Liquidity risk premia in unsecured interbank money markets*

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Abstract

Unsecured interbank money market rates such as the Euribor increased strongly with the start of the financial market turbulences in August 2007. There is clear evidence that these rates reached levels that cannot be explained alone by higher credit risk. This article presents this evidence and provides a theoretical explanation which refers to the funding liquidity risk of lenders in unsecured term money markets.

Keywords: *Liquidity premium, Interbank money markets, unsecured lending, 2007/2008 financial market turmoil*

JEL Codes: G01, G10, G21

1 Introduction

Unsecured interbank money market rates such as the Libor (London Interbank Offer Rate) and the Euribor (Euro Interbank Offered Rate) increased strongly with the start of the financial market turmoil in August 2007 and even further after the default of Lehman Brothers in September 2008. There is clear evidence that unsecured money market rates reached levels during the turmoil that cannot be explained alone by higher borrower default probabilities. Indeed, in the absence of liquidity frictions approximate no-arbitrage conditions require that the spread of for example the (one-year) Euribor over the (one-year) general collateral repo rate (i.e. the riskfree rate) should not be much above spreads of (one-year) credit default swaps (CDS) on banks. This was the case before August 2007. But since August 2007 Euribor spreads have been significantly higher than spreads on bank CDSs.

*The views expressed in this paper are our own and do not necessarily reflect the view of the European Central Bank.

This "liquidity risk premium" in money market rates has at some occasions been attributed to liquidity hoarding. Liquidity hoarding in this context may mean that banks that have a surplus of funds are not ready to lend them to other (private) banks. Anecdotal evidence suggests in this context that term unsecured money market volumes declined significantly with the start of the turbulences. However, some observations indicate that banks did not stop lending to other banks in August 2007:

- Spreads of EONIA (Euro Overnight Index Average) over the ECB minimum bid rate did not increase in the turmoil. EONIA volumes even increased slightly after the start of the turmoil. Thus, banks continued to lend unsecured to other banks overnight, although not for longer time horizons.
- Spreads of general collateral repo rates (over overnight interest rate swap rates) did not increased in the turmoil. Repo market volumes remained by and large stable for high-quality collateral. This holds also for term repo markets.[1] Thus, banks continued to lend collateralised to other banks even for longer time horizons.

This paper describes these observations in detail and provides a theoretical explanation which refers to the funding liquidity risk of lenders in unsecured interbank term money markets.

To understand the basic argumentation we offer, consider a bank with a cash surplus. This bank can offer unsecured funds either in the overnight money market (at the Eonia) or in the term money market (e.g. at the Euribor). With a certain probability, it receives a liquidity shock (liquidity outflow) at some point in the future. If the bank lends out in the term money market and receives such a shock before the loan matures, then it will have to raise funds itself when the shock arrives. It may not need to do so if it lends only repeatedly overnight until it receives a liquidity shock and then uses the repayment of the loan to satisfy its obligations. If the bank fears that it could only borrow at relatively bad conditions at the time of the shock (e.g. because it may not have enough collateral to borrow in the repo market and other banks may assume that the bank is relatively risky), then it will be ready to lend in the term money market only at elevated rates. Banks with a cash shortage may thus prefer to borrow repeatedly overnight rather than once for a longer term even if they need cash for a longer period.[2] As a consequence, term money market trades will be rare, overnight money markets will be liquid. Term money market rates as measured by the Euribor will not reflect effective rates, but only levels at which banks with a surplus would be ready to lend for a longer period.

On the basis of this argumentation, it can be concluded that interbank term money market spreads increase if, among others, (a) the risk that the lenders

[1] See ICMA (2008a) and ICMA (2008b).

[2] This, of course, does not take into account that some banks may need to borrow once for a longer period for regulatory or other reasons.

receive a liquidity shock before term loans mature increases and/or (b) the probability that the lenders face higher funding costs when such a liquidity shock arrives increases. These future funding costs go up for example when the lenders' default probabilities rise. Thus, term money market rates do not only increase when the probabilities of default of the borrowers increase, but also when the lender default probabilities go up. Future funding costs are also pushed higher if lenders may be hit by a deterioration of the quality of available collateral so that they cannot raise funds in the repo market anymore. Thus, unsecured term money market rates will be higher if lenders may face a lack of high-quality collateral before term loans mature.[3]

This last point may have played a specific role during the financial market turbulences. Before the turmoil, many banks had set up special investment vehicles (SIVs). The SIVs raised short-term loans and invested them in for example asset-backed securities (ABS). When the sub-prime crisis hit, investors stopped providing short-term loans to SIVs. The risk that a bank would need to bail out its SIVs increased. Bailing out an SIV however implies purchasing from the SIVs in particular ABSs which however are hardly accepted as collateral in repo markets.[4] Thus, with the start of the sub-prime crisis, the risk of a major furture liquidity shock and a simultaneous deterioration of available collateral soared.

Our paper contributes not only to the policy discussion on the financial market turmoil, but also to various strands of academic literature.

The empirical part of our paper relates to the very recent and growing body of empirical research that tries to decompose money market rates during the turmoil into credit risk, liquidity and other components. Examples are Wu (2008), Michaud and Upper (2008) and Taylor and Williams (2008). These authors apply regression techniques for the decomposition. In all studies, money market spreads are used as dependent variable. As independent variables, credit components are measured by CDS spreads (and other indicators) and liquidity components are measured by (dummies for) central bank interventions (Taylor and Williams (2008), Wu (2008)) or as a residual (Michaud and Upper (2008)). Wu (2008) and Michaud and Upper (2008) identify a significant impact of liquidity measures on money market rates, while Taylor and Williams conclude that liquidity does not play a role.[5] Based on a simple arbitrage argument, we measure in the empirical part of this paper the liquidity component in money market rates simply as the difference between the one-year Euribor spread and the spread of one-year CDS contracts on major banks. We find that liquidity has been a key driver of money market spreads since the start of the turmoil.

Our theoretical model contributes to another very recent strand of literature,

[3] It is noted here that loans granted in the unsecured money market usually cannot be used by the creditor as collateral to borrow in the repo market.

[4] See for example ICMA (2008) that reports that more than 80% of European repo market collateral is government bonds. For a comparison, note that less than 50% of all euro-denominated bonds are government bonds.

[5] For a discussion of the difference between Wu (2008) and Taylor and Williams (2008), see Wu (2008).

the literature on funding liquidity and collateral. A recent example is Brunnermeier and Pedersen (2006) who analyse the concept of funding liquidity and the relation between funding liquidity and market liquidity. There, funding liquidity is essentially defined in terms of collateral margin levels and therefore closely relates to institutional factors. Our model also discusses funding liquidity, funding liquidity risk and the role of collateral, with however a very different and more general focus. In our model, funding liquidity refers to the spread between interbank money market spreads and CDS spreads as well as the money market term spread. Acharya and Viswanathan (2008) discuss the role of collateral on funding liquidity and credit rationing. The impact of collateral on the maturity of loans and term spreads is not discussed.

Our paper has some similarities with the literature on financial intermediation and liquidity shocks in the presence of interbank markets, for example Bhattacharya and Gale (1987) and Allen and Gale (2004). This literature typically starts from the classical paper by Diamond and Dybvig (1983), but assumes that there is not only one bank but many banks that can lend funds among each other. In a first period, banks can either hold cash or invest in a two–period project that can be liquidated after one period only at a loss. A liquidity shock after one period occurs if depositors want to withdraw deposits early. In our paper, we also consider banks that can invest (lend) funds for one or two periods and that face the risk of random (exogenous) liquidity shocks after the first period. However, in contrast to the literature on bank runs and interbank markets, funds are always invested in the interbank market (i.e. lend out to other banks). That means that we allow that banks lend to each other for one or for two periods and can therefore model a term spread. Furthermore, we model an economy without frictions, i.e. there is for example no asymmetric information.

Our modelling approach is remotely related to the literature of optimal refinancing in the presence of liquidity shocks (and moral hazard), as pioneered by Holmström and Tirole (1998). Just like in our model economic agents are faced with liquidity shocks that they can choose to refinance short term (i.e. every period) or, long term via contracting in the initial period. In contrast to Holmström and Tirole, however, in our model this decision is merely a question of the price – since liquidity shocks do not eat up capital but sum up to zero there will always be refinancing available – while in the Holmström and Tirole world ex-ante contracting of refinancing is a form of taking out insurance against forced project termination in the interim period.

Another strand of literature to which our paper may contribute is the literature on the yield curve as we consider the term spread as a result of expected funding conditions. Finally, our paper may also provide some suggestions for the valuation of deposits and bonds by means of CDS spreads.[6]

The paper is organised in three sections. Section 2 summarises the main empirical observations. Section 3 outlines the theoretical model and provides the main intuitions. Section 4 concludes.

[6] See for example Duffie (1999) and Hull and White (2000).

2 Main empirical observations

The start of the financial market turbulences was clearly marked by a strong increase of unsecured term interbank money market rates in the beginning of August 2007. Chart 2 shows the evolution of the spread between the Euribor and the Eurepo[7] for the 3-month, 6-month and 12-month maturities. Within a few days, the spreads went up from about 10 basis points (bps) to around 70 bps and remained at these levels until later summer 2008. In September 2008, after the default of Lehman Brothers, spreads increased further to reach levels above 200 bps.

To assess the reasons for the sharp increase in the Euribor, we compare Euribor spreads with spreads of credit default swaps (CDS). A (single name) CDS contract between two parties, the protection buyer and the protection seller, is characterised by a reference entity, a reference obligation, a notional amount, a CDS spread and a maturity. The reference obligation is typically a bond issued by the reference entity that matures after the CDS contract. A one-year CDS contract on reference entity 'bank β' with a notional amount q and a CDS spread ρ means that the protection buyer pays quarterly, for the first time after three months and for the last time after 12 months, a premium $\frac{1}{4}\rho q$ to the protection seller if bank β does not default on the reference obligation within one year. If bank β defaults within one year, then the premium is not paid anymore after the day of default. Instead, the protection seller pays the notional amount of the CDS to the protection buyer after the default and receives the reference obligation from the protection buyer.

Duffie (1999) and Hull and White (2000) argue that the CDS spread should approximately equal the difference between the yield of a par bond of the reference entity that matures when the CDS matures and the risk free rate, i.e. the asset swap spread. If this is not the case, then arbitrage opportunities may arise. Suppose that the CDS spread is below the asset swap spread. Then investors could realise profits by raising funds X at the risk free rate (e.g. in the repo market against high-quality collateral), buying the par bond with these funds and buying protection with a notional amount equal to X. If the CDS spread is above the asset swap spread, then investor could sell the bond and protection and invest the proceeds from selling the bond at the risk free rate.

In a similar way, it can be argued that the one-year CDS spread on some bank β should equal the difference between the one-year unsecured interbank market rate at which the bank borrows funds and the one year repo rate. In particular, if the CDS spread is below this rate differential, then investors could raise funds in the repo market, lend them unsecured to bank β and buy protection against a default of bank β through the CDS. A precise set of assumptions that supports this argument is analysed in Annex A. It is also shown in Annex A that the (risk-neutral) probabilities of default of bank β implied in CDS spreads are lower than those implied in money market rates if CDS spreads are lower than spreads of unsecured money market rates over repo rates. It can therefore be

[7] The Eurepo is an average general collateral (GC) repo rate from euro repo transactions. For more details, see www.eurepo.org.

concluded that money market spreads do not only represent borrowers' default probabilities (credit risk premia), but also other components which may be called liquidity risk premia, if money market spreads exceed CDS spreads.

This argumentation establishes a relation between the spread of a one-year CDS on a specific bank and the (spread of) the interest rate at which the bank borrows unsecured for one year in the interbank market. However, we do not have information on this interest rate. We only have the one-year Euribor. To see to which extent we can replace the bank specific borrowing rate by the one-year Euribor, it is important to understand how the Euribor is defined. The Euribor is calculated as an (unweighted) average of (up to) 43 individual rates, each rate reported by a so-called Euribor panel bank. The 43 panel banks are supposed to report "to the best of their knowledge [...] rates being defined as the rates at which euro interbank term deposits are being offered within the EMU[8] zone by one prime bank to another at 11.00 a.m. Brussels time ('the best price between the best banks')"[9].

Thus, the rate that a panel bank reports is not the rate at which other banks offer deposits to the reporting bank or the rate at which the reporting bank offers deposits to other banks. It is the rate at which the reporting bank believes one of the best banks offers deposits to another one of the best banks. Indeed, the (up to) 43 daily individual contributions to the one-year Euribor do not deviate much from one another as Chart 3 shows. The standard deviation of individual contributions remained below five basis points even during the turmoil.[10]

It is therefore plausible to consider the Euribor as a lower bound of interbank rates so that an individual bank should normally not be able to borrow below Euribor and many banks should only be able to borrow at rates above Euribor. The above argumentation therefore suggests that arbitrage could be made if the (one-year) CDS spread of a specific bank is below the spread of the (one-year) Euribor over the one-year Eurepo rate.

Chart 4 shows (i) the spread of the one-year Euribor over the one-year Eurepo (dark line) and (ii) the average one-year CDS spread over CDS contracts on 20 Euribor panel banks. These are all panel banks for which CDS data have been available from Bloomberg at least from June 2007 onwards. CDS spreads approximately equalled Euribor spreads before the start of the turbulences in August 2007. However, since August 2007, spreads of the Euribor over repo rates have been much larger than CDS spreads. The following charts present bank specific CDS spreads for the 20 panel banks and compare them with the one-year Euribor spread. Euribor spreads have been close to CDS spreads before the start of the turbulences, but much higher than CDS spreads for virtually all banks and all days since the start of the turbulences. The only exceptions are a few banks with elevated CDS spreads in March 2008 at the peak of the Bear Stearns crisis. It appears plausible that these banks could borrow only well above Euribor at that time so that most likely the bank specific borrowing

[8] EMU stands for European Monetary Union.

[9] Quoted from the Euribor Code of Conduct, see www.euribor.org.

[10] For a comparison, the chart also provides the standard deviation of spreads of one-year CDS contracts on 20 Euribor panel banks.

rate was above the respective CDS spread also for these banks.

Before we continue to explaining these observations with our model, we report a few other observations that will turn out to be relevant in the context of the model. First, it is interesting that only spreads in term money markets widened, while spreads in overnight unsecured money markets remained unchanged. Chart 5 shows the spread of the Eonia over the ECB's minimum bid rate. Obviously Eonia spreads remained on average on pre-turmoil levels although they became much more volatile.

It is important to note that the Euribor and the Eonia are defined in very different ways. The Eonia is the volume weighted average of rates from unsecured overnight transactions of a panel of 43 prime banks (the same panel as for Euribor). The Eonia is thus based on transactions that effectively took place. The Euribor is the average of rates at which panel banks believe prime banks can borrow from other prime banks. The Euribor therefore does not necessarily refer to real transactions.

Against this background, it is interesting to look at how volumes have evolved during the turbulences. Unfortunately, there are hardly any reliable data on volumes in term money markets. However, anecdotal evidence suggests a sharp decline in unsecured term money market volumes in paralell to the strong increase of rates. For example, based on interviews with several market participants, BearingPoint (2008) finds: "In previous editions of BearingPoint's repo study we pointed out that the unsecured money market still offers banks a simple and attractive way of getting liquidity in and out of the market. However, this has changed quite dramatically since mid 2007. Today, banks only selectively provide cash on an unsecured basis. For example, borrowing money from other banks or even between different departments of the same bank for more than a day has become very difficult." and "(...) liquidity in the unsecured market is currently concentrated on 'Overnight ' transactions". Indeed, volumes in unsecured overnight markets even increased in the first year of the financial turbulences as Chart 6 on Eonia volumes shows.

3 The model

We consider a model with three periods $t = 0, 1, 2$ and N banks. There are two types of banks in $t = 0$, $\frac{1}{2}N$ banks are α-banks and $\frac{1}{2}N$ banks are β-banks. We assume that at the beginning of period 0 all banks have exactly as much cash (central bank deposits) as they need to hold in all periods. However, already in period 0 there are exogenous cash transfers d_0 from (customers of) β-banks to (customers of) α-banks. That is, each β-bank sends d_0 and each α-bank receives d_0.

In period $t = 1$, α-banks are divided into three subgroups: $(1-\gamma)\frac{1}{4}N$ banks are of type $\alpha\alpha$, $(1-\gamma)\frac{1}{4}N$ banks are of type $\alpha\beta$, and $\gamma\frac{1}{2}N$ banks are of type $\alpha\gamma$. There are exogenous cash transfers d_1 from (customers of) $\alpha\beta$-banks to (customers of) $\alpha\alpha$-banks, i.e. each $\alpha\beta$-bank sends d_1 and each $\alpha\alpha$-bank receives d_1. Banks of type $\alpha\gamma$ do not send or receive cash. We assume that the size of cash transfers d_1 is random with density f and distribution function F and that α-banks do not know in period $t = 0$ of which type they will be in period $t = 1$.

Period $t = 2$ is the termination period. Banks default only in $t = 2$. The probabilities of default are p_α for α-banks and p_β for β-banks. These probabilities are the same in $t = 0$ and in $t = 1$, i.e. all α-banks have the same probability of default independent of whether they turn out to be $\alpha\alpha$, $\alpha\beta$ or $\alpha\gamma$. Moreover, probabilities of default do not depend on the size of the (random) liquidity shock d_1. Recovery rates of default are assumed to be zero.

There are three types of markets. Banks can borrow and lend riskfree in repo markets against perfect collateral. There is a one-period and a two-period repo market in $t = 0$ and a one-period repo market in $t = 1$. The one-period repo rates $r_R^{0,1}$ in $t = 0$ and $r_R^{1,2}$ in $t = 1$ are exogenous and non-random. For simplicity we assume $r_R^{0,1} = r_R^{1,2}$. The two-period repo rate in $t = 0$ is denoted r_R and we assume $(1 + r_R) = (1 + r_R^{0,1})^2$, i.e. the risk free yield curve is flat.[11] Banks have perfect collateral C_α and C_β in $t = 0$ and the value of collateral increases at the repo rate (accrued interest). Collateral can be reused. If an α-bank receives collateral from a β-bank in $t = 0$ for a two-period loan, then the α-bank can use the collateral for example to secure a one-period loan that it receives in period $t = 1$. We will however assume most of the time that β-banks do not have collateral ($C_\beta = 0$) and discuss the role of collateral available to β-bank only briefly later on.

Banks can also borrow and lend in an unsecured interbank money market. The rate $r_\beta^{0,1}$ in the one-period unsecured market of period $t = 0$ equals of course $r_R^{0,1}$ as there is no default in period $t = 1$. For simplicity we assume that there is no one-period repo trading in $t = 0$ as the one-period unsecured trading in $t = 0$ is a perfect substitute because default cannot occur in $t = 1$. The rate in the one-period unsecured market of $t = 1$ depends on the probability of default of the borrower. Type $\alpha\beta$-banks borrow unsecured in $t = 1$ at rate $r_{\alpha\beta}^{1,2}$ with $(1 - p_\alpha)(1 + r_{\alpha\beta}^{1,2}) = (1 + r_R^{0,1})$ and type β-banks borrow unsecured in $t = 1$ at rate $r_\beta^{1,2}$ with $(1 - p_\alpha)(1 + r_\beta^{1,2}) = (1 + r_R^{0,1})$. Thus, all one-period rates are quasi exogenous. The only endogenous rate is the two-period unsecured market rate of $t = 0$, which we denote r. Interest in repo markets and in unsecured money markets is to be paid when a loan matures.

Finally, there is a credit default swap (CDS) market in $t = 0$. The underlying reference entities are β-banks. A protection buyer will receive the notional amount of the CDS contract if the underlying β-bank defaults and will pay ρ-times the notional amount otherwise. We assume that the banks of our model always trade CDS contracts with institutions that are not modelled explicitly, that never default and that ignore counterparty risk. Thus, the CDS premium that an α-bank will receive if it sells protection and the CDS premium that it has to pay when it buys protection do not depend on p_α. We will discuss counterparty risk in CDS contracts briefly in Section 4.

[11] Note that in the present model with two different maturities and a discrete time setting the flat yield curve does not imply arbitrage opportunities.

All random variables are assumed to be independently distributed. This would also imply that one β-bank defaults independently of another β-bank. However, we consider the β-banks as one representative β-bank instead of many, which implies for example that CDS contracts are on the same β-bank and that all loan trades of an α-bank with a β-bank, no matter whether the trade takes place in period 0 or in period 1 and whether it matures after one or two periods, will be with the same β-bank. All banks are risk neutral, price takers and maximize expected utility in the termination period. Utilities equal profits unless the bank defaults in which case the utility is normalized to zero. We assume that $(1 + r_R^{0,1})d_0 < d_1$. This means that $\alpha\beta$-banks will in any case need to borrow in $t = 1$, even if they lend in $t = 0$ only for one period.

Because we assume risk neutrality, banks do not trade in the CDS market to hedge the risk of losses from default in the interbank market. Moreover, the expected return from a CDS portfolio does not depend on a bank's activities in the interbank markets. Finally, activities in the CDS market are not constrained by activities in interbank markets. For example, as we assume that CDS contracts are not collateralised, borrowing in the repo market does not reduce the size of CDS trades that a bank can conduct. For these reasons, there is a clear dichotomy of the CDS market on the one hand an the interbank markets on the other. Banks build up a CDS portfolio if there is a CDS portfolio that has a positive expected return. This is the case whenever $\rho \neq \frac{p_\beta}{1-p_\beta}$. We therefore get the following result:

Proposition 1 *In equilibrium, we have* $\rho = \frac{p_\beta}{1-p_\beta}$.

If $\rho > \frac{p_\beta}{1-p_\beta}$, then α-banks would offer as much protection as possible, in our model infinitely many contracts. If $\rho < \frac{p_\beta}{1-p_\beta}$, then α-banks would demand as much protection as possible, in our model infinitely many contracts. Both is not compatible with an equilibrium.[12]

We now turn to the interbank markets and first look at cases without funding liquidity risk ($\gamma = 1$ or $p_\alpha = 0$ or sufficient endowment of collateral). Let $X_u^{0,2}$ be the equilibrium amount of unsecured two-period loans granted in $t = 0$ by an α-bank.

Proposition 2 *Let* $C_\beta = 0$. *Let* $p_\alpha = 0$ *or* $\gamma = 1$ *(or both). Then any* $X_u^{0,2} \in [0, d_0]$ *is an equilibrium and the unique two-period unsecured interbank market rate is*

$$1 + r = \frac{1 + r_R}{1 - p_\beta} \tag{1}$$

[12] It is easy to understand at this point that we could model the CDS market in a more realistic way without changing the results. It would require however a more complex notation. We could assume that there is a (large) number of big institutional investors who are active in the CDS market but have no access to the interbank market (as they are not banks). CDS contracts need to be collateralised and the institutional investors do have enough high-quality collateral for this. Given the size of the institutional investors and the banks' potential lack of collateral, the activities of the institutional investors alone determine the CDS spread, which will thus be $\rho = \frac{p_\beta}{1-p_\beta}$. Given this spread and given the lack of collateral, banks will decide not to trade CDS contracts.

Thus, if there is no funding liquidity risk or α-banks cannot default, then two-period unsecured loans are possible as the rate for such loans is low. The same holds if α-banks have sufficient collateral to borrow any amount needed in $t = 1$ in the repo:

Proposition 3 *Let $C_\beta = 0$. Let $p_\alpha > 0$ and $\gamma < 1$. Assume that there is some number $\overline{a} \in]0, d_0]$ such that $F[\sqrt{(1 + r_R)} \cdot (a + C_\alpha)] = 1$ if and only if $a \in [\overline{a}, d_0]$. Then any $X_u^{0,2} \in [0, d_0 - \overline{a}]$ is an equilibrium and if $X_u^{0,2} > 0$, then the unique two-period unsecured interbank market rate is again given by equation 1.*

In other words, if the shock d_1 cannot exceed the value of the collateral in the hands of α-banks plus the amount of money that an α-bank receives back in $t = 1$ from a one-period loan granted in $t = 0$, provided that this loan had a volume of at least $\overline{a}$, then two-period unsecured loans are possible and the rate for such loans is low.

From these propositions, we immediately get

$$\rho(1 + r_R) = r - r_R \tag{2}$$

whenever the conditions of proposition 2 or proposition 3 are fulfilled. It is easy to show that equation 2 is the arbitrage-free condition under this set-up. Thus, the approximate arbitrage-free condition $\rho = r - r_R$ can be made precise in the context of our model. Equation 2 needs to hold exactly to ensure that arbitrage cannot be made under the assumptions of the model and the conditions of proposition 2 or 3.

We now come to our main result. It describes the situation in the presence of funding liquidity risk. Funding liquidity risk is defined in our context as the risk that a liquidity shock materializes ($\gamma < 1$) and that funding costs at the time of the liquidity shock exceed the risk-free rate because (i) default probabilities are positive ($p_\alpha > 0$) and (ii) there is a lack of (high quality) collateral.

Proposition 4 *Let $C_\beta = 0$. Let $p_\alpha > 0$, $\gamma < 1$ and $F[\Psi] < 1$ with $\Psi \equiv \sqrt{(1 + r_R)} \cdot (d_0 + C_\alpha)$. Then any*

$$1 + r \in [\frac{1 + r_R}{1 - p_\beta}; \frac{1 + r_R}{1 - p_\beta} \cdot [1 + (1 - F[\Psi]) \frac{1 - \gamma}{2} \frac{p_\alpha}{1 - p_\alpha}]]$$

is an equilibrium and $X_u^{0,2} = 0$. Only if

$$1 + r \leq \frac{1 + r_R}{1 - p_\beta}$$

then β-banks would have positive demand for unsecured two-period loans in $t = 0$. Only if

$$1 + r \geq \frac{1 + r_R}{1 - p_\beta} \cdot [1 + (1 - F[\Psi]) \frac{1 - \gamma}{2} \frac{p_\alpha}{1 - p_\alpha}]$$

then α-banks would have positive supply of unsecured two-period loans in $t = 0$.

Thus, the two-period unsecured market breaks down if funding liquidity risk is present. All unsecured loans are one-period loans.

As explained in Section 2, the Euribor is defined as the (average) rate at which reporting banks believe (prime) banks offer interbank (term) deposits to other prime banks. Suppose that the banks in our model are considered prime banks. As α-banks offer two-period interbank deposits to β-banks in our model at the rate

$$r_E \equiv \frac{1+r_R}{1-p_\beta} \cdot [1 + (1-F[\Psi])\frac{1-\gamma}{2}\frac{p_\alpha}{1-p_\alpha}] - 1 \tag{3}$$

we can argue that r_E is a model representation of the Euribor. With propositions 1 and equation 3, we get

$$\rho(1+r_R) = \frac{1+r_E}{1+(1-F[\Psi])\frac{1-\gamma}{2}\frac{p_\alpha}{1-p_\alpha}} - (1+r_R) \leq r_E - r_R \tag{4}$$

Thus, the two-period CDS spread (multiplied by 1 plus the two-period repo rate) is now smaller then the spread of the two-period Euribor r_E over the two-period risk free rate. These results are in line with the empirical findings presented in Section 2.

The money market liquidity risk premium can be defined as the spread of r_E over r as given in equation 1, i.e.

$$LRP = \frac{1+r_R}{1-p_\beta} \cdot [(1-F[\Psi])\frac{1-\gamma}{2} \cdot \frac{p_\alpha}{1-p_\alpha}] \tag{5}$$

The liquidity risk premium can also be written as

$$LRP = r_E - r_R - \rho(1+r_R) \tag{6}$$

It is easy to show that equations 5 and 6 are identical.

It should be noted, however, that the liquidity risk premium essentially refers to a considerable extent to default risk, namely the risk of a default of the respective α-bank.

Up until now, we have discussed cases with $C_\beta = 0$. We merely note that the case of $C_\beta > 0$ is not very exiting. If β-banks have (perfect) collateral, then they will use it. They will simply borrow an amount of C_β in the repo market in $t = 0$ for two periods. For the remaining deficit $\widetilde{d}_0 \equiv d_0 - C_\beta$, the above argumentation holds without changes. We have assumed that repo rates are exogenous so that α-banks cannot require a funding liquidity risk premium in repo markets. There is indeed no funding risk that may result from two-period loans granted in the repo market as the lender receives high-quality collateral that can be reused. If the lender is hit by a liquidity shock before the loan matures, he can use the collateral to borrow funds in the repo market at the risk-free rate.

4 Discussion and conclusions

The above model provides a simple explanation for the rise of interest rates in unsecured interbank term money markets in the wake of the financial market turmoil that started in August 2007. It can explain two major observations: (i) (one-year) unsecured interbank market spreads have significantly exceeded CDS spreads since August 2007 (while overnight interest rate spreads have remained low); and (ii) volumes in unsecured interbank term money markets have reportedly been at extraordinarily low levels since the start of the turmoil while volumes in unsecured overnight markets remained high.[13]

Another observation that fits well to our model refers to the end-of-quarter effects on money market rates during the turmoil. For example the one-month Euribor spread has been clearly higher in the last month of each quarter (in particular in the last month of 2007) than in other months since the start of the turmoil. Similarly, the one-week Euribor spread has been higher in the last week of each quarter than in other weeks.[14] Our model suggests as a possible explanation a higher risk of liquidity shocks or of a collateral shortage at the end of the quarter so that lending money for a term that ends only after the end of the quarter is particularly risky.

As (spreads of) overnight interest rates (Eonia) have remained low, overnight interest rate swap rates have remained low, too. It is remarkable that banks do not seem to raise funds repeatedly overnight at Eonia, lend them out at Euribor and hedge the interest rate risk with an overnight interest rate swap .[15] If banks did this to a large extent, then term money market volumes should most likely be higher than anecdotal evidence suggest. Our model provides a simple explanation: Funds can only be borrowed at the Euribor, but lending funds at Euribor is hardly possible as prime banks prefer to borrow repeatedly overnight at the low overnight rate rather than for a longer period at the high Euribor.

In our argumentation, we have assumed that CDS spreads are determined by the probability of default of the reference entity and can therefore be regarded as a measure of credit risk. Accordingly, we have interpreted the difference between interest rate spreads and CDS spreads as a liquidity risk premium in unsecured money market rates. In reality, CDS spreads may also be influenced by other factors. For example, CDS spreads may go down if probabilities of default of protection sellers go up. However, this effect should be relatively limited as CDS contracts are, as most derivative contracts, collateralised. If the probability of a default of the reference entity increases, then the protection seller has to provide additional collateral to mitigate the protection buyer's counterparty

[13] The low overnight deposit spreads may not only be due to the absence of funding liquidity risk premia in overnight markets, but also due to central bank interventions which have been an important factor during the turmoil. It is also noted in this context that volumes in euro overnight markets remained high only until October 2008 when the ECB introduced full allotment fixed rate tenders and replaced with this policy to a significant extent interbank market activities.

[14] See ECB (2008), box No. 8.

[15] The interest rate risk of this strategy could be hedge through an Eonia swap.

risk. Moreover, discussions with market participants confirm that CDS spreads are still commonly used for credit risk modelling purposes, suggesting that CDS spreads indeed represent mainly the probability of default of the reference entity.

We have presented a stylized model to stress our main points. Accordingly, there are several extensions of our model that may be worth looking at. As a first step, it may be interesting to allow for two types of players with a surplus in period $t = 0$: α-banks will as before be hit with a given probability by a liquidity shock in period $t = 1$; and $\widehat{\alpha}$-banks will with certainty not be hit by such a shock (or have plenty of high-quality collateral or zero default risk). It is plausible that in the framework of our stylized model, $\widehat{\alpha}$-banks will offer two-period unsecured loans at the rate r as defined in equation 1. This rate would be acceptable for β-banks so that we would now observe two-period unsecured loans at low rates in equilibrium even if the new group of players will be small.

However, another modification in addition to the introduction of a second group of institutions would make the model more realistic and would probably broadly confirm the main results of our paper. Suppose that β-banks have a strong preference for borrowing once for two periods rather than twice for one period, for example to reduce the mismatch between long-term assets and short-term liabilities for regulatory reasons. Then there is some demand for two-period loans even if two-period rates are relatively high. In this situation, α-banks may still not be ready to lend for two periods. But $\widehat{\alpha}$-banks will and β-banks will compete for two-period loans from $\widehat{\alpha}$-banks. Thus, two-period interest rates will increase and volumes in the two-period money market will decrease when the total surplus of $\widehat{\alpha}$-banks (or the number of $\widehat{\alpha}$-banks) decreases.

In this context, it should be noted that our empirical findings and theoretical argumentation refer to the relation between CDS spreads and interbank money market spreads. There is a growing literature on the relation between CDS spreads and bonds spreads (spreads of risky bonds over the riskfree rate) which has motivated our methodology. No-arbitrage conditions in principle imply that the spread of a CDS on some reference entity equals the swap spread of a par bond issued by the same entity. Empirical research has found that this was indeed broadly the case before the start of the turmoil.[16] However, to our knowledge, it has not yet been analysed whether this has changed in the turmoil. We would expect that the difference between CDS spreads and bond spreads has been much smaller than that between CDS spreads and interbank money market spreads during the turmoil. Interbank money markets are open only to banks and primarily banks have faced high risks of liquidity shocks since August 2007. Bond markets are open also to many other types of investors, in particular to institutional investors, which might have been less affected by the turmoil.[17] Moreover, it is not impossible to use bank bonds as collateral while interbank loans are not used as collateral in repo markets. For that reason, investors in bonds should be able to raise funds in case of a need cheaper than lenders in unsecured interbank markets.

[16] See for example Zhu (2004).

[17] This may even be true for hedge funds as investors in hedge funds cannot withdraw money from hedge funds at very short notice.

Finally, we consider the influence of central banks' implementation policy on term money market rates in the context of our argumentation. Our analysis suggests that the central bank could mitigate funding liquidity strains and thus trim down term money market rates through at least two different measures. First, it could offer unlimited short-term credit at the central bank rate against a broad range of collateral. In the context of our model, this means that banks that face a liquidity shock could borrow at the relatively low central bank rate in $t = 1$, provided that they have enough central bank eligible collateral.[18] Second, the central bank could lend very liquid high-quality assets against a broader range of collateral including less liquid instruments. Banks that face a liquidity shock can borrow high-quality collateral in $t = 1$ and use this collateral to borrow in the interbank repo market at the low repo rate, again provided that these banks have enough central bank eligible collateral. It should however be noted that our model does not provide any suggestions on whether such interventions should or should not be done from a welfare perspective.

5 Annex A: no-arbitrage condition in CDS and money markets

Consider a CDS that matures after a period of length 1 (e.g. one year) which is divided into T sub-periods as in Figure 1 (for the case of $T = 4$). Call the t-th sub-period t' and the end of this sub-period t. Let 0 be the beginning of the sub-period $1'$. Time is continuous and bank β may default at any point in time so that default in each single point in time occurs with zero probability.

Assume that bank β borrows X in the unsecured interbank market in $t = 0$ at (bank β specific) borrowing rate $r_\beta^{0,T}$ for T sub-periods. Assume that this requires bank β to pay to the lender $r_\beta^{0,T}\frac{1}{T} \cdot X$ in 1, 2,..., T and additionally X in T if it does not default. If it defaults in t', then it pays $r_\beta^{0,T}\frac{1}{T} \cdot X$ in 1, 2, ..., $t-1$ and the remaining obligations $v_\beta^t(X, r_\beta^{0,T})$ (which includes interest accrued between $t-1$ and t) multiplied by the recovery rate R in t, i.e. $R \cdot v_\beta^t(X, r_\beta^{0,T})$. Note that interest is paid in real unsecured money markets only at maturity so that our assumption of interest payments before maturity is only approximately correct.

If an investor buys protection against a default of bank β through a CDS contract with notional amount q, then he will have to pay in 1, 2,..., T a premium $\rho\frac{1}{T} \cdot q$ if no default occurs. If bank β defaults in t', then the investor pays the premium $\rho\frac{1}{T} \cdot q$ in 1, 2, ..., $t-1$ and receives q in t (i.e. with some delay after the default). We assume that the protection seller also pays interest at the risk free interest rate on the notional q for the time from the last premium payment

[18] The ECB started to allocate unlimited credit to banks at a fixed rate (through full allotment fixed rate tenders) in October 2008. It was announced that this policy will be maintained until at least March 2009. In line with our argumentation, Euribor spreads for loans that mature before or shortly after March 2009 decreased significantly while Euribor spreads for loans that mature long after March 2009 remained broadly unchanged.

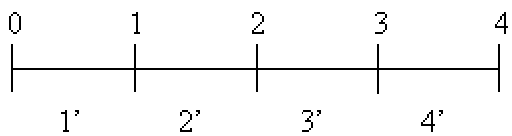

Figure 1: Time structure

$t-1$ to the time of the payment of the notional t. Finally, the investor has to deliver the reference obligation with a nominal value q. If we assume that the reference obligation yields $r_\beta^{0,T}$ and pays interest in 1, 2, ..., $t-1$ in case of a default in t', then it will trade in t at price $R \cdot v_\beta^t(q, r_\beta^{0,T})$.

Finally, there is a repo market. We assume that the repo rate for a repo from $t-1$ to t is $r_R^{t-1,t} = \frac{1}{T} r_R^{0,T}$, i.e. constant over all sub-periods.

Assume that $\rho < r_\beta^{0,T} - r_R^{0,T}$. Then an investor with sufficient assets that can be used as collateral in repo markets could make arbitrage. He could raise an amount X in the repo market in 0 for T sub-periods. He then lends X to bank β unsecured at rate $r_\beta^{0,T}$ from 0 to T and buys protection with a notional value of X. His cash flows if no default occurs are as follows: In 0, the investor raises X and invests X so that the cash flow is zero. In $t = 1, ..., T$, the investor receives $r_\beta^{0,T} \frac{1}{T} \cdot X$ from bank β, pays $(1 + r_R^{0,T} \frac{1}{T}) \cdot X$ in the repo market, raises X in the repo market and pays $\rho \frac{1}{T} \cdot X$ on the CDS. This gives a cash flow of

$$(r_\beta^{0,T} - r_R^{0,T} - \rho) \frac{1}{T} \cdot X > 0$$

If a default occurs in t', then the investor receives this amount in 1, 2, ..., $t-2$ and $t-1$. In t, he receives $X \cdot (1 + \frac{1}{T} r_R^{0,T})$ from the protection seller and has to deliver bonds with a value of $R \cdot v_\beta^t(X, r_\beta^{0,T})$. Moreover, he receives $R \cdot v_\beta^t(q, r_\beta^{0,T})$ from bank β and pays $(1 + r_R^{0,T} \frac{1}{T}) \cdot X$ in the repo market. This gives a cash flow of zero.

Thus, $\rho < r_\beta^{0,T} - r_R^{0,T}$ implies arbitrage opportunities for investors with sufficient high quality collateral. Markets are arbitrage free only if $\rho = r_\beta^{0,T} - r_R^{0,T}$.

As usual, the arbitrage free pricing approach can be translated into a risk-neutral probability approach. In a risk-neutral world, the expected present value of the CDS contract should be zero, i.e. the probabilities of default $p_{\tau'}$ of bank β in period τ' have to satisfy

$$
\begin{aligned}
0 \quad &= \quad [1 - \sum_{\tau'=1'}^{T'} p_{\tau'}] \cdot \sum_{t=1}^{T} \frac{\rho \frac{1}{T} X}{1 + \frac{t}{T} r_R^{0,T}} \\
&\quad + \sum_{\tau'=1'}^{T'} p_{\tau'} [\sum_{t=1}^{\tau-1} \frac{\rho \frac{1}{T} X}{1 + \frac{t}{T} r_R^{0,T}} + \frac{R \cdot v_\beta^t(X, r_\beta^{0,T}) - (1 + \frac{1}{T} r_R^{0,T}) \cdot X}{1 + r_R^{0,T}}]
\end{aligned}
$$

Here, the first term represents the present value of the CDS (from the perspective of the protection seller) if bank β does not default, multiplied by the probability of no default. The other terms give the present value if bank β defaults in period $1'$, ..., T', multiplied by the repsctive probability.

Similarly, the expected present value of raising X repeatedly in the repo market and lending X unsecured to bank β for T sub-periods should in a risk-neutral world equal zero:

$$
\begin{aligned}
0 \quad &= \quad [1 - \sum_{\tau'=1'}^{T'} p_{\tau'}] \cdot \sum_{t=1}^{T} \frac{(r_\beta^{0,T} - r_R^{0,T}) \frac{1}{T} X}{1 + \frac{t}{T} r_R^{0,T}} \\
&\quad + \sum_{\tau'=1'}^{T'} p_{\tau'} [\sum_{t=1}^{\tau-1} \frac{(r_\beta^{0,T} - r_R^{0,T}) \frac{1}{T} X}{1 + \frac{t}{T} r_R^{0,T}} + \frac{R \cdot v_\beta^t(X, r_\beta^{0,T}) - (1 + \frac{1}{T} r_R^{0,T}) \cdot X}{1 + r_R^{0,T}}]
\end{aligned}
$$

It is apparent that these two equations can hold simultaneously only if $\rho = r_\beta^{0,T} - r_R^{0,T}$. Moreover, if markets are complete, i.e. if there is a CDS market and an unsecured interbank market for all maturities 1,..., T and if payment days in all these markets are $1, 2, ...$ until maturity, then markets are arbitrage free if and only if the implied risk-neutral probabilities of default in CDS markets are the same as the implied risk-neutral probabilities of default in unsecured money markets.

Note that we have derived the no-arbitrage condition $\rho = r_\beta^{0,T} - r_R^{0,T}$ under a set of assumptions that are not fully realistic. The most important of these assumptions is that interest payments on interbank loans are to be made on the same days as CDS premium payments. In reality however, the CDS premium is paid quarterly while interest on interbank loans is paid when the loan matures. For the protection seller, the frequency of premium payments is of course an advantage. In order to compensate the protection buyer, one would therefore expect that the CDS spread is a bit smaller than the interest rate spread. For that reason, no arbitrage only requires that the CDS spread equals *approximately* the interest rate spread.

6 Annex B: proofs

We use the following notation:

$S_\alpha^{0,2}$: Repo loan supply of an α-bank in $t = 0$ for two periods.

$S^{1,2}_{\alpha\alpha}$: Repo loan supply of an $\alpha\alpha$-bank in $t = 1$ for one period.
$S^{1,2}_{\alpha\gamma}$: Repo loan supply of an $\alpha\gamma$-bank in $t = 1$ for one period.
$U^{0,1}$: Unsecured loan supply of an α-bank to a β-bank in $t = 0$ for one period.
$U^{0,2}$: Unsecured loan supply of an α-bank to a β-bank in $t = 0$ for two periods.
$U^{1,2}_{\alpha\alpha,\beta}$: Unsecured loan supply of an $\alpha\alpha$-bank to a β-bank in $t = 1$ for one period.
$U^{1,2}_{\alpha\alpha,\alpha\beta}$: Unsecured loan supply of an $\alpha\alpha$-bank to a $\alpha\beta$-bank in $t = 1$ for one period.
$U^{1,2}_{\alpha\gamma,\beta}$: Unsecured loan supply of an $\alpha\gamma$-bank to a β-bank in $t = 1$ for one period.
$U^{1,2}_{\alpha\gamma,\alpha\beta}$: Unsecured loan supply of an $\alpha\gamma$-bank to a $\alpha\beta$-bank in $t = 1$ for one period.
q: CDS protection demand of an α-bank in $t = 0$.
$s^{0,2}_{\beta}$: Repo loan demand of a β-bank in $t = 0$ for two periods.
$s^{1,2}_{\alpha\beta}$: Repo loan demand of an $\alpha\beta$-bank in $t = 1$ for one period.
$s^{1,2}_{\beta}$: Repo loan demand of a β-bank in $t = 1$ for one period.
$u^{0,1}$: Unsecured loan demand of a β-bank in $t = 0$ for one period.
$u^{0,2}$: Unsecured loan demand of a β-bank in $t = 0$ for two periods.
$u^{1,2}_{\alpha\beta}$: Unsecured loan demand of an $\alpha\beta$-bank in $t = 1$ for one period.
$u^{1,2}_{\beta}$: Unsecured loan demand of a β-bank in $t = 1$ for one period.

We implicitly assume here that banks cannot give loans to banks of the same type. We also assume that $\alpha\alpha$-banks cannot trade with $\alpha\gamma$-banks and $\alpha\beta$-banks cannot trade with β-banks in $t = 1$. All period 1 variables are assumed to be non-negative and all unsecured loan period 0 variables are assumed to be non-negative. Repo loan period 0 variables may be positive or negative or zero.

With this notation, we can define the profit of an α-bank for various cases. For example, if the α-bank turns out to be an $\alpha\alpha$-bank and there is no default of this bank and also no default of any of its trading partners, then

$$\pi_\alpha = (1+r_R)S^{0,2}_{\alpha}+(1+r)U^{0,2}-\rho q+(1+r^{0,1}_R)S^{1,2}_{\alpha\alpha}+\frac{1+r^{0,1}_R}{1-p_\beta}U^{1,2}_{\alpha\alpha,\beta}+\frac{1+r^{0,1}_R}{1-p_\alpha}U^{1,2}_{\alpha\alpha,\alpha\beta}$$

This event has a probability of $(1-p_\alpha)^2(1-p_\beta)(1-\gamma)\frac{1}{2}$. Here, $(1-p_\alpha)^2$ is the probability that the α-bank does not default and the $\alpha\beta$-bank with which it trades does not default. If the α-bank turns out to be an $\alpha\alpha$-bank and there is no default of this bank and of the β-bank, but default of the $\alpha\beta$-bank with which it trades, then its profit is given as above but the last term is to be replaced by zero. The probability of that event is $(1-p_\alpha)p_\alpha(1-p_\beta)(1-\gamma)\frac{1}{2}$. Taking into account that utility is zero in case of an own default, we can write the expected utility of an α-bank in $t=0$ conditional on some d_1 as

$$
\begin{aligned}
\frac{E[\pi_\alpha|d_1]}{1-p_\alpha} \quad = \quad & (1-p_\alpha)(1-p_\beta)(1-\gamma)\frac{1}{2}[(1+r_R)S_\alpha^{0,2}+(1+r)U^{0,2} \\
& -\rho q+(1+r_R^{0,1})S_{\alpha\alpha}^{1,2}+\frac{1+r_R^{0,1}}{1-p_\beta}U_{\alpha\alpha,\beta}^{1,2}+\frac{1+r_R^{0,1}}{1-p_\alpha}U_{\alpha\alpha,\alpha\beta}^{1,2}] \\
& +p_\alpha(1-p_\beta)(1-\gamma)\frac{1}{2}[(1+r_R)S_\alpha^{0,2}+(1+r)U^{0,2}-\rho q+(1+r_R^{0,1})S_{\alpha\alpha}^{1,2}+\frac{1+r_R^{0,1}}{1-p_\beta}U_{\alpha\alpha,\beta}^{1,2}] \\
& +(1-p_\alpha)p_\beta(1-\gamma)\frac{1}{2}[(1+r_R)S_\alpha^{0,2}+q+(1+r_R^{0,1})S_{\alpha\alpha}^{1,2}+\frac{1+r_R^{0,1}}{1-p_\alpha}U_{\alpha\alpha,\alpha\beta}^{1,2}] \\
& +p_\alpha p_\beta(1-\gamma)\frac{1}{2}[(1+r_R)S_\alpha^{0,2}+q+(1+r_R^{0,1})S_{\alpha\alpha}^{1,2}] \\
& +(1-p_\alpha)(1-p_\beta)(1-\gamma)\frac{1}{2}[(1+r_R)S_\alpha^{0,2}+(1+r)U^{0,2}-\rho q-(1+r_R^{0,1})s_{\alpha\beta}^{1,2}-\frac{1+r_R^{0,1}}{1-p_\alpha}u_{\alpha\beta}^{1,2}] \\
& +p_\alpha(1-p_\beta)(1-\gamma)\frac{1}{2}[(1+r_R)S_\alpha^{0,2}+(1+r)U^{0,2}-\rho q-(1+r_R^{0,1})s_{\alpha\beta}^{1,2}-\frac{1+r_R^{0,1}}{1-p_\alpha}u_{\alpha\beta}^{1,2}] \\
& +(1-p_\alpha)p_\beta(1-\gamma)\frac{1}{2}[(1+r_R)S_\alpha^{0,2}+q-(1+r_R^{0,1})s_{\alpha\beta}^{1,2}-\frac{1+r_R^{0,1}}{1-p_\alpha}u_{\alpha\beta}^{1,2}] \\
& +p_\alpha p_\beta(1-\gamma)\frac{1}{2}[(1+r_R)S_\alpha^{0,2}+q-(1+r_R^{0,1})s_{\alpha\beta}^{1,2}-\frac{1+r_R^{0,1}}{1-p_\alpha}u_{\alpha\beta}^{1,2}] \\
& +(1-p_\alpha)(1-p_\beta)\gamma[(1+r_R)S_\alpha^{0,2}+(1+r)U^{0,2} \\
& -\rho q+(1+r_R^{0,1})S_{\alpha\gamma}^{1,2}+\frac{1+r_R^{0,1}}{1-p_\beta}U_{\alpha\gamma,\beta}^{1,2}+\frac{1+r_R^{0,1}}{1-p_\alpha}U_{\alpha\gamma,\alpha\beta}^{1,2}] \\
& +p_\alpha(1-p_\beta)\gamma[(1+r_R)S_\alpha^{0,2}+(1+r)U^{0,2}-\rho q+(1+r_R^{0,1})S_{\alpha\gamma}^{1,2}+\frac{1+r_R^{0,1}}{1-p_\beta}U_{\alpha\gamma,\beta}^{1,2}] \\
& +(1-p_\alpha)p_\beta\gamma[(1+r_R)S_\alpha^{0,2}+q+(1+r_R^{0,1})S_{\alpha\gamma}^{1,2}+\frac{1+r_R^{0,1}}{1-p_\alpha}U_{\alpha\gamma,\alpha\beta}^{1,2}] \\
& +p_\alpha p_\beta\gamma[(1+r_R)S_\alpha^{0,2}+q+(1+r_R^{0,1})S_{\alpha\gamma}^{1,2}]
\end{aligned}
$$

Rearranging gives

$$
\begin{aligned}
\frac{E[\pi_\alpha|d_1]}{1-p_\alpha} \quad = \quad & (1+r_R)S_\alpha^{0,2}+(1-p_\beta)(1+r)U^{0,2}-(1-p_\beta)\rho q+p_\beta q \\
& +(1-\gamma)\frac{1}{2}(1+r_R^{0,1})[S_{\alpha\alpha}^{1,2}+U_{\alpha\alpha,\beta}^{1,2}+U_{\alpha\alpha,\alpha\beta}^{1,2}] \\
& -(1-\gamma)\frac{1}{2}[(1+r_R^{0,1})s_{\alpha\beta}^{1,2}+\frac{1+r_R^{0,1}}{1-p_\alpha}u_{\alpha\beta}^{1,2}] \\
& +\gamma(1+r_R^{0,1})[S_{\alpha\gamma}^{1,2}+U_{\alpha\gamma,\beta}^{1,2}+U_{\alpha\gamma,\alpha\beta}^{1,2}]
\end{aligned}
$$

The α-bank has the following contraints:

$$\begin{aligned} S_\alpha^{0,2} + U^{0,2} + U^{0,1} &= d_0 \\ d_0 - S_\alpha^{0,2} &\geq U^{0,1} \geq 0 \\ -S_\alpha^{0,2} &\leq C_\alpha \\ d_1 + (1 + r_R^{0,1})U^{0,1} &= S_{\alpha\alpha}^{1,2} + U_{\alpha\alpha,\alpha\beta}^{1,2} + U_{\alpha\alpha,\beta}^{1,2} \\ d_1 - (1 + r_R^{0,1})U^{0,1} &= s_{\alpha\beta}^{1,2} + u_{\alpha\beta}^{1,2} \\ s_{\alpha\beta}^{1,2} &\leq (1 + r_R^{0,1})C_\alpha + (1 + r_R^{0,1})S_\alpha^{0,2} \\ (1 + r_R^{0,1})U^{0,1} &= S_{\alpha\gamma}^{1,2} + U_{\alpha\gamma,\alpha\beta}^{1,2} + U_{\alpha\gamma,\beta}^{1,2} \end{aligned}$$

Moreover, it is clear that an $\alpha\beta$-bank will always borrow in period 1 as much as possible against collateral, i.e.

$$\begin{aligned} s_{\alpha\beta}^{1,2} &= \min\{d_1 - (1 + r_R^{0,1})U^{0,1}; (1 + r_R^{0,1})C_\alpha + (1 + r_R^{0,1})S_\alpha^{0,2}\} \\ u_{\alpha\beta}^{1,2} &= d_1 - (1 + r_R^{0,1})U^{0,1} - \min\{d_1 - (1 + r_R^{0,1})U^{0,1}; (1 + r_R^{0,1})C_\alpha + (1 + r_R^{0,1})S_\alpha^{0,2}\} \\ &= \max\{0; d_1 - (1 + r_R^{0,1})C_\alpha - (1 + r_R^{0,1})(U^{0,1} + S_\alpha^{0,2})\} \end{aligned}$$

Thus

$$\begin{aligned} \frac{E[\pi_\alpha|d_1]}{1 - p_\alpha} &= (1 + r_R)S_\alpha^{0,2} + (1 - p_\beta)(1 + r)[d_0 - U^{0,1} - S_\alpha^{0,2}] \qquad (7) \\ &\quad -(1 - p_\beta)\rho q + p_\beta q + (1 - \gamma)\frac{1}{2}(1 + r_R^{0,1})[d_1 + (1 + r_R^{0,1})U^{0,1}] \\ &\quad -(1 - \gamma)\frac{1}{2}(1 + r_R^{0,1}) \cdot \min\{d_1 - (1 + r_R^{0,1})U^{0,1}; (1 + r_R^{0,1})(C_\alpha + S_\alpha^{0,2})\} \\ &\quad -(1 - \gamma)\frac{1}{2}(1 + r_R^{0,1})\frac{1}{1 - p_\alpha} \cdot \max\{0; d_1 - (1 + r_R^{0,1})(C_\alpha + U^{0,1} + S_\alpha^{0,2})\} \\ &\quad +\gamma(1 + r_R)U^{0,1} \end{aligned}$$

We now assume $C_\beta = 0$ so that $S_\alpha^{0,2} = 0$. For any $d_1 \leq (1 + r_R^{0,1})(U^{0,1} + C_\alpha)$, we get from equation 7

$$\frac{E[\pi_\alpha|d_1 \leq (1 + r_R^{0,1})(U^{0,1} + C_\alpha)]}{1 - p_\alpha} = (1 - p_\beta)(1 + r)(d_0 - U^{0,1}) + (1 + r_R)U^{0,1}$$

and for any $d_1 \geq (1 + r_R^{0,1})(U^{0,1} + C_\alpha)$, we get from equation 7

$$\begin{aligned} &\frac{E[\pi_\alpha|d_1 \leq (1 + r_R^{0,1})(U^{0,1} + C_\alpha)]}{1 - p_\alpha} \\ =\; &(1 - p_\beta)(1 + r)(d_0 - U^{0,1}) + (1 - \gamma)\frac{1}{2}(1 + r_R)\frac{2 - p_\alpha}{1 - p_\alpha}U^{0,1} \\ &+\gamma(1 + r_R)U^{0,1} + (1 - \gamma)\frac{1}{2}(1 + r_R^{0,1})\frac{p_\alpha}{1 - p_\alpha}[(1 + r_R^{0,1})C_\alpha - d_1] \end{aligned}$$

Let $\overline{d_1}$ be the highest value in the support of d_1. Thus, α-banks maximise the unconditional expected profit

$$\frac{E[\pi_\alpha]}{1-p_\alpha} = \int_{(1+r_R^{0,1})d_0}^{(1+r_R^{0,1})(U^{0,1}+C_\alpha)} \frac{E[\pi_\alpha | d_1 \leq (1+r_R^{0,1})(U^{0,1}+C_\alpha)]}{1-p_\alpha} + \int_{(1+r_R^{0,1})(U^{0,1}+C_\alpha)}^{\overline{d_1}} \frac{E[\pi_\alpha | d_1 \leq (1+r_R^{0,1})(U^{0,1}+C_\alpha)]}{1-p_\alpha} \tag{8}$$

with respect to $U^{0,1}$ and q subject to $d_0 \geq U^{0,1} \geq 0$. Using Leibniz's rule, we get after some rearrangements

$$\frac{\partial \frac{E[\pi_\alpha]}{1-p_\alpha}}{\partial U^{0,1}} = -(1-p_\beta)(1+r) + (1+r_R)[1+(1-F[(1+r_R^{0,1})(U^{0,1}+C_\alpha)])\frac{1-\gamma}{2}\frac{p_\alpha}{1-p_\alpha}] \tag{9}$$

For a β-bank, the (unconditional) expected profit is

$$\frac{E[\pi_\beta]}{1-p_\beta} = -(1+r_R)s_\beta^{0,2} - (1+r)u^{0,2} - (1+r_R^{0,1})s_\beta^{1,2} - \frac{1+r_R^{0,1}}{1-p_\beta}u_\beta^{1,2}$$

The related constraints are

$$\begin{aligned} s_\beta^{0,2} + u^{0,1} + u^{0,2} &= d_0 \\ d_0 - s_\beta^{0,2} &\geq u^{0,1} \geq 0 \\ s_\beta^{0,2} &\leq C_\beta \\ s_\beta^{1,2} + u_\beta^{1,2} &= (1+r_R^{0,1})u^{0,1} \\ s_\beta^{1,2} &\leq (1+r_R^{0,1})C_\beta - (1+r_R^{0,1})s_\beta^{0,2} \end{aligned}$$

A β-bank will always borrow in period 1 as much as possible against collateral, i.e.

$$\begin{aligned} s_\beta^{1,2} &= \min\{(1+r_R^{0,1})u^{0,1}, (1+r_R^{0,1})(C_\beta - s_\beta^{0,2})\} \\ u_\beta^{1,2} &= (1+r_R^{0,1})u^{0,1} - \min\{(1+r_R^{0,1})u^{0,1}, (1+r_R^{0,1})(C_\beta - s_\beta^{0,2})\} \\ &= \max\{0; (1+r_R^{0,1})(u^{0,1} - C_\beta + s_\beta^{0,2}\} \end{aligned}$$

Thus, a β-bank maximizes

$$\begin{aligned} \frac{E[\pi_\beta]}{1-p_\beta} = & -(1+r_R)s_\beta^{0,2} - (1+r)(d_0 - u^{0,1} - s_\beta^{0,2}) \\ & -(1+r_R^{0,1})\min\{(1+r_R^{0,1})u^{0,1}, (1+r_R^{0,1})(C_\beta - s_\beta^{0,2})\} \\ & -\frac{1+r_R^{0,1}}{1-p_\beta}\max\{0; (1+r_R^{0,1})(u^{0,1} - C_\beta + s_\beta^{0,2}\} \end{aligned} \tag{10}$$

with respect to $u^{0,1}$ and $s_\beta^{0,2}$ and subject to $d_0 - s_\beta^{0,2} \geq u^{0,1} \geq 0$, $s_\beta^{0,2} \leq C_\beta$.

For $C_\beta = 0$, i.e. $s_\beta^{0,2} = 0$, we get

$$\frac{E[\pi_\beta]}{1-p_\beta} = -(1+r)(d_0 - u^{0,1}) - \frac{1+r_R}{1-p_\beta}u^{0,1}$$

so that

$$\frac{\partial \frac{E[\pi_\beta]}{1-p_\beta}}{\partial u^{0,1}} = (1+r) - \frac{(1+r_R)}{1-p_\beta} \tag{11}$$

■

Proof of proposition 1:

This follows immediately from equations 7 and 8

■

Proof of proposition 2:

With equation 9 we get from $\gamma = 1$ or $p_\alpha = 0$

$$\frac{\partial \frac{E[\pi_\alpha]}{1-p_\alpha}}{\partial U^{0,1}} = -(1-p_\beta)(1+r) + (1+r_R)]$$

If this equals zero, i.e. if

$$(1+r) = \frac{(1+r_R)}{(1-p_\beta)}$$

then α-banks are indifferent between lending for two periods or for one period. If $(1+r)$ is greater, then $U^{0,1} = 0$ and $U^{0,2} = d_0$. If $(1+r)$ is smaller, then $U^{0,1} = d_0$ and $U^{0,2} = 0$. If equation 11 is zero, i.e. if

$$(1+r) = \frac{(1+r_R)}{1-p_\beta}$$

then β-banks are indifferent between borrowing for two periods or for one period. If $(1+r)$ is smaller, then $u^{0,1} = 0$ and $u^{0,2} = d_0$. If $(1+r)$ is greater, then $u^{0,1} = d_0$ and $u^{0,2} = 0$. It follwos that an equilibrium requires $1+r = \frac{(1+r_R)}{(1-p_\beta)}$. As both sides of the market are now indifferent, any $X_u^{0,2} \in [0; d_0]$ is an equilibrium.

■

Proof of proposition 3:

The proof is very similar to that of proposition 2 and therefore omitted here.

■

Proof of proposition 4:

An α-bank will offer two-period funds (i.e. $U^{0,1} < d_0$) only if equation 9 is zero or negative for $U^{0,1} = d_0$. It is zero for $U^{0,1} = d_0$ iff

$$(1+r) = \frac{(1+r_R)}{(1-p_\beta)}[1 + (1-F[\Psi])\frac{1-\gamma}{2}\frac{p_\alpha}{1-p_\alpha}]$$

If $(1+r)$ is greater, then $U^{0,1} < d_0$ and $U^{0,2} > 0$. If $(1+r)$ is smaller, then $U^{0,1} = d_0$ and $U^{0,2} = 0$. If equation 11 is zero, i.e. if

$$(1+r) = \frac{(1+r_R)}{1-p_\beta}$$

then β-banks are indifferent between borrowing for two periods or for one period. If $(1+r)$ is smaller, then $u^{0,1} = 0$ and $u^{0,2} = d_0$. If $(1+r)$ is greater, then $u^{0,1} = d_0$ and $u^{0,2} = 0$.

It follwos that an equilibrium with $X_u^{0,2} > 0$ is impossible and that $X_u^{0,2} = 0$ requires that

$$1+r \in [\frac{1+r_R}{1-p_\beta}; \frac{1+r_R}{1-p_\beta} \cdot [1 + (1 - F[\Psi])\frac{1-\gamma}{2}\frac{p_\alpha}{1-p_\alpha}]]$$

■

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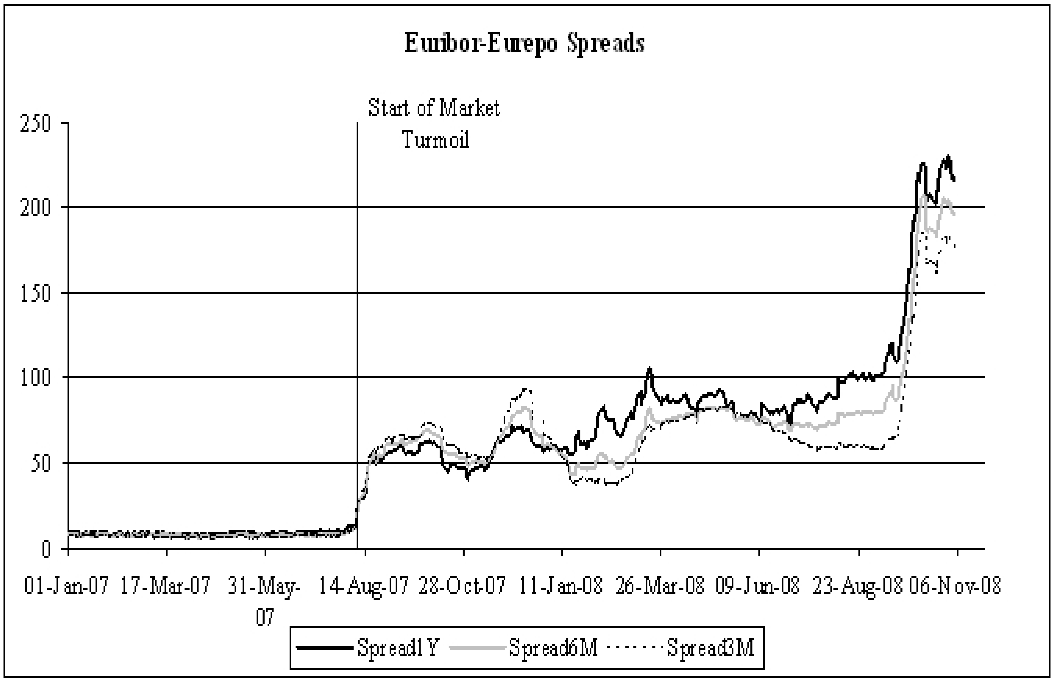


Figure 2: Spread of Euribor rates over Eurepo rates for 3-month, 6-month and 12-month maturity. Sources: Bloomberg and www.eurepo.org.

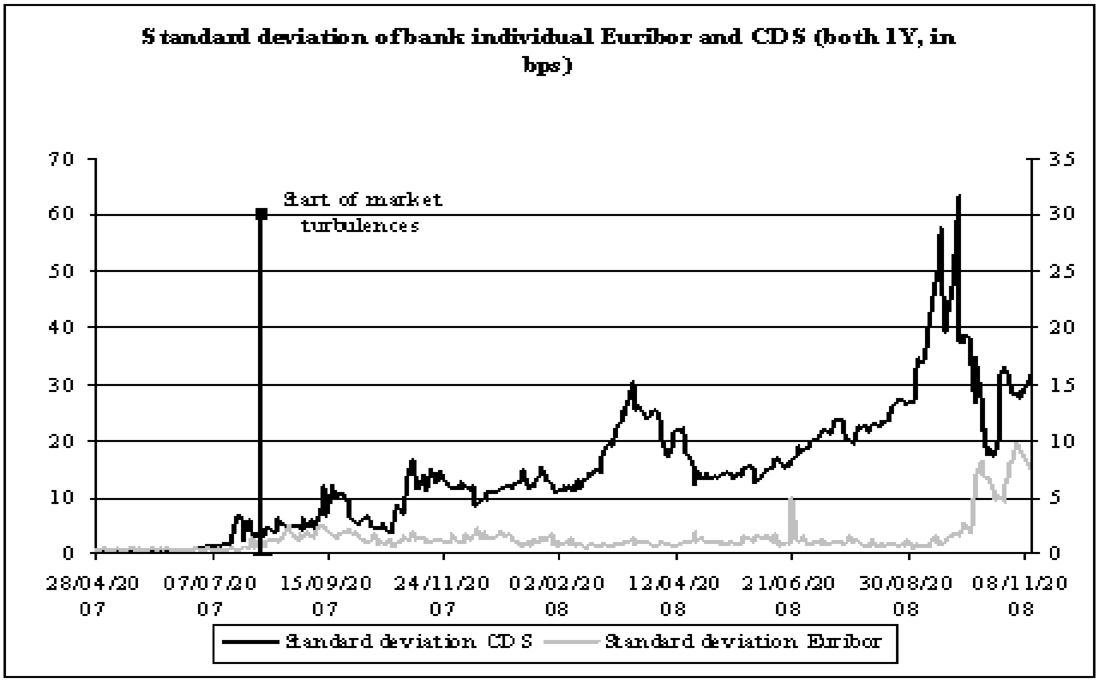


Figure 3: Standard deviation of individual contributions to the one-year Euribor (grey). Standard deviation of one-year CDS spreads for 20 Euribor panel banks.

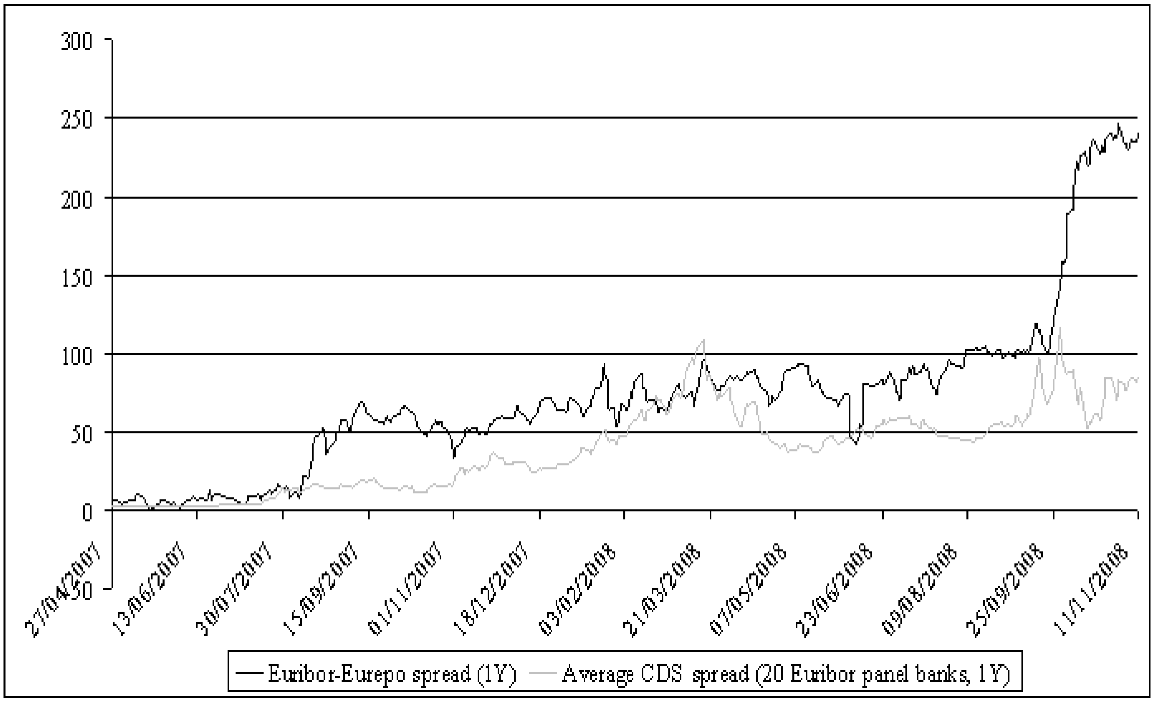
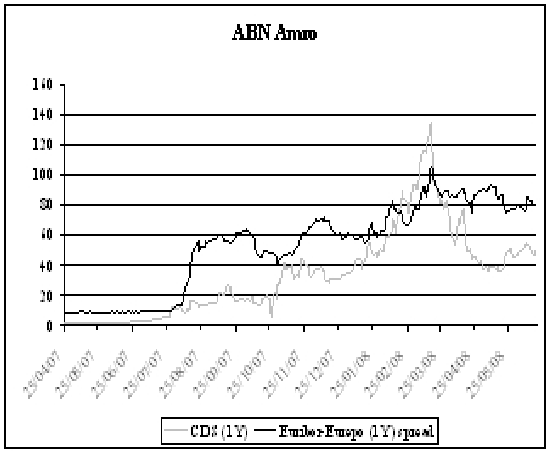
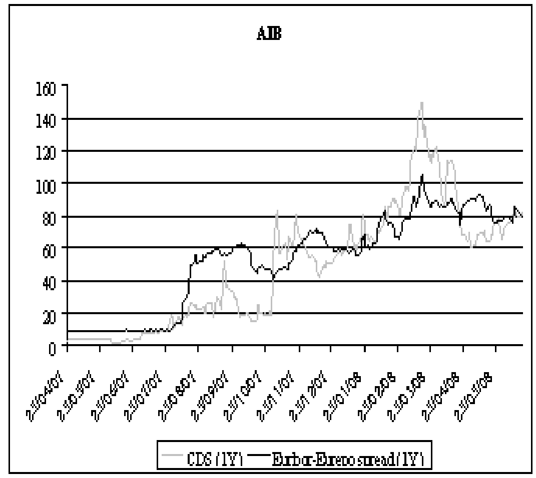
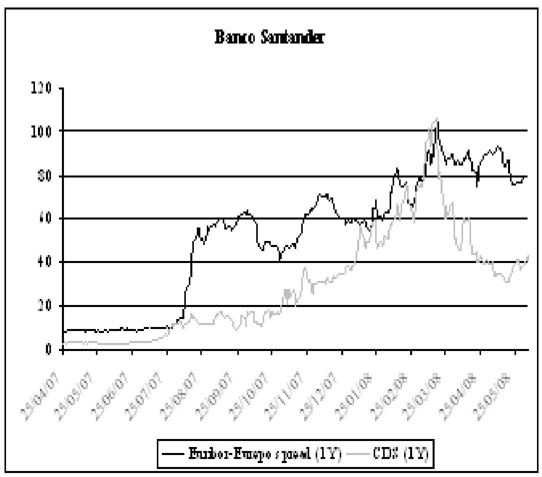
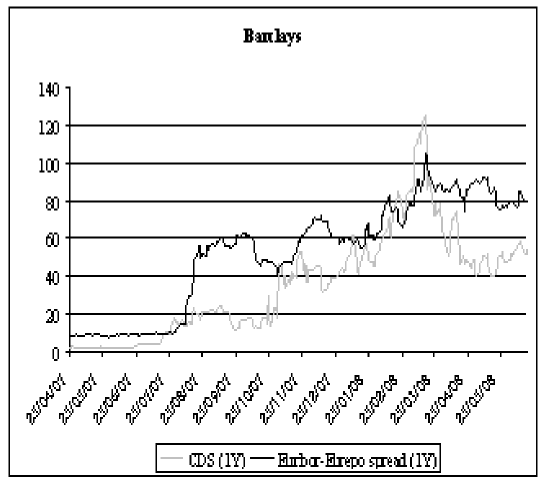


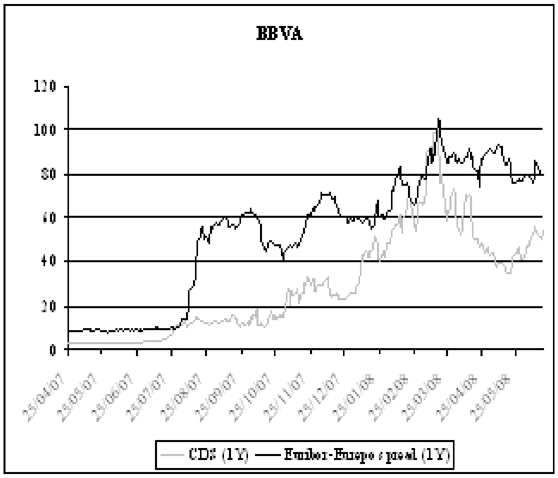
Figure 4: One year Euribor spread and average of 20 one-year CDS spreads from CDS contracts on Euribor panel banks. Sources: Bloomberg and www.eurepo.org.

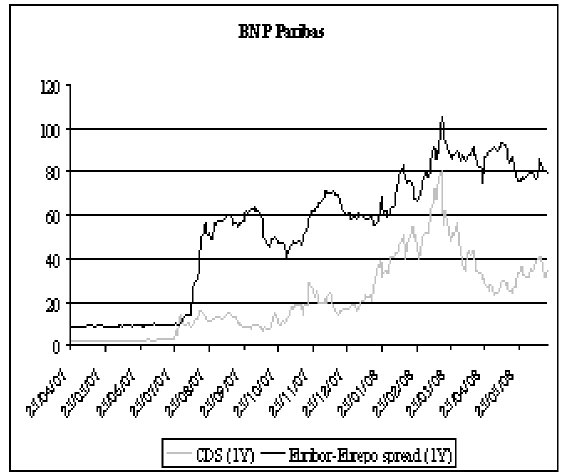

ABN Amro
0
20
40
60
80
100
120
140
CDS (1Y)

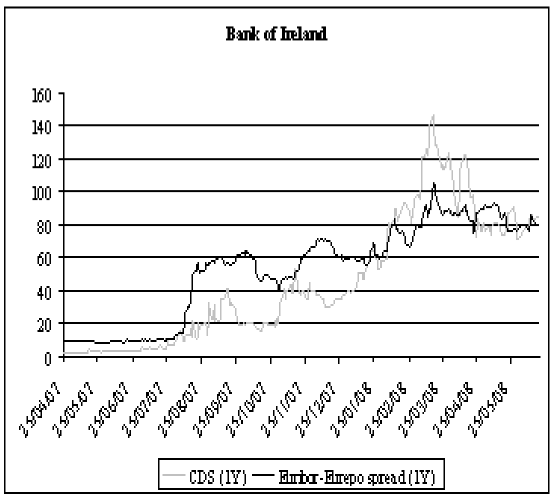

AIB
0
20
40
60
80
100
120
140
160
CDS (1Y)

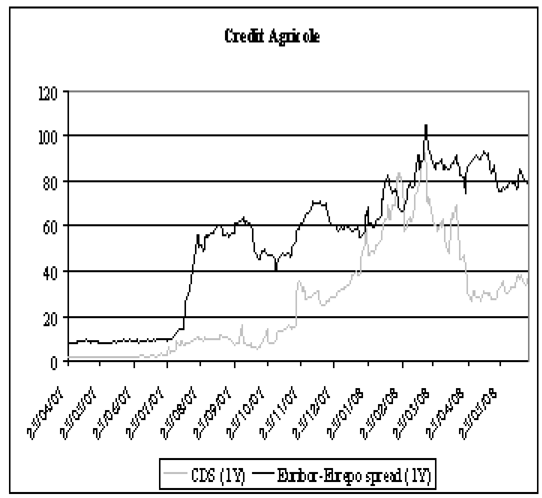

Banco Santander
0
20
40
60
80
100
120
CDS (1Y)

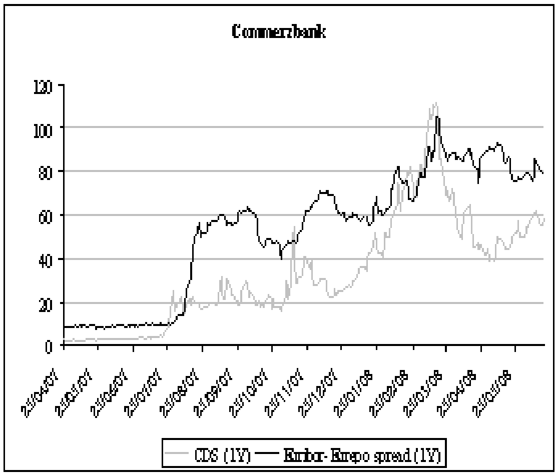

Barclays
0
20
40
60
80
100
120
140
CDS (1Y)

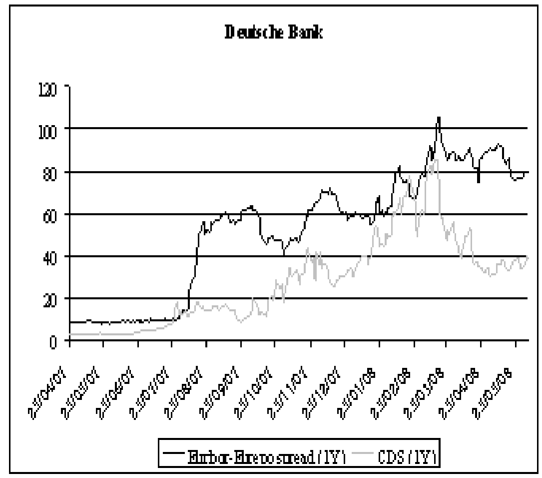

BBVA
CDS (1Y)
Euribor-Eurepo spread (1Y)

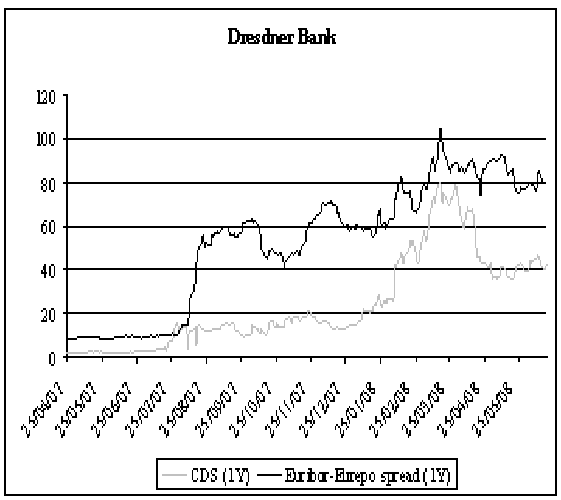

BNP Paribas
CDS (1Y)
Euribor-Eurepo spread (1Y)

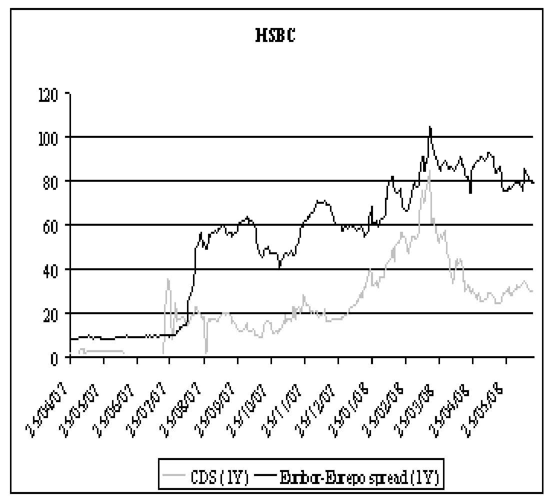

Bank of Ireland
CDS (1Y)
Euribor-Eurepo spread (1Y)

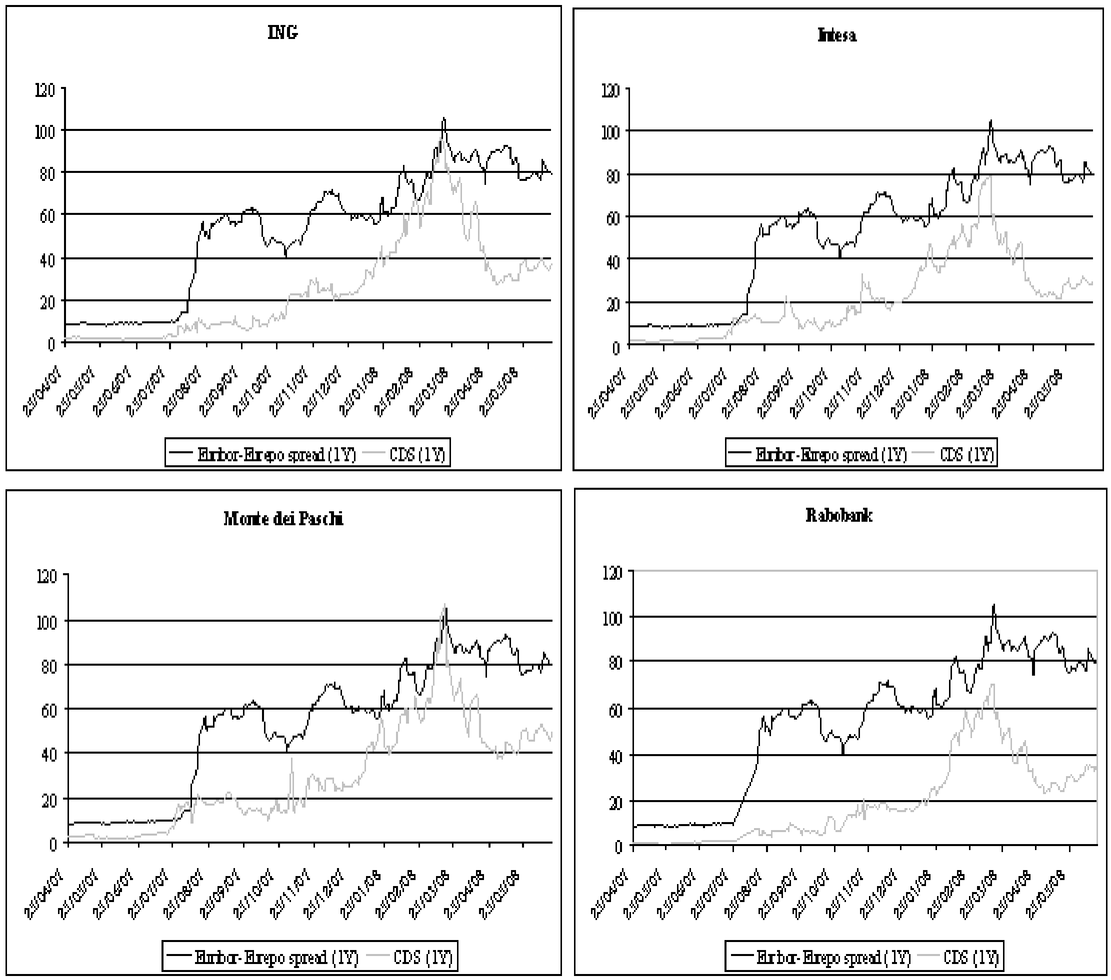

Credit Agricole
CDS (1Y)
Euribor-Eurepo spread (1Y)

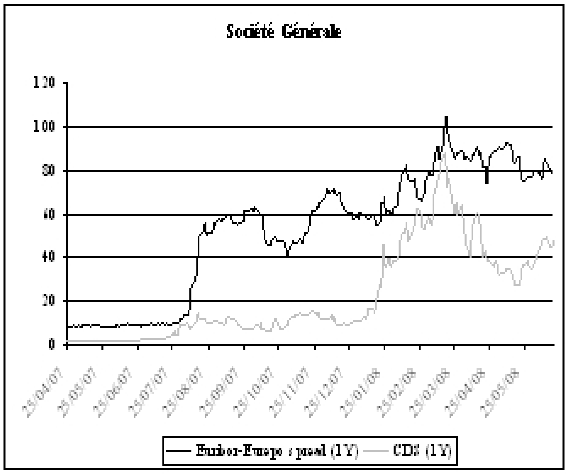

Commerzbank
CDS (1Y) Euribor-Eurepo spread (1Y)

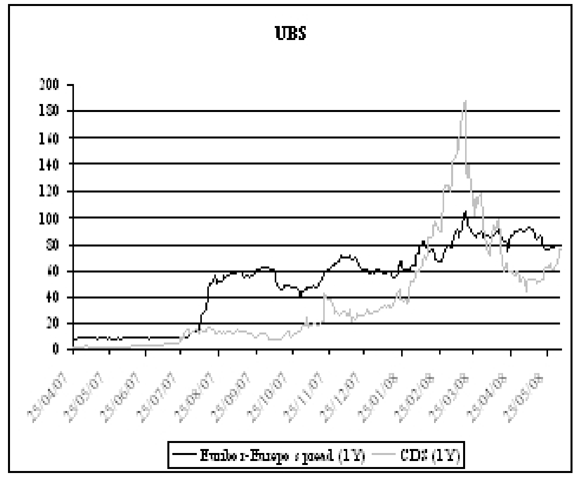

Deutsche Bank
Euribor-Eurepo spread (1Y) CDS (1Y)

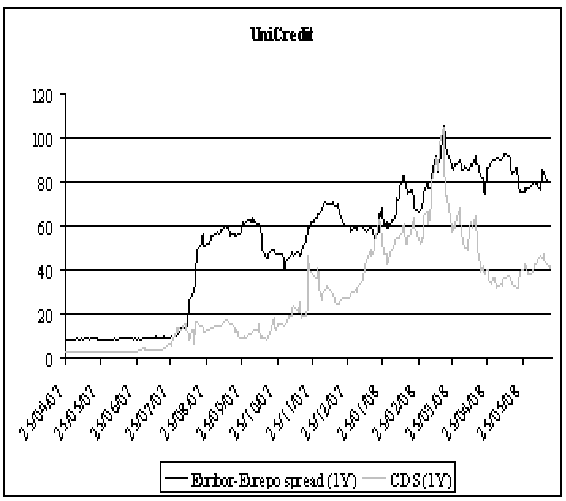

Dresdner Bank
CDS (1Y) Euribor-Eurepo spread (1Y)

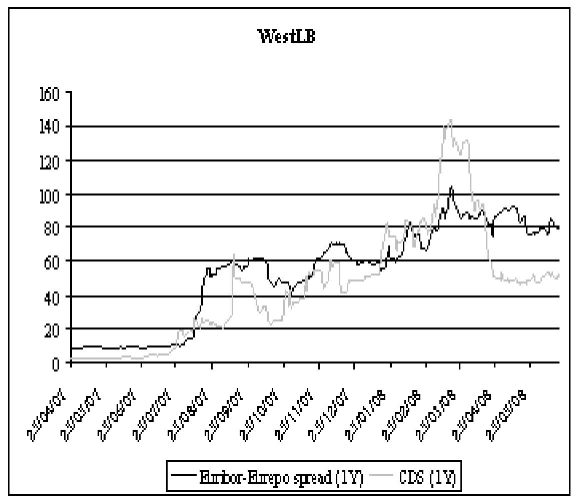

HSBC
CDS (1Y) Euribor-Eurepo spread (1Y)


ING
Intesa
Monte dei Paschi
Rabobank
0
20
40
60
80
100
120
Eribor-Erepo spread (1Y)
CDS (1Y)


Société Générale
120
100
80
60
40
20
0
25/04/07
25/05/07
25/06/07
25/07/07
25/08/07
25/09/07
25/10/07
25/11/07
25/12/07
25/01/08
25/02/08
25/03/08
25/04/08
25/05/08
Euribor-Eurepo spread (1Y)
CDS (1Y)


UBS
200
180
160
140
120
100
80
60
40
20
0
25/04/07
25/05/07
25/06/07
25/07/07
25/08/07
25/09/07
25/10/07
25/11/07
25/12/07
25/01/08
25/02/08
25/03/08
25/04/08
25/05/08
Euribor-Eurepo spread (1Y)
CDS (1Y)


UniCredit
120
100
80
60
40
20
0
25/04/07
25/05/07
25/06/07
25/07/07
25/08/07
25/09/07
25/10/07
25/11/07
25/12/07
25/01/08
25/02/08
25/03/08
25/04/08
25/05/08
Euribor-Eurepo spread (1Y)
CDS (1Y)


WestLB
160
140
120
100
80
60
40
20
0
25/04/07
25/05/07
25/06/07
25/07/07
25/08/07
25/09/07
25/10/07
25/11/07
25/12/07
25/01/08
25/02/08
25/03/08
25/04/08
25/05/08
Euribor-Eurepo spread (1Y)
CDS (1Y)

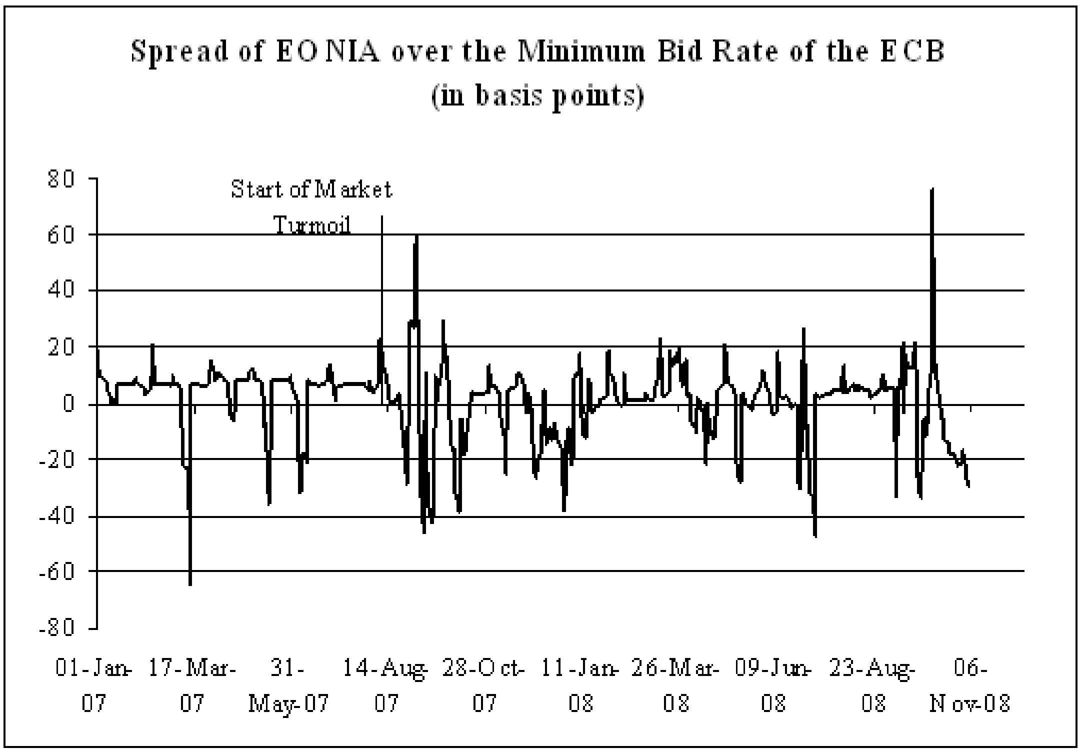


Figure 5: Eonia spread

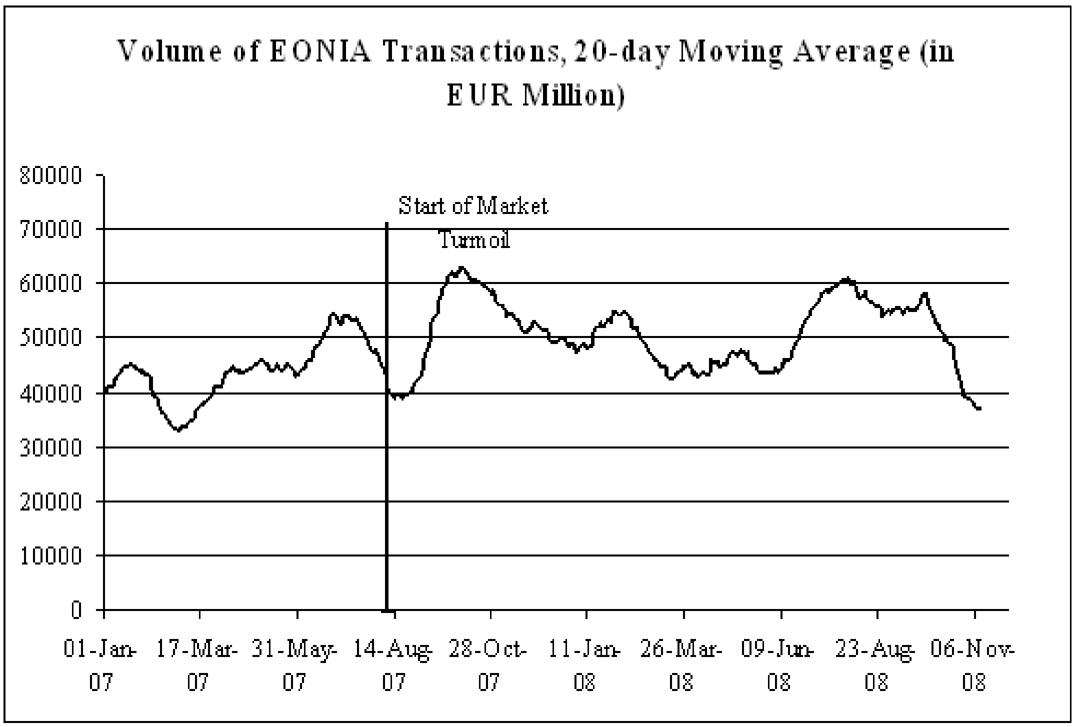


Figure 6: Eonia volume