Option-implied Distance-to-Default:

An indicator to assess banking fragility using information in option prices

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Abstract

This paper develops the Option-implied Distance-to-default (OIDD), a market-based indicator to assess individual bank distress. This measure is a modified version of the traditional Distance-to-default using information embedded in option prices through the estimation of risk-neutral probability density functions with constant maturity. The paper also evaluates the properties of the resulting OIDD series vis-à-vis traditional DD measures. Equilibrium relationships and price discovery tests are conducted using pairwise cointegration analysis and Granger causality tests. The results prove existence of dynamic equilibrium relationships between the OIDD indicator and the traditional DD measures. Granger causality tests suggest that relevant changes in DD are observable earlier in the OIDD measure, illustrating quick reaction of market expectations of banks' fragility in option markets. Results also reveal sensitivity of OIDD to trading irregularities, due to sudden stops in trading for some contracts and strike prices. This feature is interpreted as episodes of extreme market uncertainty and changes in market trends, specially witnessed since Summer 2007 and for some large financial institutions.

Keywords: distance-to-default, implied probability density functions, bank fragility

JEL classification: G13, G14, G21

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1. Introduction

The purpose of this paper is to contribute to the growing empirical literature of quantitative tools for assessing financial stability with the introduction of the Option-implied Distance-to-default (OIDD henceforth). The OIDD is a single quantitative and market-based indicator designed to assess vulnerability of individual banks in the short-run. Following the methodology of KMV-Moody's (Bohn and Crosbie, 2003), this indicator is a modified version of the traditional Distance-to-default (DD) measure using information embedded in option prices. DD measures combine financial statements and security prices information. The OIDD adds the market expectations on the underlying banks' stock prices and volatilities from estimated risk-neutral probability density functions.

Since its introduction in 2003, the use of the DD indicator has spread among market observers, as well as financial regulators and supervisors. This indicator can be used to detect bank distress and to complement the assessment of possible bank interventions and closures by supervisors (Chan Lau and Sy, 2006). Empirical literature supports the idea that this indicator covers most elements of bank risk – asset returns, asset risk and leverage – and constitutes a measure not affected by the presence of explicit or implicit safety nets (Gropp et al., 2006).

This paper is structured as follows. Section 2 summarizes the theory supporting the OIDD. First, the literature on option-implied probability density functions (PDF hereafter) is reviewed together with some remarks regarding the case of PDF applied to American-style dividend-paying equity options. The second subsection summarizes recent research on DD and its properties and applications as a market-based indicator of bank fragility. In section 3, the dataset is described in detail and the OIDD indicator is constructed using a two-step approach. First, option-implied risk-neutral PDF are estimated on a daily basis with 45-day and 60-day constant horizons using the mixture of two lognormals. In the second step, the estimates of forward-looking stock prices and volatilities entered together with the rest of balance-sheet information into the calibration of the OIDD series. Section 4 summarizes results of PDF estimates and OIDD series, highlighting stylized facts of the series and methodological limitations. Section 5 concludes and gives recommendations for future research.

2. Theoretical Underpinnings

2.1. Constant-maturity Option-implied PDF

Option-implied PDF are the input of the OIDD indicator in this study. They will make available a set of parameters used to produce alternative specifications of the OIDD series and assess the potential use of option prices information for bank default risk. PDF literature has developed extensively in the mid-nineties and has been applied to many different financial instruments but scarcely to equity options. This sub-section aims to provide a brief review of the literature and to underline the necessary adjustments made to meet the objectives of this paper.
Option implied risk-neutral probability density functions are analytic tools designed to exploit the information potential of these derivative instruments. They represent and allow retrieving the whole probability distribution of different outcomes of underlying assets and tracking expectations of market participants of these outcomes. The information embedded in PDF has proven very useful in several applications, ranging from assessment of monetary policy actions and case studies to analysis of developments in energy and currency markets. Bu and Hadri (2007), Liu et al. (2007) and Lynch and Panigirtzoglou (2008) provide a comprehensive review of PDF methods and applications.

Most techniques to estimate option-implied risk-neutral PDF in the literature are based on the Cox and Ross (1976) pricing model for European style options, which defines call option prices as the risk-neutral expected payoff of the option at maturity, discounted back at the risk-free rate[1].

$$C = e^{-rt} \int_{S_T=K}^{\infty} \left(S_T - K\right) \cdot f\left(S_T\right)\, dS_T \qquad (1)$$

where S_T is the underlying asset price at maturity T; $f(S_T)$ is the risk-neutral PDF; K is the strike price and r is the continuously compounded risk-free rate.

Several parametric and non-parametric techniques have been developed to approximate the PDF. Non-parametric techniques use (1) and Breeden and Litzenberger's (1978) result[2] and do not imply strong assumptions on the functional form of the PDF or the stochastic process of the underlying. The most commonly non-parametric method is the spline method, introduced by Shimko (1993) and extensively described in Bliss and Panigirtzoglou (2002 and 2004), Cooper (2000) and Malz (1995 and 1997). It is a smoothing technique and it stands out because of its flexibility and easy implementation. Constant-maturity PDF were first explored using this technique[3].

The most popular parametric technique is the mixture of log-normal functions discussed in Bahra (1997) and Melick and Thomas (1997). This technique assumes a weighted average of two or three log-normal PDF as the functional form and fits the observed put and call prices to predicted prices via non-linear least squares. This method is however flexible and captures the features of distributions, adapting to different stochastic processes of the underlying asset and thus different shapes, e.g. fat tails, skewness and bi-modality. In addition, this technique produces parsimonious estimates and it is computationally efficient in terms of the need of optimization techniques.

This paper follows Glatzer and Scheicher (2005), where the mixture of two log-normal density functions is used to extract a set of five parameters that are then used to generate statistics to enter into to the DD calibration and obtain alternative OIDD series. The mixture of two log-normal density functions is defined as follows:

$$f(S) = \phi L\left(S_T \middle| S_T, \alpha_1, \beta_1\right) + (1-\phi) L\left(S_T \middle| S_T, \alpha_2, \beta_2\right) \qquad (2)$$

$$\alpha_i = \log(S_t) + \left(\mu_i - \tfrac{1}{2}\sigma_i^2\right) \cdot t \qquad (3)$$

$$\beta_i = \sigma_i \sqrt{t} \quad \text{for } i=1,2 \qquad (4)$$

where $L\left(S_T \middle| S_T, \alpha_i, \beta_i\right)$ is the i-th log-normal density function with parameters α_i and β_i.[4]

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[1] A similar expression can be derived for put options: $P = e^{-rt} \int_{-\infty}^{S_T=K} \left(K - S_T\right) \cdot f\left(S_T\right)\, dS_T$ See Bahra (1997) for details.

[2] Breeden and Litzenberger (1978) established the relation between option prices and probability density functions, showing that, if there is a set of option prices across a continuum of strikes, the second derivative of a European call price with respect to the strike price delivers a risk-neutral PDF. $\frac{\partial^2 C}{\partial K^2} = e^{-rt} f\left(S_T\right)$

[3] See Bliss and Panigirtzoglou (2002 and 2004), Bødskov Andersen and Wagener (2002), Bu and Hadri (2007) and Cincibuch (2004) for a review.

[4] The underlying stock prices of the banks in the sample are considered to be log-normally distributed and each density function is defined as $L\left(S_T\right) = \frac{1}{S_t \beta \sqrt{2\pi}} e^{-\frac{(\log S_T - \alpha)^2}{2\beta^2}}$

The call and put pricing functions (see Bahra, 1997 for details) are defined as follows:

$$\hat{C}(S,K,t) = e^{-rt}\left\{\phi\left[e^{\alpha_1+\frac{1}{2}\beta_1^2}\cdot N(d_1) - K\cdot N(d_2)\right] + (1-\phi)\left[e^{\alpha_2+\frac{1}{2}\beta_2^2}\cdot N(d_3) - K\cdot N(d_4)\right]\right\} \quad (5)$$

$$\hat{P}(S,K,t) = e^{-rt}\left\{\phi\left[-e^{\alpha_1+\frac{1}{2}\beta_1^2}\cdot N(-d_1) + K\cdot N(-d_2)\right] + (1-\phi)\left[-e^{\alpha_2+\frac{1}{2}\beta_2^2}\cdot N(-d_3) + K\cdot N(-d_4)\right]\right\} \quad (6)$$

where $d_1 = \frac{-\log(K)+(\alpha_1-\beta_1^2)}{\beta_1}; \quad d_2 = d_1 - \beta_1; \quad d_3 = \frac{-\log(K)+(\alpha_2-\beta_2^2)}{\beta_2}; \quad d_4 = d_3 - \beta_2$

Finally, the loss function to be minimized and obtain the PDF parameters is:

$$\min_{\mu_1,\sigma_1,\mu_2,\sigma_2,\phi} \sum_{i=1}^{N_C}\left(\hat{C}_{i,t} - C_{i,t}\right)^2 + \sum_{j=1}^{N_P}\left(\hat{P}_{j,t} - P_{j,t}\right)^2 \quad (7)$$
$$\text{s.t.} \quad \sigma_1,\sigma_2 > 0 \quad \phi\in[0,1]$$

where $\hat{C}_{i,t}$, $\hat{P}_{j,t}$, and $C_{i,t}$, $P_{j,t}$ are predicted and actual option prices at trading day t and μ_1, σ_1, μ_2, σ2, ϕ are the five parameters that define the PDF and summary statistics, with μ_i and σ_i are the mean and standard deviation of the i-th log-normal density function and ϕ is the weight parameter.

The first issue to be tackled when dealing with PDF in practice is the time-to-maturity effect, whereby uncertainty about the underlying price decreases as maturity approaches and estimates of volatility in PDF decrease without real changes in uncertainty about the asset (Clews et al., 2000). Most techniques developed to eliminate the maturity effect rely on the spline method. The way to deal with the time-to-maturity effect in a parametric approach follows the strategy presented in Glatzer and Scheicher (2005). This strategy is explained in detail in section 3.2.

A second important issue is the fact that equity options in this paper are American-style and there is therefore an early-exercise premium. This implies that the relationship between distribution and option price derived by Cox and Ross (1976) is less direct, because it depends on the entire stochastic process of the underlying prices and not only on that at expiration date. Since most liquid options on exchange-traded financial instruments are American-style, the literature has adopted ad-hoc ways to tackle this issue. In Melick and Thomas (1997), the pricing function has been adapted and includes lower and upper bounds for option value. However, application in this case is not appropriate because this especification adds parameters to estimate and therefore reduces the degrees of freedom in a relatively limited dataset for equity options.

The strategy followed in this paper is also ad-hoc and lies in between the possibility of simply ignoring altogether the early-exercise premium, as in Carlson et al. (2005) or Healy et al. (2007), and adjusting option prices to create artificial European prices as in Bliss and Panigirtzoglou (2004) and Cincibuch (2004). The strategy here uses Barone-Adesi and Whaley (1987) (BAW) approximation to compute implied volatilities, both to filter the data set and then to create artificial constant-maturity options. In addition, Dupont (2001) argues that early-exercise premium only affects deep-in-the-money options, which are excluded from the sample in most studies.

A final remark concerns the existence of dividends in the options of banks in the sample. In this case, the daily dividend yields were included in the calculation of BAW implied volatilities estimation and constant-maturity prices, but then neglected in the parameter estimation of PDF. As a result, the constant-maturity PDF parameters estimated account for both early exercise and dividends.

2.2. Distance-to-Default

Distance-to-Default (DD) is a measure of company default risk developed by Moody's KMV (Bohn and Crosbie, 2003), which includes information from both financial statements and equity market prices. This indicator was designed to assess the likelihood of financial distress and default and it is based on the structural valuation model of corporate debt by Black and Scholes (1973) and Merton (1974). DD indicates the number of standard deviations from a default point at a fixed time horizon and is used more and more frequently by company analysis. Details of its derivation are explained in Box 1.

BOX 1. DERIVATION OF DISTANCE-TO-DEFAULT

In this model, a company value (represented by its assets A) is the sum of its debt D and equity E. Since debt is senior to equity, the latter can be expressed as a standard call option on the assets with strike price equal to the face value of the debt.

$$E = \max(0, V - D) \tag{8}$$

Applying the Black-Sholes option pricing formula, the Distance-to-Default t periods ahead is given by:

$$DD_t = \frac{\ln\frac{V_t}{D} + \left(\mu_A - \frac{1}{2}\sigma_A^2\right)t}{\sigma_A\sqrt{t}} \tag{9}$$

Where μ_A is the rate of growth of the company value (assets) and σ_A is the asset volatility[5].

In practice, asset value and asset volatility are not observable and must be estimated solving the following system of simultaneous equations:

$$E_t = V_t \cdot N(d_1) - e^{-rt} \cdot D \cdot N(d_2) \tag{10}$$

$$\sigma_E = \frac{V_t}{E_t}\sigma_A \cdot N(d_1) \tag{11}$$

where E_t is the value of equity, σ_t is the equity price return volatility and D is the face value of liabilities. The values of d_1 and d_2 are expressed as:

$$d_1 = \frac{\ln\frac{V_t}{D} + \left(r - \frac{1}{2}\sigma_A^2\right)t}{\sigma_A\sqrt{t}}, \text{ and } d_2 = d_1 - \sigma_A\sqrt{t}$$

The empirical implementation of DD in the literature uses market value as the value of equity; historical or at-the-money (ATM) implied volatility as equity price return volatility; government bond yields as the risk-free interest rate and the face value of short-term liabilities plus half of that of long-term liabilities as the face value of liabilities D. The time horizon t is usually set at one year.

Research on DD has several useful properties applicable to the analysis of bank distress. For instance, Bohn and Crosbie (2003) and Arora and Sellers (2004) show that it is a flexible tool that can be adjusted to analyze companies with dual-business lines such as financial institutions. Gropp et al. (2002) conclude DD is a complete and unbiased leading indicator to signal bank fragility, because it encompasses most elements of bank risk –asset returns, volatility (i.e. asset risk) and leverage– and constitutes a measure not affected by the presence of explicit or implicit safety nets. Gropp et al. (2004) found that DD has a predictive power of ratings' downgrading and it complements the information of other market-based indicators, such as bond spreads or CDS spreads.

[5] Assuming normal distribution of the company assets, the default probability can be computed directly as $p_t = N(-DD_t)$, where N() is the cumulative normal distribution.

In other recent financial stability literature, the potential of DD has been explored in analysis of financial integration in banking (Brasili and Vulpes, 2006), domestic and international bank contagion (Čihák and Ong, 2007), Ong et al., 2007), regulatory intervention (Chan-Lau and Sy, 2006) and even analysis of bailout size during the current financial crisis (Gapen, 2009). Currently, DD has become an important tool in financial stability analysis and is part of renown financial stability reports, such as the IMF's Global Financial Stability Report or ECB's Financial Stability Review.

3. Empirical Methodology

3.1. The Dataset

Given that the estimation of OIDD follows a two-step approach, this paper used one dataset with options to construct constant-maturity PDF and summary statistics and a complementary dataset to compute the DD series.

The sample used to compute constant-maturity PDF consists of six sets of large European banks' equity options, traded daily at Eurex[6] between January 3, 2005 and December 28, 2007. Eurex is one of the largest options and futures exchanges and banks' equity options are among the most liquid equity derivatives. All contracts are American options with maturity dates on every third Friday of the expiration month. The option cycle is March, June, September, and December and all banks in the sample have up to 60 contract months[7], the contract size is 100 and the minimum price change is 0.01 EUR. Table 1 summarizes the main features of each equity option set used in this paper.

Table 1. Dataset features.

	Bank	Eurex Code	Underlying Stock ISIN	Original Sample	Filtered Sample	Volume*	Open Interest*
1	**Deutsche Bank (DE)**	DBK	DE0005140008	351828	25137	328	5535
2	**Credit Suisse Group (CH)**	CSGN	CH0012138530	379707	38865	148	3133
3	**Société Générale (FR)**	SGE	FR0000130809	270995	10808	8	176
4	**UBS (CH)**	UBSN	CH0024899483	321752	36368	158	3168
5	**BNP Paribas (FR)**	BNP	FR0000131104	298921	38159	9	214
6	**Crédit Agricole (FR)**	XCA	FR0000045072	256730	7616	6	153

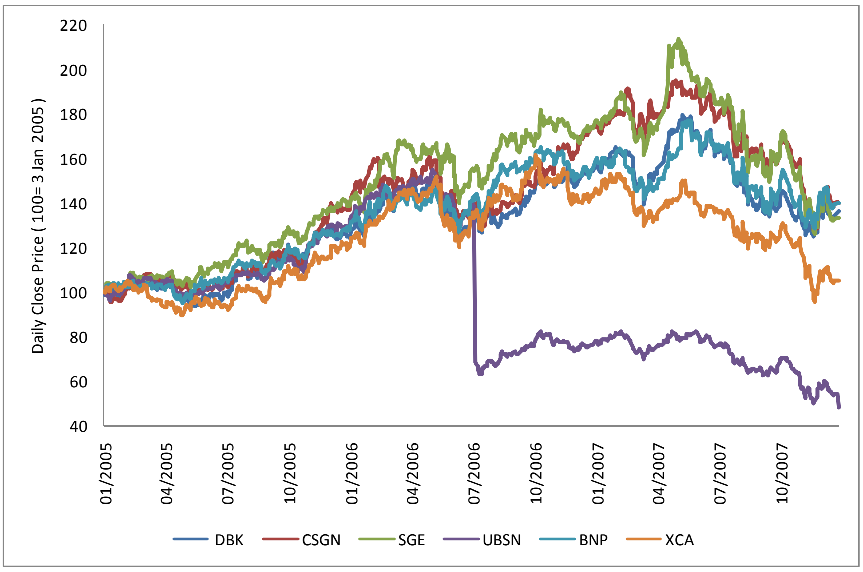
* Daily averages of filtered sample.

Underlying equities' data are daily closing prices, retrieved from Thomson Datastream, traded at exchanges and denominated in domestic currencies (see Figure 1 below). In contrast to other financial instruments, the estimation of PDF from equity options is affected by multiple listings and corporate actions. As large companies, the underlying stocks of banks in the sample may trade in several exchanges, but Eurex provides information on the underlying to ensure PDF are obtained using the appropriate underlying series. In addition to dividend payments, other corporate actions affect the option series, such as corporate restructuring and equity splits or reverse splits. For instance, UBS made effective a share split 2-for-1 on 10 July 2006, changing permanently the underlying price and making necessary an adjustment in the strike prices and other characteristics of existing option contracts.

[6] Contract specifications are available at www.eurexchange.com/trading/products/EQU/OPT/index/SX600/specifications.html

[7] This means that on any given trading day, option contracts are available for the three nearest successive calendar months, the three following quarterly months of the cycle thereafter, and the four following semi-annual months of the June and December cycle thereafter, and the two following annual months of the December cycle thereafter.

Figure 1. Banks' share prices 03/01/2005 – 28/12/2007.



The interest rates used to estimate PDF are 3-month money market rates, i.e. 3-month EURIBOR for euro area banks and CHF-denominated 3-month LIBOR for Swiss banks[8]. A maturity mismatch is inevitable when using these rates instead of a term structure of interest rates. Option market makers usually price options with term structures customized by their treasury departments or generated by financial data providers. However, 3-month money market rates were chosen because they fit the time horizon of the PDF and DD series; they are good proxies of borrowing costs and have shown low volatility across the term structure (European Central Bank, 2008). In addition, Bliss and Panigirtzoglou (2004) argue that these rates are less prone to be affected by monetary policy actions, are highly liquid and have little effect in the PDF estimation methodology.

Raw data must be filtered because daily trading is concentrated around a narrow set of near-the-money strikes (see Clews et al., 2000) and settlement prices are less informative of market expectations if strikes deviate too much, especially in the case of too deep in-the-money options. Options with less frequently traded options only may reflect previous days' traded prices and models from which the notional prices are derived[9]. Therefore, this dataset has been filtered to ensure informative constant maturity PDF. The exclusion criteria followed in this paper lead to exclude: 1) options with greater than 15% absolute moneyness[10], 2) options with fewer than five and more than 120 days to maturity; 3) options for which Barone-Adesi Whaley implied volatilities were not possible to compute, and 4) options with numeric deltas equal or greater than 0.99 or less than or equal to 0.01. See Table 1 for details.

To second set of data required to compute DD series includes balance sheet and market data. As all banks in the sample are blue-chip shares, interim and annual balance sheets are available and were retrieved from Thomson Worldscope. Remaining market data include risk-free interest rates, market value, number of shares outstanding, dividend yields and ATM implied volatilities[11]. The risk-free

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[8] EURIBOR and LIBOR historical data were obtained from the websites of the European Banking Federation (www.euribor.org) and the British Bankers Federation (www.bba.org.uk).

[9] Cox-Ross-Rubistein binomial model in this case.

[10] Absolute moneyness is defined as $\left|\frac{K}{S}-1\right|$. The thresholds in Bliss and Panigirtzoglou (2004) and Dumas et al. (1998) is 10% and in Glatzer and Scheicher (2005) is 25%. Other papers filter by time-adjusted moneyness $\left|\frac{K}{S}-1\right|\cdot\frac{1}{\sqrt{t}}$

[11] Average of put and call ATM implied volatilities based on the Cox- Ross-Rubistein binomial model, computed by Thomsom Datastream.

interest rate, which represents the expected rate of growth in the asset value over the analyzed horizon, is the 10-year government bond yield in the country of origin of each bank. These series were retrieved from Thomson Datastream along with the rest of market data.

3.2. OIDD Estimation Strategy

As discussed in section 2.1, the estimation of option-implied PDF entails tackling the time-to-maturity effect and dividend payments. This paper follows the strategy presented in Glatzer and Scheicher (2005) and the ad-hoc approach described in 2.1. For each trading day, the filtered dataset contains call and put contracts expiring between 5 and 120 days. A cross section of contracts by strike price is selected every trading day. The BAW implied volatilities estimated for data filtering are interpolated linearly across contract/maturity for horizons of 45 and 60 days. These implied volatilities are converted back into option prices using the BAW pricing function.

A new set of option prices with constant horizon constitutes the new dataset used to estimate the two log-normal mixture model described in section 2.1. using constrained non linear least squares. All cross-sections with insufficient number of strikes were eliminated. Table 2 summarizes the descriptive statistics of the constant horizon dataset.

Table 2. Constant-maturity dataset.

	Bank	Eurex Code	Sample Size		Trading Days	
			45 days	60 days	45 days	60 days
1	**Deutsche Bank (DE)**	DBK	13941	13380	755	716
2	**Credit Suisse Group (CH)**	CSGN	11441	11693	750	717
3	**Société Générale (FR)**	SGE	1069	1533	162	219
4	**UBS (CH)**	UBSN	11315	11096	761	715
5	**BNP Paribas (FR)**	BNP	11397	11491	704	681
6	**Crédit Agricole (FR)**	XCA	553	903	102	159

Table 2 shows that the sample size has been considerably reduced with respect to the original and filtered samples. Besides the necessary loss of data due to interpolation, the relatively smaller sample sizes and especially the number of trading days of Societé Générale (SGE) and Crédit Agricole (XCA) options show their relatively less liquidity at Eurex compared to other banks. Although this affects negatively the comparison of the OIDD series with other DD specifications in this paper, the increasing liquidity in equity options will enable the exercise in the future. Surprisingly, BNP Paribas (BNP) has a reasonable number of trading days in spite of just slightly larger average liquidity than those two banks, which can be interpreted as less liquidity per strike but a wider range of strikes per trading day. SGE and XCA are excluded from the OIDD assessment due to fragmented and short sample.

The parameters estimated in the previous step are used to compute daily comparable 45-day and 60-day constant-maturity PDF series and time series of forward-looking statistics. Selected option implied statistics[12] – standard deviation, mean– enter calibration of equation (9) to compute alternative OIDD specifications[13]. In particular, since equity value in equations (10) and (11) is represented by the market value, the latter can be expressed as a function of the number of shares

[12] See Annex 1 for details of the summary statistics of option-implied distributions obtained from a mixture of two log-normals.

[13] OIDD series were constructed using median, first quartile and third quartiles in addition to the benchmark series using mean. Results do not differ substantially and are therefore not reported.

outstanding (Q_t) multiplied by the current market price (P_t). The mean enters (10) and (11) as the market price (P_t^{OI}) and the estimated standard deviation σ_{OI} as the equity volatility:

$$OIDD_t = \frac{\ln\frac{V_t}{D} + \left(\mu_A - \frac{1}{2}\sigma_A^2\right)t}{\sigma_A\sqrt{t}} \tag{9'}$$

$$P_t^{OI}Q_t = V_t \cdot N(d_1) - e^{-rt} \cdot D \cdot N(d_2) \tag{10'}$$

$$\sigma_{OI} = \frac{V_t}{P_t^{OI}Q_t}\sigma_A \cdot N(d_1) \tag{11'}$$

where $d_1 = \frac{\ln\frac{V_t}{D} + \left(r - \frac{1}{2}\sigma_A^2\right)t}{\sigma_A\sqrt{t}}$, and $d_2 = d_1 - \sigma_A\sqrt{t}$

Following the literature, which states that the default point lies in between total liabilities and short-term liabilities (Bohn and Crosbie, 2003), the value of the debt D is the sum of short-term liabilities plus half of long term liabilities[14]. Balance sheet information in the sample is reported either quarterly or half-yearly and has been kept constant during the respective quarter/semester to generate daily series instead of being interpolated (see Gapen, 2009 for details).

The DD literature usually fixes the time horizon to one year. The OIDD series are computed with a time horizon of 45 and 60 days. A longer horizon is not feasible due to data limitations, i.e. maturity cycle and liquidity long time before maturity, as seen in section 3.1. This also justifies the application of the OIDD indicator to short-term analysis, trying to overcome the limitations noted in Gropp et al. (2004), given a stronger weight of market expectations in their calculation.

4. Results

In this section, we report PDF and OIDD results for four banks – DBK, CSGN, UBSN and BNP – and provide stylized facts about the PDF for selected trading days and then for OIDD vis-à-vis DD series constructed using 45-day and 60-day historical volatilities and with ATM implied volatilities. The joint analysis presents useful analytic properties of PDF and OIDD.

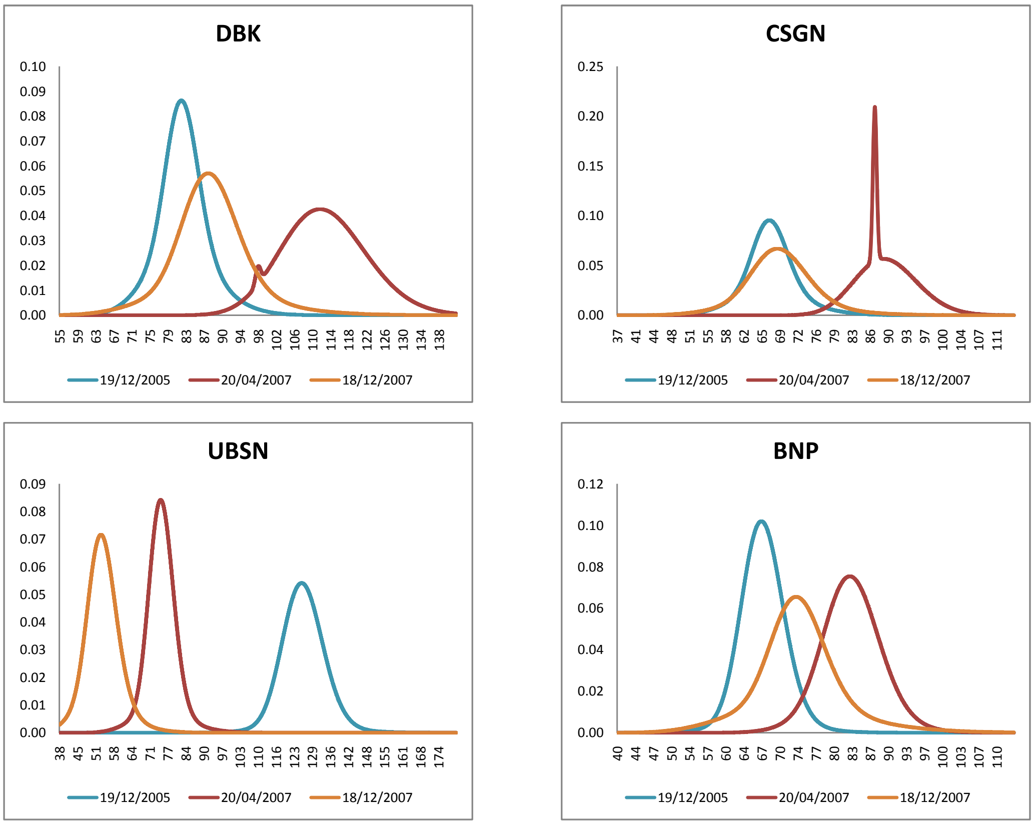
Three dates were chosen to comment on the PDF results and are depicted in Figure 2. The first trading day, 19 December 2005, is an example of the bull market during most of the sample, until Summer 2007. Even though that trading day reported a correction of previous day gains, the shape of the PDF in that period does not show significant skews but a slightly bullish one, nor fat tails and exhibits a normal-like appearance.

The PDF shapes on 20 April[15] show two relevant features. There is a clear bimodal distribution, especially for the longer term PDF, as a reflection of the change in trend of the market. Bi-modal distributions illustrate uncertainty about the direction of the market and those weeks set an end to the long bull market following the dot-com burst. Since one could argue that that correction could have been followed by a bullish recovery, options show that market participants were pondering both scenarios. Another interesting feature is the spikes in the PDF, which signal low trade volumes and force the mixture of lognormals to exhibit overshooting volatilities.

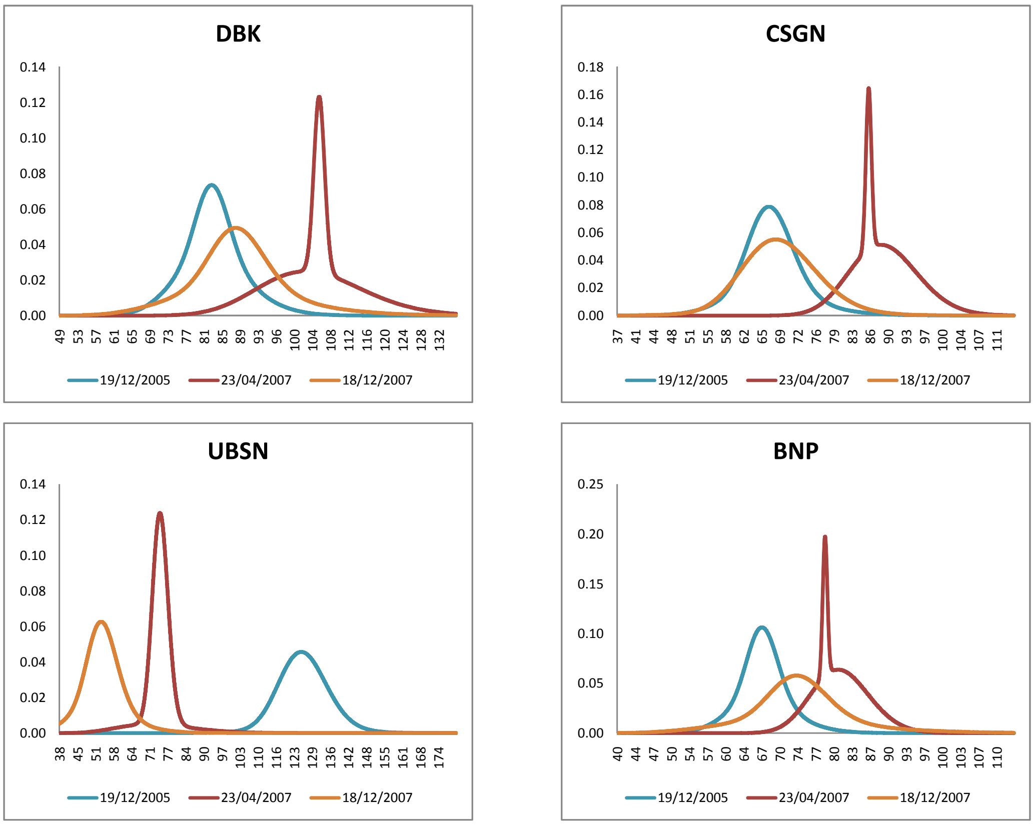
[14] Using Thomson Worldscope data, short term liabilities is the sum of items "Deposits" and "Short Term Debt and Current Portion of Long Debt" and long term liabilities is the sum of items "Long Term Debt", "Deferred taxes" and "Other Liabilities".
[15] April 20, 2007 for the 45-day PDF, April 23, 2007 for the 60-day PDF.

Figure 2. PDF for selected trading days.

45-day Constant-maturity



60-day Constant-maturity

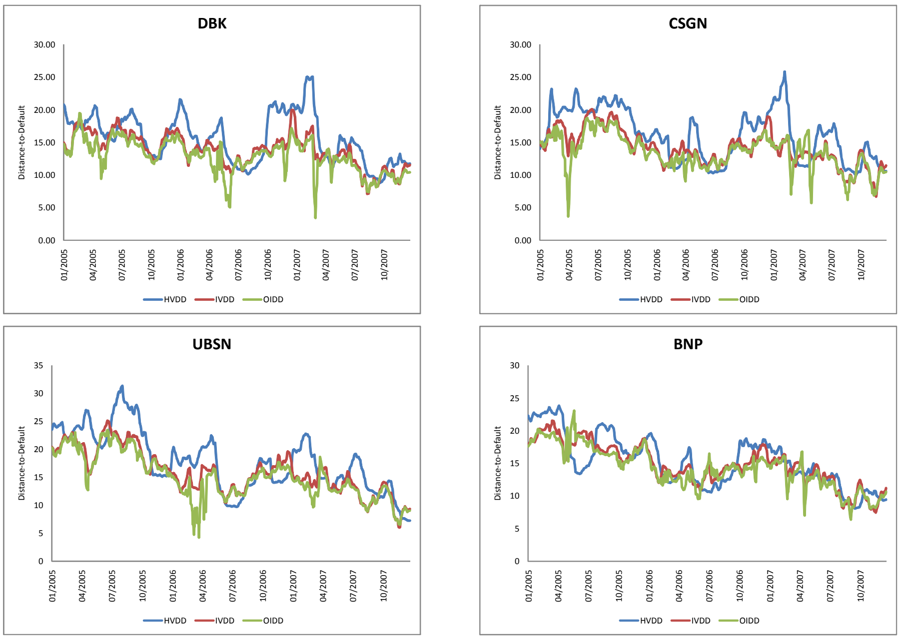


The inconvenient arising from these trading irregularities, which could be particularly high for banks with low trading volumes (typically small banks or banks in a troubled situation), is that it affects the estimation of OIDD series and either render them hard to interpret or impede their estimation.

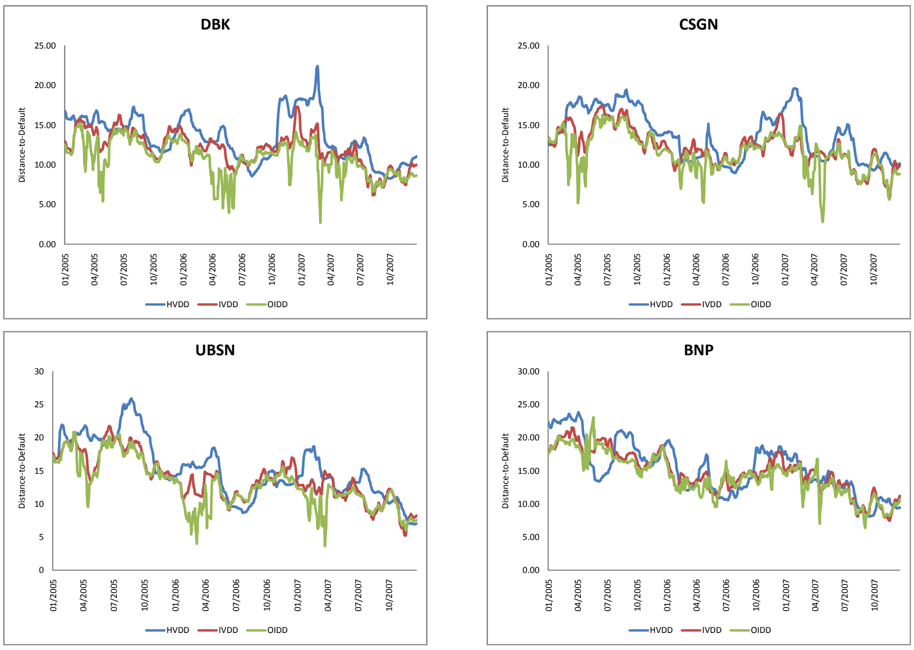
Turning to OIDD series, the charts in Figure 3 show some features of the series using option-implied information. First, the OIDD series in all cases and time horizons follow the trends of series based on historical—volatility-based (HVDD) and implied-volatility-based (IVDD). In fact, the benchmark specification of OIDD uses the mean of the distribution, which by construction, does not differ significantly from the underlying prices and the difference lies in the volatility estimates.

Figure 3. OIDD vis-à-vis alternative Distance-to-Default measures.

45-day Constant-maturity (weekly moving average)



60-day Constant-maturity (weekly moving average)



As mentioned lines above, alternative specifications were assessed, using median, first and third quartiles as market prices to calibrate option-implied asset values. The results are almost identical to the benchmark model and are therefore not reported. Compared to the HVDD, OIDD series lead in all cases and regardless of the time window used to compute HV. The difference is less clear when compared to IVDD. This similarity may stem from the fact that, all other things equal, IV used in to compute the series has implicitly a constant horizon up to the maturity date that may coincide at some points in time with the time horizon of option-implied PDF volatilities.

From the graphs can also be noticed frequent periods of noise in the data that produce higher than normal volatility. This can be understood as periods with high uncertainty in the markets that affect liquidity in option markets. On one hand, the option-implied volatilities from PDF estimation tend to be visibly and naturally higher than the previous observations and on the other, they overreaction can lead to simply lack of observations to compute robust PDF on that trading day. In any case it is a result of a slump in liquidity.

5. Comparison with Alternative DD Specifications.

To conclude the analysis of the OIDD properties vis-à-vis the alternative DD specifications, we conducted a series of tests of pairwise cointegration and causality tests on the series, following standard Johansen and Granger approaches (see Chan-Lau and Kim, 2004 for a review). First, unit roots in the series were tested using Augmented Dickey-Fuller and Phillips-Perron tests. Table 3 summarizes the results, suggesting that DD series do have unit roots in levels at regular confidence levels[16]. As a result, equilibrium relationship

Table 3. Unit Root Tests.

45-DAY CONSTANT MATURITY						
	Augmented Dickey-Fuller			**Phillipps-Perron**		
	HDD	**IVDD**	**OIDD**	**HDD**	**IVDD**	**OIDD**
Deutsche Bank	-2.922	-2.498	-2.995	-2.561	-2.603	-3.797
Crédit Suisse	-2.810	-2.859	-2.575	-2.248	-2.498	-3.519
UBS	-2.359	-2.078	-2.243	-1.560	-1.887	-2.219
BNP Paribas	-2.834	-2.150	-1.900	-2.060	-1.961	-1.928

60-DAY CONSTANT MATURITY						
	Augmented Dickey-Fuller			**Phillipps-Perron**		
	HDD	**IVDD**	**OIDD**	**HDD**	**IVDD**	**OIDD**
Deutsche Bank	-2.785	-2.311	-2.992	-2.311	-2.565	-3.321
Crédit Suisse	-2.411	-2.561	-2.614	-1.689	-2.490	-3.199
UBS	-1.969	-2.042	-2.023	-1.113	-1.848	-2.134
BNP Paribas	-3.016	-1.481	-1.535	-2.176	-1.449	-1.608

Critical Values: -3.441 (1%) ; -2.866 (5%); -2.569 (10%)

The Johansen cointegration trace test was conducted between OIDD series and HVDD/IVDD results are reported in Table 4. Coitegration between OIDD series and other DD specifications was found for all banks and time horizon, without exception. This means that OIDD has strong equilibrium relationships to the DD measures previously developed in the literature.

[16] Unit roots have been tested in first differences and all variables are stationary.

Table 4. Johansen's Cointegration Rank Tests.

45-DAY CONSTANT MATURITY		
	OIDD and HVDD	**OIDD and IVDD**
Deutsche Bank	49.509	72.617
Crédit Suisse	42.399	85.027
UBS	24.976	49.309
BNP Paribas	29.781	76.365
60-DAY CONSTANT MATURITY		
	OIDD and HVDD	**OIDD and IVDD**
Deutsche Bank	48.907	51.901
Crédit Suisse	49.739	63.211
UBS	20.607	43.744
BNP Paribas	27.742	71.577

Finally, the third test run between these series is a simple price discovery test, aimed at assessing whether the OIDD dominates as leading indicator of bank stress. Table 5 reports the results of these tests for different lags, with their respective F statistics and p-values (in italic). The results using 60-day constant-maturity are the same and therefore not reported here.

Table 5. Granger Causality Test.

45-DAY CONSTANT MATURITY

	Null Hypothesis			
Lag	OIDD does not cause HVDD	HVDD does not cause OIDD	OIDD does not cause IVDD	IVDD does not cause OIDD
	Deutsche Bank			
1	48.844	0.315	10.854	5.750
	0.000	*0.575*	*0.001*	*0.017*
5	4.227	7.944	1.898	23.945
	0.001	*0.000*	*0.092*	*0.000*
10	2.882	3.775	1.626	10.647
	0.002	*0.000*	*0.095*	*0.000*
20	2.179	3.089	1.429	6.623
	0.002	*0.000*	*0.101*	*0.000*
	Crédit Suisse			
1	23.531	0.807	2.688	7.365
	0.000	*0.369*	*0.102*	*0.007*
5	3.519	2.853	2.685	17.641
	0.004	*0.015*	*0.021*	*0.000*
10	3.016	3.024	2.150	8.325
	0.001	*0.001*	*0.019*	*0.000*
20	1.806	2.465	1.232	4.476
	0.017	*0.000*	*0.220*	*0.000*

	Null Hypothesis			
Lag	OIDD does not cause HVDD	HVDD does not cause OIDD	OIDD does not cause IVDD	IVDD does not cause OIDD
	UBS			
1	17.799	0.331	3.970	11.874
	0.000	*0.566*	*0.047*	*0.001*
5	2.348	2.291	2.330	18.210
	0.040	*0.044*	*0.041*	*0.000*
10	1.666	1.587	2.324	11.220
	0.085	*0.106*	*0.011*	*0.000*
20	1.262	1.268	2.261	5.671
	0.197	*0.193*	*0.001*	*0.000*
	BNP Paribas			
1	15.363	2.828	10.299	18.615
	0.000	*0.093*	*0.001*	*0.000*
5	2.154	2.048	2.214	20.919
	0.057	*0.070*	*0.051*	*0.000*
10	2.499	1.479	3.378	13.757
	0.006	*0.143*	*0.000*	*0.000*
20	2.004	1.453	2.249	7.181
	0.006	*0.091*	*0.002*	*0.000*

In general, results from Table 5 suggest that OIDD clearly dominates the price discovery process when compared to HVDD, as Figure 3 suggested. The exception is Deutsche Bank, where only in lag 1 there is clear evidence of lead. If OIDD is compared to IVDD, there is no clear pattern of a leading indicator, because rejection of the null hypothesis depends on the confidence level, meaning that there is alternate lead between indicators or no dominant indicator. However, the OIDD series contain noise from liquidity shocks and this could explain the lack of a clearer pattern. In any case, volatility in OIDD suggests alone stress in the markets, pointing out at uncertainty and episodes of distress in banks in the short term.

6. Conclusions.

The research conducted in this paper has served as a user's guide to explore the information potential of equity option prices to assess bank distress with the introduction of the Option-Implied Distance-to-Default. As a sub product, option-implied PDF complement the information of indicator, especially in episodes of higher volatility in stock markets and sudden liquidity stops, where the information of OIDD series becomes more difficult to interpret and its calculation can even become unfeasible.

However, the tests of equilibrium relationships between OIDD series and alternative DD measures show that OIDD shares the properties of completeness and absence of bias discussed in Gropp et al. (2004) with the other DD measures. In addition, OIDD is a leading indicator with respect to DD measures derived from historical volatilities and is at least as forward looking as DD measures incorporating information of option prices via implied volatilities.

In future versions of this paper, improvements of OIDD will include: 1) additional filtering of data to avoid overshooting volatility; 2) robustness checks using alternative methodologies (e.g. splining) to estimate PDF taking into account we are dealing with American-style equity options that pay dividends; 3) alternative inputs for the DD calibration, e.g. interpolated balance-sheet information or other definitions of market value; and 4) more sophisticated price discovery tests to precisely assess the feature of leading indicator, after correction of liquidity-driven anomalies.

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Annex 1. Moments of the two log-normal distribution

Because the mixing, the moments of the mixed distribution of the option prices are not straightforward expressions. McManus (1999) derived the following expressions:

Cumulative distribution function

$$F(S)=\hat{\phi}\left[\int_0^S L\left(S\middle|S,\alpha_1,\beta_1\right)\,dS\right]+\left(1-\hat{\phi}\right)\left[\int_0^S L\left(S\middle|S,\alpha_2,\beta_2\right)\,dS\right]$$

$$\alpha_i=\log(S)+\left(\mu_i-\tfrac{1}{2}\sigma_i^2\right)\cdot t\,,\ \beta_i=\sigma_i\sqrt{t}\quad \text{for i=1,2}$$

Mean

$$\hat{\hat{S}}=\hat{\phi}e^{\hat{\alpha}_1+\frac{1}{2}\hat{\beta}_1^2}+\left(1-\hat{\phi}\right)e^{\hat{\alpha}_2+\frac{1}{2}\hat{\beta}_2^2}$$

Variance

$$\hat{\sigma}^2=\hat{\phi}\hat{\sigma}_1^2+\left(1-\hat{\phi}\right)\hat{\sigma}_2^2+\hat{\phi}\left(1-\hat{\phi}\right)\cdot\left(\hat{\mu}_1-\hat{\mu}_2\right)^2$$