeoff: As in the model without capital, optimal policy induces temporary inflation by persistently reducing the nominal interest rate below its natural rate. The benefit of this policy is an improved stabilization of output. Rather, it is the path of the natural interest rate which is different in our model: The natural interest rate falls in response to a cost-pushing shock. Second, we find that introducing capital accumulation worsens the performance of simple rules: The optimal simple rule induces welfare cost exceeding those under Ramsey policy by 9.2%. The reason is that the optimal simple rule stabilizes inflation by aggressive active policy. The investment channel magnifies the output cost of this policy: Any reduction in output is now associated with a reduction in future output or an increase in future marginal cost. Third, when introducing investment or capital adjustment cost, our results resemble the standard model with a fixed capital stock. In this case, the natural interest rate increases as a response to a cost-pushing shock and accordingly, the response of optimal monetary policy is to increase the interest rate above its steady state. Further, the excess welfare cost of the optimal simple rule falls to 6.4% under capital adjustment cost and 2.8% under investment adjustment cost.

McCallum and Nelson (1999) argue that fluctuations in the capital stock over the business cycle can be neglected because capital and output movements are not strongly correlated at cyclical frequencies. However, the authors themselves name analytical simplicity as the main justification for the fixed capital assumption. Accordingly, many studies have analysed New Keynesian models with endogenous capital accumulation, but none has exposed its impact on optimal monetary policy explicitly. The existing literature generally considers two different assumptions: Under the rental market assumption households accumulate capital and rent capital to firms. Another possibility is to assume that each firm owns a firm-specific capital stock, which implies that any short-run adjustment in production must be done by adjusting labour demand. As demonstrated by Galí, Gertler, and López-Salido (2001), this implies that marginal cost differ across firms. The present paper assumes an economy-wide rental market for capital. Carlstrom and Fuerst (2005) contribute to the discussion that monetary policy can generate endogenous fluctuations. They find that in a rental-market model, forward-looking interest rate rules are likely to generate equilibrium indeterminacy. Sveen and Weinke (2005, 2007) focus on the determinacy properties of simple interest rate rules in a model with firm-specific capital accumulation. They find that firm-specific capital generates endogenous price stickiness which implies that the Taylor principle can be insufficient to guarantee equilibrium determinacy.[1] Sveen and Weinke (2006) analyse the welfare implications of optimized simple interest rate rules in a model with sticky wages. They find that a central bank which ignores the endogenous price-stickiness implied by firm-specific capital puts too less weight on price inflation and too much weight on wage inflation. However, these authors confine to analyzing simple rules and do not compute Ramsey optimal policy. Similarly, in an early contribution, Yun (1996) includes capital accumulation but does not analyze optimal policy. Further, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) estimate large-scale models which include capital. However, these models contain simple interest rate rules designed to capture actual monetary policy. Among the literature on optimal monetary policy, the setup of Schmitt-Grohé and Uribe (2007) is probably closest to the present paper. Following them, we analyze Ramsey optimal policy in an economy with sticky prices where households invest into capital and rent it to firms. However, Schmitt-Grohé and Uribe analyze the optimal policy in response to shocks to government expenditures and factor productivity. The optimal policy in their paper allows fluctuations in inflation because any positive interest rate distorts households' cash holding decision. In contrast, our model is a cashless economy which faces cost-push shocks. These are modelled as wage markup shocks arising from imperfect substitutability of different labour types. A markup increase implies an inefficient increase in wages and a reduction of economic activity.[2] Such

[1] Altig, Christiano, Eichenbaum, and Linde (2005) show that the endogenous price stickiness implied by models with firm-specific capital helps to reconcile the different estimates of price stickiness from macro and micro models: To match the data, macro models need large price stickiness implying that prices are adjusted in average every six quarters. However, microeconomic estimations suggest much more frequent price adjustment.

[2] The wage increase is inefficient in the sense that it does not reflect a change in the social cost of labour,

shocks play an important role in models aimed at replicating empirical dynamics, such as Smets and Wouters (2007). Further, Galí, Gertler, and López-Salido (2007) measure an inefficiency gap in the labour market, defined as the gap between the marginal product of labour and the marginal rate of substitution between leisure and consumption. They find that this gap can be decomposed into a wage and a price markup and find the wage markup to be the predominant source of fluctuations in the gap. We calibrate our wage markup shock to create an inefficiency gap identical to that measured by Galí, Gertler, and López-Salido (2007). According to Chari, Kehoe, and McGrattan (2009), shocks producing such a gap, which they call the labour wedge, are an important driver of business cycle fluctuations.[3]

The present paper is structured as follows. We start by presenting the model in the following. Section 3 presents the first-best allocation, which provides a useful benchmark and is used to expose distortions in the model economy. We further derive the problem of the Ramsey planner in the third section. In section 4 we present our calibration and the welfare measure we use. Section 5 show the impulse response of the Ramsey policy and optimal simple rules to a cost-push shock. In this section, we evaluate welfare under the alternative policies and characterize their stabilization properties with reference to the distortions identified in the analysis of the first-best equilibrium. Following Casares and McCallum (2006), who argue that a model with frictionless capital accumulation yields an implausible variability of investment, we introduce adjustment cost into our model in section 6. We analyse both capital adjustment cost and the investment adjustment cost introduced into the New Keynesian literature by Christiano, Eichenbaum, and Evans (2005).

2 The model

The model is a standard New-Keynesian model with Calvo-type sticky prices, flexible wages and monopolistically competitive firms. We introduce a cost-push shock by allowing for imperfect substitutability of labour supply. For purposes of illustration, we will here expose the model without adjustment cost.

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i.e. the wage does not increase because the marginal rate of substitution between leisure and consumption increases. Therefore, a social planner would leave the allocation unchanged as will be made explicit in section 3.1.

[3]Note there is debate on the microfoundation of these shocks and to what extent they represent inefficient fluctuations: Chari, Kehoe, and McGrattan (2009) argue that New Keynesian models are not yet ready for policy analysis because there is no microeconomic evidence on the nature of the shock. Therefore, in principle, the observed inefficiency gap might for instance be the result of a fluctuating utility value of leisure. Such a shock would imply efficient fluctuations in the sense that a social planner would accommodate the shock.

2.1 Households and labour supply

There exists a continuum of households with total mass 1, indexed over j. The households derive utility from consuming $c_{j,t}$ and enjoying leisure $1 - n_{j,t}$, with the utility function given by $u(c_{j,t}, n_{j,t}) = \frac{c_{j,t}^{1-\sigma}-1}{1-\sigma} - \chi \frac{n_{j,t}^{1+\eta}}{1+\eta}$. They further invest into capital $k_{j,t}$, which they rent out to firms at the (real) rental rate r_t^k. In nominal terms, letting P_t denote the price level, their revenue is $P_t r_t^k$. The capital stock evolves according to $k_{j,t+1} = (1-\delta)\, k_{j,t} + i_{j,t}$, where $i_{j,t}$ denotes investment and δ the depreciation rate. Households can buy both investment and consumption goods from retailers at an identical price P_t, so that a household's expenditures for consumption and investment goods amount to $P_t(i_{j,t} + c_{j,t})$. Households further have access to nominally state-contingent claims which deliver a certain payoff of one unit of currency in a particular state. Let $D_{j,t}$ denote the amount of claims purchased by a household j in period t. $r_{t,t+1}^d$ denotes the period-t price of such a claim for a particular state, divided by the probability of occurrence of that state conditional on time t information so that $E_t r_{t,t+1}^d D_{j,t}$ are a household's expenditures for acquiring claims. Note that this setup implies incomplete markets because households do not have access to *real* state-contingent claims. Further, households receive the profits earned by firms $P_t \Psi_{j,t}$ and need to pay a lump sum tax $P_t \tau_{j,t}$. Households further supply a differentiated type of labour $n_{j,t}$ for which they earn the real wage w_{jt}. For reasons explained later, the wage is subsidized by the factor γ^w. Thus, the household's budget constraint is given by

$$\gamma^w P_t w_{j,t} n_{j,t} + D_{j,t-1} + P_t \Psi_{j,t} + k_{j,t} P_t r_t^k = c_{j,t} P_t + [k_{j,t+1} - k_{j,t}(1-\delta)]\, P_t + P_t \tau_{j,t} + E_t r_{t,t+1}^d D_{j,t}$$

Defining $d_{j,t} = \frac{D_{j,t}}{P_t}$, so that $E_t r_{t,t+1}^d d_{j,t}$ denotes real expenditures on state-contingent claims, the budget constraint can be expressed in real terms as

$$\gamma^w w_{j,t} n_{j,t} + \frac{d_{j,t-1}}{\pi_t} + \Psi_{j,t} + k_{j,t}(r_t^k + 1 - \delta) = c_{j,t} + k_{j,t+1} + \tau_{j,t} + E_t r_{t,t+1}^d d_{j,t}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ denotes inflation. Before describing optimal behaviour, the structure of the labour market, which resembles Erceg, Henderson, and Levin (2000) and Smets and Wouters (2007), will be explained. Households supply labour not directly to firms but to labour packers who pay them the real wage $w_{j,t}$. Each household supplies a differentiated type of labour which is aggregated to a composite labour good by the labour packers using the production function

$$n_t = \left[\int_0^1 n_{j,t}^{\psi_t} dj\right]^{\frac{1}{\psi_t}}$$

The labour packers sell the aggregate labour good n_t to intermediate firms at the real wage w_t in a perfectly competitive market. Thus, the labour packers' demand for the good

$n_{j,t}$ is given by

$$n_{jt} = \left(\frac{w_{j,t}}{w_t}\right)^{-\zeta_t} n_t \quad \text{where} \quad \zeta_t = \frac{1}{1-\psi_t}$$

We assume that the log of the substitution elasticity ζ_t follows an AR(1) process with mean $\bar{\zeta}$. Because labour packers are perfectly competitive, we can use the zero-profit condition to write the aggregate wage as

$$w_t = \left[\int_0^1 w_{j,t}^{\frac{\psi_t}{\psi_t - 1}} dj\right]^{\frac{\psi_t - 1}{\psi_t}}$$

This implies that in absence of wage dispersion $w_t = w_{jt}$. We can now to derive the households' optimal behaviour. The first order conditions for consumption, capital and state contingent claims read

$$\begin{aligned}
\frac{\partial L}{\partial c_{j,t}} = 0 \quad &\Longleftrightarrow \quad \lambda_{j,t} = c_{j,t}^{-\sigma} \\
\frac{\partial L}{\partial k_{j,t+1}} = 0 \quad &\Longleftrightarrow \quad c_{j,t}^{-\sigma} = \beta E_t \left[c_{j,t+1}^{-\sigma}(r_{t+1}^k + 1 - \delta)\right] \\
\frac{\partial L}{\partial d_{j,t}} = 0 \quad &\Longleftrightarrow \quad r_{t,t+1}^d = \beta \frac{\lambda_{j,t+1}}{\lambda_{j,t}\pi_{t+1}} \qquad (1)
\end{aligned}$$

Optimal wage setting requires

$$\begin{aligned}
\frac{\partial L}{\partial w_{j,t}(s^t)} &= 0 \\
\Longleftrightarrow \quad & -\chi\zeta_t \left(\frac{w_{j,t}}{w_t}\right)^{-\zeta_t(1+\eta)} n_t^{1+\eta} \frac{1}{w_{j,t}} = -\gamma^w c_{j,t}^{-\sigma} (1-\zeta_t) \left(\frac{w_{j,t}}{w_t}\right)^{-\zeta_t} n_t \\
\Longleftrightarrow \quad & w_{j,t} = \frac{\zeta_t}{1-\zeta_t} \frac{1}{\gamma^w} \chi n_t^\eta \left(\frac{w_{j,t}}{w_t}\right)^{-\zeta_t \eta} c_{j,t}^\sigma \\
\Longleftrightarrow \quad & w_{j,t} = \frac{\zeta_t}{1-\zeta_t} \frac{1}{\gamma^w} \chi n_{j,t}^\eta c_{j,t}^\sigma \qquad (2)
\end{aligned}$$

This implies that households charge a markup over their marginal rate of substitution between leisure and consumption. The government subsidy, γ^w, is set so that in the steady state is efficient (see section 2.3). To simplify notation, we denote the net wage markup as

$$\mu_t^w = \frac{\zeta_t}{\zeta_t - 1} \frac{1}{\gamma^w}$$

Note that the only ex ante reason for household heterogeneity is their market power in the labour market. However, wages are perfectly flexible and households have identical market power, so that all households will charge the same wage given that their marginal rates of substitution do not differ. There is no reason for such a difference and thus households will all behave identically and we can work with a representative household. This further implies that there is no dispersion across household's labour supply which would be inefficient under the given labour aggregator, similar to the inefficiencies caused by price dispersion we will encounter later on. To summarize, the representative household's first order conditions are given by

$$\frac{\partial L}{\partial w_t} = 0 \iff w_t = \mu_t^w \chi n_t^\eta c_t^\sigma \tag{3}$$

$$\frac{\partial L}{\partial k_{t+1}} = 0 \iff c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} (r_{t+1}^k + 1 - \delta) \right] \tag{4}$$

$$\frac{\partial L}{\partial d_t} = 0 \iff r_{t,t+1}^d = \beta \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} \tag{5}$$

We can further use the asset pricing equation to construct the (nominally) risk free portfolio which pays an interest of $R_t = \frac{1}{E_t r_{t,t+1}^d}$. We thus obtain a standard Euler equation

$$c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right] \tag{6}$$

For our analysis, it will mostly be sufficient to consider equation (6) instead of the complete set of claim prices in (5) as the central bank controls the rate of the risk-free portfolio. Further, (6) can be used to derive an arbitrage-freeness between state-contingent claims and capital investment, which reads

$$R_t E_t \left[\frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} \right] = E_t \left[c_{t+1}^{-\sigma} (r_{t+1}^k + 1 - \delta) \right]$$

This equation provides one of the central mechanisms of our model. The central bank, who sets the nominal interest rate R_t, has an impact on capital accumulation: An increase in the real interest rate leads households to adapt investment until this investment promises an expected return equal to the state-contingent claims. This link is substantially weakened by the introduction of investment adjustment cost in later sections. A second, important feature of the model is the time-varying markup of labour supply, which will generate cost-push effects.

2.2 Firms

In the model, there are two types of firms: Producers and retailers. Monopolistically competitive producers rent labour and capital to produce the final good y_{it}. Retailers assemble these goods without incurring further cost and produce y_t according to

$$y_t = \left[\int_0^1 y_{it}^q di \right]^{1/q}$$

where $0 < q < 1$ is a function of the elasticity of substitution ε between two input goods, $q = \frac{\varepsilon - 1}{\varepsilon}$. In contrast to the labour sector, there is no time-varying market power here.

Retailers

Retail firms operate in a perfectly competitive market and maximize profits given a price level P_t and aggregate demand y_t :

$$\max_{y_{it}} P_t y_t - \int_0^1 P_{it} y_{it} di$$
$$\iff \quad \max_{y_{it}} P_t \left[\int_0^1 y_{it}^q di \right]^{1/q} - \int_0^1 P_{it} y_{it} di$$

Analogue to the labour sector, retailers demand the following quantity from each individual producer

$$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} y_t$$

Due to perfect competition in the retail sector, the retailers' profits are zero and thus the equilibrium price level is given by

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

Producers : Pricing decision

Producers face two decisions: They minimize cost given a certain level of production. Further, they choose optimal prices, which determine their production. Let us first focus on the pricing decision. We assume standard Calvo pricing where each producer faces a constant probability ϕ that he may not reset his price in a given period. Thus, producers maximize expected (nominal) profits subject to the Calvo pricing scheme. They receive a subsidy γ^p which is designed to render the steady state efficient. Using again that demand is given by $y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} y_t$, nominal profits are

$$\Pi_{it} = \gamma^p P_{it} y_{it} - mc_t P_t y_{it} = \gamma^p \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} y_t P_t - mc_t P_t \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} y_t$$

where mc denotes real marginal cost of firms. As is shown later on, these are constant across producers. Thus, the producers solve

$$\max_{Z_t} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \phi^s \left[\gamma^p \left(\frac{Z_t}{P_{t+s}} \right)^{1-\varepsilon} P_{t+s} y_{t+s} - mc_{t+s} \left(\frac{Z_t}{P_{t+s}} \right)^{-\varepsilon} P_{t+s} y_{t+s} \right]$$

The first order condition to this problem is

$$\sum_{s=0}^{\infty} \Lambda_{t,t+s} \phi^{s} \left[\gamma^{p} \left(1-\varepsilon\right) \left(\frac{Z_t}{P_{t+s}}\right)^{-\varepsilon} y_{t+s} + \varepsilon \left(\frac{Z_t}{P_{t+s}}\right)^{-\varepsilon-1} y_{t+s} \right] = 0$$

With a discount factor of $\Lambda_{t,t+s} = \beta^{s} \frac{c_{t+s}^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+s}}$, this gives the standard optimal prices

$$Z_t = \frac{\varepsilon}{\varepsilon-1} \frac{1}{\gamma^{p}} \frac{E_t \sum_s \left(\beta\phi\right)^s c_{t+s}^{-\sigma} y_{t+s} P_{t+s}^{\varepsilon} mc_{t+s}}{E_t \sum_s \left(\beta\phi\right)^s c_{t+s}^{-\sigma} y_{t+s} P_{t+s}^{\varepsilon-1}}$$

Prices are thus set as a markup on a weighted sum of expected future marginal cost, reflecting the positive probability of not being able to adjust prices. We assume the government to set the subsidy to $\gamma^{p} = \frac{\varepsilon}{\varepsilon-1}$, which implies that if the steady state features a constant price level, each firm sets its price equal to nominal marginal cost, $Z = \frac{\frac{1}{1-\beta\phi} c^{-\sigma} y P^{\varepsilon} mc}{\frac{1}{1-\beta\phi} c^{-\sigma} y P^{\varepsilon-1}} = Pmc$. As marginal cost are identical across firms, no firm has an incentive to change its price and the price level will indeed remain constant so that we obtain a zero inflation steady state. Further, in the steady state, the subsidy eliminates the inefficiency implied by monopolistic competition and the economy efficiently produces at a real marginal cost of unity.

Further, we eliminate the price level by rewriting the optimal price setting equation in terms of the real price $\tilde{Z}_t = Z_t / P_t$. $\tilde{Z}_t$ can be written as a

$$\tilde{Z}_t = \frac{E_t \left[c_t^{-\sigma} y_t mc_t + \beta\phi c_{t+1}^{-\sigma} y_{t+1} \pi_{t+1}^{\varepsilon} mc_{t+1} + \left(\beta\phi\right)^2 c_{t+2}^{-\sigma} y_{t+2} \pi_{t+1}^{\varepsilon} \pi_{t+2}^{\varepsilon} mc_{t+2} + ... \right]}{E_t \left[c_t^{-\sigma} y_t + \beta\phi c_{t+1}^{-\sigma} y_{t+1} \pi_{t+1}^{\varepsilon-1} + \left(\beta\phi\right)^2 c_{t+2}^{-\sigma} y_{t+2} \pi_{t+2}^{\varepsilon-1} \pi_{t+1}^{\varepsilon-1} + ... \right]}$$

We can rewrite the denominator and the numerator recursively and thus obtain a system of three equations which describe optimal price setting:

$$\tilde{Z}_t = \frac{Z_t^1}{Z_t^2}$$

$$Z_t^1 = c_t^{-\sigma} y_t mc_t + \beta\phi E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1 \tag{7}$$

$$Z_t^2 = c_t^{-\sigma} y_t + \beta\phi E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2 \tag{8}$$

To find a law of motion for inflation can be derived by using the definition of the price index[4]

$$
\begin{aligned}
P_t^{1-\varepsilon} &= \int_0^1 P_{it}^{1-\varepsilon} di \\
&= (1-\phi) Z_t^{1-\varepsilon} + \phi P_{t-1}^{1-\varepsilon} \\
&\iff \\
1 &= (1-\phi)\tilde{Z}_t^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1} \\
1 &= (1-\phi)\left(\frac{Z_t^1}{Z_t^2}\right)^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1} \qquad (9)
\end{aligned}
$$

This equation, together with the above two equations describing the evolution of Z_1 and Z_2 constitute the non-linearized version of the New Keynesian Phillips curve.

Producers : Cost minimization

Producers further minimize cost given demand y_{it} (which is determined by their pricing decision), taking as given real factor prices w_t and r_t^k. Their production function is given by $y_{it} = A_t n_{it}^{\alpha} k_{it}^{1-\alpha}$, where total factor productivity A_t is an exogenously given constant. Note that the multiplier on the constraint equals nominal marginal cost, MC. Further, we denote real marginal cost by $mc_{it} = MC_{it}/P_t$. Producers thus solve

$$\min_{n,k} \; P_t w_t n_{it} + P_t r_t^k k_{it} + MC_{it}(y_{it} - A_t n_{it}^{\alpha} k_{it}^{1-\alpha})$$

The producers' first orders read

$$
\begin{aligned}
P_t w_t = MC_{it} f_{n,t} &\iff w_t = mc_t A_t \alpha \left(\frac{k_{it}}{n_{it}}\right)^{1-\alpha} \\
P_t r_t^k = MC_{it} f_{k,t} &\iff r_t^k = mc_t A_t (1-\alpha) \left(\frac{n_{it}}{k_{it}}\right)^{\alpha}
\end{aligned}
$$

We can combine these equations to yield an expression giving the optimal capital-labour ratio and real marginal cost

$$\frac{w_t}{r_t^k} = \frac{\alpha}{(1-\alpha)} \frac{k_{it}}{n_{it}} \qquad (10)$$

$$mc_t = \frac{1}{A_t} \left(\frac{r_t^k}{1-\alpha}\right)^{1-\alpha} \left(\frac{w_t}{\alpha}\right)^{\alpha} \qquad (11)$$

Note that the capital-labour ratio is identical across firms as all have access to the same factor market. Thus, marginal cost are equal across firms, $mc_{it} = mc_t$.

Tracking price dispersion

[4] Details can be found in (A.3.1)

As the aim of this paper is to compute Ramsey optimal policy, and the derivation of a quadratic loss function is complicated by the introduction of capital, the model's equilibrium conditions will not be linearized. Thus, we have to take into account that the production of the final good production is affected by price dispersion which implies that capital and labour are inefficiently allocated across firms. To capture this effect, define intermediate output IO_t as the sum of produced intermediate goods,

$$IO_t \equiv \int_0^1 y_{it} di = \int_0^1 A_t k_{it} \left(\frac{n_{it}}{k_{it}} \right)^{\alpha} di = A_t k_t \left(\frac{n_t}{k_t} \right)^{\alpha}$$

where the last equality uses that k/n is constant across firms. Further, we know that firm demand is given by $y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} y_t$ and thus

$$IO_t = \int_0^1 y_{it} di = y_t \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} di$$

An inefficient distribution of production across firms implies that $IO_t > y_t$. We can find a recursive law of motion for the above integral so that we do not have to track individual prices of firms (following Schmitt-Grohé and Uribe (2004a)). We define the newly set price in real terms, $\tilde{Z} = \frac{Z}{P}$, and use that Z is identical across all firms which set a new price. Let $s_t = \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} di$. The law of motion for s, whose derivation can be found in A.3.2, is given by

$$s_t = (1 - \phi) \tilde{Z}_t^{-\varepsilon} + \phi s_{t-1} \pi_t^{\varepsilon} \tag{12}$$

Schmitt-Grohé and Uribe (2004a) show that s_t is limited below by 1. It increases above unity whenever firms reset their prices, i.e. $\tilde{Z}_t \neq 1$. Thus, any price dispersion implies an inefficient allocation of aggregate resources which is evident by $y_t = \frac{A_t k_t^{1-\alpha} n_t^{\alpha}}{s_t}$.

2.3 Government

Fiscal policy

We assume that the government does not issue bonds which implies that its budget must be balanced in every period. The government collects lump sum taxes to finance the production subsidy to firms and the employment subsidy to workers, $\tau_t = \tau_t^{prod} + \tau_t^{labour}$. In section A.3.3, we show that the households' "transfer", i.e. the difference of firm profits minus lump sum taxes (in real terms) equals

$$\Psi_t - \tau_t^{prod} - \tau_t^{labour} = y_t \left(1 - mc_t s_t\right) - \left(\gamma^w - 1\right) w_t n_t$$

The first term of the RHS reflects firm profits and the tax required to finance the production subsidy. These cancel out in the steady state where $s = 1$ and $mc = 1$, because the steady state is efficient. The second term reflects the tax collected to finance the subsidy to workers.

2.4 Summarizing the model's equilibrium conditions

The model's equilibrium conditions consist of the optimality conditions (3)-(4) and (6)-(12) as well as an aggegrate resource constraint which can be derived from the household's budget constraint (see A.3.4).

$$
\begin{aligned}
&\text{Households}\\
w_t &= \chi n_t^{\eta} c_t^{\sigma} \mu_t^{\omega}\\
c_t^{-\sigma} &= \beta E_t \left[c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right]\\
c_t^{-\sigma} &= \beta E_t \left[c_{t+1}^{-\sigma} (r_{t+1}^k + 1 - \delta) \right]\\
&\text{Firms}\\
r_t^k &= \frac{(1-\alpha)}{\alpha} \frac{w_t n_t}{k_t}\\
mc_t &= \frac{1}{A_t} \left(\frac{r_t^k}{1-\alpha} \right)^{1-\alpha} \left(\frac{w_t}{\alpha} \right)^{\alpha}\\
&\text{Pricing}\\
Z_t^1 &= c_t^{-\sigma} y_t mc_t + \beta\phi E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1\\
Z_t^2 &= c_t^{-\sigma} y_t + \beta\phi E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2\\
1 &= (1-\phi) \left(\frac{Z_t^1}{Z_t^2} \right)^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}\\
s_t &= (1-\phi) \left(\frac{Z_t^1}{Z_t^2} \right)^{-\varepsilon} + \phi \pi_t^{\varepsilon} s_{t-1}\\
&\text{Aggregate resources}\\
A_t k_t^{1-\alpha} n_t^{\alpha} &= \left[c_t + k_{t+1} - (1-\delta) k_t \right] s_t
\end{aligned}
\tag{13}
$$

3 Ramsey optimal policy

As a benchmark, we first present the first best allocation for this economy. These optimality conditions expose the potential distortions in our economy and thus provide a framework for our subsequent analysis of second-best policy. We then proceed to derive the Ramsey optimal policy for our model.

3.1 The social planner equilibrium

The social planner maximizes household utility subject to the economy's technology restrictions. As our labour aggregator, $n_t = \left[\int_0^1 n_{j,t}^{\psi_t} dj \right]^{1/\psi_t}$, is a concave function, and all households' preferences are identical, it is optimal for each type of labour to be used to the same extent, i.e. for any two households j, k, optimality requires

$$n_{j,t} = n_{k,t}$$

We now derive the remaining optimality conditions. As mentioned before, because household preference are identical, we can work with a representative agent. Thus, the social planner's problem reads

$$\max_{\{c,n,n_i,k,k_i,y,y_i\}} G = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right] + \lambda_t^1 \left[y_t - \left(\int_0^1 y_{it}^q di \right)^{1/q} \right] + \lambda_t^2 \left[A_t n_{it}^{\alpha} k_{it}^{1-\alpha} - y_{it} \right] \right.$$
$$\left. + \lambda_t^3 \left[y_t - c_t - k_{t+1} + (1-\delta) k_t \right] + \lambda_t^4 \left[k_t - \int_0^1 k_{it+1} di \right] + \lambda_t^5 \left[n_t - \int_0^1 n_{it+1} di \right] \right.$$

where i is an index of firms. The first-order conditions to this problem show that the social optimum is characterized by

$$u_{c,t} = \beta E_t \left[u_{c,t+1} \left(mpk_{t+1} + 1 - \delta \right) \right] \tag{14}$$
$$mpn_t = -\frac{u_{n,t}}{u_{c,t}} \tag{15}$$
$$y_{it} = y_t \tag{16}$$
$$\frac{k_{it}}{n_{it}} = \frac{k_t}{n_t} \tag{17}$$

and the aggregate resource constraint $y_t = c_t + k_{t+1} - (1-\delta)k_t$. The social planner employs both production factors up to the point where their marginal products equal their social cost. Further, he will not allow any inefficiency in production, which can be caused by dispersion in output or factor usage across firms.

Distortions in the sticky price economy

The social planner's optimality conditions provide a framework for our further analysis in the sense that they clarify the potential distortions in the competitive equilibrium of the sticky price economy. In that economy, as shown in section 2.2, there is no dispersion in the capital labour ratio across firms so that (17) holds in all economies analysed in this paper. Equation (16) represents the well-known distortion caused by price dispersion: (16) does not hold in our economies whenever $\pi_t \neq 1$. This distortion is the reason why any deviation in inflation from its steady state causes welfare losses. The second well-established distortion concerns deviations from the optimality condition (15). We here give this distortion an interpretation in terms of the economy's total markup, defined as $\mu^{tot} = \frac{mpn_t}{mrs_t}$. As in Galí (2008), let us define the firms' average price markup as $\mu_t^P = \frac{P_t}{MC_t} = \frac{1}{mc_t}$. Further, in our model the households' wage markup, which is exogenous in our model, is given by, $\mu_t^w = \frac{w_t}{mrs_t}$. As shown in section (2.2), firms labour demand satisfies $w_t = mc_t mpn_t$. We can thus write the markup distortion, i.e. the deviation from the optimality condition $\frac{mpn_t}{mrs_t} = 1$, as

$$\mu^{tot} = \frac{mpn_t}{mrs_t} = \frac{w_t}{mc_t}\frac{\mu_t^w}{w_t} = \mu_t^w \mu_t^P$$

Intuitively, whenever $\mu^{tot} > 1$, the competitive equilibrium leads to a inefficiently low usage of labour at the aggregate level. The two distortions described so far are present in any model without capital. The distortion implied by price dispersion provides the rationale for stabilizing inflation at $\pi_t = 1$ whereas the markup distortion is the reason why stabilizing output prevents welfare losses.

In the model with endogenous capital accumulation, (14) exposes a potential distortion related to capital accumulation. To be clear, we use the term "distortion" in the sense that, whenever (14) is violated, the capital accumulation decision is distorted.[5] In our economy, where intermediate firms' capital demand satisfies $r_t^k = mc_t mpk_t$, the household's first-order condition for capital accumulation reads

$$\begin{aligned} u_{c,t} &= \beta E_t \left[u_{c,t+1} \left(mc_{t+1} mpk_{t+1} + 1 - \delta \right) \right] \\ &\Longleftrightarrow \\ u_{c,t} &= \beta E_t \left[u_{c,t+1} \left(\frac{mpk_{t+1}}{\mu_{t+1}^P} + 1 - \delta \right) \right] \end{aligned} \tag{18}$$

Because firms are demand constrained, they do not generally rent capital up to its marginal product: Firms who cannot optimize prices face real marginal costs different from unity, i.e. their price markup $\frac{P_{it}}{MC_t}$changes. This causes the average price markup to deviate from its steady state value of $\mu_t^P = 1$.This influences households capital accumulation: If they expect a positive average markup, i.e. real marginal cost below unity, households will reduce capital accumulation below the level a social planner would choose, all other things equal.

The nature of the wage markup shock

We observe from the social planner's first order conditions that the wage markup shock has no impact on the first-best equilibrium: Although the shock technically is a change in technology, it represents no constraint to the social planner. The reason is that the shock concerns the process of wage formation while the social planner can just choose the amount of worked hours he prefers. Because the wage markup shock is the only shock, the social

[5] Note that this definition does not imply that a distortion arises whenever the path of capital chosen by a social planner deviates from that of the competitive equilibrium. This definition would imply that even a lump-sum tax to finance government expenditures would "distort" capital accumulation. Rather, our terminology implies that capital accumulation is distorted whenever, *all other things equal*, the representative household accumulates an amount of capital different from that implied by the social planner's optimality condition.

planner would choose constant paths for all model's variables. Thus, the efficient level of output is constant and welfare losses accrue whenever output is not in its steady state. Further, the shock also distorts the flexible price equilibrium. Thus, it is not necessarily optimal for the central bank to stabilize output at its natural rate. Rather, as in Adao, Correia, and Teles (2003), the presence of the sticky price friction equips the central bank with an additional instrument which optimal policy can use to improve upon the flexible price allocation and stabilize output closer to its efficient level, the steady state.

The central bank's tradeoff

From the above considerations, we observe that the social planner equilibrium can be achieved by completely stabilizing the total markup μ^{tot} and inflation π at their steady state values. However, this is not possible whenever our shock is present: A positive wage markup can in principle be eliminated by a negative average price markup. However, a negative average price markup requires price dispersion, so that, whenever $\mu^{w} \neq 1$, monetary policy cannot achieve $\pi_t = 1$ and $\mu^{tot} = 1$ at the same time. Intuitively, as a reaction to a positive wage markup shock, the central bank can create a negative average price markup by setting the nominal interest rate below its natural rate[6], which implies that demand and thus output will exceed their natural rates. This implies that marginal cost exceed their natural level. This policy stabilizes output at the expense of increased inflation. So far, this is the familiar tradeoff of a central bank in a sticky price model with monopolistic competition. The introduction of capital accumulation affects the central bank's tradeoff because firms' *expected* average price markup $E_t mc_{t+1}$ has an impact on capital accumulation and thus on the future path of the economy. If households expect the central bank to pursue a policy which generates a negative average price markup, they will reduce capital accumulation below the level a social planner would choose. Thus, a policy aimed at stabilizing output by creating price markups which offset the exogenous wage markup potentially increases the fluctuations of the capital stock. However, as we analyse a general equilibrium in which other distortions are present, nothing can be said about the welfare implications of a policy which distorts capital accumulation at this stage.

Other authors have derived loss functions by using a second-order approximation of household utility. For the model with capital, Edge (2003) and Sveen and Weinke (2006) derive such a loss function. However, as these contain standard deviations and, in the case of Edge (2003) also correlations among all the model's variables, they are not straightforward to interpret, either. Moreover, the welfare function derived by Sveen and Weinke (2006) has positive coefficients on the variances of consumption, output and investment while the coefficients on the variance of labour and those relating to price and wage dispersion. It is obvious that such a loss function cannot be used to answer the question if stabilizing the capital stock is beneficial. The reason is that it cannot be interpreted

[6] To be exact, it is the path of future interest rates that matters for consumption demand.

in a *ceteris paribus* manner: If we did so, we would have to conclude that increasing the consumption variance would increase welfare. Rather, the general equilibrium effects of such an increase (for instance an associated decrease in the labour variance) would have to be considered as well. Therefore, such a loss function does not provide more intuition than the strategy we apply: We calibrate the model's parameters and use second-order approximations to solve the model around its deterministic steady state. We then evaluate welfare under different policies using a welfare measure developed by Schmitt-Grohé and Uribe (2006). The distortions identified in this section are used to build an intuition for our results.

3.2 The Ramsey problem

By analysing Ramsey optimal policy, we assume that the central bank can credibly commit to its monetary policy and thus manipulate agents' expectations. As mentioned before, we do not derive a quadratic loss function from a second approximation to household utility but set up a full fledged Ramsey problem and derive its first-order conditions without linearizing the model's equilibrium conditions.[7] Due to the complexity of the model, it is not possible to condense the model's competitive equilibrium into a single implementability constraint. Thus, we apply a dual Ramsey approach: We derive an intertemporal budget constraint by iterating forward the household's budget constraint. This allows us to ensure that the transversality conditions for capital and state-contingent bonds hold. Further, we substitute out as many variables as possible in the model's equilibrium conditions. The Ramsey planner then maximizes household utility subject to the intertemporal budget constraint and the "condensed" version of the model's equilibrium conditions. Again, we restrict the derivation to the case without adjustment cost. However, the general strategy in the models with adjustment cost is the same.

Deriving the intertemporal budget constraint[8]

The representative household's budget constraint can be written as

$$w_t n_t + \frac{d_{t-1}}{\pi_t} + y_t\left(1 - mc_t s_t\right) + k_t(r_t^k + 1 - \delta) = c_t + k_{t+1} + E_t r_{t,t+1}^d d_t$$

where we used that the government collects the subsidy to workers by raising lump sum taxes, so that $y_t\left(1 - mc_t s_t\right)$ is the residual left from firm profits after paying the part of the lump-sum tax which finances the production subsidy, τ_t^{prod}. Iterating over state-contingent claims and using the household's first order conditions, the intertemporal budget constraint can be written as

[7] The solution to the resulting system of optimality conditions will be solved by second-order approximation methods using the software dynare.

[8] We will here focus on the main steps while (A.4.1) contains the detailed derivation.

$$c_t^{-\sigma}\left[\frac{d_{t-1}}{\pi t}+k_t(r_t^k+1-\delta)\right]=E_t\sum_{s=0}^{\infty}\beta^t c_{t+s}^{-\sigma}\left(c_{t+s}-w_{t+s}n_{t+s}-y_{t+s}\left(1-mc_{t+s}s_{t+s}\right)\right)$$

Substituting out the wage and the capital rental rate, the intertemporal budget constraint becomes

$$A_t=E_t\sum_{s=0}^{\infty}\beta^s\left[c_{t+s}^{1-\sigma}+\left(\frac{1-\alpha}{\alpha}\right)\chi n_{t+s}^{1+\eta}\mu_{t+s}^w-c_{t+s}^{-\sigma}\frac{IO_{t+s}}{s_{t+s}}\right] \tag{19}$$

where $A_t=c_t^{-\sigma}\frac{d_{t-1}}{\pi_t}+\frac{(1-\alpha)\chi}{\alpha}n_t^{\eta+1}\mu_t^w+c_t^{-\sigma}k_t\left(1-\delta\right).$

Writing the intertemporal budget constraint recursively

The Ramsey planner faces an intertemporal budget constraint in every period because the household cannot by trading in period zero insure against all risks. The reason for this is that he has no set of real state contingent claims so that HH solvency cannot be ensured by period zero trading but must be guaranteed in every period. As described in Ljungqvist and Sargent (2004), the sequence of intertemporal budget constraints can be written recursively:

$$\begin{aligned}&E_0\sum_{t=0}^{\infty}\beta^t\gamma_t\left\{E_t\sum_{j=0}^{\infty}\beta^{t+j}\left[c_{t+j}^{1-\sigma}+\left(\frac{1-\alpha}{\alpha}\right)\chi n_{t+j}^{1+\eta}\mu_{t+j}^w-c_{t+j}^{-\sigma}\frac{IO_{t+j}}{s_{t+l}}\right]-A_t\right\}\\&=E_0\sum_{t=0}^{\infty}\beta^t\left\{\Gamma_t\left[c_t^{1-\sigma}+\left(\frac{1-\alpha}{\alpha}\right)\chi n_t^{1+\eta}\mu_t^w-c_t^{-\sigma}\frac{IO_t}{s_t}\right]-(\Gamma_t-\Gamma_{t-1})A_t\right\}\end{aligned}$$

where γ_t denotes the multiplier on the intertemporal budget constraint and where $\Gamma_t=\Gamma_{t-1}+\gamma_t$, with $\Gamma_{-1}=0$.

The Ramsey problem

We can reduce the system of equilibrium conditions (13) by eliminating w, r^k, and mc which yields a system of 7 equations. The Ramsey planner maximizes household utility subject to the intertemporal budget constraint and the remaining equilibrium conditions, so that the Ramsey planner's problem reads

$$\begin{aligned}
\max_{\{c,n,k,R,s,\pi,\tilde{Z},Z^1,Z^2\}} J = E_0 \sum_{t=0}^{\infty} \beta^t & \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right] \\
& + \Gamma_t \left[c_t^{1-\sigma} + \frac{1-\alpha}{\alpha} \chi n_t^{1+\eta} \mu_t^w - c_t^{-\sigma} \frac{A_t k_t^{1-\alpha} n_t^{\alpha}}{s_t} \right] \\
& - (\Gamma_t - \Gamma_{t-1}) \left[\frac{(1-\alpha)\chi}{\alpha} n_t^{\eta+1} \mu_t^w + k_t c_t^{-\sigma} (1-\delta) \right] \\
& + \lambda_t^1 \left[c_t^{-\sigma} - E_t \beta c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right] \\
& + \lambda_t^2 \left[c_t^{-\sigma} - E_t \beta (1-\alpha) \frac{\chi}{\alpha} \frac{n_{t+1}^{\eta+1}}{k_{t+1}} \mu_{t+1}^w - E_t \beta (1-\delta) c_{t+1}^{-\sigma} \right] \\
& + \lambda_t^3 \left[\tilde{Z}_t - \frac{Z_t^1}{Z_t^2} \right] \\
& + \lambda_t^4 \left[\pi_t - \left(\frac{1 + (\phi - 1) \tilde{Z}_t^{1-\varepsilon}}{\phi} \right)^{\frac{1}{\varepsilon - 1}} \right] \\
& + \lambda_t^5 \left[s_t - (1-\phi) \tilde{Z}_t^{-\varepsilon} + \phi \pi_t^{\varepsilon} s_{t-1} \right] \\
& + \lambda_t^6 \left[Z_t^1 - \frac{n_t^{1+\eta}}{s_t} \frac{\chi}{\alpha} \mu_t^w - \phi \beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1 \right] \\
& + \lambda_t^7 \left[Z_t^2 - c_t^{-\sigma} \frac{A_t k_t^{1-\alpha} n_t^{\alpha}}{s_t} - \phi \beta E_t \pi_{t+1}^{\varepsilon - 1} Z_{t+1}^2 \right]
\end{aligned} \tag{20}$$

As the first order conditions to this problem are not straightforward to interpret, we proceed by describing the response of optimal policy to a cost-push shock, i.e. an increase in the markup μ_t^w, in a calibrated model. The interested reader can find the complete system of first order conditions in the appendix.

4 Calibration and welfare measure

4.1 Calibration and functional form

The model's calibration is standard and can be found in the appendix. We here confine to a description of our single shock, the wage markup shock.

Our shock is based on Galí, Gertler, and López-Salido (2007) who measure an inefficiency gap, defined as $\log \frac{mrs}{mpn}$ based on a simple model. Using wage data, they decompose this gap into a price and a wage markup, finding that the wage markup accounts for nearly all of the variation in the inefficiency gap. We calibrate our shock to match the standard deviation of their gap but set the autocorrelation at $\rho^{\varsigma} = 0.9$ instead of their estimate of 0.95. As is shown in the appendix, so as to yield $\sigma(\mu_t^w) = 0.051$, we set standard deviation of the driving innovation to $sd(\varepsilon^{\zeta}) = 0.1177$.[9] This implies that a positive shock of one standard deviation increase the households markup, $\mu_t^w = \frac{\zeta_t}{\zeta_t - 1}\frac{\bar{\zeta}-1}{\bar{\zeta}}$, by 2.35%. A note on the consistency of using their measured inefficiency gap in our model is in order: First, if Galí, Gertler, and López-Salido (2007) had measured the gap based on the flexible price version of our model, this would not have affected their results.[10] Of course, consistency requires us to use their preference specification, $\sigma = \eta = 1$. Second, price stickiness generates an average price markup which implies that the measured gap of our model will not be entirely identical to Galí, Gertler, and López-Salido (2007). However, under all simple rules we analyse, the exogenous wage markup quantitatively dominates the endogenous fluctuations in the price markup: The standard deviation of the price markup never exceeds 4% of that of the total markup. In contrast, Ramsey optimal policy generates large procyclical price markups which reduce the standard deviation of the inefficiency gap implied by our model by up to 15%. However, it is unlikely that optimal Ramsey policy was in place during the measurement period of Galí, Gertler, and López-Salido (2007).

Further, under simple rules, monetary policy follows a rule of the form[11]

$$R_t = \frac{1}{\beta}\left(\frac{\pi_t}{\bar{\pi}}\right)^{w_\pi} \quad \text{with} \quad w_\pi > 1$$

Thus, the central bank increases the nominal rate more than one-for-one whenever inflation increases above its steady state value (which is unity, as argued above).

4.2 Welfare measure

Our welfare measure is based on the representative household's expected lifetime utility, $W = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$. Following Schmitt-Grohé and Uribe (2006), we measure welfare conditional on the initial state being the deterministic steady state. The reason is that all policies imply the same deterministic steady state. The central idea of Schmitt-Grohé and Uribe (2006) is to use the policy function of the variable $V_t = E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}, n_{t+s})$ and to evaluate it at the initial state. With V denoting this policy function, what we

[9] Their wage markup has nearly identical properties: Its autocorrelation parameter is identical to the gap and the standard deviation of the wage markup is 0.054 instead of 0.051 for the gap. We calibrate our model to their gap so as to capture the total inefficiency they measure.

[10] The reason is that they have an identical utility function and measure the marginal rate of substitution between consumption and leisure as $\log mrs_t = \sigma \log c_t + \eta \log n_t - \xi_t$, where ξ_t denotes low frequency preference shifts. Further, the marginal product of labour is measured as $\log mpn_t = \log y_t - \log n_t$, so that the inclusion of capital accumulation does not affect the measurement of the gap.

[11] Smets and Wouters (2007) use a similar interest rate rule in the non-linearized model presented in the appendix to their paper.

seek is $V(x_0, \omega)$, where x_0 refers to the vector of initial states and ω is a parameter scaling the degree of uncertainty in the economy. Because we analyze welfare under wage markup shocks only, ω equals the standard deviation of this shock. If we based our welfare measure on the average value of V_t obtained from a simulation of the model, we would actually evaluate unconditional welfare and ignore the transition path involved after a policy switch. Here, such transition paths are not caused by different deterministic steady states but result from different stochastic steady states around which the model's variables fluctuate in a simulation.[12]

As is common in the literature, we measure welfare in terms of the percentage of consumption that would leave the representative household indifferent between the superior policy A and the inferior policy B. Denoting this welfare measure with λ and letting superscripts refer to a particular policy, λ is implicitly defined by

$$W^B = E_0 \sum_{t=0}^{\infty} \beta^t u\left((1-\lambda)\, c_t^A, n_t^A\right)$$

Schmitt-Grohé and Uribe (2006) show that λ, which is a function $\Lambda(x_t, \omega)$ of the initial state and ω, can be computed accurately up to second order from the second-order approximated policy functions for V. With our utility function,

$$\begin{aligned} \Lambda(x_t, \omega) &\approx \Lambda(\bar{x}, 0) + \Lambda_\omega(\bar{x}, 0)\omega + \Lambda_{\omega\omega}(\bar{x}, 0)\frac{\omega^2}{2} \\ &\Longleftrightarrow \\ \Lambda(x_t, \omega) &\approx \frac{V_{\omega\omega}^B(\bar{x}, 0) - V_{\omega\omega}^A(\bar{x}, 0)}{(\sigma-1)\left[V^A(\bar{x}, 0) - VN^A(\bar{x}, 0)\right]}\frac{\omega^2}{2} \end{aligned}$$

VN here denotes the policy function of the variable $VN_t = -E_t \sum_{s=0}^{\infty} \beta^s \left(\chi \frac{n_{t+s}^{1+\eta}}{1+\eta} + \frac{1}{1-\sigma}\right)$. Further, $V_{\omega\omega}^A(\bar{x}, 0)$ and $V_{\omega\omega}^B(\bar{x}, 0)$ are steady state values of the second derivatives of the policy function V under policy A (or B) in a second-order approximation around the model's deterministic steady state $(\bar{x}, 0)$:

$$\begin{aligned} V(x_t, \omega) \approx\, & V(\bar{x}, 0) + V_x(\bar{x}, 0)(x_t - \bar{x}) + V_\omega(\bar{x}, 0)\omega + V_{\omega x}(\bar{x}, 0)\omega(x_t - \bar{x}) \\ & + \frac{1}{2}V_{xx}(\bar{x}, 0)(x_t - \bar{x})^2 + \frac{1}{2}V_{\omega\omega}(\bar{x}, 0)\omega^2 \end{aligned}$$

In the following analyses, we will evaluate the welfare loss of each policy relative to the deterministic model. We report these results as well as the additional welfare loss implied by simple rules, expressed as percentages of the welfare loss under Ramsey optimal

[12] For a detailed description of the welfare measure used here, the interested reader is referred to the appendix, section A.2.

policy. For our purpose of comparing simple rules to optimal policy, we prefer using the latter measure. The reason is that it is independent of the shock variance which varies considerably across studies.[13]

5 Optimal monetary policy

Before interpreting the policy response, a few words on the nature of our cost-push shock are in order. The shock puts upward pressure on wages and firms' marginal cost and thus leads to both price increases and a contraction in output. It is important to note that the shock constitutes an inefficiency: As can be seen in section 3.1, the wage markup does not affect the level of output a social planner would choose, i.e. the first-best level of output stays constant. Further, the shock also distorts the flexible price equilibrium so that the central bank cannot achieve the first-best allocation by eliminating fluctuations in inflation entirely. Therefore, our shock confronts the central bank with a true tradeoff between stabilizing inflation and the total markup, as is exposed in more detail in section 3.1. All variables are defined in terms of percentage deviations from their steady state. Variables with superscript n denote a variable's value under flexible prices.

5.1 Model without capital

Figure 1 shows the impulse response function to a wage markup shock under Ramsey optimal policy in the model without capital. This model serves as a benchmark for our subsequent analysis. To illustrate the central bank's tradeoff, the last row shows the total markup μ^{tot},the exogenous wage markup μ^{w} and the endogenous average price markup $\mu^{P} = \frac{1}{mc}$ which are presented in section 3.1. The shock causes a rise in wages and a decline in output. The central bank reduces the nominal interest rate on impact but increases it above its steady state value for all periods thereafter. To clarify how this policy improves upon the flexible price allocation, observe that the central bank persistently sets the interest rate below its natural rate. This policy creates a negative average price markup and thus stabilizes output more successfully than the flexible price allocation, as indicated by the positive output gap. The cost of this policy is temporary inflation, which generates welfare losses through price dispersion. However, the central bank pursues this policy of output stabilization only for the first 2-3 quarters. The reason is the forward-looking nature of our Phillips curve: The central bank does not allow inflation to persist because any inflation in later periods would lead to pre-emptive price increases due to staggered price setting. Thus, stabilizing output in "early" periods is less costly than doing so in later periods. We will see this pattern of short-run output stabilization re-emerge under all models we analyse.

[13]For instance, Smets and Wouters (2007) estimate a one s.d. wage markup shock to increase wages by 24 percent. In our case, this figure is 2.35 percent.

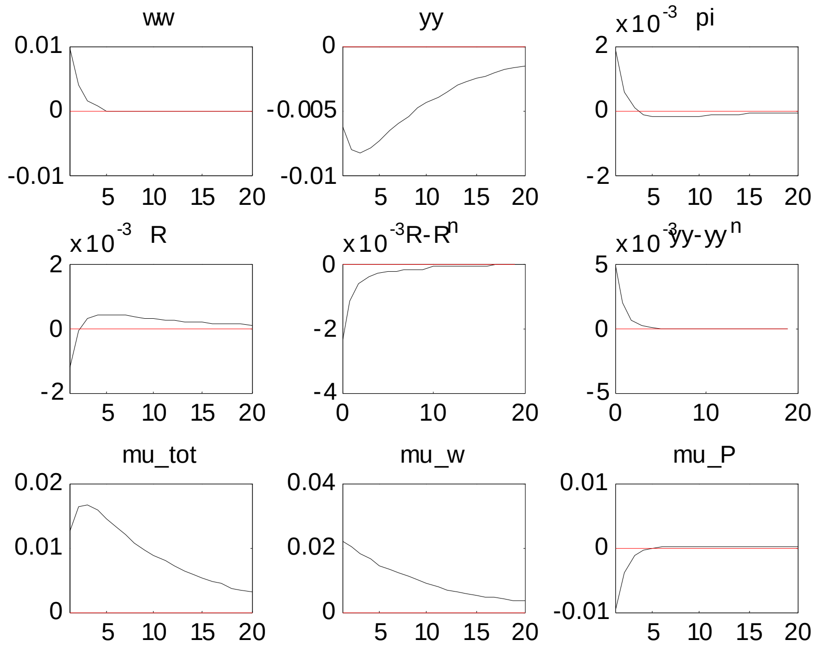

Figure 1: Ramsey policy in the model without capital

5.2 Model without adjustment cost

5.2.1 Ramsey policy

Figure 2 shows the response of Ramsey monetary policy to a wage markup shock. Output, working time and consumption decrease as a response to the shock. Investment is also reduced, which lowers the capital stock in subsequent periods. In terms of the output gap and inflation, the pattern from the model without capital re-emerges: The central bank opts to generate a negative price markup, thereby generating a positive output gap at the cost of temporary inflation. Thus, the inclusion of endogenous capital formation has not substantially altered the central bank's tradeoff in the sense that optimal policy chooses to resolve the tradeoff in a similar way. However, the path of its instrument has changed. As before, the central bank chooses a path of the nominal interest rate which is persistently below the natural interest rate, except for the first period. This generates the positive output gap and the associated inflation. The main difference we observe is that the natural interest rate now falls below its steady state for 8 quarters. This leads the central bank to reduce the nominal rate below its steady state for at least 10 quarters. This pattern is inconsistent with the usual intuition of central banks curbing inflation by applying active policy, i.e. increasing the real interest rate in response to positive inflation. We thus analyse in the next section how such a policy, which is recommended as a virtually perfect approximation of optimal policy, for instance by Schmitt-Grohé and Uribe (2007), performs well in our model.

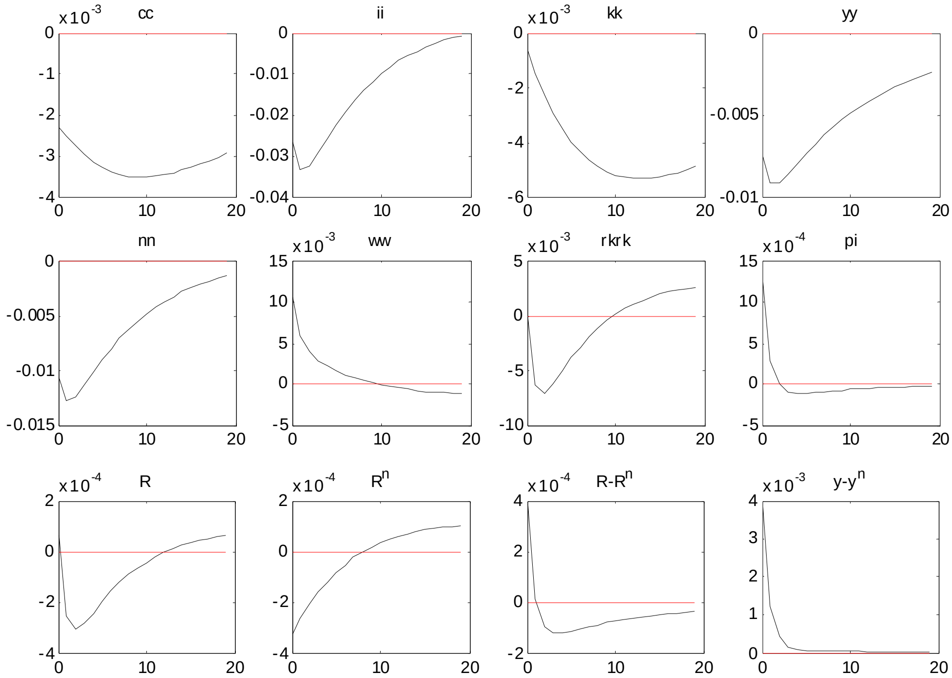

Figure 2: Ramsey policy, model with capital, without adjustment cost

5.2.2 The optimal simple rule

Figure 3 shows the impulse response functions under the optimal simple rule, which features an aggressive response to inflation, $w_\pi = 8$. This policy is very successful in stabilizing inflation and thus induces an allocation virtually identical to the flexible price economy: The output gap is close to zero at all times. The central bank achieves this by setting the interest rate very close to its natural rate. We now assess the welfare implications of this policy, which are summarized in Table 1, which gives the welfare loss λ in percent of steady state consumption under both the Ramsey policy and the optimal simple rule. The welfare loss of the optimal simple rule is 9.19% higher than that of the Ramsey policy. However, due to the rather low welfare cost of the shock, the simple rule looses only 0.0057% of steady state consumption compared to Ramsey optimal policy.[14] As suggested by the impulse response functions, the simple rule stabilizes the average markup less successfully than the Ramsey policy. The benefit of the simple rule is a reduction in the variance of

[14] However, as shown by Galí, Gertler, and López-Salido (2007), welfare cost can be moderate in average but may amount to several percentages of steady state consumption in specific recessions. Further, the absolute welfare difference $\lambda^{simple} - \lambda^{Rams}$ depends on shock variance. Therefore, we consider our relative measure $\frac{\lambda^{simple}}{\lambda^{Rams}}$ a more useful measure of the performance of simple rules.

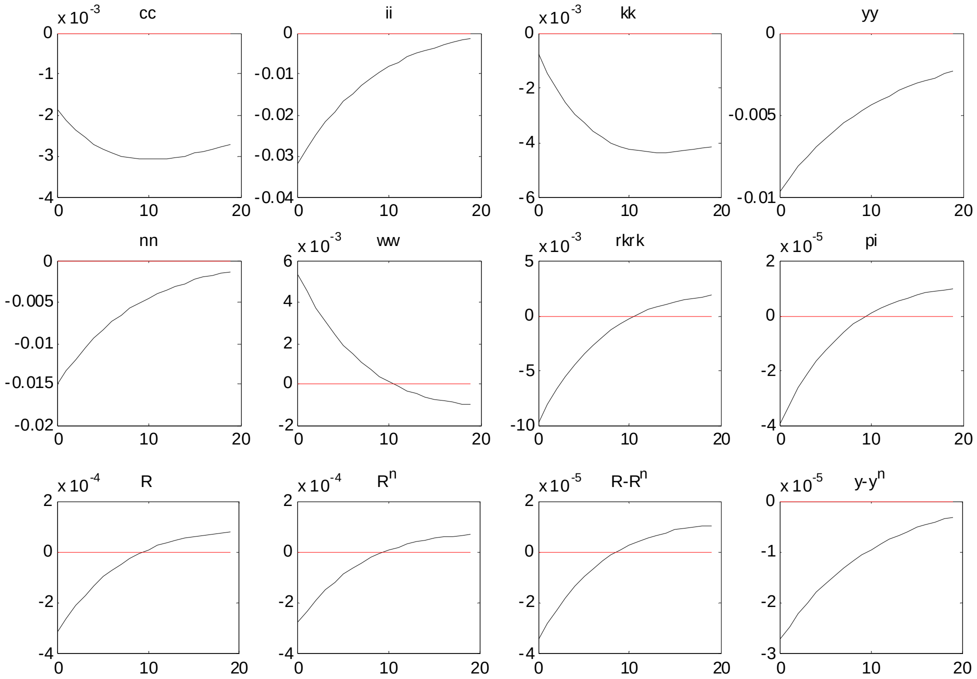

Figure 3: Optimal simple rule, no adjustment cost

inflation. However, this comes at the cost of increased fluctuations in output, the capital stock, consumption and labour.

Welfare comparison	Welfare loss λ	$sd(\hat{\pi})$	$sd(\mu^{tot})$	$sd(\hat{y})$	$sd(\hat{k})$	$sd(\hat{c})$	$sd(\hat{n})$
Optimal simple rule	0.0682%	0.0085%	0.0511	0.0256	0.0234	0.0162	0.0326
Ramsey policy	0.0625%	0.1142%	0.0477	0.0239	0.0223	0.0155	0.0302
Standard deviations refer to variables defined as $\mu_t = \log(\mu_t/\bar{\mu})$.							
Due to its low values, the s.d. of π is expressed in terms of % points.							
The optimal simple rule features a coefficient of $w_\pi = 8$.							

Table 1: Welfare comparison without adjustment cost

6 Optimal policy with positive adjustment cost

Intuition suggests that capital accumulation is likely to entail adjustment cost: When a firm desires to increase its capital stock, there is a cost of installing the investment goods purchased. Such cost lead the capital stock to behave in a more sluggish manner, which is considered empirically realistic. For instance, Casares and McCallum (2006) argue that adjustment cost are needed in order for models with endogenous capital accumulation to match cyclical data. We thus consider a version of our model under capital adjustment cost as well as the specification of investment adjustment cost used by Christiano, Eichenbaum, and Evans (2005). First, we present impulse responses to a wage markup shock for a baseline calibration for each type of adjustment cost. Subsequently, we evaluate the performance of simple rules - in terms of replicating the allocation achieved by Ramsey policy - across a range of plausible calibrations.

6.1 Capital adjustment cost

As in Chari, Kehoe, and McGrattan (2007), adjustment cost are given by the function $F(\frac{i}{k})$ so that the law of motion for capital reads

$$k_{t+1} = (1-\delta)k_t + i_t - F\left(\frac{i_t}{k_t}\right)k_t$$

$$\text{where } F = \frac{\psi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2$$

The first order conditions of the representative household with respect to investment and capital change to

$$c_t^{-\sigma} q_t = \beta c_{t+1}^{-\sigma} \left\{ q_{t+1}(1-\delta) + r_{t+1}^k - q_{t+1}\frac{\psi}{2}\left[\frac{i_{t+1}}{k_{t+1}} - \delta\right]^2 + q_{t+1}\frac{i_{t+1}}{k_{t+1}}\psi\left[\frac{i_{t+1}}{k_{t+1}} - \delta)\right]\right\}$$

$$q_t = \left[1 - \psi(\frac{i_t}{k_t} - \delta)\right]^{-1}$$

where q_t represents the value of capital relative to the consumption good. As marginal adjustment cost are zero in the steady state, q equals unity in the steady state.[15] In our benchmark scenario, we calibrate ψ to $\psi = 10$ so as to obtain a steady state elasticity of the price of capital with respect to the investment-capital ratio is 0.25, which is in the midst of the range considered reasonable by Bernanke, Gertler, and Gilchrist (1999).

6.1.1 Ramsey policy

Figure 4 shows the impulse responses under positive capital adjustment cost. The patterns of inflation and the output gap are unchanged from the previous analysis: It is again optimal to stabilize output by allowing inflation in early periods and stabilize inflation in later ones. The natural interest rate increases above its steady state and optimal policy

[15] Note that, under adjustment cost, it is not possible to derive an intertemporal budget constraint. The reason is that the above first order conditions become more complicated. Therefore, the Ramsey planner here maximizes utility subject to the aggregate resource constraint in its period-by-period form, and the (reduced) system of the model's equilibrium conditions.

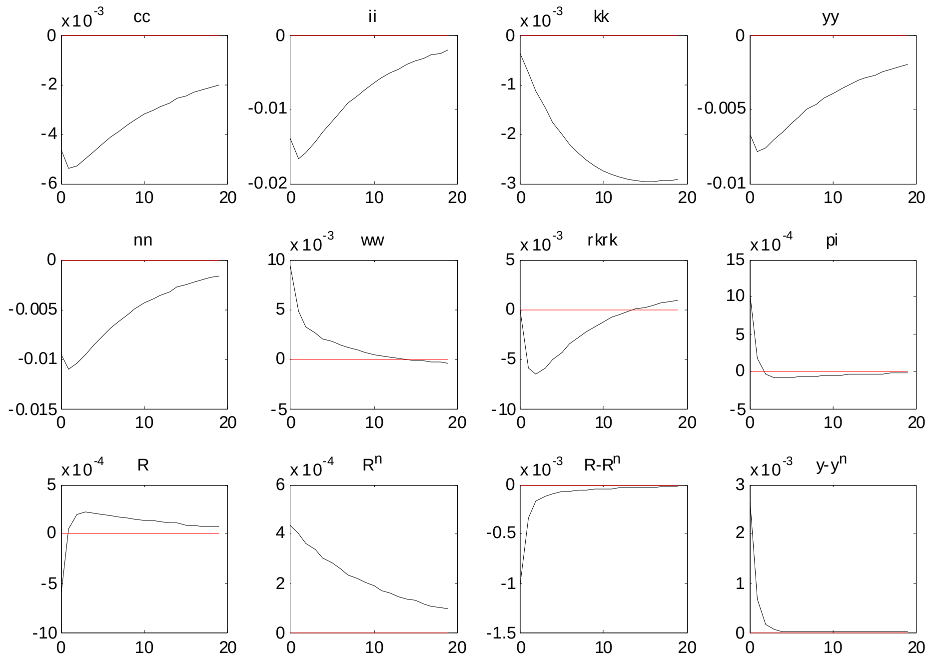

Figure 4: Ramsey optimal policy, model with capital adjustment cost

reduces its instrument below the natural rate to generate the familiar output-inflation pattern. Thus, the impulse responses resemble those of the model without capital closely.[16] The reason is that adjustment cost weaken the link between the real interest rate and investment because capital cannot be invested and disinvested at zero cost. Thus, the arbitrage equation derived earlier does not hold any more. We further observe that the central bank here increases the nominal interest rate in all periods but the first. Thus, the next section asks the question if active policy performs better in the model with capital adjustment.

6.1.2 The optimal simple rule

Figure 5 shows the impulse responses to a cost push shock under the optimal simple rule, which is again very aggressive, $w_\pi = 8$. Similar to the model without adjustment cost, the optimal simple rule stabilizes inflation more successfully than the Ramsey policy. Again, this requires that the central bank sets its instrument close to the natural interest rate. Table 2 presents the welfare cost of both policies, together with selected variables' second moments. The simple rule stabilizes output only slightly more successfully than the flexible-price allocation, which is reflected in the fact that the standard deviation of the average markup is reduced only slightly below that of the exogenous markup shock (which is 0.0510). The benefit of the simple rule is the reduced variance of inflation. As

[16] This is in line with Casares and McCallum (2006) who find an analogous result for simple rules.

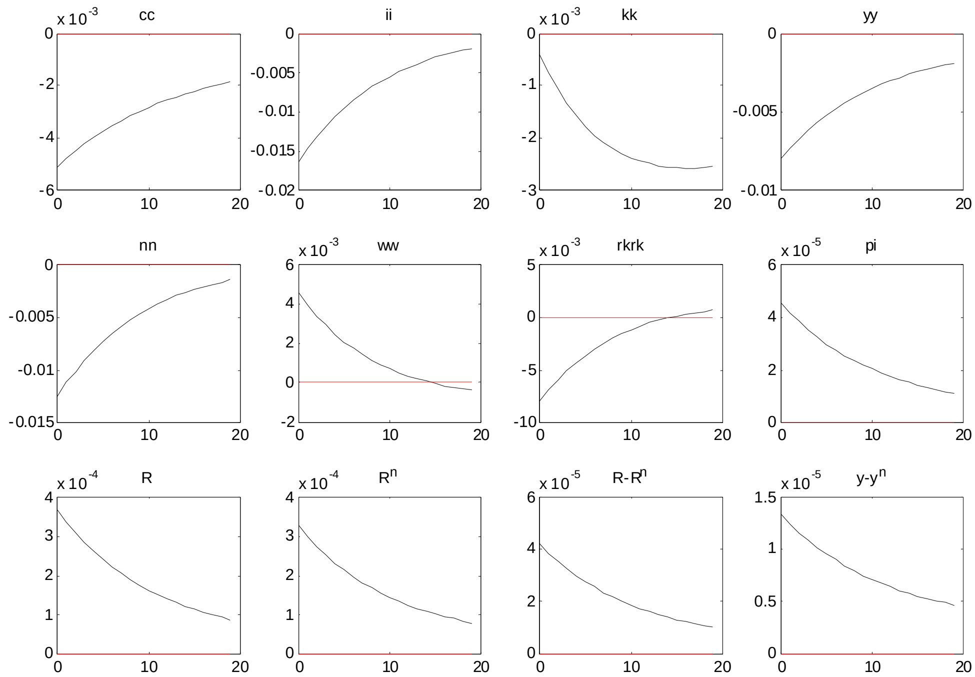

Figure 5: Optimal simple rule, model with capital adjustment cost

before, this comes at the cost of increased fluctuations in labour, consumption, capital and output, compared to the Ramsey policy. However, the welfare loss of the simple rule is only 6.41% higher than that of the Ramsey policy. Thus, the moderate adjustment cost we calibrate imply that active policy more closely resembles optimal policy.

Welfare comparison	Welfare loss λ	$sd(\pi)$	$sd(\mu^{tot})$	$sd(\hat{y})$	$sd(\hat{k})$	$sd(\hat{c})$	$sd(\hat{n})$
Optimal simple rule	0.0548%	0.0119%	0.0509	0.0210	0.0156	0.0162	0.0283
Ramsey policy	0.0515%	0.0936%	0.0482	0.0200	0.0152	0.0155	0.0268
Standard deviations refer to variables defined as $\mu_t = \log(\mu_t/\bar{\mu})$.							
Due to its low values, the s.d. of π is expressed in terms of % points.							
The optimal simple rule features a coefficient of $w_\pi = 8$.							

Table 2: Welfare comparison with capital adjustment cost

6.2 Investment adjustment cost

This section analyses optimal monetary policy and the performance of simple rules under investment adjustment cost. This specification is used in many large-scale models, most prominently by Christiano, Eichenbaum, and Evans (2005).[17] Under investment

[17] Among others, Beaudry and Portier (2006) and Burnside, Eichenbaum, and Fisher (2004) employ this specification. For a survey of models assuming investment adjustment cost, see Khan and Groth (2006).

adjustment cost, the capital law of motion takes the following form

$$k_{t+1} = (1-\delta)\,k_t + i_t S(\frac{i_t}{i_{t-1}})$$
$$\text{with } S(\frac{i_t}{i_{t-1}}) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$$

so that the households first order conditions with respect to investment and capital read

$$c_t^{-\sigma} q_t = \beta E_t \left[c_{t+1}^{-\sigma} q_{t+1}(1-\delta) + c_{t+1}^{-\sigma} r_{t+1}^k\right]$$
$$c_t^{-\sigma} = c_t^{-\sigma} q_t \left[S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}} S'\left(\frac{i_t}{i_{t-1}}\right)\right] - \beta E_t c_{t+1}^{-\sigma} q_{t+1}\left[\left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right)\right]$$

where q_t is the value of capital.[18] With respect to the calibration, there is considerable disagreement in the literature, which is discussed further in the appendix. We here analyse as a baseline case the value estimated by Christiano, Eichenbaum, and Evans (2005), $\kappa = 2.48$, which serves to generate empirically reasonable impulse responses to a monetary policy shock in their model.

6.2.1 Ramsey policy

Figure 6 shows the impulse responses under optimal policy in the model with positive investment adjustment cost. Again , we observe a pattern of temporary inflation and a positive output gap. Thus, under investment adjustment cost, optimal monetary policy resolves its tradeoff similar to the model without capital. The familiar short-run stabilization of output is achieved by setting the nominal interest rate below its natural rate. As was the case under capital adjustment cost, we observe that the central bank rises the nominal interest above its steady state value, reflecting the increased natural interest rate. Thus, both the way the central bank resolves its tradeoff, and the instrument path required to implement this policy are similar to the model without capital. The next section asks how closely a simple rule can imitate this optimal policy.

6.2.2 Optimal simple rule

The optimal simple rule in the model with investment adjustment cost is less aggressive here, with an optimal coefficient of $w_\pi = 4$. Similar to the simple rule under capital adjustment cost, the simple rule achieves a slightly positive output gap. This is achieved by allowing some inflation which drives down the real interest rate below its flexible-price

[18] The Ramsey policy is computed by maximizing utility with respect to the set of equilibrium conditions, including the above derived Euler equations for capital and investment, and an aggregate resource constraint in period-by-period form.

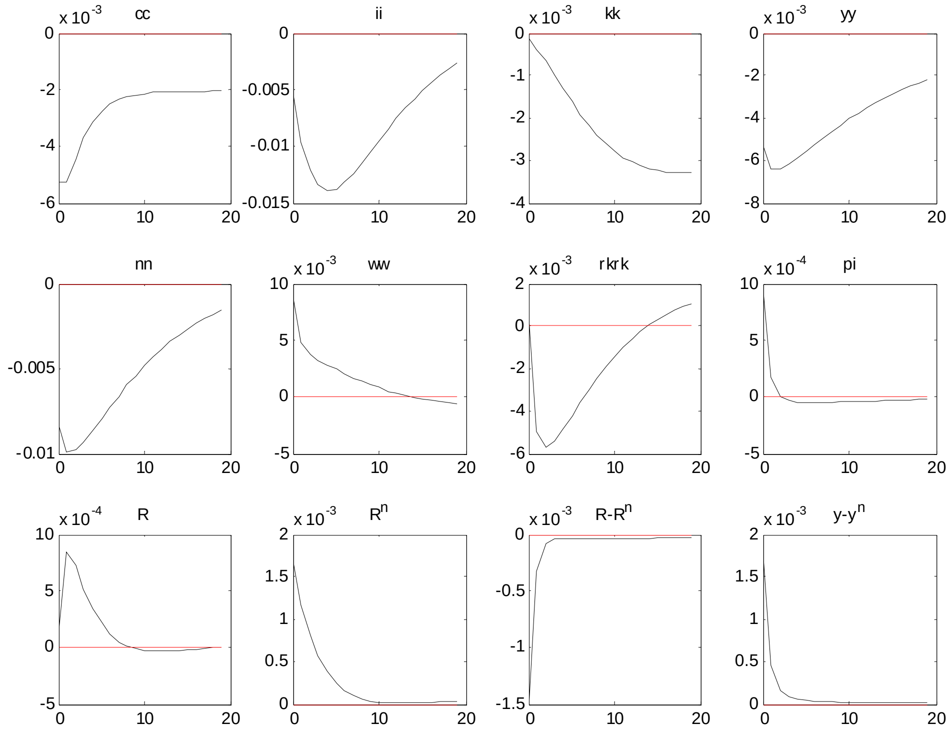

Figure 6: Ramsey policy, model with investment adjustment cost

counterpart. Comparing the optimal simple rule to Ramsey policy, our conclusions are alike to those under capital adjustment cost: The simple rule stabilizes inflation more successfully at the expense of (slightly) increased fluctuations in consumption, labour, output and the capital stock. The welfare losses under the simple rule exceed those of the Ramsey policy by only 2.83%. Thus, active policy achieves an allocation very close to the Ramsey policy under positive investment adjustment cost.

Welfare comparison	Welfare loss λ	$sd(\pi)$	$sd(\mu^{tot})$	$sd(\hat{k})$	$sd(\hat{y})$	$sd(\hat{c})$	$sd(\hat{n})$
Optimal simple rule	0.0581%	0.0648%	0.0497	0.0177	0.0213	0.0162	0.0282
Ramsey policy	0.0565%	0.0931%	0.0482	0.0178	0.0210	0.0156	0.0276
Standard deviations refer to variables defined as $\mu_t = \log(\mu_t/\bar{\mu})$.							
Due to its low values, the s.d. of π is expressed in terms of % points.							
The optimal simple rule features a coefficient of $w_\pi = 4$.							

Table 3: Welfare comparison with investment adjustment cost

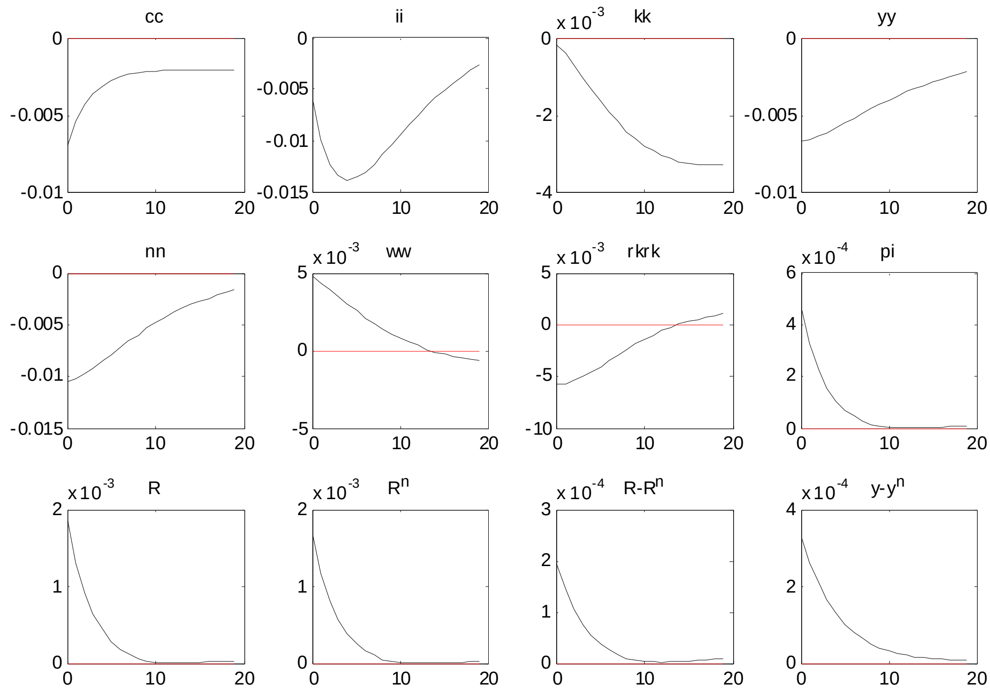

Figure 7: Optimal simple rule, model with investment adjustment cost

6.3 Welfare cost of active policy under alternative calibrations

The previous sections have shown that the ability of simple rules to imitate Ramsey optimal policy depends on the specification of adjustment cost. Because empirical estimates of adjustment cost are very heterogeneous, we here repeat the above welfare evaluation of simple rules under alternative calibrations. Beside the moderate adjustment cost analysed above ($\psi = 10$), we also analyse the cases of $\psi = 40$ and $\psi = 80$, which correspond to the relatively high adjustment cost found by the q literature.[19] Concerning investment adjustment cost, our baseline scenario was calibrated as Christiano, Eichenbaum, and Evans (2005), who use $\kappa = 2.48$. However Khan and Groth (2006) estimate much lower adjustment cost from industry data. The average elasticity of investment with respect to q they estimate implies a value of $\kappa = 0.0827$.

Table 4 shows the excess welfare losses of the optimal simple rule compared to the Ramsey optimal policy for each model. We observe that increasing capital adjustment cost to larger values does not reduce the excess cost of optimal simple rules substantially: The

[19] See Groth (2006) for an overview. Groth (2006) argues that the q literature is likely to exaggerate adjustment cost because not all variations in rents on capital must necessarily be the result of adjustment cost.

Excess welfare loss of optimal simple rule	Absolute loss	Relative loss
No adjustment cost	0.0057%	9.19%
Capital adjustment cost ($\psi = 10$)	0.0033%	6.41%
Capital adjustment cost ($\psi = 40$)	0.0027%	5.76%
Capital adjustment cost ($\psi = 80$)	0.0029%	5.63%
Investment adjustment cost ($\kappa = 0.0827$) (Groth Khan 2006)	0.0026%	4.20%
Investment adjustment cost ($\kappa = 2.48$) (CEE05)	0.0016%	2.83%
Note: The absolute loss is expressed in % of steady state consumption.		
Absolute Loss: $\lambda^{simple} - \lambda^{Ramsey}$. Relative loss: $\lambda^{simple} / \lambda^{Ramsey}$		

Table 4: Welfare evaluation of active policy for varying adjustment cost specifications

simple rules always imply welfare cost at least 5% higher than those implied by Ramsey optimal policy. Allowing for investment adjustment cost, even to a very moderate degree, lessens the excess welfare cost of the optimal simple rule to 4.20%. When increasing these to the value used by Christiano, Eichenbaum, and Evans (2005), the excess welfare cost decrease to 2.83%. Thus, under investment adjustment cost, active policy performs best and reaches an allocation close to that under Ramsey policy.

7 Conclusion

This paper analyses how endogenous capital formation affects the central bank's tradeoff and the performance of simple policy rules. We introduce a tradeoff in form of a wage markup shock calibrated so as to match the inefficiency gap measured by Galí, Gertler, and López-Salido (2007). This shock represents a distortion of the competitive equilibrium and consequently leaves the first-best allocation unaffected. Our analysis of the first-best equilibrium further shows that endogenous investment adds a third potential distortion to our New-Keynesian economy: Beside an inefficient level of worked hours and inefficient dispersion of production across firms, the intertemporal tradeoff is potentially distorted in competitive equilibrium. However, as we deal with more than one potential distortion, our analysis of the first-best equilibrium conditions does not allow us to answer the question how the central bank should influence capital accumulation so as to enhance welfare. Therefore, we calibrate the model and analyse the welfare implications of Ramsey optimal policy and simple policy rules.

We show three main results. First, introducing capital accumulation does not change the central bank's tradeoff in the following sense: In the models with and without capital, optimal policy induces similar patterns of inflation and the output gap. In both cases, it is optimal to allow temporary inflation. This creates a procyclical price markup which serves to stabilize output. However, introducing endogenous capital formation changes the instrument path required to achieve this pattern: In the model with capital, the natural rate of interest falls for 7-8 quarters. This implies that, in response to a cost-pushing shock, optimal policy reduces the nominal interest rate for at least 10 quarters (with the exception of the shock period) so as to lift output above its inefficiently low natural rate. As this interest rate pattern is not consistent with the common intuition of central banks fighting inflation by setting interest rates in an active way, we ask to what extent a simple, active policy rule can replicate the optimal allocation. As a second result, we find that, in our basic model without adjustment cost, the optimal simple policy leads to welfare losses exceeding those of the Ramsey policy by 9.2%.[20] With positive adjustment cost, these are much smaller. Our intuition for the relatively bad performance of simple rules is that, by construction, they focus on stabilizing inflation. However, when capital can be accumulated absent adjustment cost, this magnifies the output cost of simple rules: Stabilization of marginal cost after a cost-pushing shock requires a reduction in investment which puts upward pressure on future marginal cost. In subsequent periods, the central bank reduces demand further in order to counter this endogenous cost-push pressure. For this reason, the simple rule causes more severe fluctuations in output when capital can be accumulated at no cost. Our third result is identical to the findings of Casares and McCallum (2006): When allowing for positive adjustment cost, our model becomes very similar to the standard model which assumes a fixed capital stock. First, the natural interest rate increases in response to a cost-pushing shock and optimal policy increases the nominal interest rate (except in the shock period) above its steady state. Second, active policy induces an allocation closer to the Ramsey optimal policy: The welfare cost of optimal simple rules exceed those of Ramsey optimal policy by 2.83% to 6.41%, depending on the calibration and specification of adjustment cost.

To summarize, our findings are in line with Casares and McCallum (2006) who find that models which allow for endogenous capital accumulation resemble the standard model when we include adjustment cost. Similarly, we show that simple policy rules with positive adjustment cost provide a reasonably close approximation of optimal policy. This is true independent of the specification of adjustment cost as investment adjustment cost or capital adjustment cost. Further, our analysis clarifies the implications of including, or neglecting, adjustment cost for optimal policy. This is especially interesting as there is substantial disagreement concerning the magnitude of these cost. We show that adjustment cost potentially affects optimal monetary policy in two ways: When adjustment cost are low, commonly used simple interest rate rules can lead to non-negligible welfare losses. Further, endogenous capital formation potentially affects the behaviour of the natural interest rate: When adjustment cost are absent or low, the natural interest rate can decrease in response to a cost-pushing shock. From the opposite perspective, our results imply that ignoring adjustment cost can substantially affect welfare results and might lead to misleading estimates of the natural interest rate. In all our models, the natural interest

[20] The extent to which this relative measure implies absolute welfare cost depends on the welfare cost of business cycles. Although these are low in average, they can be high in specific recessions, as shown by Galí, Gertler, and López-Salido (2007).

rate is an important indicator for monetary policy: Across models, the response of optimal policy to a cost-pushing shock is a persistent reduction of the nominal interest rate below its natural rate. However, the question which simple rule can implement optimal policy exactly, is left to future research.

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A Appendix

A.1 Calibration

Calibration of the shock

The wage markup shock is the single shock we analyse here. We assume that the substitution elasticity ζ between the labour varieties evolves according to

$$\ln \zeta_t = (1-\rho_\zeta)\ln\bar{\zeta} + \rho^{\varsigma}\ln\zeta_{t-1} - \varepsilon_t^{\zeta}$$

where $\varepsilon_t^{\zeta} \sim iid$ with standard deviation σ_ζ. The resulting wage markup is given by $\mu_t^w = \frac{\zeta_t}{\zeta_t - 1}\frac{1}{\gamma^w}$, where $\gamma^w = \frac{\bar{\zeta}}{\bar{\zeta}-1}$. A first-order approximation of μ^w around $\bar{\mu}^w = 1$ yields

$$\hat{\mu}^w = \frac{1}{\bar{\zeta}-1}\hat{\zeta}_t$$

where hat variables denotes percentage deviations from steady state, $\hat{x} = \frac{x_t - \bar{x}}{\bar{x}}$. Thus, we see that the markup inherits the autocorrelation parameter of the stochastic process for the substitution elasticity, ρ^{ς}. We set this value to $\rho^{\varsigma} = 0.9$ slightly below the value of 0.95 estimated by Galí, Gertler, and López-Salido (2007). Concerning the variance, we know from a first-order approximation that $\hat{\zeta}_t = \rho_\zeta\hat{\zeta}_{t-1} - \varepsilon_t^{\zeta}$ which implies that Var $\hat{\zeta}_t = \frac{\sigma_\zeta^2}{1-\rho_\zeta^2}$. Therefore, the variance of our markup depends on the variance of the driving shock, σ_ζ^2, as follows

$$Var\hat{\mu}^w = \frac{\sigma_\zeta^2}{\left(\bar{\zeta}-1\right)^2\left(1-\rho_\zeta^2\right)}$$

We calibrate σ_ζ^2 to match the standard deviation of the inefficiency gap estimated by Galí, Gertler, and López-Salido (2007) at 0.051. Given our parameters, this implies setting $\sigma_\zeta = 0.1112$. Thus, a shock of one standard deviation reduces the substitution elasticity ζ from 6 to 5.33 and raises the markup by $\frac{0.1112}{\bar{\zeta}-1} = 2.22\%$.[21]

Parameter calibration	
Discount Factor	$\beta = 0.9901$
Inverse of intertemporal substitution elasticity	$\sigma = 1$
Inverse of Frisch elasticity of labour supply	$\eta = 1$
Substitution elasticity intermediate goods (StSt)	$\varepsilon = 6$
Substitution elasticity labour goods (StSt)	$\bar{\zeta} = 6$
Scale parameter for disutility of labour	$\chi = 10$
Labour share	$\alpha = 0.64$
Depreciation rate	$\delta = 0.025$
Calvo price stickiness	$\phi = 0.7$
Wage markup shock	$\rho_\zeta = 0.9;\ \sigma^\zeta = 0.1112$
(shock calibrated to match)	$\sigma(\mu_t^w) = 0.051$
Capital adjustment cost	$\psi = [10;\ 40;\ 80]$
Investment adjustment cost	$\kappa = [0.0827;\ 2.48]$

Calibration of the remaining parameters

We apply a standard calibration of the supply side of the economy where a model period is taken to represent a quarter of a year. α is calibrated to labour income share of 0.64, δ to a (yearly) steady state investment/capital ratio of around 0.1 and β is calibrated to a quarterly SS real interest rate of 1%, i.e. $R = \frac{1}{\beta} = 1.01$. Thus, $\frac{k}{y}$ is determined by the capital Euler equation,

$$1 = \beta\left(r^k + 1 - \delta\right) \Longleftrightarrow \frac{k}{y} = \frac{1-\alpha}{\frac{1}{\beta} - 1 + \delta}$$

which yields a value of 10.29 given our parameters. Note that this is in line with empirical observations: Annually, $\frac{k}{y}$ is around 2.5-3 and thus its quarterly value is between 7.5 and 12 because annual GDP is a flow variable whose value depends on the period

[21] We also experimented modelling the markup directly as a process of the form $\mu_t^w = \rho_u \mu_{t-1}^w + \varepsilon_t^u$ with $\rho = 0.9$. So as to get $sd(\mu) = 0.051$, the standard deviation of is set ε^u to $\sigma_u = \sqrt{0.051^2(1-\rho_u^2)} = 0.0222$. This shock has welfare effects virtually identical to those of the shock to ζ: The welfare cost of fluctuations under the Ramsey policy are identical up to 6 digits.

length. Following Galí, Gertler, and López-Salido (2007), we set the inverse of the labour supply elasticity η, to 1. Card (1994) suggests a range of 0.2 to 0.5 based on microeconomic estimates. Further, this is in line with Smets and Wouters (2007) who estimate an elasticity of 1.92. Thus, the value of unity represents a reasonable choice. We also follow Galí, Gertler, and López-Salido (2007) in the choice of σ, the intertemporal substitution elasticity of consumption goods and set $\sigma = 1$, i.e. utility logarithmic in consumption. Barsky, Kimball, Juster, and Shapiro (1997) estimate an elasticity of 0.18 using microdata, implying a a value of around 5 for σ. However, the macroeconomic literature tends to use lower risk aversion coefficients: For instance, Smets and Wouters (2007) estimate $\sigma = 1.39$. Thus, $\sigma = 1$ is a choice rather close toward the lower bound of available estimates. The parameter χ affects disutility from work and is set to 10 so as to generate a reasonable steady state share of time spent working of 0.29.

A Calvo parameter of $\phi = 0.7$ is in line with price stickiness in the euro area, implying that on average prices are reset every 3-4 quarters: For the euro area, Alvarez, Dhyne, Hoeberichts, Kwapil, Bihan, Lünnemann, Martins, Sabbatini, Stahl, Vermeulen, and Vilmunen (2006) estimate a mean duration of prices of 13 months, i.e. 3.25 quarters. For the U.S., average price duration is estimated much lower, at 6.7 months. The steady state substitution elasticities between the differentiated labour and intermediate goods are both set to 6 so that in the steady state, markups of firms and workers are 20%. The price markup is in line with Galí, López-Salido, and Vallés (2004) who assumes $\varepsilon = 6$ and Galí, Gertler, and López-Salido (2007) who suggest steady state price markups of 0.15 to 0.20. Justiniano and Primiceri (2008), in a Bayesian estimation, find posterior means of 0.17 for the wage markup and 0.22 for firm markups on goods prices. Concerning the wage markup, Galí, Gertler, and López-Salido (2007) suggests values around 0.30 to 0.35, i.e. a value of 4 for the steady state substitution elasticity. Sveen and Weinke (2006) use $\bar{\zeta} = 6$ as well. Note that both markups are eliminated by steady state subsidies. The parameter ε mainly affects the importance of price dispersion (and thus inflation) for welfare: A low ε implies higher market power and consequently less responsive demand. Thus, a given inflation rate implies less dispersion in factor input across firms and thus lower welfare losses. Further, the capital adjustment cost are calibrated to the steady state elasticity of the market value of capital with respect to the investment-capital ratio x . Using $q_t = \left[1 - \psi(\frac{i_t}{k_t} - \delta)\right]^{-1}$, this elasticity is given by

$$\eta_{q,x} = \frac{\partial q}{\partial x}\frac{x}{q} = \frac{\psi}{\left[1 - \psi(\frac{\bar{\imath}}{\bar{k}} - \delta)\right]^2}\frac{\frac{\bar{\imath}}{\bar{k}}}{\bar{q}} = \psi\delta$$

so that $\psi = 10$ implies $\eta_{q,x} = 0.25$, which is reasonable according to Bernanke, Gertler, and Gilchrist (1999). We use a value of $\psi = 10$ as a baseline calibration. However, there is substantial disagreement regarding this elasticity, ranging from a value of 4 (Dotsey (2002)) to $\frac{1}{15}$ (Baxter and Crucini (1993)). According to Groth (2006), $\eta_{q,x}$ is estimated at around 1-2 in the q literature, which implies up to 8 times higher adjustment cost. However, Groth (2006), following Hall (2004) and Shapiro (1986) believes that the q literature exaggerates adjustment cost as not all variation in firms stock prices (their value) is likely to come from rents on capital due to adjustment cost.[22] Based on the estimate by Shapiro (1986), Groth (2006) suggests a value of $\psi = 17$. Woodford (2003, chapter 5) assumes rather low adjustment cost and sets ψ around 3. Galí, López-Salido, and Vallés (2004), following King and Watson (1996) sets $\eta_{q,x} = 1$, implying higher adjustment cost of $\psi = 40$.

Concerning the specification of adjustment cost in the change of investment, we follow Christiano, Eichenbaum, and Evans (2005) and Schmitt-Grohé and Uribe (2006) and set $\kappa = 2.48$.in our baseline scenario. Note that only Schmitt-Grohé and Uribe (2006) assume the exact functional form I use while Christiano, Eichenbaum, and Evans (2005) only specify that the second derivative of the cost function $S(i_t/i_{t-1})$ be equal to κ at the SS $[S''(1) = \kappa]$. According to Khan and Groth (2006), who give an overview of estimated elasticities in models with investment adjustment cost, the value used here is well inside the range estimated by other authors. However, the industry-level estimation of Khan and Groth (2006) suggests an elasticity of investment with respect to the shadow price of capital around thirty times larger than that implied by the calibration used here. This implies a value of $\kappa = 0.0827$ which we use as an alternative calibration.

A.2 The Welfare measure

Our welfare measure is based on Schmitt-Grohé and Uribe (2006) and is derived from the representative household's expected lifetime utility

$$W = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right)$$

Conditional vs. unconditional welfare

To correctly evaluate welfare, we have to be clear on the definition of the expectation operator E_0. We will here follow Schmitt-Grohé and Uribe (2006) and evaluate welfare conditional on the initial state being the deterministic steady state.[23] This is sensible because all models analysed in this paper have the same deterministic steady state independent of policy. For this reason, we do not face the problem of having to evaluate transition periods from one to another deterministic steady state, which is not straightforward when using local approximation methods. However, we face a similar problem

[22] Hall (2004) and Shapiro (1986) estimate adjustment cost directly from Euler equations whereas the q literature relates a firm's value to the replacement cost of its assets and thus assigns any fluctuations in the measured q to the presence of adjustment cost.

[23] This approach is also recommended by Kim, Kim, Schaumburg, and Sims (2008).

which stems from the fact that the models' stochastic steady states generally depend on monetary policy. Therefore, an unconditional welfare measure will be biased because it ignores the transition to a particular stochastic steady state.

Our welfare measure

When comparing two policies, our welfare measure is the percentage λ of steady state consumption that a household would be willing to give up under the superior policy A and still be as well of as under the inferior policy B (equivalent variation). Note that we can only interpret λ as percentage points of steady state consumption when A refers to the deterministic model.[24] Let superscripts A and B denote welfare, consumption and worked hours under a given policy A or B. Thus, we need to solve

$$W^B = E_0 \sum_{t=0}^{\infty} \beta^t u\left(\left(1-\lambda\right) c_t^A, n_t^A\right)$$

for λ. Given our period utility function, $u(c,n) = \frac{c^{1-\sigma}-1}{1-\sigma} - \chi \frac{n^{1+\eta}}{1+\eta}$, we can rewrite the preceding equation as

$$\begin{aligned} W^B &= E_0 \sum_{t=0}^{\infty} \beta^t \frac{(1-\lambda)^{1-\sigma}\left(c_t^A\right)^{1-\sigma}-1}{1-\sigma} - \chi \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \\ &= (1-\lambda)^{1-\sigma} W^A - \left[1-(1-\lambda)^{1-\sigma}\right]\left\{E_0 \sum_{t=0}^{\infty} \beta^t \left[\chi \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} + \frac{1}{1-\sigma}\right]\right\} \end{aligned}$$

Defining $WN = -E_0 \sum_{t=0}^{\infty} \beta^t \left(\chi \frac{n_t^{1+\eta}}{1+\eta} + \frac{1}{1-\sigma}\right)$, we obtain

$$\begin{aligned} W^B &= (1-\lambda)^{1-\sigma} W^A + \left[1-(1-\lambda)^{1-\sigma}\right] WN^A \\ &= (1-\lambda)^{1-\sigma}\left[W^A - WN^A\right] + WN^A \end{aligned}$$

Solving for λ yields

$$\lambda = 1 - \left(\frac{W^B - WN^A}{W^A - WN^A}\right)^{\frac{1}{1-\sigma}}$$

Evaluating welfare from a second-order approximated policy function

Now define $V_t = E_t \sum_{s=0}^{\infty} \beta^s u\left(c_{t+s}, n_{t+s}\right)$. The idea of Schmitt-Grohé and Uribe (2006) is that by determining the solution of V_t in terms of its policy function, W is equal to the value of this policy function at the initial state. It is important to note that we are not interested in V_t but only in V_0. Nevertheless, we can use standard methods to solve for the policy function of V_t and evaluate it at the initial state. Schmitt-Grohé

[24] This is the reason why we compute a welfare loss separately for each policy and do not directly compare simple rules to optimal policy.

and Uribe (2004b) show that the solution to an equation system which can be written as $E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0$ is given by a policy function $y_t = g(x_t, \omega)$. y here denotes non-predetermined variables (such as V_t) while x includes endogenous and exogenous state variables. The parameter ω summarizes the uncertainty in the model. As we evaluate welfare under wage markup shocks only, ω equals the standard deviation of that shock. As mentioned above, we wish to evaluate welfare conditional on the deterministic steady state. Thus, the policy function we are interested in is that for V, evaluated at the initial state $V(\bar{x}, 0)$, where $\bar{x}$ denotes the deterministic steady state. Algebraically, our approach implies

$$W = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) = V(x_0, \omega)$$

Analogously, WN can be evaluated by using the policy function VN for the variable $VN_t = -E_t \sum_{s=0}^{\infty} \beta^s \left(\chi \frac{n_{t+s}^{1+\eta}}{1+\eta} - \frac{1}{1-\sigma} \right)$. For our welfare analysis, we will thus evaluate WN by using $WN = VN(x_0, \omega)$.

Computing λ from the approximated policy function for V

Schmitt-Grohé and Uribe (2004b) demonstrate the application of perturbation methods to solve for the policy functions. A second-order approximation of the policy function for V around the deterministic steady state yields

$$\begin{aligned} V(x_t, \omega) \approx & V(\bar{x}, 0) + V_x(\bar{x}, 0)(x_t - \bar{x}) + V_\omega(\bar{x}, 0)\omega + V_{\sigma x}(\bar{x}, 0)\omega(x_t - \bar{x}) \\ & + \frac{1}{2} V_{xx}(\bar{x}, 0)(x_t - \bar{x})^2 + \frac{1}{2} V_{\omega\omega}(\bar{x}, 0)\omega^2 \end{aligned}$$

where we have used that in the deterministic steady state, $\omega = 0$. We now proceed along the lines of Schmitt-Grohé and Uribe (2006) to show that our welfare measure λ can be approximated correctly up to second order from a second-order approximation of the policy functions V and VN. Our welfare measure λ can be viewed as a function of states x_t and the uncertainty parameter, $\Lambda(x_t, \omega)$

$$\Lambda(x_t, \omega) = 1 - \left(\frac{V^B(x_t, \omega) - VN^A(x_t, \omega)}{V^A(x_t, \omega) - VN^A(x_t, \omega)} \right)^{\frac{1}{1-\sigma}}$$

Because we wish to measure welfare conditional on the initial state being the deterministic steady state, it is sufficient to consider the derivatives of Λ w.r.t ω :

$$\Lambda(x_t, \omega) \approx \Lambda(\bar{x}, 0) + \Lambda_\omega(\bar{x}, 0)\omega + \Lambda_{\omega\omega}(\bar{x}, 0)\frac{\omega^2}{2}$$

Because the deterministic steady states are identical across policies, $V^A(\bar{x}, 0) = V^B(\bar{x}, 0)$ and thus $\Lambda(\bar{x}, 0) = 0$. Differentiating $\Lambda(x_t, \omega)$ once w.r.t. ω, we obtain (where I write $V^B(x_t, \omega) = V^B$ for simplification)

$$
\begin{aligned}
&\Lambda_\omega(x_t,\omega)\\
=&-\frac{1}{1-\sigma}\left(\frac{V^B-VN^A}{V^A-VN^A}\right)^{\frac{1}{1-\sigma}-1}\left\{\frac{\left[V^B_\omega-VN^A_\omega\right]\left[V^A-VN^A\right]-\left[V^A_\omega-VN^A_\omega\right]\left[V^B-VN^A\right]}{\left[V^A-VN^A\right]^2}\right\}
\end{aligned}
$$

We know from the analysis of Schmitt-Grohé and Uribe (2004b) that the first derivative of a policy function w.r.t the uncertainty parameter must be zero, i.e. $V_\omega(\bar{x},0)=0$ and $VN_\omega(\bar{x},0)=0$. [25] This implies $\Lambda_\omega(\bar{x},0)=0$. To simplify the next steps, let us define the numerator and the denominator

$$
\begin{aligned}
Z&=Z1+Z2\\
Z1&=\left[V^B_\omega(x_t,\omega)-VN^A_\omega(x_t,\omega)\right]\left[V^A(x_t,\omega)-VN^A(x_t,\omega)\right]\\
Z2&=-\left[V^A_\omega(x_t,\omega)-VN^A_\omega(x_t,\omega)\right]\left[V^B(x_t,\omega)-VN^A(x_t,\omega)\right]\\
N&=\left[V^A(x_t,\omega)-VN^A(x_t,\omega)\right]^2
\end{aligned}
$$

To evaluate $\Lambda_{\omega\omega}(\bar{x},0)$, we need to differentiate $\Lambda(x_t,\omega)$ twice w.r.t. ω. This yields .

$$
\begin{aligned}
&\Lambda_{\omega\omega}(x_t,\omega)\\
=&\left\{-\left(\frac{1}{(1-\sigma)^2}-\frac{1}{1-\sigma}\right)\left(\frac{V^B-VN^A}{V^A-VN^A}\right)^{\frac{1}{1-\sigma}-2} *\right.\\
&\left.\frac{\left[V^B_\omega-VN^A_\omega\right]\left[V^A-VN^A\right]-\left[V^A_\omega-VN^A_\omega\right]\left[V^B-VN^A\right]}{\left[V^A-VN^A\right]^2}\right\}\\
&-\frac{1}{1-\sigma}\left(\frac{V^B-VN^A}{V^A-VN^A}\right)^{\frac{1}{1-\sigma}-1}\left\{\frac{\frac{dZ1}{d\omega}*N+\frac{dZ2}{d\omega}*N-\frac{dN}{d\omega}*Z}{\left[V^A-VN^A\right]^4}\right\}
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{dZ1}{d\omega}*N&=\left\{\left[V^B_{\omega\omega}-VN^A_{\omega\omega}\right]\left[V^A-VN^A\right]+\left[V^B_\omega-VN^A_\omega\right]\left[V^A_\omega-VN^A_\omega\right]\right\}\left[V^A-VN^A\right]^2\\
\frac{dZ2}{d\omega}*N&=\left\{-\left[V^A_{\omega\omega}-VN^A_{\omega\omega}\right]\left[V^B-VN^A\right]-\left[V^A_\omega-VN^A_\omega\right]\left[V^B_\omega-VN^A_\omega\right]\right\}\left[V^A-VN^A\right]^2\\
\frac{dN}{d\omega}*Z&=2\left[V^A-VN^A\right]\left[V^B_\omega-VN^A_\omega\right]\left[V^A-VN^A\right]-\left[V^A_\omega-VN^A_\omega\right]\left[V^B-VN^A\right]
\end{aligned}
$$

We need to evaluate this derivative at the deterministic steady state, so that many of the terms involved drop out: Because $V_\omega(\bar{x},0)=0$, the numerator in the first term, is zero

[25] This is the technical reason why certainty equivalence holds in a first-order approximation.

in the deterministic steady state:

$$\left[V_\omega^B - VN_\omega^A\right]\left[V^A - VN^A\right] - \left[V_\omega^A - VN_\omega^A\right]\left[V^B - VN^A\right] = 0$$

Further, in the deterministic steady state,

$$\frac{dZ1}{d\omega} * N = \left[V_{\omega\omega}^B(\bar{x},0) - VN_{\omega\omega}^A(\bar{x},0)\right]\left[V^A(\bar{x},0) - VN^A(\bar{x},0)\right]\left[V^A(\bar{x},0) - VN^A(\bar{x},0)\right]^2$$

$$\frac{dZ2}{d\omega} * N = -\left[V_{\omega\omega}^A(\bar{x},0) - VN_{\omega\omega}^A(\bar{x},0)\right]\left[V^B(\bar{x},0) - VN^A(\bar{x},0)\right]\left[V^A(\bar{x},0) - VN^A(\bar{x},0)\right]^2$$

and

$$\frac{dN}{d\omega} * Z = 0$$

Also, $\frac{V^B - VN^A}{V^A - VN^A} = 1$ in the deterministic steady state, so that

$$\begin{aligned}&\Lambda_{\omega\omega}(\bar{x},0)\\ &= -\frac{1}{1-\sigma}\left\{\frac{\left\{\left[V_{\omega\omega}^B(\bar{x},0) - VN_{\omega\omega}^A(\bar{x},0)\right] - \left[V_{\omega\omega}^A(\bar{x},0) - VN_{\omega\omega}^A(\bar{x},0)\right]\right\}\left[V^A(\bar{x},0) - VN^A(\bar{x},0)\right]^3}{\left[V^A(\bar{x},0) - VN^A(\bar{x},0)\right]^4}\right\}\\ &= \frac{V_{\omega\omega}^B(\bar{x},0) - V_{\omega\omega}^A(\bar{x},0)}{(\sigma-1)\left[V^A(\bar{x},0) - VN^A(\bar{x},0)\right]}\end{aligned}$$

Therefore, our welfare measure can be computed accurately up to second-order from

$$\begin{aligned}\Lambda(x_t,\omega) &\approx \Lambda_{\omega\omega}(\bar{x},0)\frac{\omega^2}{2}\\ &= \frac{V_{\omega\omega}^B(\bar{x},0) - V_{\omega\omega}^A(\bar{x},0)}{(\sigma-1)\left[V^A(\bar{x},0) - VN^A(\bar{x},0)\right]}\frac{\omega^2}{2}\end{aligned}$$

We obtain $V_{\sigma\sigma}^B(\bar{x},0)\frac{\omega^2}{2}$ and $V_{\sigma\sigma}^A(\bar{x},0)\frac{\omega^2}{2}$ from the "correction term" in dynare, because the second-order derivatives of policy functions wrt ω lead to this constant correction term. $V^A(\bar{x},0)$ and $VN^A(\bar{x},0)$ are the deterministic steady state values of the variables V^A and VN^A.

Interpretation

Because both policies have identical steady states, the term in the numerator $V_{\omega\omega}^B(\bar{x},0) - V_{\omega\omega}^A(\bar{x},0)$ is the only welfare relevant difference between the two policies.[26] Thus, the heart of our welfare measure (the term in the numerator) implies that we compare the solution

[26] All other terms in the policy functions for V^A and V^B are zero in the deterministic steady state when variables are expressed as deviations from their deterministic steady state - except for the constant term $V^A(\bar{x},0)$ which is equal across any two policies with an identical deterministic steady state. Thus, computing $V_{\omega\omega}^B(\bar{x},0) - V_{\omega\omega}^A(\bar{x},0)$ is equivalent to computing $V^B(\bar{x},0) - V^A(\bar{x},0)$.

to the value functions V under two policies, conditional on the same (deterministic) steady state. In principle, we could use the approximate solutions for the policy functions of V conditional on any other steady state, as long as we condition on the same steady state for both policies. Here, we use the deterministic steady state so that we include the welfare effects implied by the transition from the deterministic steady state to the (different) stochastic steady states.

Case of logarithmic utility

The base case scenario analysed in this paper sets $\sigma = 1$ which corresponds to logarithmic utility in consumption. The welfare measure in this case is much simpler to compute. As above, λ is implicitly defined by

$$\begin{aligned} W^B &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left[(1-\lambda)\, c_t^A\right] - \chi \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \right\} \\ &= (1-\lambda)\, W^A - \left[1-(1-\lambda)\right] E_0 \sum_{t=0}^{\infty} \beta^t \chi \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \end{aligned}$$

Rearranging terms,

$$\begin{aligned} W^B &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left[(1-\lambda)\, c_t^A\right] - \chi \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \right\} \\ &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(1-\lambda) + \log c_t^A - \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \right\} \\ &= \frac{\log(1-\lambda)}{1-\beta} + E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t^A - \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \right\} \\ &= \frac{\log(1-\lambda)}{1-\beta} + W^A \end{aligned}$$

Solving for λ yields

$$\begin{aligned} \log(1-\lambda) &= \left(W^B - W^A\right)(1-\beta) \\ &\iff \\ 1-\lambda &= \exp\left[\left(W^B - W^A\right)(1-\beta)\right] \\ &\iff \\ \lambda &= 1 - \exp\left[\left(W^B - W^A\right)(1-\beta)\right] \end{aligned}$$

We now evaluate welfare by using the policy functions of the variable

$$V_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left[c_t\right] - \chi \frac{n_t^{1+\eta}}{1+\eta} \right\}$$

which we denote $V(x_t, \omega)$. Thus, λ is a function of states and uncertainty ω,

$$\Lambda(x_t, \omega) = 1 - \exp\left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\}$$

As before, we approximate λ up to second order,

$$\Lambda(x_t, \omega) \approx \Lambda(\bar{x}, 0) + \Lambda_\omega(\bar{x}, 0)\omega + \Lambda_{\omega\omega}(\bar{x}, 0)\frac{\omega^2}{2}$$

Computing the derivatives yields

$$\begin{aligned}
\Lambda_\omega(x_t, \omega) =& - \exp\left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\} * \left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\} \\
\Lambda_{\omega\omega}(x_t, \omega) =& - \exp\left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\} * \left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\}^2 \\
& - \exp\left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\} * \left\{\left[V_{\omega\omega}^B(x_t, \omega) - V_{\omega\omega}^A(x_t, \omega)\right](1-\beta)\right\}
\end{aligned}$$

As before, identical (deterministic) steady states imply $V^A(\bar{x}, 0) = V^B(\bar{x}, 0)$ and thus $\Lambda(\bar{x}, 0) = 0$. From Schmitt-Grohé and Uribe (2004b), we know that $V_\omega^A(\bar{x}, 0) = V_\omega^B(\bar{x}, 0) = 0$, which implies $\Lambda_\omega(\bar{x}, 0) = 0$ and can be used to simplify the second derivative: $\Lambda_{\omega\omega}(\bar{x}, 0) = -\left\{\left[V_{\omega\omega}^B(\bar{x}, 0) - V_{\omega\omega}^A(\bar{x}, 0)\right](1-\beta)\right\}$ .Thus, our welfare measure is given by

$$\Lambda(x_t, \omega) \approx (1-\beta)\left[V_{\omega\omega}^A(x_t, \omega) - V_{\omega\omega}^B(x_t, \omega)\right]\frac{\omega^2}{2}$$

Hence, we can again evaluate welfare using the correction terms given by dynare, which equal $V_{\omega\omega}^A(\bar{x}, 0)\frac{\omega^2}{2}$.

A.3 Model equilibrium

This section contains derivations of the model's equilibrium conditions concerning the law of motions for inflation and price dispersion. We further present the model's fiscal policy and derive the aggregate resource constraint.

A.3.1 Law of motion for inflation

In section 2.2 , we write the definition of the price index recursively so as to find a law of motion for inflation.

$$\begin{aligned}
P_t^{1-\varepsilon} &= \int_0^1 P_{it}^{1-\varepsilon} di \\
&\Longleftrightarrow \\
1 &= (1-\phi)\tilde{Z}_t^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}
\end{aligned}$$

To show this, use that reoptimizing firms all set identical prices Z. Further, the share of firms which were allowed to reoptimize k periods ago is $\phi^k(1-\phi)$ which allows us to write

$$
\begin{aligned}
P_t^{1-\varepsilon} &= \int_0^1 P_{it}^{1-\varepsilon} di \\
&= (1-\phi)\left[Z_t^{1-\varepsilon} + \phi Z_{t-1}^{1-\varepsilon} + \phi^2 Z_{t-2}^{1-\varepsilon} + ...\right]
\end{aligned}
$$

Lagging this expression by one period and pre-multiplying with ϕ gives

$$
\phi P_{t-1}^{1-\varepsilon} = (1-\phi)\left[\phi Z_{t-1}^{1-\varepsilon} + \phi^2 Z_{t-2}^{1-\varepsilon} + \phi^3 Z_{t-3}^{1-\varepsilon} + ...\right]
$$

so that the price index follows

$$
P_t^{1-\varepsilon} = (1-\phi) Z_t^{1-\varepsilon} + \phi P_{t-1}^{1-\varepsilon}
$$

Dividing by $P_t^{1-\varepsilon}$, this implies

$$
1 = (1-\phi)\tilde{Z}_t^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}
$$

which is the law of motion used in the main body of this paper.

A.3.2 Tracking price dispersion

We can find a recursive law of motion for the above integral so that we do not have to track individual prices of firms (following Schmitt-Grohé and Uribe (2004a)). Note that we denote the newly set price in real terms: $\tilde{Z} = \frac{Z}{P}$ and use that Z is identical across all firms which set a new price:

$$
\begin{aligned}
s_t &\equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di \\
&= (1-\phi)\left(\frac{Z_t}{P_t}\right)^{-\varepsilon} + (1-\phi)\phi\left(\frac{Z_{t-1}}{P_t}\right)^{-\varepsilon} + (1-\phi)\phi^2\left(\frac{Z_{t-2}}{P_t}\right)^{-\varepsilon} + ... \\
&= (1-\phi)\left(\frac{Z_t}{P_t}\right)^{-\varepsilon} + (1-\phi)\phi\left(\frac{Z_{t-1}}{P_t}\frac{P_{t-1}}{P_{t-1}}\right)^{-\varepsilon} + (1-\phi)\phi^2\left(\frac{Z_{t-2}}{P_t}\frac{P_{t-2}}{P_{t-2}}\frac{P_{t-1}}{P_{t-1}}\right)^{-\varepsilon} \\
&= (1-\phi)\tilde{Z}_t^{-\varepsilon} + (1-\phi)\phi\tilde{Z}_{t-1}^{-\varepsilon}\pi_t^{\varepsilon} + (1-\phi)\phi^2\tilde{Z}_{t-2}^{-\varepsilon}\pi_t^{\varepsilon}\pi_{t-1}^{\varepsilon} + ... \\
&= (1-\phi)\sum_{j=0}^{\infty}\phi^j \tilde{Z}_{t-j}^{-\varepsilon}\prod_{s=1}^{j}\pi_{t+1-s}^{\varepsilon}
\end{aligned}
$$

Lagging this expression by one period

$$
\begin{aligned}
s_{t-1} &= (1-\phi)\sum_{j=0}^{\infty}\phi^j \tilde{Z}_{t-1-j}^{-\varepsilon}\prod_{s=1}^{j}\pi_{t-s}^{\varepsilon} \\
&= (1-\phi)\tilde{Z}_{t-1}^{-\varepsilon} + (1-\phi)\phi\tilde{Z}_{t-2}^{-\varepsilon}\pi_{t-1}^{\varepsilon} + (1-\phi)\phi^2\tilde{Z}_{t-3}^{-\varepsilon}\pi_{t-1}^{\varepsilon}\pi_{t-2}^{\varepsilon} + ...
\end{aligned}
$$

Multiplying this by $\pi_t^{\varepsilon}\phi$, we get

$$\begin{aligned}
s_{t-1}\pi_t^{\varepsilon}\phi &= (1-\phi)\phi\tilde{Z}_{t-1}^{-\varepsilon}\pi_t^{\varepsilon} + (1-\phi)\phi^2\tilde{Z}_{t-2}^{-\varepsilon}\pi_t^{\varepsilon}\pi_{t-1}^{\varepsilon} + (1-\phi)\phi^3\tilde{Z}_{t-3}^{-\varepsilon}\pi_t^{\varepsilon}\pi_{t-1}^{\varepsilon}\pi_{t-2}^{\varepsilon} \\
&= (1-\phi)\sum_{j=0}^{\infty}\phi^{j+1}\tilde{Z}_{t-1-j}^{-\varepsilon}\prod_{s=0}^{j}\pi_{t-s}^{\varepsilon} \\
&= (1-\phi)\sum_{j=1}^{\infty}\phi^{j}\tilde{Z}_{t-j}^{-\varepsilon}\prod_{s=1}^{j}\pi_{t+1-s}^{\varepsilon}
\end{aligned}$$

Thus, we can eliminate all lagged Z terms by taking the difference

$$\begin{aligned}
s_t - \phi s_{t-1}\pi_t^{\varepsilon} &= (1-\phi)\tilde{Z}_t^{-\varepsilon} \\
&\Longleftrightarrow \\
s_t &= (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}
\end{aligned}$$

which is the law of motion used in the main body of this paper, (12).

A.3.3 Fiscal policy

Expenditures for the production subsidy amount to[27]

$$\begin{aligned}
\tau_t^{prod} &= \int_0^1 \left(\frac{\varepsilon}{\varepsilon-1} - 1\right)\frac{P_{it}}{P_t} y_{it} di \\
&= \frac{1}{\varepsilon-1}\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} y_t di \\
&= \frac{1}{\varepsilon-1} y_t
\end{aligned}$$

where we used the demand condition of firms, $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t$ and the definition of the price index, $\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} = 1$. Expenditures for the subsidy to workers are given by

$$\tau_t^{labour} = (\gamma^w - 1)\, w_t n_t$$

[27] Note that we analyse taxes and profits in real terms, consistent with the notation introduced in the main part of the paper.

Note that the sum of firm profits Ψ_t is

$$\begin{aligned}\Psi_t =& \int_0^1 \frac{\varepsilon}{\varepsilon-1}\frac{P_{it}}{P_t} y_{it} - mc_t y_{it} di \\ =& y_t \left[\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} di - mc_t \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} di \right] \\ =& \frac{\varepsilon}{\varepsilon-1} y_t \left(1 - mc_t s_t\right)\end{aligned}$$

Therefore, the households' "transfer", i.e. the difference of firm profits minus lump sum taxes (in real terms) equals

$$\Psi_t - \tau_t^{prod} - \tau_t^{labour} = y_t \left(1 - mc_t s_t\right) - \left(\gamma^w - 1\right) w_t n_t$$

Firm profits and the production subsidy cancel out in the steady state where $s = 1$ and $mc = 1$, because the steady state is efficient.

A.3.4 Derivation of the aggregate resource constraint

In the following that the aggregate resource constraint $A_t k_t^{1-\alpha} n_t^{\alpha} = [c_t + k_{t+1} - (1-\delta)k_t] \, s_t$ can be derived from the representative household's budget constraint which reads

$$w_t n_t + \frac{d_{t-1}}{\pi_t} + y_t \left(1 - mc_t s_t\right) + k_t(r_t^k + 1 - \delta) = c_t + k_{t+1} + E_t r_{t,t+1}^d d_{j,t}$$

where we have used that $\gamma^w w_t n_t + \Psi_t - \tau_t = w_t n_t + y_t \left(1 - mc_t s_t\right)$. In other words, because the government taxes away the subsidy to workers, the net revenue the household gains from receiving firm profits and the labour subsidy after paying the lump-sum tax equals $y_t \left(1 - mc_t s_t\right)$. Because the representative household cannot hold state-contingent claims (which are traded only among households), $d_t = 0 \; \forall \; t$. Thus, the constraint simplifies to

$$y_t \left(1 - mc_t s_t\right) + w_t n_t + k_t r_t^k = c_t + k_{t+1} - k_t(1-\delta)$$

Now use that the producers' first order conditions are $w_t = mc_t mpn_t$ and $r_t^k = mc_t mpk_t$,[28] which implies that

$$\begin{aligned}w_t n_t + k_t r_t^k =& mc_t \alpha \frac{IO_t}{n_t} n_t + k_t mc_t (1-\alpha) \frac{IO_t}{k_t} \\ =& mc_t IO_t\end{aligned}$$

so that

$$y_t \left(1 - mc_t s_t\right) + mc_t IO_t = c_t + k_{t+1} - k_t(1-\delta)$$

[28] Given our production function, the marginal products of labour and capital depend only on the capital labour ratio which is constant across firms as was shown above.

Because $y_t = \frac{IO_t}{s_t}$, this simplifies to

$$\frac{IO_t}{s_t} = c_t + k_{t+1} - k_t(1-\delta)$$

which by using the definition of intermediate output IO_t is equivalent to

$$A_t k_t^{1-\alpha} n_t^{\alpha} = [c_t + k_{t+1} - (1-\delta)k_t]\, s_t$$

A.4 The Ramsey problem without adjustment cost

A.4.1 Deriving the intertemporal budget constraint

We now write states explicitly with s^t denoting the history of states up to period t. Denoting the price of a state-contingent claim to a unit of currency with $Q_{t,t+1}(s^{t+1})$, the representative household's budget constraint can be written as

$$\begin{aligned} & w_t(s^t)n_t(s^t) + \frac{d_{t-1}(s^t)}{\pi_t(s^t)} + y_t(s^t)\left(1 - mc_t(s^t)s_t(s^t)\right) + k_t(s^{t-1})(r_t^k(s^t) + 1 - \delta) \\ = & c_t(s^t) + k_{t+1}(s^t) + \sum_{s^{t+1}} Q_{t,t+1}(s^{t+1})d_t(s^{t+1}) \end{aligned}$$

which uses $\gamma^w w_t n_t + \Psi_t - \tau_t = w_t n_t + y_t\left(1 - mc_t s_t\right)$ as was derived in section 2.3. We will derive the intertemporal budget constraint by iterating the households budget constraint forward for d, the state-contingent bond. It will be useful to define $q_{0,t}(s^t)$ as

$$q_{0,t}(s^t) = \beta^t pr(s^t|s^0)\frac{\lambda_t(s^t)}{\lambda_0(s^0)}$$

Using the first order condition for state contingent claims, $Q_{t,t+1}(s^{t+1}) = \beta\frac{pr(s^{t+1})}{pr(s^t)}\frac{\lambda_{t+1}(s^{t+1})}{\lambda_t(s^t)\pi_{t+1}(s^{t+1})}$, we can express $q_{0,t}(s^t)$ as

$$q_{0,t}(s^t) = \prod_{i=0}^{t} Q_{i,i+1}(s^{i+1})\pi_{i+1}(s^{i+1})$$

Rearranging the budget constraint yields

$$\begin{aligned} d_{-1}(s^0) &= \pi_0(s^0)\Omega_0(s^0) + \pi_0(s^0)\sum_{s^1} Q_{0,1}(s^1)d_0(s^1) \\ d_{-1}(s^0) &= \pi_0\left(\Omega_0\right) + \pi_0\sum_{s^1}\frac{q_{0,1}(s^1)}{\pi_1(s^1)}d_0(s^1) \end{aligned}$$

where $\Omega_t(s^t) = c_t(s^t) - w_t(s^t)n_t(s^t) - y_t(s^t)\left(1 - mc_t(s^t)s_t(s^t)\right) + k_{t+1}(s^t) - k_t(s^{t-1})(r_t^k(s^t) +$

$1-\delta$). Iterating forward for d, we get

$$
\begin{aligned}
d_0(s^1) &= \pi_1(s^1)\Omega_1(s^1) + \pi_1 \sum_{s^2} \frac{q_{1,2}(s^2)}{\pi_2(s^2)} d_1(s^2) \\
d_{-1}(s^0) &= \pi_0(s^0)\Omega_0(s^0) + \pi_0(s^0) \sum_{s^1} q_{0,1}(s^1) \left(\Omega_1(s^1) + \frac{q_{1,2}(s^2)}{\pi_2(s^2)} d_1(s^2) \right) \\
&= \pi_0(s^0)\Omega_0(s^0) + \pi_0(s^0) \sum_{s^1} q_{0,1}(s^1)\Omega_1(s^1) + \pi_0 \sum_{s^1} q_{0,1}(s^1) q_{1,2}(s^2)\Omega_2(s^2) + ... \\
&= \pi_0(s^0)\Omega_0(s^0) + \pi_0(s^0) \sum_{s^1} q_{0,1}(s^1)\Omega_1(s^1) + \pi_0 \sum_{s^2} q_{0,2}(s^2)\Omega_2(s^2) + ... \\
d(s^0)/\pi_0(s^0) &= \Omega_0(s^0) + \sum_{s^1} q_{0,1}(s^1)\Omega_1(s^1) + \sum_{s^2} q_{0,2}(s^2)\Omega_2(s^2) + ...
\end{aligned}
$$

Now we can obtain an arbitrage freeness equation by first rewriting the capital Euler equation and then using $\frac{Q_{t,t+1}(s^{t+1})}{p(s^{t+1}|s^t)} c_t^{-\sigma}(s^t)\pi_{t+1}(s^{t+1})\frac{1}{\beta} = c_{t+1}^{-\sigma}(s^{t+1})$

$$
\begin{aligned}
c_t^{-\sigma}(s^t) &= \beta E_t \left[c_{t+1}^{-\sigma}(q_{t+1}^k + 1 - \delta) \right] \\
c_t^{-\sigma}(s^t) &= \beta \sum_{s^{t+1}} pr(s^{t+1}|s^t) c_{t+1}^{-\sigma}(s^{t+1})(r_{t+1}^k(s^{t+1}) + 1 - \delta) \\
1 &= \sum_{s^{t+1}} Q_{t,t+1}(s^{t+1})\pi_{t+1}(s^{t+1})(r_{t+1}^k(s^{t+1}) + 1 - \delta) \\
1 &= \sum_{s^{t+1}} q_{t,t+1}(s^{t+1})(r_{t+1}^k(s^{t+1}) + 1 - \delta)
\end{aligned}
$$

This arbitrage freeness condition can be used to eliminate capital in the intertemporal budget constraint whose capital terms are

$$
\begin{aligned}
& k_1(s^0) - k_0(s^{-1})(r_0^k(s^0) + 1 - \delta) + \sum_{s^1} q_{0,1}(s^1) \left[k_2(s^1) - k_1(s^0)(r_1^k(s^1) + 1 - \delta) \right] \\
& + \sum_{s^2} q_{0,2}(s^2) \left[k_3(s^2) - k_2(s^2)(r_2^k(s^2) + 1 - \delta) \right] + ... \\
&= -k_0(s^{-1})(r_0^k(s^0) + 1 - \delta) + \lim_{t \longrightarrow \infty} \sum_{s^t} q_{0,t}(s^t) k_{t+1}(s^t)
\end{aligned}
$$

where we have used that $k_1(s^0)$ is independent of s^1. The intertemporal budget constraint thus reads

$$\frac{d_{-1}(s^0)}{\pi_0(s^0)} + k_0(s^{-1})(r_0^k(s^0)+1-\delta) = E_0 \sum_{t=0}^{\infty} \sum_{s^t} q_{0,t}(s^t)\left(c_t(s^t) - w_t(s^t)n_t(s^t) - y_t(s^t)\left(1 - mc_t(s^t)s_t(s^t)\right)\right)$$
$$+ \lim_{t\longrightarrow\infty} \sum_{s^t} q_{0,t}(s^t)k_{t+1}(s^t) + \lim_{t\longrightarrow\infty} \sum_{s^t} q_{0,t}(s^t)d_t(s^t)$$

We now substitute out $q_{0,t}$ by using $q_{0,t}(s^t) = \beta^t pr(s^t|s^0)\frac{c_t^{-\sigma}(s^t)}{c_0^{-\sigma}(s^0)}$. Further, the two latter terms are zero by the transversality conditions. Thus, the intertemporal budget constraint is given by

$$c_0^{-\sigma}\left[\frac{d_{-1}}{\pi_0} + k_0(r_0^k+1-\delta)\right] = E_0 \sum_{t=0}^{\infty} \beta^t c_t^{-\sigma}\left(c_t - w_t n_t - y_t\left(1 - mc_t s_t\right)\right)$$

Now, we eliminate the wage, the capital rental rate and real marginal cost

$$\begin{aligned} mc_t &= \frac{1}{A_t}\left(\frac{r_t^k}{1-\alpha}\right)^{1-\alpha}\left(\frac{w_t}{\alpha}\right)^{\alpha} \\ r_t^k &= \frac{(1-\alpha)}{\alpha}\frac{w_t n_t}{k_t} \\ w_t &= \chi n_t^{\eta} c_t^{\sigma} \mu_t^w \\ \Longrightarrow mc_t &= \frac{1}{A_t n_t^{\alpha} k_t^{1-\alpha}}\frac{\chi}{\alpha}\left(n_t^{1+\eta} c_t^{\sigma}\mu_t^w\right) \end{aligned}$$

Further using $y_t = \frac{A_t n_t^{\alpha} k_t^{1-\alpha}}{s_t}$, we get

$$\begin{aligned} c_t^{-\sigma} y_t\left(1 - mc_t s_t\right) &= c_t^{-\sigma}\frac{A_t n_t^{\alpha} k_t^{1-\alpha}}{s_t} - c_t^{-\sigma} A_t n_t^{\alpha} k_t^{1-\alpha} mc_t \\ &= c_t^{-\sigma}\frac{A_t n_t^{\alpha} k_t^{1-\alpha}}{s_t} - \frac{\chi}{\alpha} n_t^{1+\eta}\mu_t^w \end{aligned}$$

Thus, the intertemporal budget constraint can be written as

$$\begin{aligned} A_0 &= E_0 \sum_{t=0}^{\infty} \beta^t \left[c_t^{1-\sigma} - \chi n_t^{1+\eta}\mu_t^w - c_t^{-\sigma}\frac{A_t n_t^{\alpha} k_t^{1-\alpha}}{s_t} + \frac{\chi}{\alpha} n_t^{1+\eta}\mu_t^w\right] \\ A_0 &= E_0 \sum_{t=0}^{\infty} \beta^t \left[c_t^{1-\sigma} + \left(\frac{1-\alpha}{\alpha}\right)\chi n_t^{1+\eta}\mu_t^w - c_t^{-\sigma}\frac{A_t n_t^{\alpha} k_t^{1-\alpha}}{s_t}\right] \end{aligned}$$

where $A_t = c_t^{-\sigma}\left[\frac{d_{t-1}}{\pi_t} + k_t(r_t^k+1-\delta)\right]$. As households trade state-contingent claims among each other, in the aggregate, no state contingent bonds can be held so that in equilibrium $d_t = 0 \; \forall \; t$. Thus, the LHS simplifies to $A_t = c_t^{-\sigma}\left[k_t(r_t^k+1-\delta)\right]$.

A.4.2 Writing the intertemporal budget constraint recursively

The Ramsey planner faces an intertemporal budget constraint in every period because the household cannot by trading in period zero insure against all risks. The reason for this is that he has no set of real state contingent claims so that HH solvency cannot be ensured by period zero trading but must be guaranteed in every period. Letting γ_t denote the multiplier on the intertemporal budget constraint, the Ramsey planner's Lagrange takes the form

$$\begin{aligned}
L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right] \\
+\gamma_t \left\{ E_t \sum_{j=0}^{\infty} \beta^{t+j} \left[c_{t+j}^{1-\sigma} + \left(\frac{1-\alpha}{\alpha} \right) \chi n_{t+j}^{1+\eta} \mu_{t+j}^{w} - c_{t+j}^{-\sigma} \frac{IO_{t+j}}{s_{t+l}} \right] - A_t \right\} \\
+\lambda_t^1 [] \\
+...
\end{aligned}$$

As described in Ljungqvist and Sargent (2004), the sequence of intertemporal budget constraints can be written recursively:

$$\begin{aligned}
& E_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \left\{ E_t \sum_{j=0}^{\infty} \beta^{t+j} \left[c_{t+j}^{1-\sigma} + \left(\frac{1-\alpha}{\alpha} \right) \chi n_{t+j}^{1+\eta} \mu_{t+j}^{w} - c_{t+j}^{-\sigma} \frac{IO_{t+j}}{s_{t+l}} \right] - A_t \right\} \\
= & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Gamma_t \left[c_t^{1-\sigma} + \left(\frac{1-\alpha}{\alpha} \right) \chi n_t^{1+\eta} \mu_t^{w} - c_t^{-\sigma} \frac{IO_t}{s_t} \right] - (\Gamma_t - \Gamma_{t-1}) A_t \right\}
\end{aligned}$$

where $\Gamma_t = \Gamma_{t-1} + \gamma_t$ and $\Gamma_{-1} = 0$.

A.4.3 First order conditions to the Ramsey problem without adjustment cost

The Ramsey problem (20) reads

$$
\begin{aligned}
\max_{\{c,n,k,R,s,\pi,\tilde{Z},Z^1,Z^2\}} J = E_0 \sum_{t=0}^{\infty} \beta^t & \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right] \\
& + \Gamma_t \left[c_t^{1-\sigma} + \frac{1-\alpha}{\alpha} \chi n_t^{1+\eta} \mu_t^w - c_t^{-\sigma} \frac{A_t k_t^{1-\alpha} n_t^{\alpha}}{s_t} \right] \\
& - (\Gamma_t - \Gamma_{t-1}) \left[\frac{(1-\alpha)\chi}{\alpha} n_t^{\eta+1} \mu_t^w + k_t c_t^{-\sigma} (1-\delta) \right] \\
& + \lambda_t^1 \left[c_t^{-\sigma} - E_t \beta c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right] \\
& + \lambda_t^2 \left[c_t^{-\sigma} - E_t \beta (1-\alpha) \frac{\chi}{\alpha} \frac{n_{t+1}^{\eta+1}}{k_{t+1}} \mu_{t+1}^w - E_t \beta (1-\delta) c_{t+1}^{-\sigma} \right] \\
& + \lambda_t^3 \left[\tilde{Z}_t - \frac{Z_t^1}{Z_t^2} \right] \\
& + \lambda_t^4 \left[\pi_t - \left(\frac{1 + (\phi - 1) \tilde{Z}_t^{1-\varepsilon}}{\phi} \right)^{\frac{1}{\varepsilon - 1}} \right] \\
& + \lambda_t^5 \left[s_t - (1-\phi) \tilde{Z}_t^{-\varepsilon} + \phi \pi_t^{\varepsilon} s_{t-1} \right] \\
& + \lambda_t^6 \left[Z_t^1 - \frac{n_t^{1+\eta}}{s_t} \frac{\chi}{\alpha} \mu_t^w - \phi \beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1 \right] \\
& + \lambda_t^7 \left[Z_t^2 - c_t^{-\sigma} \frac{A_t k_t^{1-\alpha} n_t^{\alpha}}{s_t} - \phi \beta E_t \pi_{t+1}^{\varepsilon - 1} Z_{t+1}^2 \right]
\end{aligned}
$$

The first order conditions to this problem are

$$
\begin{aligned}
&\frac{\partial J}{\Gamma_t}=0 \quad\Leftrightarrow\quad c_t^{1-\sigma}-c_t^{-\sigma}\frac{IO_t}{s_t}\frac{(1-\alpha)\chi}{\alpha}+\beta\frac{(1-\alpha)\chi}{\alpha}n_{t+1}^{\eta+1}\mu_{t+1}^{w}+\beta k_{t+1}c_{t+1}^{-\sigma}(1-\delta)=0\\
&\frac{\partial J}{R_t}=0 \ \Leftrightarrow \lambda_t^1=0\ \forall\ t \quad \text{(which is used in all further equations)}\\
&\frac{\partial J}{c_t}=0 \quad\Leftrightarrow\ 0=\ c_t^{-\sigma}\left[1+\Gamma_t(1-\sigma)\right]+\sigma(1-\delta)\lambda_{t-1}^2c_t^{-\sigma-1}\\
&\qquad +\sigma c_t^{-\sigma-1}\left[\Gamma_t\frac{A_tk_t^{1-\alpha}n_t^{\alpha}}{s_t}+(1-\delta)k_t-\lambda_t^1+\frac{R_{t-1}}{\pi_t}\lambda_{t-1}^1-\lambda_t^2+\lambda_t^7\frac{A_tk_t^{1-\alpha}n_t^{\alpha}}{s_t}\right]\\
&\frac{\partial J}{n_t}=0 \ \Leftrightarrow\quad 0=(1+\eta)\frac{\chi}{\alpha}\left[-\frac{\alpha}{(1+\eta)}n_t^{\eta}+\Gamma_{t-1}(1-\alpha)\mu_t^wn_t^{\eta}-\lambda_{t-1}^2(1-\alpha)\frac{\mu_t^w}{k_t}n_t^{\eta}-\lambda_t^6\frac{\mu_t^w}{s_t}n_t^{\eta}\right]\\
&\qquad +c_t^{-\sigma}\frac{A_tk_t^{1-\alpha}\alpha n_t^{\alpha-1}}{s_t}\left[-\Gamma_t-\lambda_t^7\right]\\
&\frac{\partial J}{k_{t+1}}=0 \ \Leftrightarrow\quad 0=\ (1-\delta)c_{t+1}^{-\sigma}\left(\Gamma_t-\Gamma_{t+1}\right)+\frac{(1-\alpha)\chi}{\alpha}\lambda_t^2E_t\frac{n_{t+1}^{\eta+1}\mu_{t+1}^w}{(k_{t+1})^2}\\
&\qquad -\frac{c_{t+1}^{-\sigma}}{s_{t+1}}A_{t+1}(1-\alpha)k_{t+1}^{-\alpha}n_{t+1}^{\alpha}\left[\Gamma_{t+1}+\lambda_{t+1}^7\right]\\
&\frac{\partial J}{\pi_t}=0 \ \Leftrightarrow\quad \lambda_t^4+\phi\left[-\varepsilon s_{t-1}\lambda_t^5\pi_t^{\varepsilon-1}-\varepsilon\lambda_{t-1}^6Z_t^1\pi_t^{\varepsilon-1}-(\varepsilon-1)\,\lambda_{t-1}^7Z_t^2\pi_t^{\varepsilon-2}\right]=0\\
&\frac{\partial J}{Z_t^1}=0 \ \Leftrightarrow\quad -\frac{\lambda_t^3}{Z_t^2}+\lambda_t^6-\lambda_{t-1}^6\phi\pi_t^{\varepsilon}=0\\
&\frac{\partial J}{Z_t^2}=0 \ \Leftrightarrow\quad \lambda_t^3\frac{Z_t^1}{\left(Z_t^2\right)^2}+\lambda_t^7-\lambda_{t-1}^7\phi\pi_t^{\varepsilon-1}=0\\
&\frac{\partial J}{\tilde{Z}_t}=0 \ \Leftrightarrow\quad 0=\lambda_t^3+\varepsilon\lambda_t^5(1-\phi)\tilde{Z}_t^{-\varepsilon-1}\\
&\qquad +\lambda_t^4\left(\frac{1+(\phi-1)\tilde{Z}_t^{1-\varepsilon}}{\phi}\right)^{\left(\frac{1}{\varepsilon-1}-1\right)}\frac{(\phi-1)}{\phi}\left(1-\varepsilon\right)\tilde{Z}_t^{-\varepsilon}\\
&\frac{\partial J}{s_t}=0 \quad\Leftrightarrow\quad \Gamma_tc_t^{-\sigma}\frac{A_tk_t^{1-\alpha}n_t^{\alpha}}{s_t^2}+\lambda_t^5-\lambda_{t+1}^5\beta\phi\pi_{t+1}^{\varepsilon}+\frac{\chi}{\alpha}\lambda_t^5\frac{n_t^{\eta+1}}{s_t^2}\mu_t^w+\lambda_t^5c_t^{-\sigma}\frac{A_tk_t^{1-\alpha}n_t^{\alpha}}{s_t^2}=0
\end{aligned}
$$