Envy, Tax Equity and Taxation*

Tobias König and Andreas Wagener

Faculty of Economics and Management
Leibniz-University of Hannover
Koenigsworther Platz 1
30167 Hannover, Germany

Phone/Fax: +49 - 511 - 762 8214/4574
e-mail: koenig@sopo.uni-hannover.de, wagener@sopo.uni-hannover.de

Abstract: We augment the standard taxation model for small open economies by status and fairness concerns. We show that zero capital taxation is a knife's edge result: As soon as we endow individuals with the slightest concern for other people's income or tax equity, zero capital taxation ceases to be optimal. This holds irrespective of whether we allow envy or tax equity to impact on work incentives or on absolute levels of well-being. Moreover, the optimal tax policy is non-monotonic in the degrees of envy and tax equity. Above a certain degree of envy, higher degrees make the pivotal voter - though he is worker - want to opt for a higher level of labour taxation. Regarding the level of government spending, we show that envy and tax fairness effects do not necessarily erode the size of the public sector.

JEL-classification: H20, E62, P16, Z10
Keywords: Economics and Culture, Envy, Fairness, Taxation, Policy Mix
This version: February 26, 2009

*Preliminary Draft: Please do not quote. Comments are welcome.

1 Introduction

A fundamental theorem of taxation theory states that small open economies should not rely on capital taxation. This result, originally derived in Gordon (1986) and restated in Razin and Sadka (1991), emerges from the assumption of an infinitely elastic capital supply which small countries face. The burden of a tax on capital will then be shifted onto workers or other immobile domestic factors. But if the immobile factors bear the tax burden anyway, it is less costly to tax them directly and, by this, to avoid the excess burden associated with capital flight.

Zero capital taxation, thus, is optimal in these models – it maximizes the representative household's utility and, thus, is the policy outcome that people actually want and would vote for. However, in reality the prospect of zero taxes on capital hardly looks popular. It flies in the face of all sorts of concerns with fairness and equal treatment – which remain unmodelled in the standard framework of optimal international taxation. Given that over the past decades a large body of evidence has been gathered suggesting that people's preferences are not solely driven by material self-interest but also by other-regarding, status and fairness concerns, a theory of taxation that disregards these aspects might appear incomplete.

Several lines of reasoning call for abandoning purely self-concerned preferences in tax models. First, happiness research shows that, in addition to one's absolute level of income, well-being also depends on one's income position relative to others (e.g. Clark and Oswald, 1996; Luttmer, 2005; Layard 2006). Taxation may change these relative positions. Consequently, people who receive income only from labour may dislike tax exemptions for earners of capital income since these would worsen their income position relative to that of capital owners. Governments with (re-)election concerns anticipate this "envy effect" and therefore would not shift the whole tax burden on labour.

Secondly, people may dislike a tax system that exclusively relies taxes on labour incomes for lack of fairness.[1] Many countries pay at least lip service to the principle of horizontal tax equity – stating that equal incomes should be taxed equally rates (Kaplow, 1995). This principle also forms part of the rationale underlying the comprehensive income tax, a normative ideal to which many countries (used to) adhere. Zero taxes on capital in the presence of a positive tax rate on labour clearly violate this principle. Moreover, burdening only one subgroup of the population (i.e., workers) can also be in conflict with the benefit principle of taxation, stating that the taxes an agent pays should somehow reflect the benefits that (s)he receives from the goods and services supplied by the state.[2] Since everybody benefits from the provision of public goods, the benefit principle calls for a positive share in taxes for everyone.

To summarize, people seem to care about taxation not only to the extent that it affects their

[1]Such fairness concerns have been amply found in the experimental literature; for a survey, see, e.g., Fehr and Schmidt (2006).

[2]For a discussion of benefit and sacrifice principles of taxation see, e.g., Neill (2000).

own well-being. Status and fairness concerns might be relevant as well.

In this paper we analyze the implications of status and fairness concerns for tax structures in open economies. The economic part of our model is fairly standard: A single output is produced with labour and capital. Capital is internationally perfectly mobile. Workers are immobile but their supply of labour is endogenous. There is a positive relation between capital and wages. To finance expenditures for a government-provided good, the incomes from capital and labour can be taxed at source. However, as capital taxation generates a higher excess burden, taxation should optimally (in the standard sense) rely only on labour. This holds irrespectively of whether government expenditures are exogenous or endogenous.

We augment this model by fairness and equity concerns, which can be modelled either as preferences over the after-tax incomes of others or be embedded as explicit tax norms. We pursue both ways. First, we allow utilities to be negatively affected by the (higher) incomes of others. In a median-voter framework this is relevant when workers (the median voter) regard with envy the incomes of the richer capitalists. Tax designers then face a trade-off between the negative (economic) and positive (political) effects of capital taxation. On the one hand, higher taxes on capital drive capital out of the country and, by this, also depress gross wages. On the other hand, they reduce the income distance to capital owners and thereby dampen envy. Second, rather than caring about others' incomes, fairness and equity concerns may take the form of an egalitarian tax norm deviations from which cause utility losses. The trade-off is similar as before: The (economic) excess burden of capital taxes has to be weighed against the better compliance to the egalitarian tax norm.

Fairness and equity concerns may not just be a feel-good item in individual preferences. Anecdotal evidence suggests that tax-related envy and unfairness felt in the context of taxation may indeed affect work incentives: Envious and dissatisfied individuals spend less effort on work. We therefore allow both for a level and an incentive effect of unfairness: the former implying that unfairness reduces the level of well-being in general while the latter (only) raises the disutility from labour.

We analyze whether and how the inclusion of envy and tax equity effects (on utility and work incentives) impacts on optimal taxation: Does the result of zero capital taxation still hold? Do we need high strong degrees on envy or tax equity concerns to destroy it? How does the optimal tax mix change with varying degrees of envy or tax equity? What are the effects on the level of the provision of the public goods?

We show that zero capital taxation is indeed a knife's edge result: As soon as we endow individuals with the slightest concern for other people's income or tax fairness, zero capital taxation ceases to be optimal; this holds irrespective of whether we allow envy or tax equity to impact on work incentives or on well-being. Moreover, we show that the optimal tax policy is non-monotonic in the degrees of envy and tax equity concerns. Above a certain degree of envy, higher degrees make the pivotal voter - though he is worker - want to opt for a higher level of labour taxation. The reason is that the economy may end up on the decreasing part of the partial Laffer curve for the capital tax – a situation that would never occur within the standard framework of optimal taxation. Interestingly, if concerns for tax egalitarianism become overwhelming (i.e., the sole thing that matters), we do not necessarily end up with equal factor taxation. Similarly, even when workers are only motivated by envy they do not necessarily wish the whole tax burden to be shifted on capitalists. For the level of government spending, we show that envy and tax fairness effects may erode the size of the public sector. The intuition is that envy forces the economy to overly heavily rely on capital taxation, an economically relative inefficient instrument to generate tax revenues that increases the marginal costs of public funds. This calls for smaller government budgets.

Our paper contributes twofold to the theory of optimal taxation. First, it complements a recent literature that incorporates social preferences into optimal tax frameworks. It is well known that, in the presence of envy, taxation may be a Pigou-type instrument to close the gap between the private and social benefits from consumption (see, e.g., Alvarez-Cuarado 2007; Alonsa-Carrera and Caball 2006; Arrow and Dasgupta 2005; Liu and Turnovsky, 2005). In our framework, unequal taxation is the *source* of envy – not a remedy against it. Second, we add to the theory of an optimal mix of capital and labour taxation in open economies. This is still an ongoing issue of research, especially since empirical studies failed to establish a robust negative link between capital market integration and a relative lower tax burden on capital (see Haufler et al., 2008, for a survey). Our paper suggests an explanation why so far a race to the bottom for capital taxes has not happened to the extent predicted by the standard tax competition literature. When voters have social preferences, concerns for envy and tax equity may outweigh the economic benefits from low capital taxes.

This paper proceeds as follows: Section 2 sets out the basic model where we focus on tax equity concerns. In section 3, we analyze optimal policies and their comparative statics for the case that government spending is exogenous. In section 4, we extend the model to include endogenous government spending. Section 5 allows for concerns about relative income positions. Section 6 briefly concludes.

2 The model

Equity and fairness concerns may be held with respect to tax rates or with respect to income. We first analyze the effects of tax equity concerns; the case that individual dislike differences in after-tax incomes will be addressed in Section 5.

We consider a small open one-good-economy which is inhabited by a large number of identical individuals. For simplicity, we normalize the number of individuals to unity. Production in

the economy takes place in one single-output firm that is owned by foreigners. Production uses labour and capital as its inputs. Capital is an internationally mobile factor of production that can be purchased on world capital markets at an exogenous rental rate of $r > 0$ per unit.
Each individual in the domestic population has convex and increasing preferences over consumption c, leisure – which will be negatively represented by working hours ℓ – and a publicly provided good g. We assume that these preferences can be represented by an additively separable utility function

$$u(c, \ell) = c - E(\ell, \psi) + h(g) - \Omega, \tag{1}$$

with $h'(g) > 0 > h''(g)$. The function $e(\cdot)$ represents disutility from work.
The special feature of our model is that disutility from work depends not only of labour ℓ (with $E_\ell > 0$ and $E_{\ell\ell} > 0$) but also of a parameter, ψ, that captures the individual's feeling of being treated "unfairly" by tax policy. Thus, preferences are not exogenously given but depend on the policy choices made in the society. In particular, we shall assume that the both the absolute and the marginal disutility from labour increases whenever the individuals feel being treated "unfairly" by tax policy ($E_\psi > 0$, $E_{\ell\psi} > 0$). The unfairness of taxation will be measured by the difference between the tax rates on labour and on capital, denoted by t_ℓ and t_k. Specifically, we assume that

$$\psi = \psi(t_\ell, t_k), \tag{2}$$

with $\psi_\ell := \partial\psi/\partial t_\ell \geq 0$ and $\psi_k := \partial\psi/\partial t_k \leq 0$. Higher taxes on labour weakly lower work morale while higher taxes on capital boost it. The modelling of ψ in particular covers the case where ψ just (negatively) depends on the difference in statutory tax rates, $(t_\ell - t_k)$.
Notice that the parameter ψ is not restricted to reflect (un-)fairness concerns. It can also be interpreted in an envy fashion. Individuals are jealous of tax cuts in other people's income. Here, home workers envy tax cuts for capital owners who in our model reside abroad. Think of this as that workers cannot directly observe capitalist's income or consumption. Instead, they take capital tax cuts as a proxy for an increase in capitalist's income or consumption - and this is what they are actually jealous of.
Besides the incentive effect of tax equity or "tax envy", there is a mere level effect, represented by Ω. Assuming that Ω is a function of the tax rates with $\Omega_\ell := \partial\Omega/\partial t_\ell \geq 0$ and $\Omega_k := \partial\Omega/\partial t_k \leq 0$, we are able to seperate between two channels of tax equity or "tax envy": a work a morale effect and an effect leaving incentives unaltered.
The legal incidence of labour taxation is assumed to lie with workers. Thus, the disposable income of a worker just equals the hourly net wage $(w - t_\ell)$ times hours worked: $c = (w - t_\ell) \cdot \ell$. The (gross) wage rate w will be endogenously determined (see below).

Individuals take the wage and tax rate as parametrically given. Substituting for c in (1) and maximizing over ℓ yields

$$E_\ell(\ell, \psi(t_\ell, t_k)) = w - t_\ell \tag{3}$$

which implicitly defines the labour supply function as $\ell^S(w, t_\ell, t_k)$ with

$$\frac{\partial \ell^S}{\partial w} = \frac{1}{E_{\ell\ell}} > 0, \tag{4}$$

$$\frac{\partial \ell^S}{\partial t_\ell} = -\frac{1}{E_{\ell\ell}} \cdot (1 + E_{\ell\psi} \cdot \psi_\ell) < 0, \tag{5}$$

$$\frac{\partial \ell^S}{\partial t_k} = -\frac{E_{\ell\psi}}{E_{\ell\ell}} \cdot \psi_k > 0. \tag{6}$$

Labour supply thus increases in the gross wage, decreases in the wage tax and, in the presence of tax equity effects, decreases in the capital tax. Also observe that the tax equity effect reinforces the negative wage tax effect.

Firms maximize their profits. They pay taxes on each unit of capital they hire. Denoting by K and L, respectively, the amounts of capital and labour employed in the firm, output of the firm equals $F(K, L)$, where F is a strictly increasing, constant-returns-to-scale and strictly quasi-concave production function.[3] Since the cost of hiring an additional hour of labour are w while an additional unit of capital costs $r + t_k$, the firm's net profits amount to

$$\pi = F(K, L) - w \cdot L - (r + t_k) \cdot K = L \cdot (f(k) - w - (r + t_k) \cdot k). \tag{7}$$

Here, $k := K/L$ denotes capital per labour unit and $f(k)$ is the standard per-unit-of-labour production function; f is strictly increasing and strictly concave. The firm takes input prices and taxes as given. Profit maximization implies

$$f'(k) = r + t_k, \tag{8}$$

which implicitly defines the capital intensity $k = k(r + t_k)$ as a function of the cost of capital, with

$$k'(r + t_k) = \frac{1}{f''(k)} < 0. \tag{9}$$

Since we assume constant returns to scale, the gross wage rate is determined via the factor price frontier and is given by

$$w(r + t_k) = f(k) - (r + t) \cdot k \tag{10}$$

with

$$w'(r + t_k) = -k. \tag{11}$$

[3] CRS and quasi-concavity together imply that F is concave, too.

In equilibrium, labour supply must equal labour demand. The equilibrium employment L^* is, thus, given by

$$L^*(t_\ell, t_k) = \ell^S(w(r+t_k), t_\ell, t_k) \tag{12}$$

which decreases in the tax rate on labour but has an ambiguous response to higher capital taxation:

$$\frac{\partial L^*}{\partial t_\ell} = \frac{\partial \ell^S}{\partial t_\ell} < 0, \tag{13}$$

$$\frac{\partial L^*}{\partial t_k} = w'(r+t_k) \cdot \frac{\partial \ell^S}{\partial w} + \frac{\partial \ell^S}{\partial t_k} = -k \cdot \frac{\partial \ell^S}{\partial w} + \frac{\partial \ell^S}{\partial t_k} \lesseqgtr 0. \tag{14}$$

Notice that when fairness concerns are sufficiently high, the tax equity effect may offset the usual disincentive from higher capital taxation on labour supply. In this case, equilibrium employment would increase in the tax rate on labour.

The government provides a (public) good g (measured in terms of output) which has to be financed out of the revenues from labour and capital taxes. Hence, its budget constraint reads:

$$g = t_\ell \cdot L^* + t_k \cdot K = L^*(t_\ell, t_k) \cdot (t_\ell + t_k \cdot k(r+t_k)) =: G(t_\ell, t_k). \tag{15}$$

In what follows, we shall refer to $G(t_\ell, t_k)$ as the Laffer curve of the economy. For later use, we note that from (15) the partial derivatives of the Laffer curve with respect to the two tax rates are given by

$$\frac{\partial G}{\partial t_k} = \frac{\partial L^*}{\partial t_k}(t_\ell + t_k k) + L^*(k + t_k k') =: G_k, \tag{16}$$

$$\frac{\partial G}{\partial t_\ell} = \frac{\partial L^*}{\partial t_\ell}(t_\ell + t_k k) + L^* =: G_\ell. \tag{17}$$

3 Optimal tax policy with exogenous government spending

In this section, we assume that a given and fixed level of government revenues $\bar{g}$ has to be met; the case of endogenous government expenditures will be dealt with in Section 4.

3.1 Some taxation of capital is optimal

The government chooses t_ℓ and t_k such as to maximize individual welfare (recall that firm owners are absentee capitalists). Plugging the equilibrium levels of employment L^* and (15) into (1) and taking into account that via (10) $w = w(r+t_k)$ gives indirect utility (= social welfare) in equilibrium as follows:

$$V(t_\ell, t_k) := (w(r+t_k) - t_\ell) \cdot L^*(t_\ell, t_k) - E(L^*(t_\ell, t_k), \psi(t_\ell, t_k)) - \Omega(t_\ell, t_k). \tag{18}$$

As government expenditures g are exogenously fixed, the utility $h(g)$ derived from them does not matter here; it is omitted from (18). The government chooses tax rates t_ℓ and t_k such as to maximize V subject to the revenue constraint. The Lagrangian W for this problem reads:

$$\max_{t_\ell, t_k} \quad W(t_\ell, t_k) = V(t_\ell, t_k) + \lambda[G(t_\ell, t_k) - \bar{g}], \tag{19}$$

where λ denotes the Lagrange multiplier and $\bar{g}$ the exogenous level of the public good to be financed. Differentiating (19), with respect to tax rates (t_k, t_ℓ) and using the Envelope-Theorem gives:

$$\begin{aligned} \frac{\partial W}{\partial t_\ell} &= -L^* + \lambda \cdot G_\ell - E_\psi \cdot \psi_\ell - \Omega_\ell \\ &= L^* \cdot [\lambda - 1] + \lambda \cdot (t_\ell + t_k k) \cdot \frac{\partial L^*}{\partial t_\ell} - E_\psi \cdot \psi_\ell - \Omega_\ell \qquad (20) \\ \frac{\partial W}{\partial t_k} &= w'(r + t_k) L^* + \lambda \cdot G_k - E_\psi \cdot \psi_k - \Omega_k \\ &= kL^* \cdot [\lambda - 1] + \lambda \cdot \left((t_\ell + t_k k) \cdot \frac{\partial L^*}{\partial t_k} + t_k k' L^* \right) - E_\psi \cdot \psi_k - \Omega_k. \qquad (21) \end{aligned}$$

No fairness concerns. As a benchmark, we consider the standard case where preferences do not exhibit fairness considerations (i.e., $\psi_k = \psi_\ell = \Omega_k = \Omega_\ell = 0$). We denote the welfare-maximal solution in that case by $(t_\ell^{exog0}, t_k^{exog0})$. Observe that in this case:

$$\frac{\partial L^*}{\partial t_\ell} = -\frac{\partial \ell^S}{\partial w} \quad \text{and} \quad \frac{\partial L^*}{\partial t_k} = -k \cdot \frac{\partial \ell^S}{\partial w} \tag{22}$$

such that from (20) and (21)

$$\frac{\partial W}{\partial t_\ell} = \frac{1}{k} \cdot \frac{\partial W}{\partial t_k} - \lambda L^* \frac{t_k k'}{k} > \frac{1}{k} \cdot \frac{\partial W}{\partial t_k} \tag{23}$$

for all (t_ℓ, t_k) with $t_k > 0$. Hence, it can never be optimal to tax capital: $t_k^{exog0} = 0$.[4] Thus, without fairness consideration, we obtain the well-known result that a small country should levy no source taxes on capital. The intuition is that a small country faces a fixed rate of return on capital and, thereby, an infinitely elastic capital supply. Then, capital taxes are entirely born by the immobile factor, which makes it less costly to tax this factor directly.[5] Thus, we have $(t_k^{exog0} = 0, t_\ell^{exog0} > 0)$.

The standard result of zero capital taxation does not survive when fairness considerations come into play.

[4] Formally, if $\frac{\partial V}{\partial t_\ell} = 0$, one gets $\frac{\partial V}{\partial t_k} < 0$ for all $t_k > 0$ such that a reduction of t_k is worthwhile.

[5] See Bucovetsky and Wilson, 1991; Fuest and Huber, 2001 for the case of endogenous public spending

Disutility from unequal tax rates. First, consider the case where a feeling of "unfair" factor taxation has no incentives effects (i.e., $\psi_k = \psi_\ell = 0$) but the level effect of unfairness is present, i.e. $\Omega_k \leq 0, \Omega_\ell \geq 0$ with at least one strict inequality. In this case, (22) continues to hold and we get from (20) and (21) that

$$\frac{\partial W}{\partial t_\ell} = \frac{1}{k} \cdot \frac{\partial W}{\partial t_k} - \lambda \frac{t_k k' L^*}{k} - \Omega_\ell + \frac{1}{k}\Omega_k. \tag{24}$$

This equation only differs from (23) by $-\Omega_\ell + \Omega_k/k < 0$ implying that zero taxation of capital is no longer optimal: at $t_k = 0$ and $\frac{\partial W}{\partial t_\ell} = 0$, we get $\frac{\partial W}{\partial t_k} > 0$ instead of $\frac{\partial W}{\partial t_k} = 0$ such that a positive t_k is warranted. Intuitively, with preferences for "fair" taxation, capital taxation not only has extra costs (distortion of the capital intensity), it also has comparative advantages over wage taxation, since it reduces the psychological costs from tax differences. For later use, notice that in a welfare maximum,

$$L^* t_k k'/k = \frac{1}{\lambda}(\frac{1}{k}\Omega_k - \Omega_\ell). \tag{25}$$

Incentive effects. Suppose now that the violation of the tax equity norm does not cause a deterioration in utility *per se*, but distorts the incentives to provide labour. I.e., we shall assume that $\psi_k(t_\ell, t_k) \leq 0 \leq \psi_\ell(t_\ell, t_k)$ (with at least one strict inequality), while we reset $\Omega_k, \Omega_\ell \equiv 0$. In this case, the partial derivatives of equilibrium employment with respect to the tax rates are given by

$$\frac{\partial L^*}{\partial t_\ell} = -\frac{1}{E_{\ell\ell}} \cdot (1 + E_{\ell\psi} \cdot \psi_\ell) \quad \text{and} \quad \frac{\partial L^*}{\partial t_k} = -\frac{1}{E_{\ell\ell}} \cdot (k + E_{\ell\psi} \cdot \psi_k). \tag{26}$$

Using (26), it follows from (20) and (21) that

$$\frac{\partial W}{\partial t_\ell} = \frac{1}{k} \cdot \frac{\partial W}{\partial t_k} - \lambda L^* \frac{t_k k'}{k} + \underbrace{(\frac{1}{k}\psi_k - \psi_\ell)[E_\psi + \lambda(t_\ell + t_k k)\frac{E_{\ell\psi}}{E_{\ell\ell}}]}_{<0}. \tag{27}$$

This implies again that zero taxation of capital can never be optimal: For any $(t_\ell, t_k) = (t_\ell, 0)$, we get $\frac{\partial V}{\partial t_k} > k \cdot \frac{\partial V}{\partial t_\ell}$ such that an increase in t_k is warranted. In an interior optimum $\frac{\partial W}{\partial t_k} = \frac{\partial W}{\partial t_\ell} = 0$ and, from (27),

$$L^* \frac{t_k k'}{k} = \frac{1}{\lambda}(\frac{1}{k}\psi_k - \psi_\ell)[E_\psi + \lambda(t_\ell + t_k k)\frac{E_{\ell\psi}}{E_{\ell\ell}}]. \tag{28}$$

To sum up:

Result 1 *With fairness concerns, whether they shape incentives or just affect utility levels, zero capital taxes are never optimal.*

The rationale behind having capital taxed is that the result from standard OT theory to leave capital untaxed balances on a knife's edge. Any effect providing capital taxation with some extra marginal benefit induces the government to rely on at least some capital taxation. Here, tax equity does the job.

3.2 Comparative statics with level effects

The inclusion of tax equity considerations provide governments with incentives to levy positive capital tax rates. But precisely how do different degrees of fairness concerns affect optimal tax policy? To answer this, we first consider the case where fairness concerns do not impact on work incentives. In addition, we suppose that envy is caused only by the difference between capital and labour tax rates, i.e.,

$$\Omega = \tilde{\Omega}(\beta \cdot (t_\ell - t_k)) \tag{29}$$

with $\tilde{\Omega}' > 0$ and $\tilde{\Omega}'' \geq 0$. The parameter $\beta > 0$ serves as a parametric measure for the intensity of the envy effect. The comparative statics of (t_ℓ, t_k) with respect to β are given through:

$$\begin{pmatrix} W_{\ell\ell} & W_{\ell k} & G_\ell \\ W_{\ell k} & W_{kk} & G_k \\ G_\ell & G_k & 0 \end{pmatrix} \cdot \begin{pmatrix} \mathrm{d}t_\ell \\ \mathrm{d}t_k \\ \mathrm{d}\lambda \end{pmatrix} = \begin{pmatrix} -W_{\ell\beta} \\ -W_{k\beta} \\ 0 \end{pmatrix} \mathrm{d}\beta,$$

with $W_{xy} = \partial^2 W/(\partial t_x \partial t_y)$ and $W_{x\beta} = \partial^2 W/(\partial t_x \partial \beta)$. From (20), (21) and (29) we get that

$$W_{k\beta} = -W_{\ell\beta} = \Omega_{\ell\beta} = \tilde{\Omega}' + \beta(t_\ell - t_k) \cdot \tilde{\Omega}'' > 0. \tag{30}$$

Hence, we applying Cramer's Rule to (30) we obtain:

$$\frac{\mathrm{d}t_\ell}{\mathrm{d}\beta} = -\frac{1}{D} \cdot \Omega_{\ell\beta} \cdot (G_k^2 + G_\ell G_k) \tag{31}$$

$$\frac{\mathrm{d}t_k}{\mathrm{d}\beta} = \frac{1}{D} \cdot \Omega_{\ell\beta} \cdot (G_\ell^2 + G_\ell G_k) \tag{32}$$

$$\frac{\mathrm{d}(t_\ell - t_k)}{\mathrm{d}\beta} = -\frac{1}{D} \cdot \Omega_{\ell\beta} \cdot (G_k + G_\ell)^2. \tag{33}$$

Here, D is the determinant of the bordered Hessian on the LHS of (30).[6] In a welfare maximum, $D > 0$ as well as $W_{kk}, W_{\ell\ell} < 0$.

First, observe that the weak assumption $\tilde{\Omega}' > 0$ (envy somehow increases in the tax rate differential) suffices to make fairness concerns affect optimal tax policies – we do not need to assume that $\tilde{\Omega}'' > 0$ (the psychological costs of tax inequity increase more than proportionately with the tax gap).

[6]The coefficient matrix is given by $D = 2G_k G_\ell W_{\ell k} - (G_k^2 W_{\ell\ell} + G_\ell^2 W_{kk})$.

As can be seen immediately from (33), a stronger concern for tax equity has an unambiguous effect on the tax rate differential: $\mathrm{d}(t_\ell - t_k)$ is strictly decreasing in β, irrespective of the signs of the partial derivatives of the Laffer curve (G_ℓ, G_k). Starting from $t_\ell > t_k = 0$ at $\beta = 0$, the stronger the tax equity norm, the more the tax structure moves into the direction of equal taxation.

To determine the signs of (31) and (32), we manipulate these expressions in the following way. From (22), (16), (17), (25) and $\Omega_\ell = -\Omega_k$, it follows that, in an interior equilibrium, we have

$$G_\ell = \frac{1}{k}G_k - \frac{1}{\lambda}\Omega_k(1+\frac{1}{k}) \tag{34}$$

Substituting for G_ℓ from (34) into (31), we obtain

$$\frac{\mathrm{d}t_\ell}{\mathrm{d}\beta} = \underbrace{-\frac{1}{D}}_{<0} \cdot \Omega_{t_l\beta} \cdot G_k(1+\frac{1}{k})[G_k \underbrace{-\frac{\Omega_k}{\lambda}}_{>0}] \gtreqless 0. \tag{35}$$

Surprisingly, the effects from stronger tax equity concerns on the tax rate on wage income are unclear in sign. If $G_k > 0$, then the wage tax decreases in the degree tax fairness, which is accordance with intuition: the more envious people are about the capital tax privilege government chooses, the lower the tax burdens they are willing to accept. But, the counter-intuitive case, that a stronger desire to correct for tax unfairness is associated with higher labour taxation may also occur. This is the case, if $G_k < 0$, i.e. if the economy happens to be on the downward-sloped part of the Laffer curve of the capital tax rate (provided that $G_k - \frac{\Omega_k}{\lambda} > 0$)). Below we will show that under certain conditions government has in fact incentives to push the economy beyond the maximum of the partial capital-tax-Laffer-curve.

Likewise, one can show that

$$\frac{\mathrm{d}t_k}{\mathrm{d}\beta} = -\frac{1}{k}\frac{\mathrm{d}t_\ell}{\mathrm{d}\beta}\frac{1}{G_k}[(G_k - \frac{\Omega_k}{\lambda}) - \frac{k\Omega_k}{\lambda}]. \tag{36}$$

This expression is greater than zero, irrespective of the sign of G_k. Thus, we get a monotone behaviour of the capital tax rate.

To show that $\frac{\mathrm{d}t_\ell}{\mathrm{d}\beta} > 0$ might be an optimal policy response, we provide a numerical

Example 1. In this and all following examples, we consider a Cobb-Douglas technology where per-capita output is produced according to $y = k^\alpha$. We parameterize the disutility from labour by $E = 0.5 \cdot \psi \cdot \ell^2$. The disutility from tax rate differentials is assumed to follow $\Omega = 0.5 \cdot \beta \cdot (t_\ell - t_k)^2$. The parameter α, capital's share of output, is set equal to 0.25. The "dislove for work" parameter, ψ, is set to 0.1, and the world market's rental rate, r, to 0.25. Figure 1 illustrates different values for β. Each graph plots government iso-budget contours (solid lines) and indifference curves (dashed lines), containing all tax rate combinations (t_k, t_ℓ)

that, respectively, yield the same budget size and the same utility level (see below). The iso-budget contours are negatively sloped for moderate capital tax rates: a higher capital tax rate entails higher tax revenues and, thus, allows for a lower tax rate on labour to keep the budget size still constant. However, above a certain level of t_k the negative effect of a higher capital tax rate on tax revenues (a lower tax base induced by capital flight) dominates, such that the same amount of g can only be met with a higher tax burden on labour income. As there are no incentive effects from envy here, budget contours are identical in all four panels of Figure 1. By constrast, indifference curves vary with β, i.e., with the strength of the tax equity concern. For low values of β indifference curves are negatively sloped since individuals place high emphasis on the adverse effects of capital taxation on consumption ($w' < 0$). For $\beta = 0$ both the labour and the capital tax rate are considered as "bads" – while t_ℓ has adverse effects on consumption via lower net wages, a higher t_k depresses gross wages. Indifference curves closer to the origin denote higher utility levels. With increasing concerns for tax equity, the indifference curves bend upwards. Closer the tax gap between factors is increasingly considered as a "good". So, losses in consumption can be less easily compensated for by a lower tax burden on labour income. When the tax equity concern becomes overwhelming, i.e. the sole thing that matters, indifference curves have slope 1 and the highest utility level is represented by the 45°-line.[7]

In an optimal tax mix, the indifference curve must be tangent to the (lower leg of the) iso-budget contour representing the exogenous revenue requirement $\bar{g}$. In the standard case ($\beta = 0$), this tangential point is on the vertical axis where capital is tax exempt. Starting from such a position, the tangential point moves along the budget contour towards the 45°-line. This initially entails a reduction of t_ℓ and an increase in t_k. However, with equity concerns strong enough, eventually the upward-sloped part of the iso-budget contour might be entered. The optimal tax mix then leads the economy on the downward-sloped part of the (partial) Laffer for the capital tax rate (where $G_k < 0$). Thus, it is shown that ($\frac{\mathrm{d}t_\ell}{\mathrm{d}\beta} > 0$) is possible.[8]

If it is possible to finance the exogenous revenue requirement at equal tax rates (the iso-budget contour intersects with the diagonal), $t_\ell = t_k$ will eventually be implemented when β grows. However, for sufficiently high budget requirements an economy with stronger tax equity motives will remain stuck at that point on the iso-budget contour that is at minimal distance to the diagonal. From here onwards, $\frac{\mathrm{d}t_\ell}{\mathrm{d}\beta} = \frac{\mathrm{d}t_\ell}{\mathrm{d}\beta} = 0$.

We sum up the main general findings of this section in

Result 2 *As long as the government revenue requirement is not too high, greater concerns for tax equity call for higher taxes on capital and for closing the gap between capital and labour taxes. However, greater tax equity concerns do not necessarily imply lower tax rates on labour.*

[7]All tax combinations along the 45°-line are then considered as equally good.

[8]Formally, the tax mix (t_k, t_ℓ) that is at minimum distance to the 45°-line satisfies, on the iso-budget contour for g, the condition $-G_k/G_\ell = 1$. From (31) to (33), this implies that tax rates do no furthr vary with β.

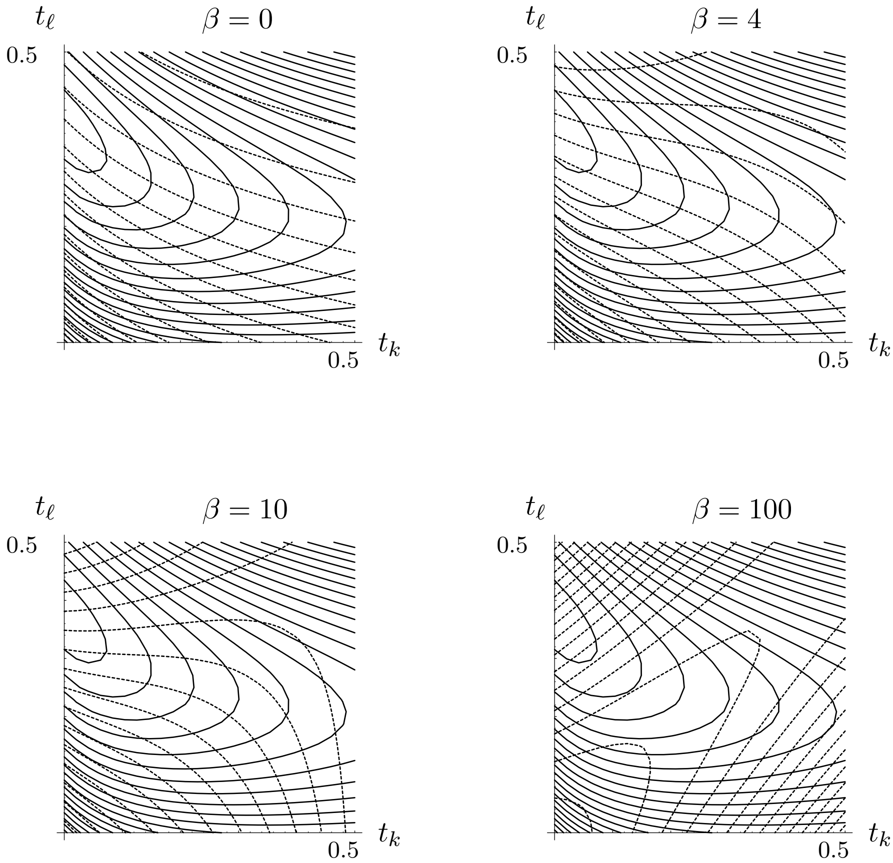

Figure 1: Tax equity without incentive effects. Government iso-budget contours (solid) and indifference curves (dashed) for varying values of β. Horizontal axis: capital tax rate (t_k), vertical axis: labour tax rate (t_ℓ).

3.3 Comparative statics with incentive effects

Now, we turn to the effects of stronger fairness concerns when tax equity considerations impact on work incentives. Here, we show numerical examples suggesting that the counter-intuitive case $G_k < 0$ (which impies that the wage tax increase in spite of higher equity concerns) does not occur in a welfare maximum. A full analytical exposition is left for future versions of this paper.

Example 2. As above, preferences are parameterized by $u = c - 0.5 \cdot \psi \cdot \ell^2$. Now ψ is not a constant but a function given by

$$\psi = \psi_0 + 0.5 \cdot \beta \cdot (t_\ell - t_k)^2. \tag{37}$$

The level of spending is again exogenous. Throughout the numerical examples, we set ψ_0 equal to 0.1; all other parameters take on the same values as in Example 1.

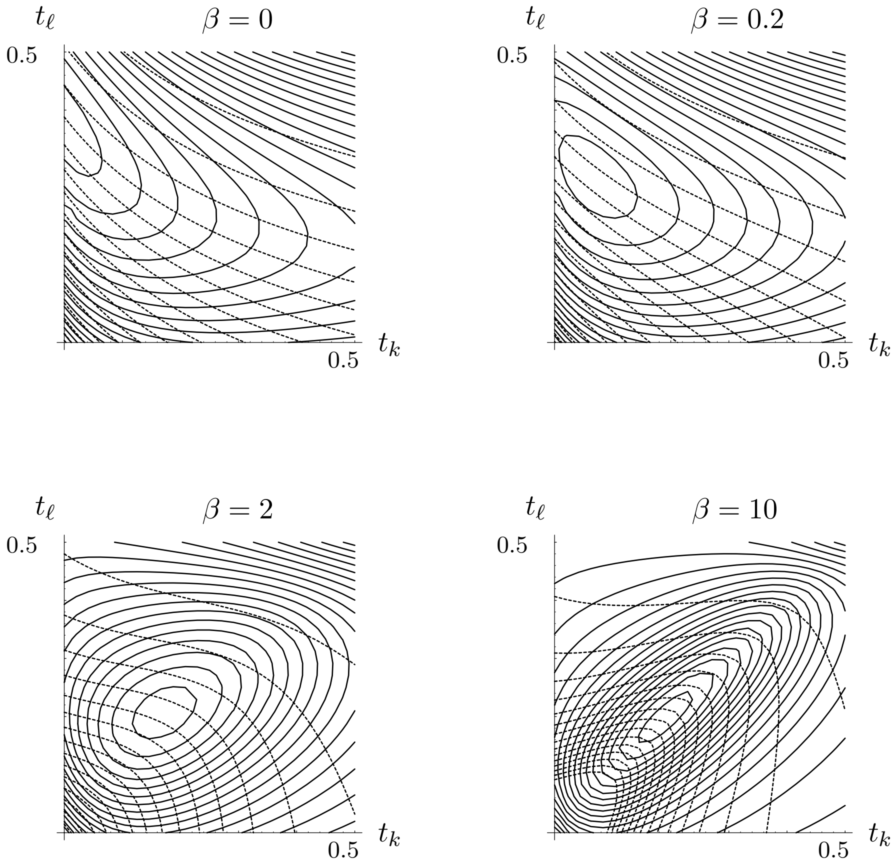

Figure 2: Tax equity with incentive effects. Government iso-budget contours (solid) and indifference curves (dashed) for varying values of β. Horizontal axis: capital tax rate (t_k), vertical axis: labour tax rate (t_ℓ).

Figure 2 depicts government iso-budget contours (solid lines) and the indifference curves (dashed lines) for different values of β. Contrary to the case where tax fairness considerations leave work incentives unaltered, the government iso-budget contours now depend on the strength of the fairness norm: similar to indifference curves, budget contours bend upwards when β increases. The reason is that (starting again from a situation in which $t_\ell > t_k$) a higher capital tax motivates people to work more. Thus, the same level of tax revenues can be generated at an ever lower wage tax than in the absence of incentive effects. Consequently, the peaks of budget surfaces (in three-dimensional space) move towards the 45°-line. In the extreme case where people only care for tax equity, tax revenues can only be earned when $t_\ell = t_k$ (for any given t_k); otherwise people would not supply any labour.

Figure 2 suggests that the capital tax rate, the tax rate differential and the wage tax rate move monotonically with β. For the wage tax, this is in contrast to Example 1.

4 Endogenous government spending

In this section we analyze the effects of tax equity when government spending is endogenous. Such an exercise is worthwhile as equity concerns make government activities less attractive; after all, they lead to the adoption of tax mixes that are excessively costly from a pure efficiency perspective. This might impact on the optimal level of government expenditures – and a first intuition would suggest that greater equity concerns call for smaller governments. But better have a closer look.

4.1 Some taxation of capital is optimal

We again employ the set-up of Section 2. Again, the government chooses t_ℓ and t_k in order to maximize social welfare (= indirect utility). Allowing g to vary rather than being pre-set, the government objective function is

$$V(t_\ell, t_k) := (w(r+t_k) - t_\ell) \cdot L^*(t_\ell, t_k) - E(L^*(t_\ell, t_k), \psi(t_\ell, t_k)) + h(G(t_\ell, t_k)) - \Omega(t_\ell, t_k) \quad (38)$$

where $L^*(\cdot)$ and $G(\cdot)$ are defined as in (12) and (15). Differentiating V, as defined in (38), with respect to tax rates (t_k, t_ℓ) and using the Envelope-Theorem gives:

$$\frac{\partial V}{\partial t_\ell} = L^* \cdot \left[h'(G) - 1\right] + h'(G) \cdot (t_\ell + t_k k) \cdot \frac{\partial L^*}{\partial t_\ell} - E_\psi \cdot \psi_\ell - \Omega_\ell \quad (39)$$

$$\frac{\partial V}{\partial t_k} = kL^* \cdot \left[h'(G) - 1\right] + h'(G) \cdot \left((t_\ell + t_k k) \cdot \frac{\partial L^*}{\partial t_k} + t_k k' L^*\right) - E_\psi \cdot \psi_k - \Omega_k \quad (40)$$

$$= k \cdot \frac{\partial V}{\partial t_\ell} + h'(G) t_k k' L^* + k\Omega_\ell - \Omega_k \quad (41)$$

These conditions give rise to

Result 3 *1. In the absence of fairness concerns capital should optimally never be taxed.*

2. In the presence of fairness concerns, whether they shape incentives or just affect utility levels, zero capital taxes are never optimal.

3. The level of the government-provided good is always inefficiently low.

The analytical results on the tax structure coincide with those of Section 4.1; and the proof of items 1 and 2 is almost identical. Underprovision of the government good in the absence of fairness concerns (i.e., when $\psi_\ell = \Omega_\ell = 0$) can be seen equating (39) to zero with $t_k = 0$; we get the Atkinson-Stern Rule:

$$h'(G) = \frac{1}{1 + \frac{\partial \ell^S}{\partial w} \cdot \frac{t_\ell}{\ell^S}} > 1. \quad (42)$$

Hence, the marginal willingness-to-pay for the government good exceeds the marginal rate of transformation (which equals 1). The reason for this underprovision is the financing through a distortionary (labour) tax. With fairness concerns ($\Omega_\ell > 0 = \psi_\ell$), the costs of public funds further increase since government expenditures will now partly be financed through the even less efficient capital tax. Formally, underprovision can be seen in (39) which, for $\partial L^*/\partial t\ell < 0$ and $\Omega_\ell > 0$, necessitates that $h'(G) > 1$.

4.2 Comparative statics with level effects

As in the previous section, let us first consider the case that the feeling of unfair taxation has no incentives effects, i.e., $\psi_k = \psi_\ell = 0$). Instead, only the level effect of unfairness is working, i.e. $\Omega_k \leq 0, \Omega_\ell \geq 0$ with at least one strict inequality. For simplicity let us (as in Section 3) assume that Ω is given by (29): $\Omega = \tilde{\Omega}(\beta \cdot (t_\ell - t_k))$. While comparative statics of (39) and (40) are quite messy, some reasonably general results are available. Our first finding is in the spirit of Result 2:

Result 4 *Suppose that fairness concerns are such that $t_\ell > t_k$ with t_k, β small, but positive. Then a more intense fairness concern (represented by an increase β) calls for a decrease in the tax rate on labour, an increase in the tax rate on capital and, consequently, a decrease in the tax rate differential.*

The **proof** is in the Appendix. While Result 4 sounds plausible, it should be noted that the qualifications of weak fairness concerns made at the opening of the proposition are indeed essential. This is shown by means of

Example 3: As in Example 1, we choose $y = k^\alpha$. To arrive at explicit solutions, we further suppose that labour supply is inelastic at some level $\bar{L} > 0$. Utility is given by $u = c - \Omega$, where $\Omega = 0.5\beta(t_\ell - t_k)^2$.
Figure 3 illustrates optimal policies when parameter values are set to $\bar{L} = 0.2$, $\alpha = 1/3$, and $r = 0.2$. The first graph shows that β and t_k are strictly positively related. The other three graphs plot, respectively, $(t_\ell - t_k)$, t_ℓ, and optimal government expenditure $G(t_\ell(\beta), t_k(\beta))$ against t_k – which translates, by the first graph, into similarly shaped plots against β. As can be seen, t_k and the tax rate differential $t_\ell - t_k$ vary monotonically with β (as predicted by Result 4), but the labour tax rate initially falls and later rises when fairness concerns intensify beyond some level. This shows that the labour tax rate is indeed generally not monotonic in the strength of fairness considerations.
The fourth graph shows that also government expenditures are non-monotonic in β, first falling, then rising. Hence, the first-order intuition that a fairness-induced increase in the marginal costs of public funds (due to greater reliance on capital taxes) always calls for smaller government is incorrect. Of course, Result 3 remains valid in that government expenditure is always inefficiently

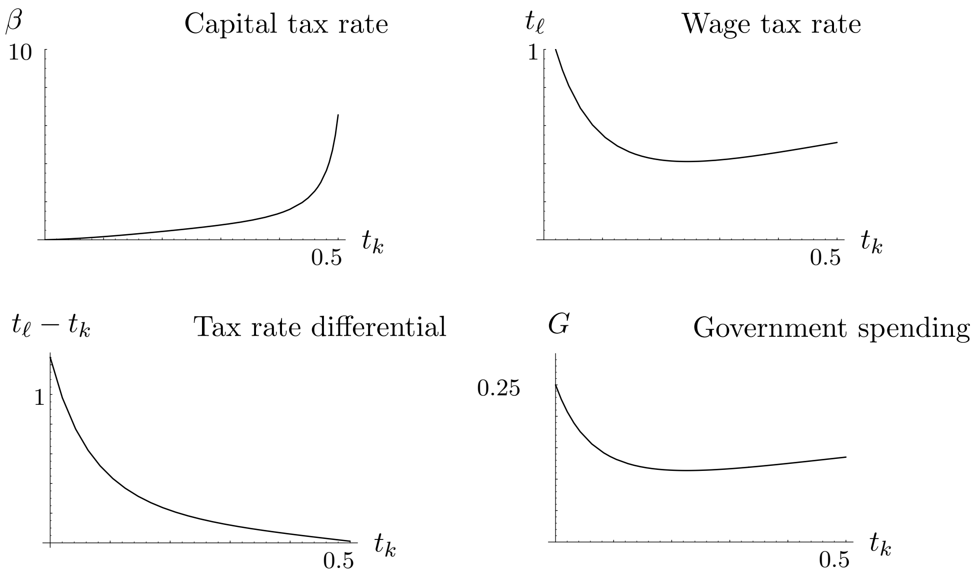

Figure 3: Optimal policies when government spending is endogenous.

low.[9]

Result 5 *Greater fairness concerns do not necessarily call for a reduction in government expenditures.*

5 Extension: Relative income positions

In this section we analyze the effects when individuals have direct preferences over relative income positions. We assume that there are workers and capitalists where the former are in the majority. Capital owners can invest their capital at home or abroad. The world exogenous interest rate is r. Let n denote the share of the total domestic capital stock that is owned by a domestic capitalist. The production sector is the same as above. Workers compare their labour income to the income of a capitalist. In this setting, the difference between a worker's income and a capitalist's income amounts to

$$\Delta(t_\ell, t_k) = ((w - t_\ell) \cdot \ell - \ell \cdot k \cdot \frac{r}{n}), \tag{43}$$

where k denotes the total capital stock per labour unit. Preferences of workers are given by

$$u(c, \ell) = c - \frac{1}{2} \cdot \psi \cdot \ell^2 + c \cdot \sqrt{g} - \frac{1}{2} \cdot \Delta^2. \tag{44}$$

[9] In the example, an inelastic labour supply is assumed. Hence, Result 3 does not (strictly) apply. Rather, in the example G is at its efficient level for $\beta = 0$: we have $G = 0.25$, which solves $1 = h'(G) = 0.5G^{-0.5}$.

The Government wants to be re-elected and therefore chooses a policy mix (t_ℓ, t_k) so as to maximize (44) s.t. to the budget constraint (15).

Here, we analyze numerical examples for the case that envy does not affect work incentives. We choose $a = 1/4$, $\psi = 0.1$, $r = 0.4$ and $n = .25$. These parameter values ensure that, starting from $t_k = 0$, a worker is poorer than a capitalist and that Δ is monotonically increasing in t_ℓ and decreasing in t_k. First, we consider the case where the government has to meet exogenous levels of government spending ($c = 0$, $g = \bar{g}$). In figure (4), we depict the government Iso-budget lines and the indifference curves for varying digress of envy. It can be seen from figure (4), that

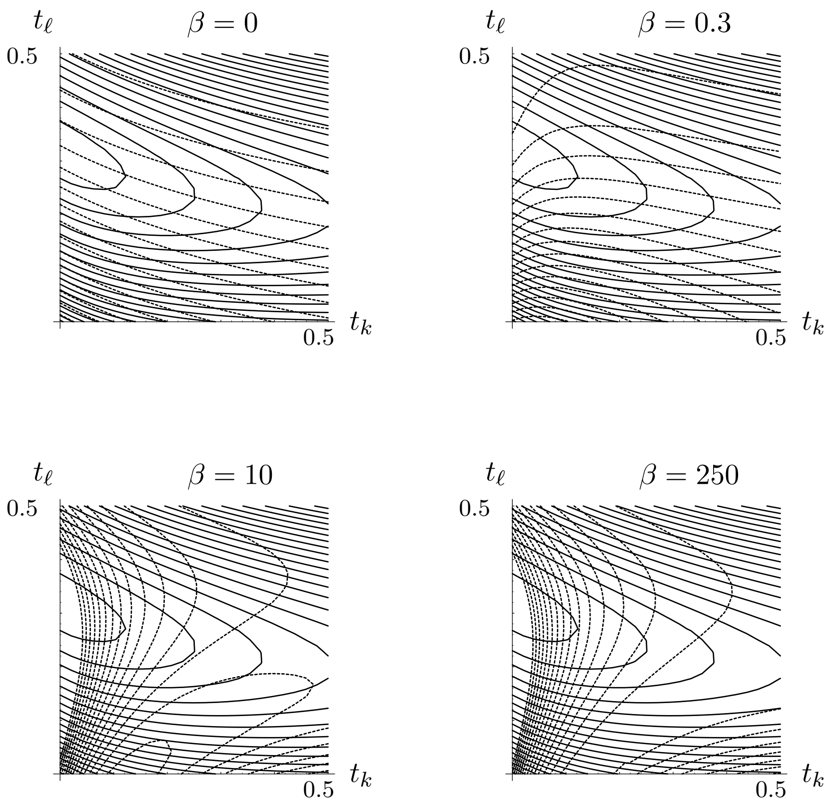

Figure 4: Envy without incentive effects. Exogenous spending. Indifference curves for varying values of β, representing the degree of envy. Horizontal axis: capital tax rate (t_k), vertical axis: labour tax rate (t_ℓ).

the government also relies on capital taxation when $\beta > 0$. With increasing envy, the tangential point curves up alongside a given Iso-budget line. In contrast to preferences for tax equity, the economy will not end up with equal tax rates but in a situation in which average income is equal between the two classes ($t_k > t_\ell$). Also notice that for sufficient high values of $\bar{g}$ and β, the government will push the economy on the downward-sloped part of the partial laffer curve for t_k.

When government spending is endogenous, we obtain results similar to the tax equity case (graphs are not shown). Here, the erosion of the size of the public sector is driven by concerns for egalitarian income.

6 Conclusion

In this paper, we augmented the standard taxation model for small open economies by status and fairness concerns. Going beyond standard neoclassical assumptions, we endowed individuals with a direct preference over tax rates. We also built on recent empirical findings suggesting that individuals care for their relative income position. In both cases, we separated between fairness considerations that may shape work incentives and those which just scale up or down absolute utility levels.

We find that even when individuals are endowed with the slightest concern for other people's income or tax equity, the standard Gordon/Razin/Sadka result of zero capital taxation ceases to be optimal. This holds irrespective of whether we allow envy or tax equity to impact on work incentives or on well-being.

Our comparative statics reveal some interesting non-monotonicities. Generally, the relationship between the optimal tax rate on labour income and the degree of tax equity (or tax envy) is U-shaped. Thus, goverments taxing labour income at higher tax rates than capital income will not necessarily end up with lower wage taxes if individuals have stronger concerns for egalitarian factor taxation. Regarding the levels of government spending, our findings are not fully in accordance with first-order intuition, either. On a priori grounds, one might expect that a stronger concern for equal tax rates calls for higher capital tax rates and, by this, for a smaller public sector (capital taxation is associated with high excess burden). However, while the former is true, the latter is not necessarily the case. Tax fairness effects erode the size of the public sector only when they are relatively weak. With a strong concerns for egalitarian taxation, further increases in the degreee of tax equity are asscociated with higher government expenditure.

These non-monotonicities are established for the case in which fairness concerns leave work incentives unaltered. It is left for future versions of this paper to analyze whether they occur in the incentive-shaping scenario. However, our graphical illustrations suggest that this is not the case.

References

Arrow, K. J., P. Dasgupta, L. Goulder, G. Daily, P. Ehrlich, G. Heal, S. Levin, K.-G. Maler, S. Schneider, D. Starrett, and B. Walker (2004). Are We Consuming Too Much? Journal of Economic Perspectives 183, 147-172.

Alonso-Carrera J., J. Caball, and X. Raurich (2006). Welfare Implications of the Interaction Between Habits and Consumption Externalities. International Economic Review 47, 557-71.

Alvarez-Cuadrado, F. (2007). Envy, Leisure and Restrictions on Working Hours. Canadian Journal of Economics 40, 1286-1310.

Clark, A. E. and A. J. Oswald (1996). Satisfaction and Comparison Income. Journal of Public Economics LXI, 359-381.

Fehr, E. and K.M. Schmidt (2006), The Economics of Fairness, Reciprocity and Altruism: Experimental Evidence, in S. Kolm and J.M. Ythier, eds. Handbook of the Economics of Giving, Altruism and Reciprocity, Vol. 1. North Holland, New York.

Gordon, R. (1986). Taxation of Investment and Savings in a World Economy. American Economic Review 76, 1086-1102.

Haufler, A., A. Klemm, and G. Schjelderup (2008). Economic Integration and the Relationship Between Profit and Wage Taxes. Oxford University Center of Business Taxation, Working Paper 2008/10. Forthcoming, Public Choice.

Kaplow, L. (1995). A fundamental objection to tax equity norms: a call for utilitarianism. National Tax Journal 48, 497-514.

Layard, R. (2006). Happiness and Public Policy: A Challange to the Profession. Economic Journal 116, 24-33.

Liu, W.-F., and S. J.Turnovsky (2005). Consumption Externalities, Production Externalities, and Long-Run Macroeconomic Efficiency. Journal of Public Economics 89, 1097-1129.

Luttmer, E. F. P. (2005). Neighbors as Negatives: Relative Rarnings and Well-Being. Quarterly Journal of Economics 120, 963-1002.

Neill, J. R. (2000). The Benefit and Sacrifice Principles of Taxation: A Synthesis. Social Choice and Welfare 17, 117-124.

Razin, A. and E. Sadka (1991). International Tax Competition and Gains from Tax Harmonization. Economic Letters 37, 69-76.

Appendix

Proof of Result 4

From (29), $\Omega_{\ell\beta} = -\Omega_{k\beta} = \tilde{\Omega}' + \beta(t_\ell - t_k)\tilde{\Omega}'' > 0$. Using (39) and (40), the comparative statics of (t_ℓ, t_k) with respect to β are given by:

$$\begin{aligned} \begin{pmatrix} V_{\ell\ell} & V_{\ell k} \\ V_{\ell k} & V_{kk} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{d}t_\ell \\ \mathrm{d}t_k \end{pmatrix} &= -\begin{pmatrix} V_{\ell\beta} \\ V_{k\beta} \end{pmatrix} \mathrm{d}\beta \\ &= \left[\tilde{\Omega}' + \beta \cdot (t_\ell - t_k) \cdot \tilde{\Omega}''\right] \cdot \begin{pmatrix} +1 \\ -1 \end{pmatrix} \mathrm{d}\beta \end{aligned}$$

(with $V_{xy} = \partial^2 V/(\partial t_x \partial t_y)$ and $V_{x\beta} = \partial^2 V/(\partial t_x \partial \beta)$). Consequently, by Cramer's Rule:

$$\begin{aligned} \frac{\mathrm{d}t_\ell}{\mathrm{d}\beta} &= \frac{1}{D} \cdot \left[\tilde{\Omega}' + \beta \cdot (t_\ell - t_k) \cdot \tilde{\Omega}''\right] \cdot (V_{kk} + V_{\ell k}) \\ \frac{\mathrm{d}t_k}{\mathrm{d}\beta} &= -\frac{1}{D} \cdot \left[\tilde{\Omega}' + \beta \cdot (t_\ell - t_k) \cdot \tilde{\Omega}''\right] \cdot (V_{\ell\ell} + V_{\ell k}) \\ \frac{\mathrm{d}(t_\ell - t_k)}{\mathrm{d}\beta} &= \frac{1}{D} \cdot \left[\tilde{\Omega}' + \beta \cdot (t_\ell - t_k) \cdot \tilde{\Omega}''\right] \cdot (V_{\ell\ell} + V_{kk} + 2V_{\ell k}) . \end{aligned}$$

Here, D is the determinant of the matrix on the LHS of (30). In a welfare maximum, $D > 0$ as well as $V_{kk}, V_{\ell\ell} < 0$. Our claim is therefore proven if (but not only if) $V_{\ell k} < 0$.[10] Lemma 1 below shows that this indeed holds. ■

Lemma 1 *Suppose that $t_\ell > t_k$, and $t_k, \beta > 0$, but small. Then $V_{\ell k} < 0$.*

Proof of Lemma: From (29), $\Omega_{\ell\ell} = \Omega_{kk} = -\Omega_{\ell k} = \beta^2 \tilde{\Omega}'' > 0$. Calculate:

$$\begin{aligned} V_{\ell\ell} &= -\frac{\partial L}{\partial t_\ell} + h''(G)\left(\frac{\partial G}{\partial t_\ell}\right)^2 + h'(G)\frac{\partial^2 G}{\partial t_\ell^2} - \Omega_{\ell\ell} \\ &= \frac{\partial \ell}{\partial w} + h''(G)\left(L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right)^2 + h'(G)\left(2\frac{\partial L}{\partial t_\ell} + (t_\ell + t_k k)\frac{\partial^2 L}{\partial t_\ell^2}\right) - \Omega_{\ell\ell} \\ &= \frac{\partial \ell}{\partial w} + h''(G)\left(L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right)^2 + h'(G)\left(-2\frac{\partial \ell}{\partial w} + (t_\ell + t_k k)\frac{\partial^2 \ell}{\partial w^2}\right) - \beta^2 \tilde{\Omega}'' \end{aligned}$$

[10] In fact. this condition is overly strict. It would suffice that $V_{\ell k} < \max\{-V_{\ell\ell}, -V_{kk}\}$.

Moreover,

$$\begin{aligned}
V_{\ell k} &= -\frac{\partial L}{\partial t_k} + h''(G)\frac{\partial G}{\partial t_\ell}\frac{\partial G}{\partial t_k} + h'(G)\frac{\partial^2 G}{\partial t_\ell \partial t_k} - \Omega_{\ell k} \\
&= k\frac{\partial \ell}{\partial w} + h''(G)\left(L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right)\left(L(k + t_k k') + (t_\ell + t_k k)\frac{\partial L}{\partial t_k}\right) \\
&\quad + h'(G)\left(\frac{\partial L}{\partial t_k} + (k + t_k k')\frac{\partial L}{\partial t_\ell} + (t_\ell + t_k k)\frac{\partial^2 L}{\partial t_\ell \partial t_k}\right) - \Omega_{\ell k} \\
&= k\frac{\partial \ell}{\partial w} + h''(G)\left(L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right)\left(k\left[L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right] + L t_k k'\right) \\
&\quad + h'(G)\left(-2k\frac{\partial \ell}{\partial w} - t_k k'\frac{\partial \ell}{\partial w} + k(t_\ell + t_k k)\frac{\partial^2 \ell}{\partial w^2}\right) - \Omega_{\ell k} \\
&= k V_{\ell\ell} + A_1 \qquad (45)
\end{aligned}$$

where we set

$$A_1 := h''(G)\frac{\partial G}{\partial t_\ell} L t_k k' - h'(G) t_k k' \frac{\partial \ell}{\partial w} + (k+1)\beta^2 \tilde{\Omega}'' \geq 0$$

for all $\beta, t_k \geq 0$. Finally,

$$\begin{aligned}
V_{kk} &= w'\frac{\partial L}{\partial t_k} + h''(G)\left(\frac{\partial G}{\partial t_k}\right)^2 + h'(G)\frac{\partial^2 G}{\partial t_k^2} - \Omega_{kk} \\
&= k^2\frac{\partial \ell}{\partial w} + h''(G)\left(k\left[L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right] + L t_k k'\right)^2 \\
&\quad + h'(G)\left(2(k + t_k k')\frac{\partial L}{\partial t_k} + (t_\ell + t_k k)\frac{\partial^2 L}{\partial t_k^2} + (2k' + t_k k'')L\right) - \Omega_{kk} \\
&= k^2\frac{\partial \ell}{\partial w} + h''(G)\left(k\left[L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right] + L t_k k'\right)^2 \\
&\quad + h'(G)\left(-2k^2\frac{\partial \ell}{\partial w} - k t_k k'\frac{\partial \ell}{\partial w} + k^2(t_\ell + t_k k)\frac{\partial^2 \ell}{\partial w^2} + (2k' + t_k k'')L - k t_k k'\frac{\partial \ell}{\partial w}\right) - \Omega_{kk} \\
&= k^2\frac{\partial \ell}{\partial w} + h''(G)\left\{k\left(L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right)\left(k\left[L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right] + L t_k k'\right)\right. \\
&\quad \left. + L t_k k'\left(L t_k k' + k\left(L - (t_\ell + t_k k)\frac{\partial \ell}{\partial w}\right)\right)\right\} \\
&\quad + h'(G)\left(-2k^2\frac{\partial \ell}{\partial w} - k t_k k'\frac{\partial \ell}{\partial w} + k^2(t_\ell + t_k k)\frac{\partial^2 \ell}{\partial w^2} + (2k' + t_k k'')L - k t_k k'\frac{\partial \ell}{\partial w}\right) - \Omega_{kk} \\
&= k V_{\ell k} + A_2 \qquad (46)
\end{aligned}$$

where

$$A_2 := \underbrace{h''(G) L t_k k'\frac{\partial G}{\partial t_k} - h'(G) k t_k k'\frac{\partial \ell}{\partial w}}_{>0} + \underbrace{h'(G) L (2k' + t_k k'')}_{?} \underbrace{-(k+1)\beta^2\tilde{\Omega}''}_{<0}$$

is unclear in sign. Recall that $V_{\ell\ell}, V_{kk} < 0$ in an optimum. Moreover, the determinant D has to be positive. From (45) and (46), calculate that $D = (A_2/k - A_1)V_{\ell k} - A_1 A_2/k$, which is positive if and only if

$$(-kA_1 + A_2)V_{\ell k} > A_1 A_2.$$

For $\beta = 0$ (and, hence $t_k = 0$) we get that $A_1 = 0 > 2h'(G)Lk' = A_2$, which then yields that $V_{\ell k}$ must be negative. The same holds for small values of β and t_k.[11] ■

[11] Also, from (45) and (46), $V_{k\ell}$ cannot be positive (or even larger than $-V_{kk}$ and $-V_{\ell\ell}$ unless t_k and β are sufficiently large. This is excluded by assumption, however.