An Estimated Dynamic Stochastic General Equilibrium Model for Estonia

Paolo Gelain and Dmitry Kulikov*

Bank of Estonia
Estonia pst 13
15095 Tallinn
Estonia
phone: +372 6680777
e-mail: dmitry.kulikov@epbe.ee

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Abstract

This paper presents the first version of an open economy Dynamic Stochastic General Equilibrium (EP DSGE) model for Estonia. The model is designed to highlight the main driving forces behind Estonian business cycle and to understand how the euro area economic shocks and monetary policy transmit into the small open economy of Estonia. EP DSGE is a two area DSGE model incorporating New Keynesian features such as the nominal price and wage rigidity, variable capital utilization, investment adjustment costs, as well as other typical features like external consumption habits — both for Estonia and the euro area part of the model. It is rich in structural shocks such as technology, preference, mark–up, etc. The ultimate goal of the new model is to be used in simulation exercises, policy advice and forecasting at the Bank of Estonia.

JEL classification: E4, E5
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Non-technical summary

This paper reports theoretical foundations and empirical results for the first version of an open economy Dynamic Stochastic General Equilibrium model for Estonia developed at the Bank of Estonia. One of the main goals of building a DSGE model of the Estonian economy is to use it for understanding the monetary policy and export–import linkages between a small open economy of Estonia and much larger economy of the euro area. The list of other potential tasks for the Bank of Estonia's Dynamic Stochastic General Equilibrium model include simulation exercises, policy advice and forecasting of the main macroeconomic aggregates.

The Bank of Estonia's DSGE model presented in this paper is a rich New Keynesian DSGE model which incorporates many important features that are found to be essential for reproducing the complex dynamics and persistence of the real–world macroeconomic time series. It incorporates the key ingredients that are needed to effectively describe the functioning of Estonian economy:

- The currency board regime, free capital mobility and resulting lack of an independent monetary policy conducted by the national central bank. The monetary policy of Estonia is effectively imported from the ECB and therefore depends on the euro area business cycle. The spread between domestic and euro area interest rates is the key for understanding the macroeconomic developments in Estonia over the last decade;

- Estonian economy is a textbook example of a small open economy in terms of the openness to foreign trade as well. The impact of the euro area business cycle on the domestic economy of Estonia via the mutual trade links is very important;

- Real and nominal convergence still features prominently in the main macroeconomic aggregates of Estonia. However, this version of the Bank of Estonia's DSGE model is specified for the business cycle frequency only, and filters out the long run dynamics of the empirical data. The future revisions of the model will address this issue with due care.

The unique feature of the Bank of Estonia's DSGE model presented in this paper is inclusion of a fully specified, calibrated DSGE model for the euro area. The economy of Estonia is considered to be a small open economy on the fringes of the euro area — its main trading partner and *de facto* implement of Estonia's monetary policy due to the currency board arrangement and free capital mobility between the two areas. The euro area part of the model is a fully articulated New Keynesian DSGE model of Smets and Wouters (2003), subject to its own set of ten structural shocks, that is designed to reproduce the monetary policy conducted by the ECB, and to act as a foreign market for Estonian exports and imports. The two area setup of the Bank of Estonia's DSGE model allows for meaningful simulations of the euro area monetary policy effects on the home economy of Estonia. The forthcoming integration of Estonian economy into the euro area makes a thorough understanding of these effects particularly important.

Empirical part of this paper reports estimation results for model's structural parameters, impulse response functions and variance decomposition of the main variables. Out of 52 structural parameters in the Bank of Estonia's DSGE model, 34 are estimated using a data sample consisting of 14 macroeconomic series for Estonia and the euro area. Statistical estimates of the main parameters are largely in line with previous studies for Estonia, when a direct comparison can be made. It is also worth mentioning that the net foreign asset position of Estonia has been found an important and statistically significant factor in explaining the country risk premium, but the results suggest that other explanatory factors may be warranted.

The empirical relevance of structural shocks is assessed using the variance decomposition. It is found that the most important domestic shocks in explaining the variability of Estonian macroeconomic series are the price mark up shock, that often dominates the other shocks contributing 50% or more of the variability of the state variables, and the technology shock. The euro area shocks also play a very significant role in driving the dynamics of Estonian macroeconomic aggregates. Among the most prominent euro area shocks that affect Estonia are the labor supply, the interest rate and the technology shock.

As mentioned previously, the first version of the Bank of Estonia's DSGE model focuses on the business cycle fluctuations of the main Estonian macroeconomic aggregates, leaving their long run trends aside. The future developments of the model are likely to incorporate the long run dynamics as well, considering that Estonia is still subject to effects of real and nominal convergence stemming from its catch-up with the developed euro area economies. Other areas of the future developments of the model include incorporation of the financial sector together with relevant frictions, adding the housing sector combined with collateral-constrated households, and expanding the government sector in the model.

1 Introduction

This paper reports theoretical foundations and empirical results for the first version of an open economy Dynamic Stochastic General Equilibrium model for Estonia developed at the Bank of Estonia. One of the main goals of building a DSGE model of the Estonian economy is to use it for understanding the monetary policy and export–import linkages between a small open economy of Estonia and much larger economy of the euro area. The list of other potential tasks for the Bank of Estonia's Dynamic Stochastic General Equilibrium model, henceforth abbreviated as EP DSGE, include simulation exercises, policy advice and forecasting of the main macroeconomic aggregates.

For now, all these jobs are carried out at Eesti Pank by EMMA model, see Kattai (2005). EMMA is a traditional medium scale backward–looking macroeconomic model estimated on an equation-by-equation basis. It incorporates a number of theory–based restrictions, but unlike a typical DSGE model is not derived from the ground up using utility and profit maximization framework of the modern macroeconomics.

Recently, a new breed of microfounded DSGE models that incorporate a large number of structural shocks, nominal and real rigidities and other features needed to describe persistence of the real–world macroeconomic time series has received a lot of attention by the leading monetary policy institutions around the world; refer to Tovar (2008) for a recent survey of DSGE modeling at the central banks. These models became possible thanks to advances in the macroeconomic theory, offering an advantage over the traditional backward-looking models in terms of clear interpretations of the main relationships among the forward-looking economic agents that are subject to the uncertainty stemming from a large number of well–motivated structural disturbances. In addition, the newly found popularity of DSGE models in many central banks comes from recent developments in powerful computational methods that permit statistical inference for many structural parameters using the real-world macroeconomic data.

Likewise, the first version of EP DSGE model presented in this paper is a step toward eventual phasing out of EMMA at the Bank of Estonia as the main tool for simulation of different macroeconomic scenarios and policy advice. However, a substantial amount of work remains to be done before the new model is sufficiently refined and ready to be used by the policy makers.

A DSGE approach to modeling Estonian economy has been previously attempted in Colantoni (2007) and Lendvai and Roeger (2008). Colantoni (2007) estimates a two area DSGE model using Estonian macroeconomic data with a goal of studying the interest rate channel of the monetary policy transmission between Estonia and the euro area. Compared to Colantoni (2007), the EP DSGE model has similar objectives, but its structure has been refined to reflect the existing monetary policy regime between the two areas, and the statistical inference has been substantially improved. The second paper by Lendvai and Roeger (2008) calibrates an open economy DSGE model with several types of households, a housing sector and separate tradable and non-tradable production sectors in order to assess the relative importance of productivity growth and credit expansion in driving the long run trends of the main Estonian macroeconomic aggregates over the last decade. In contrast to Lendvai and Roeger (2008), where a specific simulation exercise is carried out to understand the long run trends, EP DSGE model is focused on the effects of the euro area monetary policy and export–import links on the economy of Estonia at the business cycle frequency.

The first version of EP DSGE model presented in this paper is a rich New Keynesian DSGE model which incorporates many important features that are found to be essential for describing the complex dynamics and persistence of the real–world macroeconomic time series. The key references for the model are papers by Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005) and Adolfson et al. (2005). Specifically, EP DSGE model incorporates external habit formation in consumption, investment adjustment costs, price and wage rigidities and indexation to the past inflation, and variable capital utilization.

In addition to these frictions, there are eleven structural shocks that drive the dynamics of Estonian economy in the model. Among the fundamental shocks are production technology and investment–specific technology innovations, labour supply and preference shocks, an equity premium shock, and the government consumption innovation. The domestic "cost push" disturbances include a stochastic price mark–up in the production sector and a wage mark–up in the labour demand function. The interactions between the euro area and the economy of Estonia are driven by stochastic mark–ups in the export and import sectors, as well as an idiosyncratic risk premium shock in the equation linking domestic and euro area interest rates.

The open economy aspect of the EP DSGE model is based around the paper by Adolfson et al. (2005). In particular, export and import firms in the model operate by selling differentiated consumption goods on foreign and domestic markets subject to the local currency price stickiness and indexation to the past inflation. In contrast to Adolfson et al. (2005), trade between the economies of Estonia and the euro area in EP DSGE model takes place the final consumption good only. This simplification is due to unavailability of suitably disaggregated export and import price indices in the Estonian foreign trade statistics. Other differences from Adolfson et al. (2005) include omission of the unit root technology in favor of the stationary one, missing working capital channel of monetary policy, much less articulated modeling of the government sector, as well as inclusion of a fully specified, calibrated DSGE model for the euro area.

The latter feature of EP DSGE model is particularly important considering the design goals and prospective use of the model at the Bank of Estonia. The economy of Estonia is considered to be a small open economy on the fringes of the euro area — its main trading partner and *de facto* implement of Estonia's monetary policy due to the currency board arrangement and free capital mobility between the two areas. The euro area part of EP DSGE is a fully articulated New Keynesian DSGE model of Smets and Wouters (2003), subject to its own set of ten structural shocks, that is designed to reproduce the monetary policy conducted by the ECB, and to act as a foreign market for Estonian exports and imports. The two area setup of EP

DSGE model allows for meaningful simulations of the euro area monetary policy effects on the home economy of Estonia. The forthcoming integration of Estonian economy into the euro area makes a thorough understanding of these effects particularly important.

The empirical results obtained and reported in this paper can be considered satisfactory for the fist version of the model. Statistical estimates of the main structural parameters are largely in line with previous studies for Estonia, when a direct comparison can be made. However, there are a few areas that await an improvement in the future versions of the model. The external sector is of particular concern, where both the dynamics of trade links with the euro area, as well as the role of net foreign assets in picking up the spread between the domestic and euro area interest rates need further examination.

The paper is structured as follows. Section 2 provides a short summary of the main building blocks of EP DSGE model avoiding excessive technical details. Section 3 and section 4 describe the key equations of the model pertaining to the economies of Estonia and euro area respectively. The log–linearized versions of these equations are reported in Appendix 8.2 and 8.3. An overview of the estimation methodology, data series and calibrated parameters is given in section 5. Main empirical results are discussed in detail in section 6. Finally, conclusion summarizes the main findings of the paper.

2 A short summary of EP DSGE model

The EP DSGE model presented in this paper takes into account the following key features of the Estonian economy:

- The currency board regime, free capital mobility and resulting lack of an independent monetary policy conducted by the national central bank. The monetary policy of Estonia is effectively imported from the ECB and therefore depends on the euro area business cycle.[1] The spread between domestic and euro area interest rates is the key for understanding the macroeconomic developments in Estonia over the last decade;

- Estonian economy is a textbook example of a small open economy in terms of the openness to foreign trade as well. The impact of the euro area business cycle on the domestic economy of Estonia via the mutual trade links is very important;

- Real and nominal convergence still features prominently in the main macroeconomic aggregates of Estonia. However, the first version of EP DSGE model in this paper is

[1] Prior to re-pegging Estonian Kroon to Euro in 1999 it was fixed to Deutsche Mark at the rate 1 DM = 8 EEK. During the second half of 1990-s Estonian banking system was still not completely integrated with the European and Scandinavian ones. The Asian financial crisis of 1997 and the subsequent Russian financial crisis of 1998 have changed the landscape of Estonian banking sector, effectively bringing all major Estonian banks into the hands of Scandinavian owners. Since then the spreads between domestic and euro area interest rates has narrowed dramatically.

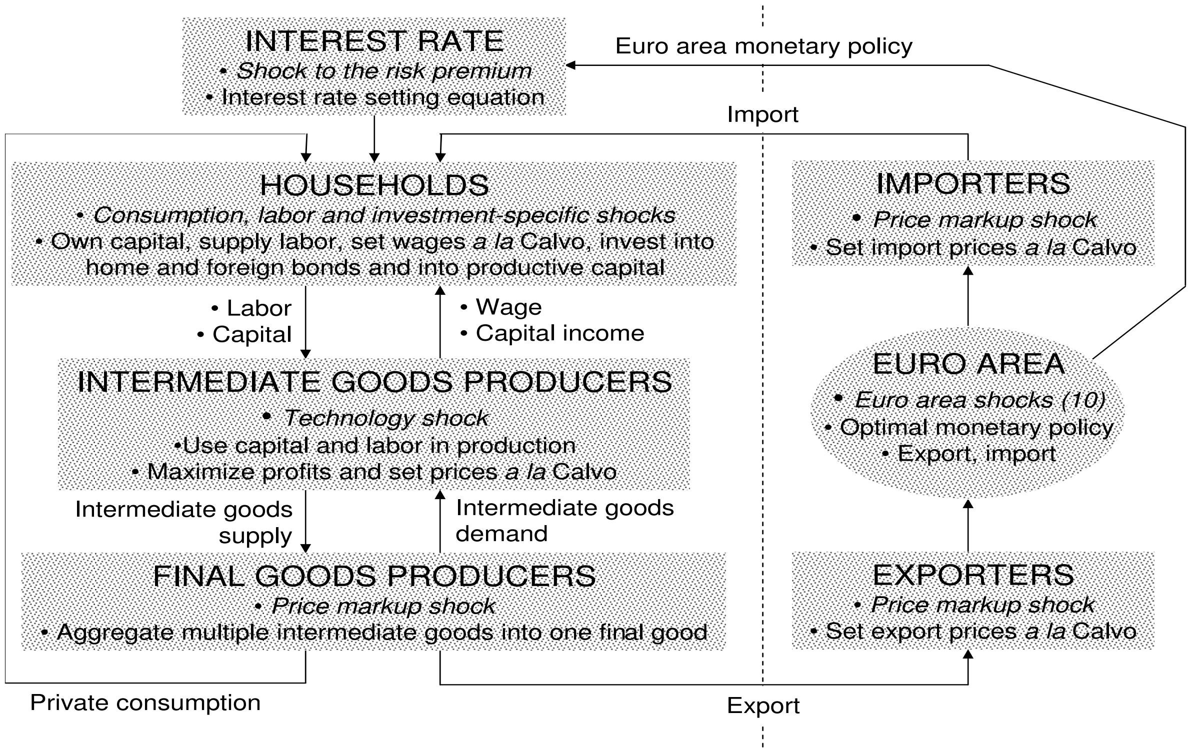

Figure 1: An overview of EP DSGE model

specified for the business cycle frequency only, and filters out the long run dynamics of the empirical data.[2]

Figure 1 provides an overview of the main building blocks and links of the EP DSGE model. It is a two area DSGE model, consisting of a small open economy DSGE model for Estonia and a large closed economy DSGE model for euro area. The two parts are linked through the monetary policy channel — one way from the euro area to Estonia — and by export–import trading links, where the euro area economy serves as a source of imports to the home economy of Estonia and generates demand for Estonian exports.[3] Foreign trade with the euro are is assumed to be in terms of the composite final consumption good only.

The Estonian part of EP DSGE is a fairy typical small open economy DSGE model that is similar to Adolfson et al. (2005). It has 24 state variables and 11 structural shocks, and consists of the following main sections:

- Households own labor and capital, optimize their consumption and supply of working hours across time, set wages in Calvo (1983) manner subject to labour demand from the

[2]The future versions of the model are likely to address this issue directly, by incorporating the unit root technology and suitable steady state inflation dynamics.

[3]The breakdown of Estonian trade statistics in 2008 reveals that 70% of the foreign trade takes place with the EU countries. However, the share of euro area countries in the foreign trade is around 25% because many of Estonia's major trading partners in the Baltic sea region, such as Latvia, Lithuania, Sweden, Denmark and Poland, are not euro area members. Since these countries are themselves highly open to the euro area trade, the assumption of EP DSGE model about export–import trading links with the euro area is a reasonable approximation.

labour aggregator, and invest into domestic and foreign bonds as well as the productive capital;

- Firms are of four types: final good producers operating in perfectly competitive market, monopolistically competitive domestic intermediate good producers that set prices in Calvo (1983) manner, and export and import firms that set prices of differentiated consumption goods in Calvo (1983) manner;
- Government sector is assumed to follow a balanced budged fiscal policy driven by an exogenous government consumption shock;
- Domestic nominal interest rates are linked to the euro area ones via the uncovered interest rate parity condition $R_t^n = \Omega(FA_t, \epsilon_t^{\text{risk}})\, R_t^{n,\text{EA}}$, where the next period expected change in the nominal exchange rate is set to zero because of the currency board.[4] Instead, the idiosyncratic part of the interest rate spread is picked up by ϵ_t^{risk}.

The euro area part of EP DSGE is a calibrated version of Smets and Wouters (2003) closed economy DSGE model with 13 state variables and 10 structural shocks. Calibrated values of all structural parameters are taken directly from Smets and Wouters (2003) study.[5]

3 Key equations: Estonian economy

3.1 Households

Household $i \in [0,1]$ maximizes its intertemporal utility function by choosing how much to consume $\{C_t^i : t \geq 0\}$, how much to invest today in order to build the capital that will be used in production tomorrow $\{I_t^i : t \geq 0\}$, the hours it wants to work $\{L_t^i : t \geq 0\}$, the utilization rate of capital $\{z_t^i : t \geq 0\}$, how much capital to rent to the firms $\{K_t^i : t \geq 0\}$, and how many domestic $\{B_t^i : t \geq 0\}$ and euro area $\{B_t^{i,EA} : t \geq 0\}$ bonds to hold:[6]

$$\max_{\{C_t^i, I_t^i, L_t^i, z_t^i, K_t^i, B_t^i, B_t^{i,EA}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^{\beta} \left[\frac{1}{1-\sigma_c} \left(C_t^i - hC_{t-1} \right)^{(1-\sigma_c)} - \frac{\varepsilon_t^L}{1+\sigma_L} \left(L_t^i \right)^{(1+\sigma_L)} \right]$$

where $\log \varepsilon_t^{\beta} = \rho_{\beta} \log \varepsilon_{t-1}^{\beta} + u_t^{\beta}$, $u_t^{\beta} \sim N(0, \sigma_{\beta}^2)$ is the discount factor shock (or preference shock) and $\log \varepsilon_t^L = \rho_L \log \varepsilon_{t-1}^L + u_t^L$, $u_t^L \sim N(0, \sigma_L^2)$ is the labor supply shock. Households' behavior

[4] In fact, the institutional arrangement of the 17 years old currency board system in Estonia rules out possibility of an unilateral Euro peg rate changes by the central bank of Estonia. All such changes must be enacted by the national parliament and therefore are likely to take some time before coming into effect.

[5] Future upgrades of the EP DSGE model are likely to move to the part-estimated – part-calibrated euro area part due to differences in the sample period between Smets and Wouters (2003) and the present paper.

[6] Household's domestic bond holdings B_t^i in the Estonian economy part of EP DSGE model can be thought of as a proxy for per capita net short-term saving/borrowing by the residents in Estonian banks; ditto for the euro area bonds $B_t^{i,EA}$ in foreign banks. There is no market for short-term government obligations in Estonia, and almost all financing needs of Estonian households and firms are met by the banking sector. The first version of EP DSGE presented in this paper does not explicitly model the banking sector, an omission that is likely to be addressed in the future versions of the model.

is characterized by external habit formation, whose degree is governed by parameter h. A household has a positive utility in period t only if it able to consume more than a fraction h of the economy-wide average per-household consumption at $t-1$. The inverse of the intertemporal elasticity of substitution in consumption (or equivalently the coefficient of relative risk aversion) and the inverse of the elasticity of work effort with respect to the real wage are denoted by σ_c and σ_L respectively.

The maximization is constrained. Firstly, at every time period $t \geq 0$ each household faces the following budget constraint in real terms:[7]

$$C_t^i + I_t^i + B_t^i + \bar{e}B_t^{i,EA} =$$
$$R_{t-1}^n \frac{B_{t-1}^i}{\pi_t^c} + \Omega(FA_{t-1}, \epsilon_{t-1}^{\text{risk}})\bar{e}R_{t-1}^{n,EA}\frac{B_{t-1}^{i,EA}}{\pi_t^c} + \frac{W_t^i}{P_t^c}L_t^i + R_t^k z_t^i K_{t-1}^i - \Psi\left(z_t^i\right)K_{t-1}^i + T_t^i + Div_t^i$$

where $R_t^n = (1 + i_t)$, T_t^i are the net transfers, Div_t^i are dividends from the final good sector firms (owned by the households), π_t^c is the gross inflation rate ($1+\frac{P_t^c - P_{t-1}^c}{P_{t-1}^c}$ or equivalently $\frac{P_t^c}{P_{t-1}^c}$, with P_t^c the CPI), $\bar{e}$ is the fix nominal exchange rate, W_t^i is the wage earned by the household, R_t^k is the return on capital, $\Psi(z_t^i)$ is the cost of capital utilization function (with $\Psi(1) = 0$) and the country specific risk premium function:

$$\Omega(FA_t, \epsilon_t^{\text{risk}}) = \exp\left(-\phi_{\text{fa}}FA_t + \log\epsilon_t^{\text{risk}}\right), \tag{1}$$

where $FA_t \equiv \frac{\bar{e}B_t^{EA,n}}{P_t^D}$ is the net foreign asset position of the economy of Estonia,[8] where P_t^D is the domestic consumption price index, and $\log\epsilon_t^{\text{risk}} = \rho_{\text{risk}}\log\epsilon_{t-1}^{\text{risk}} + u_t^{\text{risk}}$, $u_t^{\text{risk}} \sim N(0, \sigma_{\text{risk}}^2)$ is an idiosyncratic component of the country specific risk.[9] The idea behind this formulation is to capture the imperfect integration in the international financial markets. The higher the debt a country has with the rest of the world, the higher the risk of a default and then the higher the risk premium it has to pay. Moreover, the introduction of this risk premium is needed to ensure a well defined steady state in the model (See Schmitt-Grohè and Uribe (2003)).

Secondly, each household faces the capital accumulation equation at every time period $t \geq 0$:

$$K_t = (1-\delta)K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t\,x_t\,, \tag{2}$$

where δ is the depreciation rate of capital, $\log x_t = \rho_x\log x_{t-1} + u_t^x$, $u_t^x \sim N(0, \sigma_x^2)$ is a stationary investment–specific technology shock, and $S(I_t/I_{t-1})$ is the investment adjustment

[7]In nominal terms the budget constraint is

$$P_t^c C_t^i + I_t^{i,n} + B_t^{i,n} + B_t^{i.EA,n} = R_{t-1}^n B_{t-1}^{i,n} + \Omega(FA_{t-1}, \epsilon_{t-1}^{\text{risk}})\bar{e}R_{t-1}^{n,EA}B_{t-1}^{i,EA,n} + W_t L_t^i + R_t^k K_{t-1}^{i,n} + T_t^{i,n} + Div_t^{i,n}$$

Dividing by P_t^c

$$C_t^i + I_t^i + B_t^i + B_t^{i.EA} = R_{t-1}^n\frac{B_{t-1}^{i,n}}{P_t^c} + \Omega(FA_{t-1}, \epsilon_{t-1}^{\text{risk}})\bar{e}R_{t-1}^{n,EA}\frac{B_{t-1}^{i,EA,n}}{P_t^c} + \frac{W_t}{P_t^c}L_t^i + R_t^k K_{t-1}^i + T_t^i + Div_t^i$$

Then, multiplying $\frac{B_{t-1}^{i,n}}{P_t^c}$ and $\frac{B_{t-1}^{i,EA,n}}{P_t^c}$ by $\frac{P_{t-1}^c}{P_{t-1}^c}$, those two terms become $\frac{B_{t-1}^i}{\pi_t^c}$ and $\frac{B_{t-1}^{i,EA}}{\pi_t^c}$

[8]See Appendix 8.4 for details on the $\Omega(FA_t, \epsilon_t^{\text{risk}})$ function.

[9]See Lundvik (1992) and Benigno (2001).

costs function. It has the same properties assumed in many previous papers, see Christiano, Eichenbaum and Evans (2005), namely $S(1) = S'(1) = 0$ and $S''(1) > 0$.

The Lagrangian equation is as follows:

$$\max_{\{C_t^i, I_t^i, L_t^i, z_t^i, K_t^i, B_t^i, B_t^{i,EA}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\ell_t]$$

$$\begin{aligned} \ell_t \;=\; & U_t^i \left[\frac{\varepsilon_t^{\beta}}{1-\sigma_c} \left(C_t^i - hC_{t-1} \right)^{(1-\sigma_c)} - \frac{\varepsilon_t^L}{1+\sigma_L} \left(L_t^i \right)^{(1+\sigma_L)} \right] \\ & + \lambda_t \left[\begin{array}{c} R_{t-1}^n \frac{B_{t-1}^i}{\pi_t^c} + \Omega(FA_{t-1}, \epsilon_{t-1}^{\text{risk}}) \overline{e} R_{t-1}^{n,EA} \frac{B_{t-1}^{i,EA}}{\pi_t^c} + \frac{W_t^i}{P_t^c} L_t^i + R_t^k z_t^i K_{t-1}^i \\ - \Psi\left(z_t^i\right) K_{t-1}^i + T_t^i + Div_t^i - C_t^i - I_t^i - \frac{B_t^i}{P_t^c} - \frac{B_t^{i.EA}}{P_t^c} \end{array} \right] \\ & + Q_t \left[(1-\delta) K_{t-1}^i + \left[1 - S\left(\frac{I_t^i}{I_{t-1}^i} \right) \right] I_t^i x_t - K_t^i \right] \end{aligned} \tag{3}$$

The first order conditions are[10]

$$\frac{\partial \ell_t}{\partial C_t} = 0: \qquad \beta^t \varepsilon_t^{\beta} \left(C_t - hC_{t-1} \right)^{-\sigma_c} - \beta^t \lambda_t = 0 \tag{4}$$

$$\begin{aligned} \frac{\partial \ell_t}{\partial I_t} = 0: \qquad & -\beta^t \lambda_t + \beta^t Q_t x_t \left[1 - S\left(\frac{I_t^i}{I_{t-1}^i} \right) - S'\left(\frac{I_t^i}{I_{t-1}^i} \right) \frac{I_t^i}{I_{t-1}^i} \right] \\ & + \beta^{t+1} \mathbb{E}_t \left\{ Q_{t+1} x_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\} = 0 \end{aligned} \tag{5}$$

$$\frac{\partial \ell_t}{\partial z_t} = 0: \qquad R_t^k = \Psi'\left(z_t\right)$$

$$\begin{aligned} \frac{\partial \ell_t}{\partial K_t} = 0: \qquad & \beta^{t+1} \mathbb{E}_t \left\{ \lambda_{t+1} \left[z_{t+1} R_{t+1}^k - \Psi\left(z_{t+1}\right) \right] \right\} - \beta^t Q_t \\ & + \beta^{t+1} \mathbb{E}_t \left\{ Q_{t+1} (1-\delta) \right\} = 0 \end{aligned} \tag{6}$$

$$\frac{\partial \ell_t}{\partial B_t} = 0: \qquad \beta^t \lambda_t - \beta^{t+1} \mathbb{E}_t \left\{ \lambda_{t+1} R_t^n \frac{1}{\pi_{t+1}^c} \right\} = 0 \tag{7}$$

$$\frac{\partial \ell_t}{\partial B_t^{EA}} = 0: \qquad \beta^t \overline{e} \lambda_t - \beta^{t+1} \mathbb{E}_t \left\{ \lambda_{t+1} \Omega(FA_t, \epsilon_t^{\text{risk}}) \overline{e} R_t^{n,EA} \frac{1}{\pi_{t+1}^c} \right\} = 0 \tag{8}$$

The first order condition for the labour supply is derived in the next section because households are assumed to be able to supply labour monopolistically. We report here the derivative in the case in which households offer labour in a competitive way, underlying that this is also the equation which the next section one reduces to when their non-competitive nature disappears

$$\frac{\partial \ell_t}{\partial L_t} = 0: \qquad -\beta^t \varepsilon_t^{\beta} \varepsilon_t^L \left(L_t \right)^{\sigma_L} + \beta^t \lambda_t \frac{W_t^i}{P_t^c} = 0 \tag{9}$$

From the first order condition (4) it is possible to derive the consumption Euler equation:

$$\varepsilon_t^{\beta} \left(C_t - hC_{t-1} \right)^{-\sigma_c} = \lambda_t \,,$$

[10]The index i is skipped because the decentralized solution is the same as the centralized one, hence the first order conditions are the same.

which implies

$$\mathbb{E}_t\, \varepsilon^{\beta}_{t+1}\left(C_{t+1}-hC_t\right)^{-\sigma_c} = \mathbb{E}_t\, \lambda_{t+1}\,.$$

Combining the two previous equations:

$$\mathbb{E}_t\, \frac{\varepsilon^{\beta}_{t}\left(C_{t}-hC_{t-1}\right)^{-\sigma_c}}{\varepsilon^{\beta}_{t+1}\left(C_{t+1}-hC_t\right)^{-\sigma_c}} = \mathbb{E}_t\, \frac{\lambda_t}{\lambda_{t+1}}\,. \tag{10}$$

Using equation (7):

$$\lambda_t = \beta \mathbb{E}_t\, \lambda_{t+1} R^n_t \frac{1}{\pi^c_{t+1}}\,, \qquad \mathbb{E}_t\, \frac{\lambda_t}{\lambda_{t+1}} = \beta \mathbb{E}_t\, R^n_t \frac{1}{\pi^c_{t+1}}\,.$$

Equations (5) and (6) may be re-written defining the marginal Tobin Q as $q_t = \frac{Q_t}{\lambda_t}$ (the ratio of the two lagrangian multipliers, or more loosely the value of installed capital in terms of its replacement cost). They become respectively[11]

$$1 = q_t x_t \left[1 - S\left(\frac{I^i_t}{I^i_{t-1}}\right) - S'\left(\frac{I^i_t}{I^i_{t-1}}\right)\frac{I^i_t}{I^i_{t-1}}\right] + \beta \mathbb{E}_t \left\{ q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} x_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \right\} \tag{11}$$

$$q_t = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1}\left(1-\delta\right) + z_{t+1} R^k_{t+1} - \Psi\left(z_{t+1}\right) \right] \right\} \tag{12}$$

Equation (11) is nothing more then an investment Euler equation which describes the optimal path for investment. Equation (12) establishes the optimal way to determine the price of capital, taking into account its future return and its depreciation rate.

Finally, combining equations (7) and (8) a modified UIP condition is obtained, taking into account the country specific risk:

$$\beta^t \lambda_t - \beta^{t+1} \mathbb{E}_t \left\{ \lambda_{t+1} R^n_t \frac{1}{\pi^c_{t+1}} \right\} = \beta^t \bar{e} \lambda_t - \beta^{t+1} \mathbb{E}_t \left\{ \lambda_{t+1} \Omega(FA_t, \epsilon^{\text{risk}}_t) \bar{e} R^{nEA}_t \frac{1}{\pi^c_{t+1}} \right\}$$

$$R^n_t = \Omega(FA_t, \epsilon^{\text{risk}}_t) R^{n,EA}_t\,. \tag{13}$$

Aggregate consumption is assumed to be given by a CES index of domestically produced and imported goods according to

$$C_t = \left[\left(1-\alpha_c\right)^{\frac{1}{\eta_c}} \left(C^D_t\right)^{\frac{\eta_c - 1}{\eta_c}} + \left(\alpha_c\right)^{\frac{1}{\eta_c}} \left(C^F_t\right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}}$$

where C^D_t and C^F_t are consumption of the domestic and imported goods, α_c is the share of import in consumption and η_c is the elasticity of substitution between domestic and foreign consumption goods.

[11]Note that when there are not investment adjustment costs, i.e. $S(I^i_t/I^i_{t-1}) = 0$, the investment dynamics equation implies that

$$q_t = \frac{1}{x_t}$$

namely the Tobin's Q is equal to the replacement cost of capital (the relative price of capital). Furthermore, if $x_t = 1$, as in the standard neoclassical growth model, $q_t = 1$.

Household maximizes C_t subject to the two following expenditure constraint

$$P_t^D C_t^D + P_t^{F,c} C_t^F = P_t^c C_t$$

where

$$P_t^c = \left[(1-\alpha_c) \left(P_t^D\right)^{1-\eta_c} + \alpha_c \left(P_t^{F,c}\right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \tag{14}$$

Both P_t^D and P_t^F are expressed in domestic currency and are the domestic price index and the imported consumption goods index respectively.

From the maximization we obtain the following two conditions

$$C_t^D = (1-\alpha_c) \left(\frac{P_t^D}{P_t^c} \right)^{-\eta_c} C_t \tag{15}$$

$$C_t^F = \alpha_c \left(\frac{P_t^{F,c}}{P_t^c} \right)^{-\eta_c} C_t \tag{16}$$

3.1.1 Labour Supply

Each household is a monopoly supplier of a differentiated labour service required by domestic intermediate goods producers.[12] This implies that households in the economy can determine their wages subject to substitutability between different labour services governed by the parameter λ_t^w. After setting their wages, households supply the required amount of working hours inelastically at the going wage rate.

The framework is similar to the one used to derive the New Keynesian Phillips Curve in the next section. A labour aggregator is assumed to hire differentiated labour services form the households and transform them into a homogenous input good L_t using the following technology:

$$L_t = \left[\int_0^1 \left(L_t^i\right)^{\frac{1}{1+\lambda_t^w}} di \right]^{1+\lambda_t^w},$$

where L_t^i is the i-th household's labour supply, L_t is the aggregated labour, and λ_t^w is the time varying wage mark-up, governed by $\log \lambda_t^w = \log \lambda^w + u_t^w$, $u_t^w \sim N(0, \sigma_w^2)$, where λ^w is the steady state value of λ_t^w.[13]

[12] The main references are Kollmann (2001), Erceg et al. (2000), Christiano, Eichenbaum and Evans (2005). Most recent references are Adolfson et al. (2005) and Fernandez-Villaverde and Rubio-Ramirez (2007). The latter has a good mathematical appendix with detailed derivations of all relevant formulas.

[13] See Chari et al. (2008) for a discussion and criticism of the wage mark-up shock and other shocks as well.

The implied solution for L_t^i is[14]

$$L_t^i = \left(\frac{W_t^i}{W_t}\right)^{\frac{1+\lambda_t^w}{\lambda_t^w}} L_t \, ,$$

where

$$W_t = \left[\int_0^1 \left(W_t^i\right)^{-\frac{1}{\lambda_t^w}} di\right]^{-\lambda_t^w}$$

It is also assumed that not all households can optimally re-set their wages each time. Using Calvo (1983) assumptions, only a fraction $1-\theta_w$ of all households can optimally set their wages each time period. Those households who cannot re-set their wages are assumed to be able to index them to the past inflation according to the following formula:

$$W_{t+1}^i = \left(\pi_t^c\right)^{\tau_w} W_t^i \, . \tag{17}$$

Given this set up, households optimize their wages taking into account the probability of being unable to re-set their wages for some number of time periods in the future. The resulting wage equation in log–linear form is given by:[15]

$$\begin{aligned} \widehat{w}_t =& \frac{\beta}{1+\beta}\mathbb{E}_t\, \widehat{w}_{t+1} + \frac{1}{1+\beta}\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\, \widehat{\pi}_{t+1}^c - \frac{1+\beta\tau_w}{1+\beta}\widehat{\pi}_t^c + \frac{\tau_w}{1+\beta}\widehat{\pi}_{t-1}^c \\ & - \frac{1}{1+\beta}\frac{(1-\beta\theta_w)(1-\theta_w)}{\left[1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right]\theta_w}\left[\widehat{w}_t - \sigma_l \widehat{l}_t - \frac{\sigma_c}{1-h}\left(\widehat{c}_t - h\widehat{c}_{t-1}\right) + \widehat{\varepsilon}_t^L\right] + u_t^w \, . \end{aligned} \tag{18}$$

If wages are completely flexible, that is when $\theta_w = 0$, this equation reduces to (9).

[14]The maximization problem for the labour aggregator is:

$$\max_{\{L_t^i\}} W_t L_t - \int_0^1 W_t^i L_t^i di$$

s.t.

$$L_t = \left[\int_0^1 \left(L_t^i\right)^{\frac{1}{1+\lambda_t^w}} di\right]^{1+\lambda_t^w} \, .$$

[15]It is derived by solving the following maximization problem, which is a part of the lagrangian equation (3):

$$\max_{\{W_t^i\}} \mathbb{E}_0 \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[-\frac{\varepsilon_t^\beta \varepsilon_t^L}{1+\sigma_L}\left(L_{t+k}^i\right)^{(1+\sigma_L)} + \lambda_{t+k}\prod_{s=1}^{k}\frac{\left(\pi_{t+s-1}^D\right)^{\tau_w}}{\pi_{t+s}^D}\frac{W_t^i}{P_t^c}L_{t+k}^i\right]$$

s.t.

$$L_{t+k}^i = \left[\prod_{s=1}^{k}\frac{\left(\pi_{t+s-1}^D\right)^{\tau_w}}{\pi_{t+s}^D}\frac{W_t^i}{W_{t+k}}\right]^{-\frac{1+\lambda_t^p}{\lambda_t^p}} L_t \, .$$

The first order condition from this maximization problems needs to be combined with the law of motion of the aggregate wage index:

$$W_t^{-\frac{1}{\lambda_t^p}} = \theta_w \left[W_{t-1}\left(\pi_{t-1}^D\right)^{\tau_w}\right]^{-\frac{1}{\lambda_t^p}} + (1-\theta_w)\left(W_t^*\right)^{-\frac{1}{\lambda_t^p}} \, ,$$

where W_t^* is the wage set by the optimizing households.

3.2 Firms

3.2.1 Final Good Producers

The final good is produced using the intermediate goods j-s by the following technology[16]

$$Y_t = \left[\int_0^1 \left(Y_t^j \right)^{\frac{1}{1+\lambda_t^p}} dj \right]^{1+\lambda_t^p}$$

where $\log \lambda_t^p = \log \lambda^p + u_t^{\lambda^p}$, $u_t^{\lambda^p} \sim N(0, \sigma_{\lambda^p}^2)$ is the time varying price mark-up and λ^p is its steady state value. This is interpreted as a cost push shock to the inflation equation.

The cost minimization condition[17] in the final goods sector can be written as

$$Y_t^j = \left(\frac{P_t^j}{P_t^D} \right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}} Y_t \tag{19}$$

where P_t^j is the price of the intermediate good j and P_t^D is the domestic price index which can be written as

$$P_t^D = \left[\int_0^1 \left(P_t^j \right)^{-\frac{1}{\lambda_t^p}} dj \right]^{-\lambda_t^p}$$

3.2.2 Intermediate Goods Producers

Firms producing intermediate goods operate in a competitive market. They hire labour form households, paying the salary W_t, and they rent the capital they need paying a return R_t^k. Firm j produces output Y_t^j on the basis of the following Cobb-Douglas production function

$$Y_t^j = A_t \left(\widetilde{K}_{t-1}^j \right)^{\alpha} \left(L_t^j \right)^{1-\alpha} - \Phi$$

[16]In a standard set up, this technology is reported as follows

$$Y_t = \left[\int_0^1 \left(Y_t^j \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where ε is the price elasticity of demand for good j. It is known the the gross mark up $(1+\lambda^p)$ is equal to

$$\frac{\varepsilon}{\varepsilon - 1}$$

In the text we have just substituted ε with its expression in terms of the mark-up and assumed that it is time varying:

$$\varepsilon_t = \frac{1+\lambda_t^p}{\lambda_t^p}$$

[17]This condition is obtained minimizing the following cost function

$$\min_{\{Y_t^j\}} \int_0^1 P_t^j Y_t^j$$

subject to the quantity constraint

$$\left[\int_0^1 \left(Y_t^j \right)^{\frac{1}{1+\lambda_t^p}} dj \right]^{1+\lambda_t^p} \geq Y_t$$

where $\widetilde{K}_{t-1}^{j}$ is the effective capital stock given by $\widetilde{K}_{t-1}^{j} = z_t K_{t-1}^{j}$, Φ are fixed costs to assure that profits are zero in the steady state, and $\log A_t = \rho_a \log A_{t-1} + u_t^a$, $u_t^a \sim N(0, \sigma_a^2)$ is a stationary technology shock.

Firms minimize costs under the production function constraint. The objective function is

$$\min_{\{\widetilde{K}_{t-1}^{j}, L_t^j\}} \left(\frac{W_t}{P_t^D}\right) L_t^j + R_t^k \widetilde{K}_{t-1}^{j}$$

The lagrangian function is:

$$\min_{\{\widetilde{K}_{t-1}^{j}, L_t^j\}} \ell_t = \left(\frac{W_t}{P_t^D}\right) L_t^j + R_t^k \widetilde{K}_{t-1}^{j} + \zeta_t \left[Y_t^j - A_t \left(\widetilde{K}_{t-1}^{j}\right)^{\alpha} \left(L_t^j\right)^{1-\alpha}\right]$$

The first order conditions are:

$$\frac{\partial \ell_t}{\partial \widetilde{K}_{t-1}^{j}} = 0: \qquad R_t^k - \zeta_t A_t \alpha \left(\widetilde{K}_{t-1}^{j}\right)^{\alpha-1} \left(L_t^j\right)^{1-\alpha} = 0 \tag{20}$$

$$\frac{\partial \ell_t}{\partial L_t^j} = 0: \qquad \frac{W_t}{P_t^D} - \zeta_t A_t (1-\alpha) \left(\widetilde{K}_{t-1}^{j}\right)^{\alpha} \left(L_t^j\right)^{-\alpha} = 0 \tag{21}$$

where the lagrangian multiplier ζ_t represents the real marginal cost.

Solving (21) for the lagrangian multiplier and substituting the result into (20) gives:

$$R_t^k = \frac{\alpha}{1-\alpha} \frac{W_t}{P_t^D} \frac{L_t^i}{\widetilde{K}_{t-1}^{j}}$$

which implies that

$$\frac{L_t^i}{\widetilde{K}_{t-1}^{j}} = \left(\frac{W_t}{P_t^D}\right)^{-1} R_t^k \frac{1-\alpha}{\alpha}.$$

Then, using this result to substitute out $\frac{L_t^i}{\widetilde{K}_{t-1}^{j}}$ in (21) we have an expression for the real marginal cost:

$$MC_t = \frac{1}{A_t} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{W_t}{P_t^D}\right)^{1-\alpha} \left(R_t^k\right)^{\alpha}$$

Intermediate goods producers face another type of problem. Each period, only a fraction $1-\theta$ of them, randomly chosen, can optimally adjust their prices (see Calvo (1983)). For those that cannot re-optimize, prices are indexed to the past inflation as follows:

$$P_{t+1}^D = \left(\pi_t^D\right)^{\tau_\pi} P_t^D, \tag{22}$$

where τ_π is the parameter governing the degree of price indexation.

Maximizing the expected discounted profits:[18]

$$\max_{\{P_t^j\}} \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta\theta)^i \frac{\lambda_{t+i}}{\lambda_t} \left\{ \left[\prod_{s=1}^{i} \left(\pi_{t+s-1}^D\right)^{\tau_\pi} \frac{P_t^j}{P_{t+i}^D} - MC_{t+i} \right] Y_{t+i}^j \right\}$$

[18] In order to maintain the paper self-contained we do not report the derivation of the New Keynesian Phillips curve. Moreover, it has been derived in many papers and books, so we refer to them. See Walsh (2003), Adolfson et al. (2005), Fernandez-Villaverde (2007) among others.

subject to the intermediate good demand function by the final good producers, see (19):

$$Y_{t+i}^{j} = \left[\prod_{s=1}^{i} \left(\pi_{t+s-1}^{D} \right)^{\tau_\pi} \frac{P_t^j}{P_{t+i}^D} \right]^{-\frac{1+\lambda_t^p}{\lambda_t^p}} Y_{t+i} \,,$$

it is possible to derive the first order condition for the optimal price P_t^*:[19]

$$\begin{aligned} \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta\theta)^i \lambda_{t+i} \Bigg\{ \Bigg(\left(-\frac{1}{\lambda_t^p} \right) \left[\prod_{s=1}^{i} \left(\frac{\pi_{t+s-1}^D}{\pi_{t+s}^D} \right)^{\tau_\pi} \right]^{-\frac{1}{\lambda_t^p}} \frac{P_t^*}{P_t^D} \\ + \left(\frac{1+\lambda_t^p}{\lambda_t^p} \right) \left[\prod_{s=1}^{i} \left(\frac{\pi_{t+s-1}^D}{\pi_{t+s}^D} \right)^{\tau_\pi} \right]^{-\frac{1+\lambda_t^p}{\lambda_t^p}} MC_{t+i} \Bigg) Y_{t+i}^j \Bigg\} = 0 \,. \end{aligned} \tag{23}$$

Given that in each time period a fraction of firms can re-set their prices optimally, and the rest index their prices using the previous period's inflation rate, the aggregate price index evolves according to the following weighted average formula:

$$P_t^D = \left[\int_0^\theta \left[\left(\pi_{t-1}^D \right)^{\tau_\pi} P_{t-1}^D \right]^{-\frac{1}{\lambda_t^p}} + \int_\theta^1 \left(P_t^* \right)^{-\frac{1}{\lambda_t^p}} \right]^{-\lambda_t^p} ,$$

equivalently:

$$\left(P_t^D \right)^{-\frac{1}{\lambda_t^p}} = \theta \left[\left(\pi_{t-1}^D \right)^{\tau_\pi} P_{t-1}^D \right]^{-\frac{1}{\lambda_t^p}} + (1-\theta) \left(P_t^* \right)^{-\frac{1}{\lambda_t^p}} \,. \tag{24}$$

Combining the log–linearized equation (24) with the first order condition for the optimal price in (23) leads to an equation for the domestic inflation rate. It is given by the hybrid New Keynesian Phillips Curve:

$$\widehat{\pi}_t^D = \frac{\beta}{1+\beta\tau_\pi} \mathbb{E}_t \widehat{\pi}_{t+1}^D + \frac{\tau_\pi}{1+\beta\tau_\pi} \widehat{\pi}_{t-1}^D + \frac{1}{1+\beta\tau_\pi} \frac{(1-\beta\theta)(1-\theta)}{\theta} \widehat{mc}_t + u_t^{\lambda^p} \,. \tag{25}$$

When prices are completely flexible, that is when $\theta = 0$, and the price mark-up is zero, the previous equation reduces to the usual flexible price condition whereby the real marginal cost are equal to one.

3.3 Importers

The import and export sectors in EP DSGE model are based on Adolfson et al. (2005). The import sector consists of a large number of firms that buy a homogenous good in the euro area

[19] Since all firms face the same technology shock and the resulting optimal capital–output ratio is similar across all intermediate producers, the optimal price P_t^* is the same for all firms. Solving this equation for P_t^* and assuming flexible prices ($\theta = 0$) gives the standard monopolistic competition outcome that firms set their price a mark up over their nominal marginal cost:

$$P_t^* = (1+\lambda_t^p)\, MC_t$$

market and turn it into differentiated consumption goods[20] using a brand naming technology, i.e. without costs. These differentiated consumption goods are then sold to domestic households subject to price stickiness in the local currency.

Firms buy the homogenous good at a price P_t^{EA}, which is the CPI of the Euro Area. The framework in which these firms operate is identical to the one of the intermediate goods producers in terms of price setting behaviour. A fraction of importers $1-\theta_{c,F}$ are allow to optimize in each period. For those which do not optimize, prices evolve as follows:

$$P_{t+1}^{F,c} = \left(\widehat{\pi}_t^{c,F}\right)^{\tau_{c,F}} P_t^{F,c}, \tag{26}$$

where $P_t^{F,c} = \left[\int_0^1 \left(P_t^{j,F,c}\right)^{-\frac{1}{\lambda_t^{c,F}}} dj\right]^{-\lambda_t^{c,F}}$ is the imported good price index, $\log \lambda_t^{c,F} = \log \lambda^{c,F} + u_t^{\lambda_t^{c,F}}$, $u_t^{\lambda^{c,F}} \sim N\left(0, \sigma^2_{\lambda^{c,F}}\right)$ is the imported goods time varying mark-up[21], and $\tau_{c,F}$ is the degree of price indexation. The final imported good is a composite of continuum of j differentiated imported goods, each supplied by a different firm and with price $P_t^{j,F,c}$, which follows the CES function:

$$C_t^F = \left[\int_0^1 C_t^{j,F\frac{1}{1+\lambda_t^{c,F}}} dj\right]^{1+\lambda_t^{c,F}} .$$

This implies the following demand for each imported good:

$$C_t^{j,F} = \left(\frac{P_t^{j,F,c}}{P_t^{,cF}}\right)^{-\frac{1+\lambda_t^{c,F}}{\lambda_t^{c,F}}} C_t^F .$$

Importing firms maximize their profits subject to the Calvo (1983) price stickiness restric-

[20]In Adolfson et al. (2005) there is a distinction between imported consumption and investment goods. This version of EP DSGE model does not make this distinction because the statistical data related to prices of imported investment goods for Estonia is not readily available. This would make estimation of the corresponding Phillips Curve difficult. It is assumed that only consumption goods and services are imported. The same applies to the export sector in subsection 3.4.

[21]Note that the steady state value of this mark up does not enter in any of the equation we are going to estimate. It is only relevant for the steady state values of some variables.

tion.[22] The resulting inflation equation in log-linearized form is given by:

$$\widehat{\pi}_t^{c,F} = \frac{\beta}{1+\tau_{c,F}\beta}\mathbb{E}_t\,\widehat{\pi}_{t+1}^{c,F} + \frac{\tau_{c,F}}{1+\tau_{c,F}\beta}\widehat{\pi}_{t-1}^{c,F} + \frac{1}{1+\tau_{c,F}\beta}\frac{(1-\theta_{c,F})(1-\beta\theta_{c,F})}{\theta_{c,F}}\left(\widehat{mc}_t^{c,F} + u_t^{\lambda_t^{c,F}}\right).$$

3.4 Exporters

The exporting firms buy final domestic good and differentiate it by brand naming. They sell the continuum of differentiated consumption goods to the households in the euro area. The nominal marginal cost is thus the price of the domestic good P_t^D. Since only consumption goods are exported, each exporting firm j faces the following demand function for its product:

$$C_t^{j,x} = \left(\frac{P_t^{j,x,c}}{P_t^{x,c}}\right)^{-\frac{1+\lambda_t^{c,x}}{\lambda_t^{c,x}}} C_t^x\,,$$

where $P_t^{x,c}$ is the export price index expressed in the local currency of the export market, and $\log\lambda_t^{c,x} = \log\lambda^{c,x} + u_t^{\lambda^{c,x}}$, $u_t^{\lambda^{c,x}} \sim N(0,\sigma^2_{\lambda^{c,x}})$ is the stochastic mark-up on differentiated export goods. Once again, the problem that exporting firms are faced with is determined by the Calvo (1983) framework. A fraction $1-\theta_{c,x}$ of the firms can set optimal prices each period. For the remaining share of firms the evolution of prices is following:

$$P_{t+1}^{x,c} = (\pi_t^{c,x})^{\tau_{c,x}}\,P_t^{x,c}\,. \tag{27}$$

Exporting firms maximize their profits subject to the price stickiness restriction.[23] The

[22] They maximize discounted stream of profits:

$$\max_{\{P_t^{j,F,c}\}} \mathbb{E}_0\sum_{i=0}^{\infty}(\beta\theta_{c,F})^i\,\frac{\lambda_{t+i}}{\lambda_t}\left\{\left[\prod_{s=1}^{i}\left(\pi_{t+s-1}^{c,F}\right)^{\tau_{c,F}}\frac{P_t^{j,F,c}}{P_{t+i}^{F,c}} - MC_{t+i}^{c,F}\right]C_{t+i}^{j,F}\right\}$$

s.t.

$$C_{t+i}^{j,F} = \left[\prod_{s=1}^{i}\left(\pi_{t+s-1}^{c,F}\right)^{\tau_{c,F}}\frac{P_t^{j,F,c}}{P_{t+i}^{F,c}}\right]^{-\frac{1+\lambda_t^{c,F}}{\lambda_t^{c,F}}} C_{t+i}^F\,,$$

where $MC_{t+i}^{c,F} = \frac{P_t^{EA}}{\bar{e}P_t^{F,c}}$.

In addition, the usual aggregate price equation needs to be combined with the first order condition obtained from the above maximization problem:

$$\left(P_t^{F,c}\right)^{-\frac{1}{\lambda_t^{c,F}}} = \theta_{c,F}\left[\left(\pi_{t-1}^{c,F}\right)^{\tau_{c,F}}P_{t-1}^{F,c}\right]^{-\frac{1}{\lambda_t^{c,F}}} + (1-\theta_{c,F})\left(P_t^{*F,c}\right)^{-\frac{1}{\lambda_t^{c,F}}}\,.$$

[23] They maximize discounted stream of profits:

$$\max_{\{P_t^{j,x,c}\}} \mathbb{E}_0\sum_{i=0}^{\infty}(\beta\theta_{c,x})^i\,\frac{\lambda_{t+i}}{\lambda_t}\left\{\left[\prod_{s=1}^{i}\left(\pi_{t+s-1}^{c,x}\right)^{\tau_{c,x}}\frac{P_t^{j,x,c}}{P_{t+i}^{x,c}} - MC_{t+i}^{c,x}\right]C_{t+i}^{j,x}\right\}$$

s.t.

$$C_{t+i}^{j,x} = \left[\prod_{s=1}^{i}\left(\pi_{t+s-1}^{c,x}\right)^{\tau_{c,x}}\frac{P_t^{j,x,c}}{P_{t+i}^{x,c}}\right]^{-\frac{1+\lambda_t^{c,x}}{\lambda_t^{c,x}}} C_{t+i}^x\,,$$

where $MC_{t+i}^{c,x} = \frac{P_t^D}{\bar{e}P_t^{x,c}}$.

resulting inflation equation in log-linearied form is given by:

$$\widehat{\pi}_t^{c,x} = \frac{\beta}{1+\tau_{c,x}\beta}\mathbb{E}_t\,\widehat{\pi}_{t+1}^{c,x} + \frac{\tau_{c,x}}{1+\tau_{c,x}\beta}\widehat{\pi}_{t-1}^{c,x} + \frac{1}{1+\tau_{c,x}\beta}\frac{(1-\theta_{c,x})(1-\beta\theta_{c,x})}{\theta_{c,x}}\left(\widehat{mc}_t^{c,x} + u_t^{\lambda^{c,x}}\right).$$

In addition, Estonian economy is assumed to be small relative to the euro area economy in EP DSGE model, and hence it plays just a negligible part in aggregate foreign consumption. Assuming that the aggregate foreign consumption follows a CES function, the euro area demand for the aggregate domestic consumption good (and in this case also for the total export) is given by:

$$Exp_t = C_t^x = \left(\frac{P_t^{x,c}}{P_t^{EA}}\right)^{-\eta_{EA}} C_t^{EA},$$

where η_{EA} is the elasticity of substitution between the foreign and home consumption goods, and C_t^{EA} is the euro area aggregate consumption. Here it is assumed that $Y_t^{EA} = C_t^{EA}$, hence:[24]

$$Exp_t = \left(\frac{P_t^{x,c}}{P_t^{EA}}\right)^{-\eta_{EA}} Y_t^{EA}. \tag{28}$$

3.5 Policies

3.5.1 Fiscal Policy

Fiscal policy is exogenous and assumed to behaves as follows:

$$\log G_t = \rho_g \log G_{t-1} + u_t^g, \tag{29}$$

where $u_t^g \sim N(0, \sigma_g^2)$. In addition, the balanced budget condition implies that $G_t = -T_t$.

3.5.2 Monetary Policy

The monetary policy of Estonia is subject to the currency board arrangement and free capital mobility between the domestic and euro area markets. The UIP condition derived previously in (13) implies that the domestic nominal interest rate is given by:

$$R_t^n = \Omega(FA_t, \epsilon_t^{\text{risk}})R_t^{n,EA}.$$

In other words, it is determined by the monetary policy in the euro area and by the country specific risk premium.

In addition, the usual aggregate price equation needs to be combined with the first order condition obtained from the above maximization problem:

$$(P_t^{x,c})^{-\frac{1}{\lambda_t^{c,x}}} = \theta_{c,x}\left[\left(\pi_{t-1}^{c,x}\right)^{\tau_{c,x}} P_{t-1}^{x,c}\right]^{-\frac{1}{\lambda_t^{c,x}}} + (1-\theta_{c,x})\left(P_t^{*x,c}\right)^{-\frac{1}{\lambda_t^{c,x}}}$$

[24]This assumption is not properly correct. In fact, as we will see in one of the following sections, $Y_t^{EA} = C_t^{EA} + INV_t^{EA} + G_t^{EA} + \Psi(z_t)K_{t-1}^{EA}$, where $\Psi(z_t)K_{t-1}^{EA}$ is the cost associated with variations in the degree of capital utilization. This assumption does not affect the estimation results.

3.6 Aggregate Resource Constraint

The aggregate resource constraint is:

$$Y_t \equiv C_t^D + C_t^F + I_t + G_t - \Psi(z_t) K_{t-1} + Exp_t - M_t \, . \tag{30}$$

Imports are defined as follows:

$$M_t = C_t^F \, . \tag{31}$$

Now we can substitute out all the components of the domestic output into (30) using equations (15), (31) and (28):

$$Y_t = (1-\alpha_c)\left(\frac{P_t^D}{P_t^c}\right)^{-\eta_c} C_t + C_t^F + I_t + G_t - \Psi(z_t) K_{t-1} + \left(\frac{P_{c,t}^x}{P_t^{EA}}\right)^{-\eta_{EA}} Y_t^{EA} - C_t^F \, ,$$

$$Y_t = (1-\alpha_c)\left(\frac{P_t^D}{P_t^c}\right)^{-\eta_c} C_t + I_t + G_t - \Psi(z_t) K_{t-1} + \left(\frac{P_{c,t}^x}{P_t^{EA}}\right)^{-\eta_{EA}} Y_t^{EA} \, .$$

3.7 The Net Foreign Assets

The net foreign assets evolve in the following manner:

$$\bar{e}B_t^{EA} = \bar{e}P_t^{x,c} Exp_t - \bar{e}P_t^{EA} M_t + \Omega(FA_t, \epsilon_t^{\text{risk}})\bar{e}R_{t-1}^{n,EA} B_{t-1}^{EA} \, . \tag{32}$$

Dividing both sides of this equation by P_t^D and using the definitions of Exp_t, M_t, $MC_t^{c,x}$ and C_t^F from equations (28), (31), (16), and footnote 23, equation (32) can be written as:

$$FA_t = \frac{\bar{e}P_t^{x,c}}{P_t^D}\left(\frac{P_t^{x,c}}{P_t^{EA}}\right)^{-\eta_{EA}} Y_t^{EA} - \frac{\bar{e}P_t^{EA}}{P_t^D} C_t^F + \Omega(FA_t, \epsilon_t^{\text{risk}}) R_{t-1}^{n,EA} \frac{FA_{t-1}}{\pi_t^D} \, ,$$

$$FA_t = (MC_t^{c,x})^{-1}\left(\frac{P_t^{x,c}}{P_t^{EA}}\right)^{-\eta_{EA}} Y_t^{EA} - \frac{\bar{e}P_t^{EA}}{P_t^D}\alpha_c\left(\frac{P_t^{F,c}}{P_t^D}\right)^{-\eta_c} C_t + \Omega(FA_t, \epsilon_t^{\text{risk}}) R_{t-1}^{n,EA} \frac{FA_{t-1}}{\pi_t^D} \, ,$$

where $\pi_t^D = \frac{P_t^D}{P_{t-1}^D} = 1 + \frac{P_t^D - P_{t-1}^D}{P_{t-1}^D}$.

4 Key equations: Euro area economy

The euro area part of EP DSGE model is based on Smets and Wouters (2003) paper.[25] In contrast to Adolfson et al. (2005), where the rest of the world is described by a low dimensional

[25] Originally, the idea was to model euro area by a basic three equations NK DSGE model, and estimate it either separately or jointly with the Estonian economy part. This is more in line with the spirit of small open economy DSGE models, where the rest of the world is often just a three equation VAR system. However, some estimation difficulties led to a change in the approach, and the model by Smets and Wouters (2003) was chosen to describe the monetary policy and business cycles in the euro area. An interesting future extension would be to consider an open economy model for the euro area as well, allowing to study the effects of the rest of the world on the Estonian economy through the corresponding impact on the euro area. In this respect, Adolfson et al. (2005) is an excellent reference.

VAR system, this papers specifies a full fledged DSGE model as a counterpart to the Estonian economy part described in Section 3. Since one of the main interests of this paper is to examine various euro area disturbances that hit the economy of Estonia, it is necessary to have a model which incorporates a range of structural shocks with clear economic interpretations attached to them. For this reason, and due to increased dimensions of the euro area part of EP DSGE model, values of the structural parameters for the euro area economy are calibrated according to Smets and Wouters (2003).

In terms of equations, the model is similar to the Estonian economy part in Section 3, but with a few substantial differences.[26] The differences are due to the fact that it is a closed economy model having an independent monetary policy described by a monetary policy rule.

The aggregate resource constraint is given by:

$$Y_t^{EA} = C_t^{EA} + INV_t^{EA} + G_t^{EA} + \Psi\left(z_t^{EA}\right) K_{t-1}^{EA}.$$

The monetary policy rule is as follows:

$$\begin{aligned}\widehat{r}_t^{n,EA} &= \phi_m \widehat{r}_{t-1}^{n,EA} + (1-\phi_m)\left[\overline{\pi}_t + r_{\pi^{EA}}\left(\widehat{\pi}_{t-1}^{EA} - \overline{\pi}_t\right) + r_{y^{EA}}\left(\widehat{y}_{t-1}^{EA} - \widehat{y}_{t-1}^{P,EA}\right)\right] + \\ &\quad + r_{\Delta\pi}\left(\widehat{\pi}_t^{EA} - \widehat{\pi}_{t-1}^{EA}\right) + r_{\Delta y}\left[\widehat{y}_t^{EA} - \widehat{y}_t^{P,EA} - \left(\widehat{y}_{t-1}^{EA} - \widehat{y}_{t-1}^{P,EA}\right)\right] + u_t^{r^{n,EA}},\end{aligned}$$

where $\log\overline{\pi}_t = \rho_{\overline{\pi}}\log\overline{\pi}_{t-1} + u_t^{\overline{\pi}}$, $u_t^{\overline{\pi}} \sim N(0,\sigma_{\overline{\pi}}^2)$ is the inflation objective shock, $u_t^{r^{n,EA}} \sim N(0,\sigma_{r^{n,EA}}^2)$ is a idiosyncratic monetary policy shock, and $Y_t^{P,EA}$ is the potential output. In DSGE literature, the potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of the cost push shocks ($u_t^{w,EA}, u_t^{\lambda^p,EA}, u_t^{q,EA}$). As in Smets and Wouters (2003) the model reported in Appendix 8.3 is expanded with flexible prices and wages version ($\theta^{EA} = \theta_w^{EA} = u_t^{w,EA} = u_t^{\lambda^p,EA} = u_t^{q,EA} = 0$) in order to calculate the model-consistent output gap.

5 Data and estimation

5.1 Bayesian estimation methodology

Statistical inference for the structural parameters of the EP DSGE model introduced in sections 3 and 4 is obtained by Bayesian methods, the corresponding empirical results can be found in section 6 of this paper. Bayesian statistical methods have recently gained popularity in applied macro-economic modeling, for a recent overview of main literature and methods of Bayesian analysis of DSGE models refer to An and Schorfheide (2007). This subsection gives an overview of the main stages of the Bayesian statistical inference for DSGE models.

In contrast to the traditional approach to statistical estimation and testing known under the banner of "frequentist statistics" where the inference based on repeated sampling plays a pivotal role and a "true" model with an unknown but constant set of parameters is assumed to exist, Bayesian statistics adopts a view that parameters are just "mental constructs that

[26] The set of log-linearized equations is reported in Appendix 8.3 as a reference.

exist only in the mind of the researcher", see Poirier (1995). Bayesian statistics is based on a fusion of priors about the model parameters with the likelihood function based on the real–world data, where "the latter represents a "window" for viewing the observable world shared by a group of researches who agree to disagree in terms of possibly different prior distributions", see Poirier (1995). Bayesian statistics knowingly departs from the assumptions of repeated sampling experiments and underlying data generating process based on unknown and constant set of parameters — the two assumptions that are crucial to the traditional frequentist approach.

Bayesian statistics can be characterized as a learning process, where observed data collected in $\mathbf{Y}$ is used to learn about the posterior distribution $f(\boldsymbol{\theta}\,|\,\mathbf{Y})$ of a k-dimensional vector of model parameters $\boldsymbol{\theta}$, given the likelihood function $L(\boldsymbol{\theta};\mathbf{Y})$ and the prior distribution $f(\boldsymbol{\theta})$. This learning process is based on a version of the Bayes' Theorem:

$$f(\boldsymbol{\theta}\,|\,\mathbf{Y}) = \frac{f(\boldsymbol{\theta})L(\boldsymbol{\theta};\mathbf{Y})}{f(\mathbf{Y})} \propto f(\boldsymbol{\theta})L(\boldsymbol{\theta};\mathbf{Y})\,, \tag{33}$$

where $f(\mathbf{Y}) = \int_{\mathbf{R}^k} f(\boldsymbol{\theta})L(\boldsymbol{\theta};\mathbf{Y})d\boldsymbol{\theta}$ can be treated as a normalizing constant for the purpose of posterior inference. The main object of interest for Bayesian inference is the posterior distribution function $f(\boldsymbol{\theta}\,|\,\mathbf{Y})$, which summarizes information available in the data $\mathbf{Y}$ about the vector of parameters $\boldsymbol{\theta}$. The posterior distribution function may further be combined with a statistical loss function in order to arrive to point and interval inference about $\boldsymbol{\theta}$ as well as other forms of statistical decisions involving the vector of parameters.

It follows from expression (33) that Bayesian statistical inference requires both the likelihood function and the prior distribution. Remaining part of this section provides a general overview of the steps involved in construction of the likelihood function for a typical DSGE model. This discussion is applicable not only to the EP DSGE model introduced in sections 3 and 4, but also to other DSGE and real business cycle models found in the literature and estimated in the form of first-order linear or log-linear approximations around the steady state. The issues related to the particular choice of priors for the EP DSGE model are deferred to section 5.2.

In general, the likelihood function of a typical DSGE model cannot be written in closed form as a function of data $\mathbf{Y}$ and model parameters $\boldsymbol{\theta}$. However, given $\mathbf{Y}$ and $\boldsymbol{\theta}$, the procedure to evaluate the likelihood function at these values in an implicit form involves three stages. They are described below in detail.

The first stage involves writing a theoretical macro-economic model as a system of linear expectational and non-expectational equations, including exogenous stochastic processes. Appendices 8.2 and 8.3 list the corresponding system of log-linearized equations for the EP DSGE model. Let $\boldsymbol{x}_t$ denote a $m \times 1$ vector of endogenous model variables, $\boldsymbol{y}_t$ denote $n \times 1$ vector of other endogenous model variables and $\boldsymbol{z}_t$ be $k \times 1$ vector of exogenous stochastic processes. A

DSGE model can be written in linearized form as follows:

$$\begin{aligned}
\mathbf{0} &= \mathbf{A}\boldsymbol{x}_t + \mathbf{B}\boldsymbol{x}_{t-1} + \mathbf{C}\boldsymbol{y}_t + \mathbf{D}\boldsymbol{z}_t \\
\mathbf{0} &= \mathbb{E}_t\big(\mathbf{F}\boldsymbol{x}_{t+1} + \mathbf{G}\boldsymbol{x}_t + \mathbf{H}\boldsymbol{x}_{t-1} + \mathbf{J}\boldsymbol{y}_{t+1} + \mathbf{K}\boldsymbol{y}_t + \mathbf{L}\boldsymbol{z}_{t+1} + \mathbf{M}\boldsymbol{z}_t\big) \\
\boldsymbol{z}_{t+1} &= \mathbf{N}\boldsymbol{z}_t + \boldsymbol{\epsilon}_t\,,
\end{aligned} \tag{34}$$

where a $k \times 1$ vector $\boldsymbol{\epsilon}_t$ of stochastic shocks has mean zero and variance–covariance matrix $\boldsymbol{\Sigma}$. Vector of model parameters $\boldsymbol{\theta}$ is mapped into the matrices $\mathbf{A}$ to $\boldsymbol{\Sigma}$ of this system according to the theoretical model.

Linear system of expectational equations (34) is solved in the second stage of the likelihood function evaluation. The method of undetermined coefficients stipulates the following solution of the system (34):

$$\begin{aligned}
\boldsymbol{x}_t &= \mathbf{P}\boldsymbol{x}_{t-1} + \mathbf{Q}\boldsymbol{z}_t \\
\boldsymbol{y}_t &= \mathbf{R}\boldsymbol{x}_{t-1} + \mathbf{S}\boldsymbol{z}_t\,,
\end{aligned} \tag{35}$$

where matrices $\mathbf{P}$ to $\mathbf{S}$ are mappings of matrices $\mathbf{A}$ to $\mathbf{M}$ defined in (34) and therefore also functions of the vector of model parameters $\boldsymbol{\theta}$. Uhlig (1999) gives a comprehensive overview of the method of undetermined coefficients for linear systems of expectational equations like (34), including conditions on dimensions and ranks of matrices $\mathbf{A}$ to $\boldsymbol{\Sigma}$ that are necessary to obtain the solution (35). In particular, stability or lack of thereof of the system of linear expectational equations (34) depends on the vector of model parameters $\boldsymbol{\theta}$ through the matrices $\mathbf{A}$ to $\mathbf{M}$ and is reflected by the eigenvalues of $\mathbf{P}$ matrix in (35).

Having obtained the system of stochastic difference equations (35) for endogenous variables $\boldsymbol{x}_t$ and $\boldsymbol{y}_t$ and using the law of motion of the exogenous stochastic processes $\boldsymbol{z}_t$, the likelihood function $L(\boldsymbol{\theta};\mathbf{Y})$ of a DSGE model is evaluated using the Kalman filter in the third stage of the procedure. The Kalman filter is needed because endogenous variables of the model in vectors $\boldsymbol{x}_t$ and $\boldsymbol{y}_t$ usually involve some quantities for which no empirical counterparts can be observed in macro-economic statistics. Let $m+k\times 1$ dimensional vector $\tilde{\boldsymbol{x}}_t$ be defined as $\tilde{\boldsymbol{x}}_t^{\mathrm{T}} := (\boldsymbol{x}_{t-1}^{\mathrm{T}}, \boldsymbol{z}_t^{\mathrm{T}})$, and let $\tilde{\boldsymbol{y}}_t$ be a vector of observed variables at time period $1 \le t \le T$ s.t. the data matrix is given by $\mathbf{Y} := (\tilde{\boldsymbol{y}}_1^{\mathrm{T}}, \tilde{\boldsymbol{y}}_2^{\mathrm{T}}, \ldots, \tilde{\boldsymbol{y}}_T^{\mathrm{T}})$. A DSGE model can be written in the Kalman filter form using the solution (35) as follows:

$$\begin{aligned}
\tilde{\boldsymbol{x}}_{t+1} &= \begin{pmatrix} \mathbf{P} & \mathbf{Q} \\ \mathbf{0} & \mathbf{N} \end{pmatrix} \tilde{\boldsymbol{x}}_t + \begin{pmatrix} \mathbf{0} \\ \boldsymbol{\epsilon}_t \end{pmatrix} \\
\tilde{\boldsymbol{y}}_t &= \boldsymbol{\Gamma}\tilde{\boldsymbol{x}}_t\,,
\end{aligned} \tag{36}$$

where matrix $\boldsymbol{\Gamma}$ maps a subset of endogenous model variables into the observed data, and may include elements of the matrices $\mathbf{R}$ and $\mathbf{S}$ from the system (35). In some cases the measurement equation in (36) may additionally involve measurement errors, if observed data is deemed to be imperfect counterpart of endogenous model variables. In the estimation of the EP DSGE model no measurement errors are included in the Kalman filter equations (36). The value of the likelihood $L(\boldsymbol{\theta}; \mathbf{Y})$ at the vector of model parameters $\boldsymbol{\theta}$ is computed using standard Kalman filter recursions as detailed in Hamilton (1994) pp. 372–408.

It is necessary to note that the mapping of parameters $\boldsymbol{\theta}$ into the likelihood function $L(\boldsymbol{\theta}; \mathbf{Y})$ of a typical DSGE model is highly complicated, involving nonlinear transformations at the solution stage (35). This might give rise to identifiability issues which are difficult to deal with because of the lack of developed diagnostic methods. Some of the issues related to identification in DSGE models are discussed in Canova (2008) and Iskrev (2008).

Apart from the likelihood and the priors, Bayesian statistical inference based on (33) requires a set of techniques to evaluate the posterior distribution $f(\boldsymbol{\theta} \,|\, \mathbf{Y})$. Specifically, one is usually interested in at least first few moments of the posterior distribution of $\boldsymbol{\theta}$, but accepted current practice requires reporting of the kernel posterior density estimates of the model parameters. Since for a typical DSGE model $f(\boldsymbol{\theta} \,|\, \mathbf{Y})$ is not available in closed form, computationally intensive Monte Carlo sampling methods are needed to generate draws from the posterior distribution $f(\boldsymbol{\theta} \,|\, \mathbf{Y})$. For a good survey of Monte Carlo methods in Bayesian statistics refer to Robert and Casella (2004).

Metropolis–Hastings Markov chain Monte Carlo algorithm offers a general and easy-to-implement way to draw random numbers from probability distributions for which no procedural random number generators are available. A particularly simple implementation of the algorithm is called random walk Metropolis–Hastings and involves the following four steps:

1. Given the previous draw $\boldsymbol{\theta}_{i-1}$ from $f(\boldsymbol{\theta} \,|\, \mathbf{Y})$, generate a candidate draw as follows:

$$\boldsymbol{\theta}_i^* = \boldsymbol{\theta}_{i-1} + \boldsymbol{v}_i \, , \text{ where } \boldsymbol{v}_i \sim N(\mathbf{0}, \mathbf{S}) \, ;$$

2. Compute:

$$\alpha_i := \min \left[1, \frac{f(\boldsymbol{\theta}_i^* \,|\, \mathbf{Y})}{f(\boldsymbol{\theta}_{i-1} \,|\, \mathbf{Y})} \right] ;$$

3. Assign the new draw $\boldsymbol{\theta}_i$ from $f(\boldsymbol{\theta} \,|\, \mathbf{Y})$ as:

$$\boldsymbol{\theta}_i = \begin{cases} \boldsymbol{\theta}_i^* & \text{if } \alpha_i \geq u_i \\ \boldsymbol{\theta}_{i-1} & \text{if } \alpha_i < u_i \end{cases} , \text{ where } \boldsymbol{u}_i \sim U[0,1] \, ;$$

4. Repeat steps 1 to 3 until enough random draws are generated from the distribution $f(\boldsymbol{\theta} \,|\, \mathbf{Y})$.

Given N draws $\{\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_N\}$ from the posterior distribution $f(\boldsymbol{\theta} \,|\, \mathbf{Y})$ supplied by the Metropolis–Hastings or other sampling method, empirical moments, kernel density estimates and other posterior statistics can be computed in the usual fashion.

The practical implementation of the steps associated with evaluation of the likelihood function and drawing from the posterior distribution varies from low-level programming of all necessary steps in one of the mathematical programming languages, such as MATLAB, to using higher-level packages specifically written for the analysis and estimation of DSGE models, such as Dynare.[27] The latter is used for simulation and estimation of the EP DSGE model in this paper.

Posterior distributions of the model parameters reported in Section 6 of this paper have been obtained using MATLAB/Dynare toolbox as follows. The inference is based on 2 parallel chains of 800000 draws each, where the last 400000 draws are used for statistics and diagnostics. The algorithm is started by simulation–based maximization of the posterior kernel,[28] followed by evaluation of the Hessian matrix at the posterior kernel maximum, which it then used as an input parameter for the main run of Metropolis–Hastings algorithm to compute the posterior inference for the model parameters.

5.2 Data and priors

Open economy DSGE model for Estonia introduced in sections 3 and 4 is estimated in the form of log-deviations from the long-run steady-state. In other words, the model is not designed to explain long-run trends and seasonal fluctuations in macro-economic variables, but rather is focused on the business cycle frequency features of the main macro aggregates. Empirical data series are therefore required to undergo a certain treatment before being used in evaluation of the model's likelihood function. This section also includes a discussion of the priors choice for the model's structural parameters.

The likelihood function of the EP DSGE model, which includes 24 domestic and 13 euro area endogenous variables, is based on 14 data series, including 3 series that describe most important euro area macro-economic indicators. The data series used for model estimation are shown on Figures 2 to 7. All empirical variables are quarterly, covering the time interval from 1995Q1 to 2007Q4, thus giving 52 observations per data series. All series pertaining to the domestic economy are sourced from Eurostat's database[29], euro area series are taken from the AWM database, refer to Fagan et. al. (2005).

As mentioned previously, since the theoretical model is not designed to pick up seasonal fluctuations in the macro-economic data, all seasonal features of the series are removed using

[27] See Dynare's homepage at `www.cepremap.cnrs.fr/dynare` and Juillard (2004).

[28] In Dynare this procedure is implemented using the option `mode_compute=6`. It takes 350000 iterations in order to obtain the reasonable results. In fact, the complexity of the model prevented the widely used Sims' optimizer to find acceptable starting points for the Metropolis–Hastings sampling. Because of the random nature of Sims' optimizer, it stopped at different modes each time, which in most cases where not the actual modes. Using the simulation–based procedure to compute the modes helped to improve the situation, although some minor problems remained. This is also clear from a visual inspection of the posterior kernel surface graphs (available upon request) for each parameter, given that all other parameters are fixed at their modes. Those graphs show that for some parameters the mode computed is not at the minimum of the objective function, and for some parameters the surface is so flat that it is difficult to make out if the picked mode is really at the minimum. Another advantage of using `mode_compute=6` is that it automatically tunes `mh_jscale` option (the option which sets the scale parameters of the covariance matrix of the proposal distribution) in order to have the acceptance ratio of the Metropolis–Hastings algorithm at around 30%.

[29] Refer to `ec.europa.eu/eurostat` page.

either TRAMO/SEATS software package[30] or, as in the case of inflation series, year-on-year changes in the associated nominal price variables.

Given below is a detailed list of individual data series used in evaluation of the model's likelihood function:

- Upper left part of Figure 2 shows linearly de-trended real per capita output variable $\widehat{y}_t$, constructed as the sum of real per capita private consumption, real per capita investment, real per capita government consumption and real per capita trade balance;

- Lower left part of Figure 2 shows linearly de-trended real per capita private consumption variable $\widehat{c}_t$ based on the national accounts statistics;

- Upper left part of Figure 3 displays de-meaned labour supply variable $\widehat{l}_t$ which is computed as employment share of the total working age population, defined as 15 to 74 years old, because no statistics on the actual hours of work is available;

- Lower left part of Figure 3 depicts linearly de-trended per capita real wage variable $\widehat{wp}_t$ calculated using statistics on nominal wage net of social security contributions and GDP deflator;

- Upper left part of Figure 4 shows linearly de-trended real per capita investment series $\widehat{i}_t$ based on the national accounts statistics;

- Lower left part of Figure 4 depicts linearly de-trended real per capita export series $\widehat{x}_t$ based on the national accounts statistics;

- Upper left part of Figure 5 displays linearly de-trended real per capita import series $\widehat{m}_t$ based on the national accounts statistics;

- Lower left part of Figure 5 shows linearly de-trended euro area real output variable $\widehat{y}_t^{EA}$ sourced from the AWM database;

- Four panels of Figure 6 show de-meaned quarterly inflation rate variables: $\widehat{\pi}_t^d$ calculated using year-on-year change in GDP deflator series, $\widehat{\pi}_t^x$ calculated using year-on-year change in export deflator series, $\widehat{\pi}_t^m$ calculated using year-on-year change in import deflator series and $\widehat{\pi}_t^{EA}$ calculated using year-on-year change in euro area GDP deflator series. First three are sourced from Eurostat's database, the last is from the AWM database;

- Finally, two panels of Figure 7 display de-meaned nominal quarterly interest rates: $\widehat{r}_t^n$ is given by 3 months average deposit rate in Estonia, since no statistics on short-term interest rates on government obligations is available, $\widehat{r}_t^{EA}$ variable sourced from the AWM database.

[30]Developed by Victor Gómez and Agustín Maravall at the Bank of Spain, refer to `www.bde.es` for details.

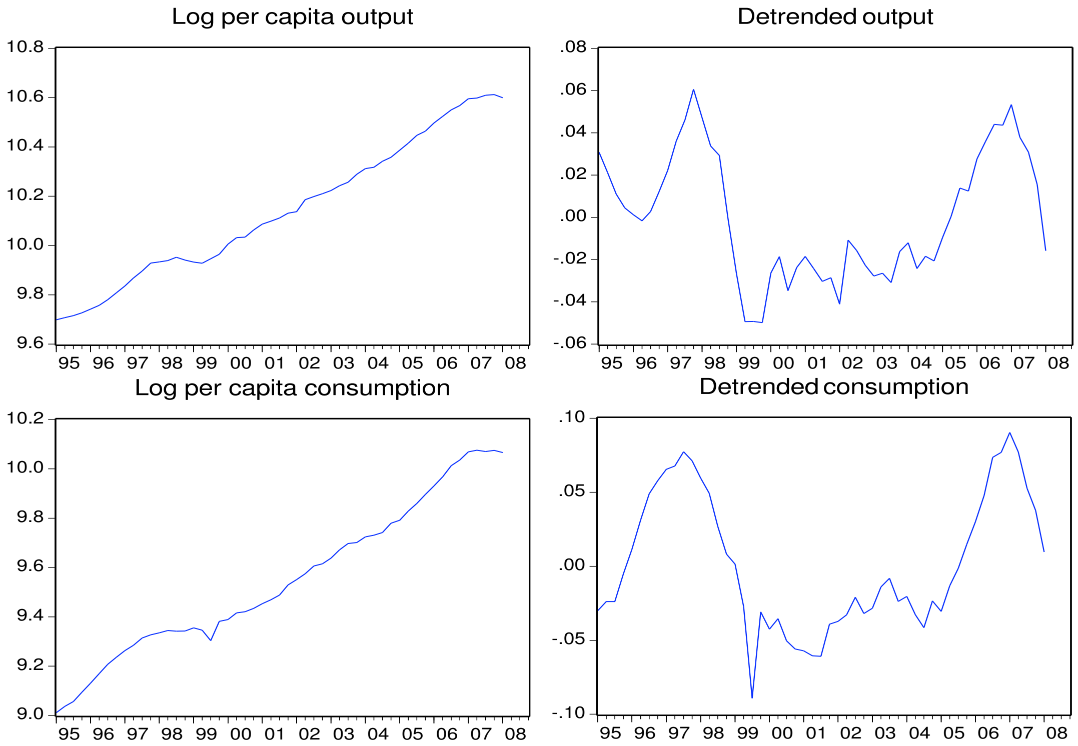

Figure 2: Real per capita output and consumption in Estonia, 1995Q1 to 2007Q4

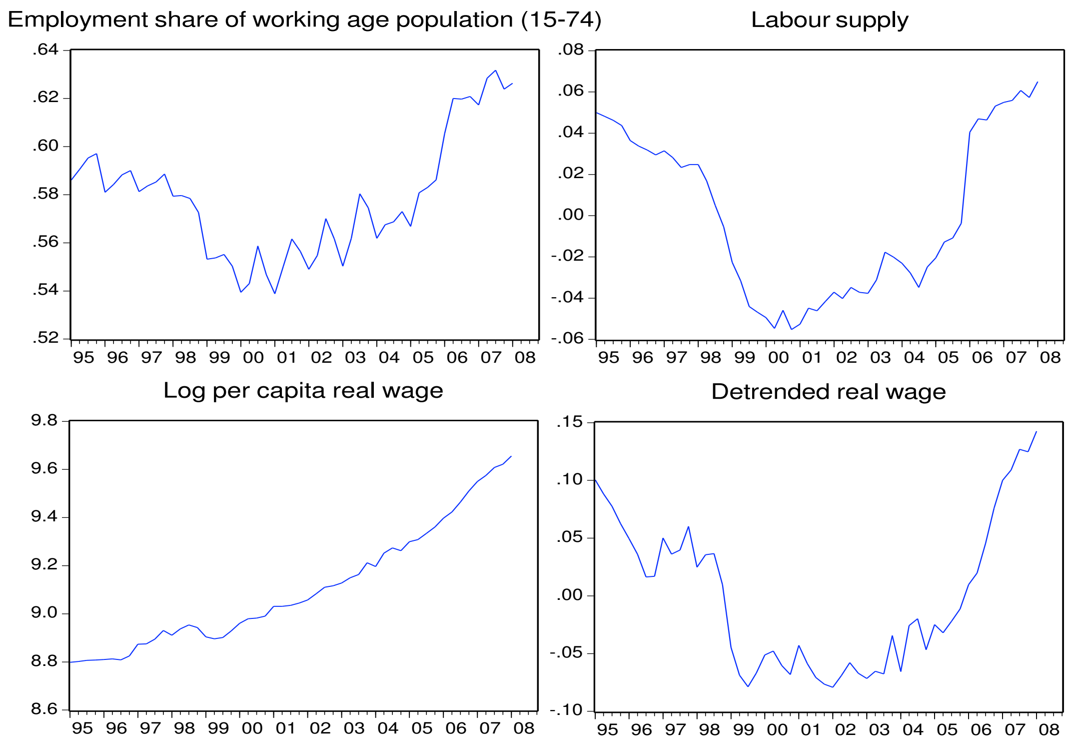

Figure 3: Labour supply and real per capita wage in Estonia, 1995Q1 to 2007Q4

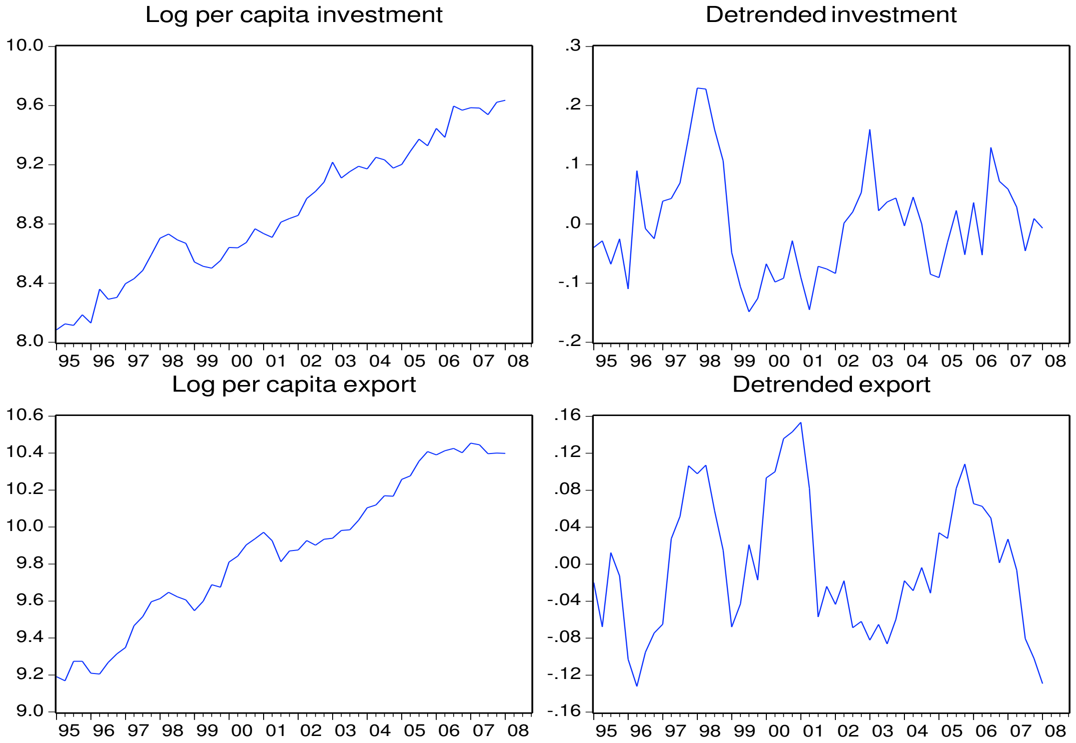

Figure 4: Real per capita investment and export in Estonia, 1995Q1 to 2007Q4

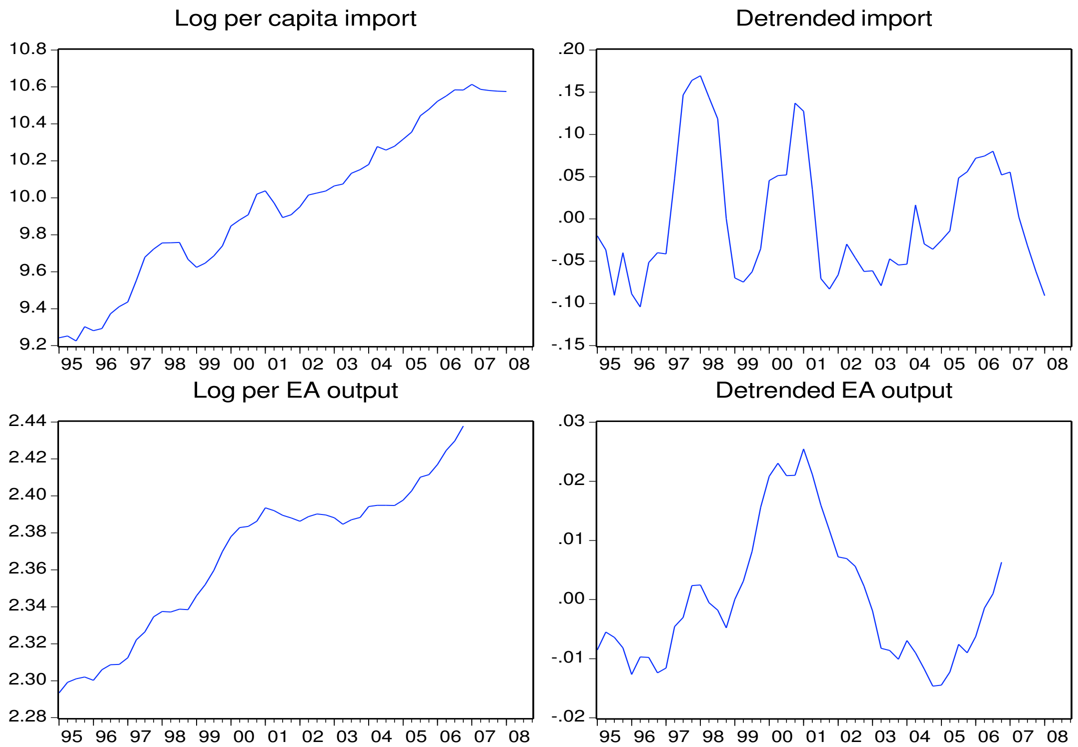

Figure 5: Real per capita import in Estonia and real per capita output in the EA, 1995Q1 to 2007Q4

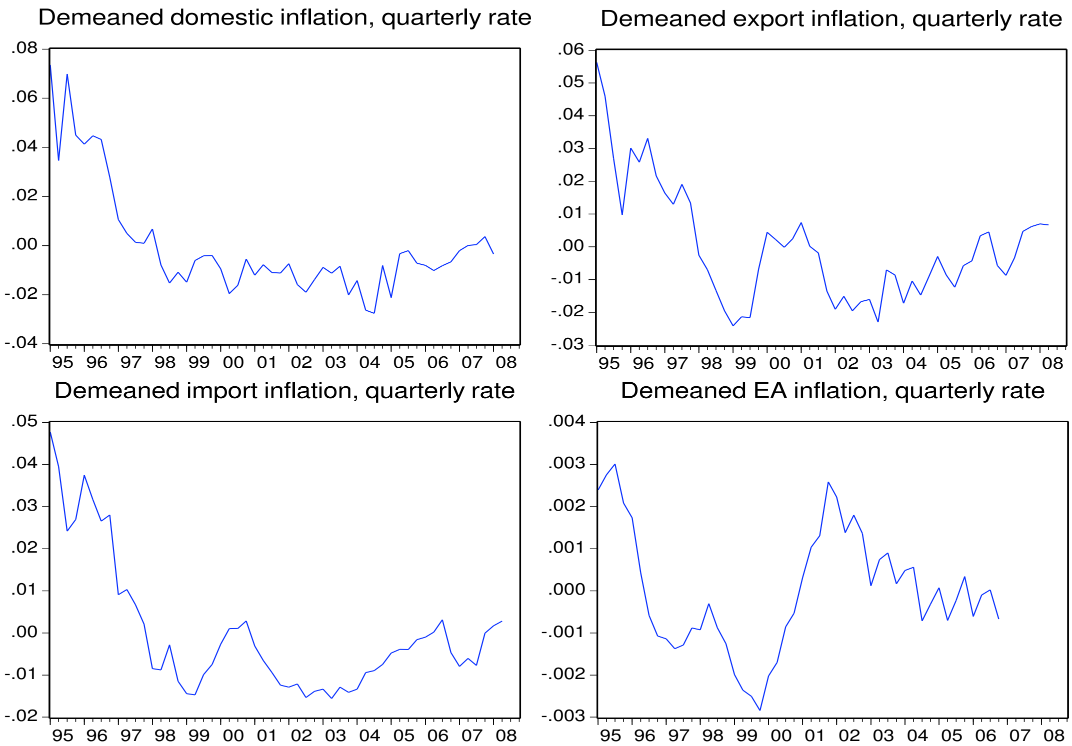

Figure 6: Quarterly domestic, export and import inflation rates in Estonia, and the EA domestic quarterly inflation rate, 1995Q1 to 2007Q4

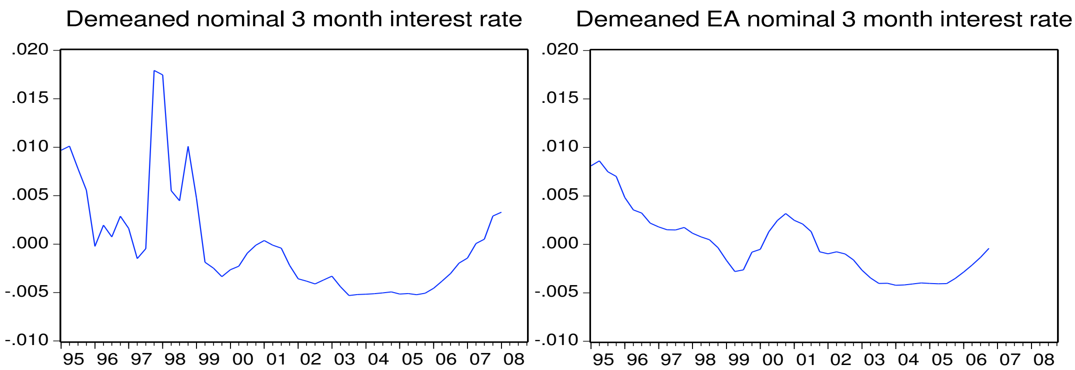

Figure 7: Nominal quarterly interest rates in Estonia and in the EA, 1995Q1 to 2007Q4

The prior distributions and associated hyper-parameters are selected according to Adolfson et. al. (2005) for the Estonian part of the model; refer to the first part of Table 2 for a detailed list of the prior distributions and their associated hyper-parameters. Recall that the euro area part is not estimated in this version of EP DSGE model, and all corresponding parameters are fixed at the values reported in Smets and Wouters (2003). Effectively, degenerate priors have been imposed on the euro area parameters of the model. In addition, a number of steady-state ratios and deep structural parameters for which a good reference value is available on the theoretical grounds are fixed during the estimation. These parameters are listed in Table 1. Among values in the table, steady-state ratios are calibrated to sample averages, the capital–output ratio α is taken from Ratto et. al. (2008), the discount rate β and the wage mark up parameter λ_w from Smets and Wouters (2003), and the remaining parameters are selected to have empirically plausible steady-state values for import–output ratio.[31]

Table 1: Values of fixed parameters

Steady state ratios		**Other parameters**		
C/Y	0.55	Capital output ratio	α	0.46
INV/Y	0.29	Capital depreciation rate	δ	0.025
G/Y	0.24	Intertemp. discount factor	β	0.9875
IMP/Y	0.85	Wage mark up	λ_w	0.5
EXP/Y	0.77	Return on capital	$R^k = (1 - \beta + \beta\delta)/\beta$	0.0378
C/IMP	0.65	Risk free interest rate	$R = 1/\beta$	0.0126
INV/IMP	0.34	Relative price exp./EA	$\gamma^{c,x}$	1
G/IMP	0.28	Share import in consump.	α_c	0.5
EXP/IMP	0.92	El. substit. goods consump/EA	$\eta_{F,c}$	5
YEA/Y	2	Price mark up imp.	$\gamma^{F,c} = \eta_{F,c}/(\eta_{F,c} - 1)$	1.25
YEA/EXP	2.6	Relative price CPI/dom.	$\gamma^c = [1 - \alpha_c(\gamma^{F,c})^{1-\eta_c}]^{1/(1-\eta_c)}$	1.11

6 Empirical results

6.1 Posterior distributions of the parameters

Figure 8 depicts three diagnostic graphs for the Metropolis–Hastings sampling algorithm runs that are used to compute posterior inference in Table 2, refer to Brooks and Gelman (1998). Given the recursive nature of the algorithm, it is required that the runs display as stable

[31] We tried to estimate the model changing the gross mark up on imported goods up to a value of 6 (which means a mark up of 500 per cent), and this does not affect results. We decided for 1.25 because 25 per cent seems a reasonable mark up and because it is usually found in other estimation (See Adolfson et al. (2005) and (2007)) that the mark up is higher in the importing sector, because the price elasticity of demand for good j is lower and firms can exploit this lower willingness to switch from a good to another one and fix a higher mark up. We also fixed the mark up for the domestic producers at 20 per cent, implied by an elasticity of 6 (See Dabušinskas and Kulikov (2007)). We did not estimate neither that mark up (or elasticity), nor any other mark up, because they created a lot of problem in terms of convergence. This is left for future development.

behavior as possible. Moreover, it is important that the convergence measures computed within and between the chains converge. Figure 8 shows that an overall converges is reached after about 200000 draws. As for individual parameters, results are in general satisfactory, although in some cases the diagnostic measures do not appear to be sufficiently stable, especially the ones based on the third moment. But in all cases converges is achieved after about 200000 draws.[32]

Table 2 reports summary statistics of prior and posterior distributions for all estimated parameters of EP DSGE model. The last two columns of the table show 90% confidence intervals; most of the estimated parameters appear to be statistically different from zero based on this measure.[33] In the rest of this subsection the parameters in Table 2 are discussed in an order corresponding to their perceived economic significance.

The parameter ϕ_{fa} that enters the country specific risk premium function $\Omega(FA_t, \epsilon_t^{\mathrm{risk}})$ in (1) has the mean of estimated posterior distribution equal to 0.0088. This value appears to be relatively low in comparison to some previously reported estimates in the literature. For example, in Adolfson et al. (2005) the mode of this parameter is 0.14 in their benchmark model for the euro area.[34] Adolfson et al. (2007) report even higher value for Finland at around 0.3, unaffected by various assumptions about structural shocks and when the UIP equation in their model is modified to include dependence on the exchange rate. The relatively low estimate of ϕ_{fa} in Table 2 may indicate that the net foreign asset position of Estonia is not a good explanatory factor for the observed interest rate spread. This is corroborated by the fact that the idiosyncratic component of $\Omega(FA_t, \epsilon_t^{\mathrm{risk}})$ is estimated to be highly persistent, with the posterior mean of ρ_{risk} equal to 0.9562, suggesting that $\epsilon_t^{\mathrm{risk}}$ captures a high share of the risk premium variation in the data. This can be partly explained by data issues: Figure 7 clearly indicates presence of two pronounced interest rate spikes in Estonian interest rates in the second half of 1990-s induced by the Asian and Russian financial crises. These events have coincided with substantial structural shifts in Estonian banking sector, and a dramatic reduction in the interest rate spread in the following years. The specification of $\Omega(FA_t, \epsilon_t^{\mathrm{risk}})$ function in (1) might be too simple to pick up these changes.

Empirical Calvo parameters reported in Table 2 carry information about the timing of price and wage setting decisions by the domestic firms and households. The posterior mean of price stickiness parameter θ of the domestic intermediate goods producers is given by 0.7080, implying

[32]Diagnostic graphs for the individual parameters are available separately on request.

[33]Note that the standard deviations shown in Table 2 are computed prior to running the Metropolis–Hastings posterior density sampling algorithm. They are based on the quadratic approximation using Hessian evaluated at the posterior kernel mode, and therefore are not fully reliable for two reasons. Firstly, the mode of the final posterior distribution may be different from the one obtained by maximizing the posterior kernel, although it should not be the case in a good estimation. Secondly, even if the two modes coincide, the standard deviations are based on the normality assumption, which is not necessary satisfied by the posterior distribution.

[34]Their estimations reveal that ϕ_{fa} is not robust to different specification of the model. In particular, one of their models estimated assuming i.i.d. mark-up shocks similar to the specification in this paper, has ϕ_{fa} equal to 0.035. This parameter needs additional robustness checks in the future versions of EP DSGE model.

a price duration around 3.5 quarters. This is in line with the recent finings by Dabušinskas and Kulikov (2007), who report price duration around 4 quarters (Calvo parameter of 0.75) from a similarly specified New Keynesian Phillips Curve for Estonia. On the other hand, estimated values of Calvo parameters are usually higher for the euro area, see Smets and Wouters (2003), Adolfson et al. (2005) and Galì and Gerlter (1999).

Turning to the export–import sector, it is worth noting that the corresponding price stickiness parameters are higher in both cases than θ for the domestic intermediate goods producers. The implied price duration ranges from over 8 quarters in the import sector to over 5 quarters in the export sector. On the other hand, Adolfson et al. (2005) report lower price stickiness in the export–import sector relative to the domestic one in their euro area model.

The posterior mean of the wage stickiness parameter θ_w is remarkably low in comparison with the price stickiness coefficients. Implied average duration of a wage contract is below 3 quarters. Both Smets and Wouters (2003) and Adolfson et al. (2005) also find lower degree of wage stickiness relative to the price stickiness for the euro area, although their wage Calvo parameters are somewhat higher than the one obtained for Estonia.

The next set of parameters in Table 2 is related to price and wage indexation, see equations (17), (22), (26), and (27). These parameters are directly linked to the weights of forward and backward looking components in the corresponding Phillips curves and the real wage equation. The posterior mean of the domestic price indexation coefficient τ_π is estimated at 0.8985, giving the weight of the forward looking component in (25) at 0.52 versus 0.48 for the backward looking one. This result is in line with Dabušinskas and Kulikov (2007). Empirical indexation parameters in the export–import sector are slightly lower, with estimated posterior means around 0.81 for both parameters. There is a high degree of wage indexation to the past inflation as indicated by the mean of estimated posterior distribution of τ_w. The implied weights of the present and past inflation rates in (18) are given by 0.92 and 0.42 respectively.

Another parameter of interest that can be calculated using empirical results in Table 2 links the real marginal costs to the domestic inflation rate in the Phillips curve equation (25). The implied value of this parameter computed at the posterior means of θ and τ_π is equal to 0.066. This is notably higher than the previously reported coefficient by Dabušinskas and Kulikov (2007), but a precise statistical comparison of the two results is infeasible due to differences in the estimation methodologies.

The remaining parameters in Table 2 are as follows. The posterior distribution of σ_c, which is the inverse of intertemporal elasticity of substitution of consumption, is centered around 1.85. This is higher than the value 1.39 reported by Smets and Wouters (2003) for the euro area, and implies that Estonian households are less responsive to the variation in the real interest rate than their european counterparts. At the same time, the posterior mean of external consumption habit parameter h is higher in Estonia than in the euro area, the respective values are 0.83 and 0.60. This result may be attributed to the "catching up with Joneses" effect that can characterize a country with high GDP growth rate. The inverse elasticity of work effort with respect to the real wage is governed by the parameter σ_L, posterior mean of which is estimated at 1.59, with the value of corresponding elasticity equal to 0.63. This result is close to the one obtained in Staehr (2008), where he finds that "1 per cent increase in after-tax hourly income would lead to 0.6 percentage point more individuals being employed".

Two parameters linked to the export–import sector in EP DSGE model are the elasticity of substitution between domestic and imported consumption goods η_c and the elasticity of substitution between exported and domestic consumption goods in the euro area η_{EA}. Their posterior means reported in Table 2 are respectively 2.05 and 1.29, both in line with empirical results in the literature, see for instance Ratto et al. (2008).[35] The inverse elasticity of the investment adjustment cost function parameter φ is centered at 7.71. The corresponding elasticity is 0.13, and according to the interpretation of this parameter in Christiano, Eichenbaum and Evans (2005) this point estimate implies that a 1 per cert permanent change in the price of capital induces about a 10 per cent change in investment.

Among estimated autoregressive parameters of the structural shocks reported in Table 2, ρ_β and ρ_x stand out as low relative to the benchmark studies for the euro area in Smets and Wouters (2003) and Adolfson et al. (2005). All other estimated autoregressive coefficients are in line with the literature, with ρ_g emerging as the largest among them. Recall that the fiscal policy in EP DSGE model in this paper is effectively exogenous w.r.t. all other sectors, refer to (29). A relatively large estimated ρ_g parameter may indicate that the persistence coming from other structural disturbances is insufficient to describe the observed dynamics of Estonian macroeconomic time series.

6.2 Fit to the data

The fit of EP DSGE model to the macroeconomic data series for Estonia and euro area described in section 5 is shown on Figure 9.[36] Specifically, the real-world data are plotted against the one-sided Kalman filter predicted values of the corresponding series. The procedure to compute one step ahead forecast of $\tilde{\boldsymbol{y}}_t$ based on the partially unobserved state vector $\tilde{\boldsymbol{x}}_t$ together with data up to the period t in the Kalman filter representation of DSGE model (36) is described in detail in Hamilton (1994).

Figure 9 shows that the fit of EP DSGE model to Estonian macroeconomic series related to the domestic sector is good, with an exception of the nominal interest rate variable. However, the model can be considered inadequate in reproducing the essential features of the main foreign sector variables, including the euro area variables. Recall that the first version of EP DSGE

[35]The typical estimates for the elasticity of substitution between home and foreign goods are around 5 to 20 using the micro data, see the references in Obstfeld and Rogoff (2000). However, the macro data–based estimates are usually a lot lower, in the range of 1.5 to 2, see e.g. Collard and Dellas (2002).

[36]There are several other methods that can be used to assess the goodness of fit of DSGE models, refer to An and Schorfheide (2007). Among them is posterior odds comparison of DSGE model with a bayesian VAR model, and comparison of the autocorrelation and cross-correlation structure of the real-world and DSGE model-generated time series. A more thorough model checking exercise is left for the future version of the EP DSGE model.

Table 2: Priors and posteriors of the EP DSGE model parameters

Parameters	Priors			Posteriors			
	distribution	mean	s.d.	mean	s.d.	90% interval	
Shocks' standard deviations							
Preference (σ_β)	*Inv. Gamma*	0.2	∞	0.0733	0.0133	0.0423	0.1062
Labour supply (σ_L)	*Inv. Gamma*	0.2	∞	0.0985	0.0455	0.0458	0.1611
Investment specific (σ_x)	*Inv. Gamma*	0.1	∞	0.0284	0.0041	0.0195	0.0359
Technology (σ_a)	*Inv. Gamma*	0.4	∞	0.0508	0.0032	0.0470	0.0549
Equity premium (σ_q)	*Inv. Gamma*	0.4	∞	0.2792	0.0889	0.1072	0.4681
Government spending (σ_g)	*Inv. Gamma*	0.3	∞	0.1855	0.0337	0.1521	0.2184
Risk premium (σ_{risk})	*Inv. Gamma*	0.08	∞	0.0134	0.0023	0.0107	0.0157
Wage (σ_w)	*Inv. Gamma*	0.25	∞	0.0338	0.0026	0.0294	0.0383
Price mark-up domestic (σ_{λ^p})	*Inv. Gamma*	0.15	∞	0.0215	0.0036	0.0176	0.0238
Price mark-up imports ($\sigma_{\lambda^{c,F}}$)	*Inv. Gamma*	0.4	∞	0.2382	0.0456	0.0894	0.3585
Price mark-up exports ($\sigma_{\lambda^{c,x}}$)	*Inv. Gamma*	0.4	∞	0.1594	0.0222	0.0745	0.1763
Auto-regressive coefficients							
Fiscal policy (ρ_g)	*Beta*	0.85	0.1	0.9695	0.0125	0.9504	0.9895
Preference (ρ_β)	*Beta*	0.85	0.1	0.5482	0.0561	0.3982	0.6976
Technology (ρ_a)	*Beta*	0.85	0.1	0.7999	0.0221	0.6538	0.9449
Investment specific (ρ_x)	*Beta*	0.85	0.1	0.4440	0.0964	0.2858	0.6177
Labour supply (ρ_L)	*Beta*	0.85	0.1	0.8078	0.0372	0.6444	0.9778
Risk premium (ρ_{risk})	*Beta*	0.65	0.1	0.9562	0.0225	0.9318	0.9817
Calvo parameters							
Inflation domestic (θ)	*Beta*	0.75	0.05	0.7080	0.0360	0.6558	0.7606
Inflation imports ($\theta_{c,F}$)	*Beta*	0.5	0.1	0.8795	0.0380	0.8323	0.9284
Inflation exports ($\theta_{c,x}$)	*Beta*	0.5	0.1	0.8161	0.0637	0.7574	0.8735
Wage (θ_w)	*Beta*	0.7	0.05	0.6311	0.0184	0.5611	0.6998
Indexation parameters							
Inflation domestic (τ_π)	*Beta*	0.75	0.15	0.8985	0.1176	0.8023	0.9940
Inflation imports ($\tau_{c,F}$)	*Beta*	0.5	0.15	0.8240	0.1281	0.7162	0.9364
Inflation exports ($\tau_{c,x}$)	*Beta*	0.5	0.15	0.8060	0.1048	0.7021	0.9152
Wage (τ_w)	*Beta*	0.75	0.15	0.8287	0.1063	0.6651	0.9927
Elasticities							
Inverse el. of intertemporal subst. (σ_c)	*Normal*	1	0.375	1.8488	0.1604	1.4264	2.2506
Inverse el. of labour supply (σ_L)	*Normal*	2	0.75	1.5917	0.7427	0.4462	2.6772
El. of substitution dom/imp goods (η_c)	*Inv. Gamma*	2	0.1	2.0508	0.1833	1.7540	2.3576
El. of substitution exp/EA goods (η_{EA})	*Inv. Gamma*	2	0.1	1.2895	0.1491	1.1991	1.3715
Inverse el. of capital utilization (ψ)	*Normal*	0.2	0.075	0.1662	0.0386	0.0498	0.2799
Inverse el. of investment adj. cost (φ)	*Normal*	4	1.5	7.7053	1.3986	5.8108	9.6105
Other parameters							
1+share of fix costs (ϕ)	*Normal*	1.45	0.2	1.1986	0.3191	0.7425	1.6277
Habit (h)	*Beta*	0.7	0.05	0.8297	0.0208	0.7867	0.8723
Risk premium (ϕ_{fa})	*Inv. Gamma*	0.5	∞	0.0088	0.0008	0.0070	0.0104

Notes: The EP DSGE model parameter name and symbol are shown in the first column. The following three columns describe the corresponding prior distribution: its type, and the first two moments. The last four columns summarize the posterior distribution: its mean, and 5% and 95% percentiles are computed using the Metropolis–Hastings sampler, its standard deviation is a quadratic approximation at the posterior mode.

model presented in this paper uses calibration for the euro area part of the model based on Smets and Wouters (2003). It is based on the sample period 1980Q2 to 1999Q4 which only partially overlaps with the newer data sample used for the EP DSGE model in this paper. Foreign sector variables in EP DSGE model, such as export, import and the corresponding prices, depend on the evolution of the main euro area variables, and therefore their fit is likely to be substantially improved by switching from calibration to estimation of the euro area part of EP DSGE model in the future.

6.3 Impulse Response Functions

The impulse response functions for most important model variables are shown on Figures 14 to 22. These are orthogonalized responses to one standard deviation of all model shocks.[37]

I want to comment first the responses to a risk premium shock (i.e. an increase in the risk premium), depicted on Figure 20. This would require to talk about the real exchange rate, because of the effects on imports and exports. Nevertheless, we do not have any real explicit exchange rate in the mode, but we have several relative prices which are indirectly related to it. We derived that relationship in appendix 8.1 and we refer to that, freely referring the the real exchange rate.

A positive risk premium shock generate a depreciation of the real exchange rate. This leads to an increase of export (Estonian goods are more competitive abroad) and to a decrease in import (Euro Area goods are more expensive).[38] This positive effect on exports together with the switch of consumption from foreign goods to presumably domestic goods should increase domestic output. Nevertheless, there is another effect of the risk premium shock which counteracts the fist one, i.e. the implied increase of the domestic interest rate. It depresses investment and consumption (the latter because of the intertemporal substitution effect) and the total effect on output is higher that the one coming from exports and imports, generating a downturn. Employment decreases as a consequence (implying a decrease of wages as well). In the end, domestic inflation decreases because the standard mechanism governing it at the basis of the NK models (incorporate in the NK Phillips Curve). If output drops above the potential, it means that also the marginal cost are below their natural level. Given that mark up is the inverse of the marginal costs, it is above the potential (or desired) level. Firms wants then to reduce prices (and hence inflation) to drive back their mark up to their desired level.

The technology shock is presented on Figure 14. The responses are in line with the all other estimations. A rise in productivity leads to an increase in output, consumption and investment. In line with what found by Galì 1999 (and confirmed by Smets and Wouters (2005) and Adolfson et al. (2005) among others), hours worked decrease after a positive technology

[37]We have chosen to report only those responses to save space. All the other responses are available on request.

[38]A one standard deviation risk premium shock leads on impact to a response of γ_t^{EA} equal -0.001201 and of γ_t^c equal to 0.0005501. The negative response is higher and, given the equation for the real exchange rate, equation (A.3), the latter increases (i.e. there is a depreciation). This generates the responses of imports and exports described in the text.

shock. In a model where there is a rule that describe monetary policy, the nominal interest rate in this situation would decrease (as in Smets and Wouters 2003) and in part compensate the reduction in marginal costs. Moreover, marginal costs are not recovered enough and then inflation decreases as well. In our model, interest rate does not react because of domestic changes but as a consequence of changes in foreign (Euro Area) related variables caused by domestic development. In fact, the technology shock causes a real exchange rate depreciation[39], and this has the standard consequences on import and export. Interest rate falls because the net foreign assets increase, due to the excess of exports on imports.

Another interesting shock is the labour supply one, it is shown on Figure 16. Its implications are very similar to the technology shock ones. Nevertheless, it is worth noting that now hours worked increase and as a consequence wages drop, forcing domestic inflation down (via the effect on marginal costs). The fall in inflation affects consumption which in turns stimulates output and investments. The increase in domestic output together with the reduction in inflation stimulate exports and reduce imports, causing the net freing assets to increase. Although an initial increase in imports, the effect on the interest rate is always negative and very low.

A positive wage mark up is supposed to have completely reverse effects. In fact after such a shock wages increase.[40] This reduction transmit to both labour and domestic inflation, decreasing the former and increasing the latter. Both those consequences generate a fall in consumption and then on output and in turns to investments. In addition, there is a real exchange appreciation (due the increase in domestic inflation) which causes exports to fall and imports to goes up. The positive effect on the interest rate due to the negative net foreign asset position is observed.

The most part of the reaming shocks are quite standard and their effects are by now extensively described in many paper. Moreover, the description above for the other shocks make clear which are the mechanism underlying the model and make the interpretation easy. Hence we leave it to the reader.

We want instead to highlight the effects of the two Euro Area monetary shocks.

The interest rate in Estonia is related one to one to the Euro Area interest rate as described by the UIP equation, with the net foreign asset positions and with the stochastic part. A positive interest rate shock in the Euro Area has the well know effects in that area on output and inflation. They are the same affect that can be found in Smets and Wouters (2003). But what happens in Estonia? There are two effects which have a direct negative impact on the Estonian output. On the one hand the domestic interest rate increases. On the other

[39] Again, the depreciation of the real exchange rate id due to the contemporaries variation of γ_t^{EA} and γ_t^c. They move in the opposite direction but the negative movement of γ_t^{EA} (-0.007599) dominates the positive movement of γ_t^c (0.00348), hence re_t increases

[40] This increases in wages may be explained in different ways. If the mark up shock is seen as an increased power of the Trade Unions, wages are higher because they have more bargaining power and then they can contract a higher wage. As a result, and as prescribed by the theories about the relationship between wages and Trade Unions power, there is a negative effect on employment. Contrary, if we consider that shock as an increased workers' preference for leisure. In this case wages drop because there is a drop in the labour supply.

hand exports fall, because of the decrease of the Euro Area output. The domestic downturn depress investments and has a negative effect on employment, which in turns negatively affects consumption and wages. The net negative effect of exports and imports (the latter decrease as a consequence of the domestic drop in consumption, but less then exports) on the net foreign asset position justifies the extra positive response of the interest rate.[41]

As for the inflation objective shock, there are many mechanisms in place. An increase in the inflation target has the first effects on inflation, which increases as well. A direct consequence is the increase of the Euro Area interest rate. Nevertheless the Euro Area output does not decrease, but increases. Why? Let's see what happens in Estonia. Interest rate goes up, but again that does not cause any reduction in output investment and consumption. The reason lays in the fact that the effect on domestic inflation is higher that the effects on the inflation of the imported goods, and this shifts the domestic consumption towards the imported goods. This is the explanation of the increase in the output in the Euro Area. In addition, the negative effect of the increased interest rate in Estonia on consumption is mode the compensated by the increase in Estonia Exports due to the increase in the Euro Area change in output. The positive effect on consumption is then the cause of the expansion in the estonia economy, with the consequences of increasing hours worked and wages (there is more demand for workers form the firms which are growing)

6.4 Variance Decomposition

The relative importance of structural shocks for the dynamics of state variables in EP DSGE is measured by the share of total variation that a particular shock helps to explain for each given state variable of the model. The variance decomposition methodology is standard in time series analysis, refer to Hamilton (1994) pp. 323–324, and can be applied to DSGE models written in the vector autoregressive form (36).

In this subsection the variance decomposition of EP DSGE model is carried for the full two–area model as well as for the Estonian economy part only. The separate decomposition for Estonia is needed because the euro area structural shocks dominate the variability of the model's state variables, making it difficult to assess the relative importance of Estonian shocks for the Estonian economy.[42] The variance decomposition for Estonia is computed by setting the standard deviations of the euro area structural shocks to zero. In addition to 10 standard euro area shocks, the risk premium innovation ϵ_t^{risk} is also shut down in the variance decomposition for Estonia, although this shock cannot be considered as *bona fide* outside shock for the Estonian economy.[43]

[41]Without this effect the domestic interest rate would have increased one by one with the Euro Area interest rate.

[42]Recall that the euro area part of the EP DSGE model is calibrated, including the standard errors of the corresponding structural shocks. The latter are notably higher than the estimated standard errors of the structural shocks for the Estonian economy part of the model, leading to an imbalance in the variance decomposition results for the full model.

[43]The variance decomposition in Table 3 is not substantially affected by the choice to shout down the

Table 3 presents the variance decomposition results for the Estonian economy part of EP DSGE model.[44] It is possible to see that the most important shock is the domestic price mark up shock, which in some cases explains almost 50% of state variables dynamics. This result is somewhat unexpected, as this shock is usually found to be of a secondary importance in explaining the variance of endogenous variables, apart inflation and wages, even at a very short time horizon. The second most important shock, as is normal in the literature, is the technology shock. It explains about the 30% of the variance of many state variables. The remaining shocks have a marginal contribution to the overall variability of the model, apart from direct effects on the variables that they are linked to. Notably, the wage mark shock, although not very dominant, appears to be more relevant that usually found in literature, see Smets and Wouters (2003).

Table 4 reports the variance decomposition for the full model that includes Estonian and euro area part with a full complement of 21 structural shocks. Ideally, one can divide the table into four quadrants corresponding to the two parts of the model in the vertical dimension, and two sets of structural shocks in the horizontal one. Contribution of the Estonian shocks to the Estonian part of the model appears in the upper left quadrant. It is easy to see, that the importance of these shocks is considerably smaller in the full model than in Table 3, although the structure of their relative contribution to the variation of domestic endogenous variables remains the same. The lower left quadrant of the table shows that Estonian shocks have no effects on the euro area state variables. The lower right quadrant is almost entirely similar to the variance decomposition reported in Smets and Wouters (2003). Finally, the upper right quadrant of Table 4 shows that the most important euro area shocks for the Estonian part of the model are the labour supply shock, the interest rate shock, and the technology shock.

idiosyncratic risk premium component. When this shock is present, it serves to decrease the effect of the technology and the domestic mark up shocks in explaining the variability of the states, without dominating the overall picture.

[44] All results in this subsection are based on 20 lags approximation to the unconditional variance–covariance matrix of the vector of state variables.

Table 3: Variance decomposition for the Estonian economy part of EP DSGE (in per cent)

States	Shocks									
	u^a	u^g	u^β	u^x	u^l	u^w	u^{λ^p}	$u^{\lambda^{c,F}}$	u^{λ^x}	u^q
y	39.05	5.63	0.49	3.17	1.78	7.87	40.04	0.05	1.63	0.28
c	33.86	3.17	16.74	2.03	1.46	6.68	35.65	0.06	0.26	0.1
inv	35.61	3.25	3.87	27.32	2.36	6.16	19.01	0.05	0.34	2.04
r	30.59	0.56	1.68	0.76	0.82	7.05	58.02	0.1	0.38	0.04
r^n	31.21	0.55	3.06	0.85	0.89	7.02	55.96	0.15	0.27	0.04
π^D	21.26	0.39	0.35	0.42	0.47	5.09	71.66	0.03	0.31	0.02
π^c	21.15	0.38	0.35	0.42	0.47	5.06	71.27	0.57	0.3	0.02
k	40.85	5.21	5.01	21.19	3.19	6.61	16.26	0.07	0.39	1.21
l	31.84	7.64	0.67	4.36	2.81	13.13	37.01	0.07	2.07	0.38
q	10.33	0.23	0.98	0.53	0.35	2.17	15.08	0.02	0.07	70.23
mc	52.97	0.41	0.4	0.34	0.42	14.64	30.56	0.01	0.22	0.03
w	33.45	0.22	0.67	0.15	0.81	26.65	37.87	0.02	0.15	0.01
r^k	34.86	3.56	0.6	3.26	1.28	7.49	47.47	0.05	1.18	0.26
fa	31.21	0.55	3.06	0.85	0.89	7.02	55.96	0.15	0.27	0.04
$\pi^{c,F}$	0	0	0	0	0	0	0	100	0	0
$mc^{c,F}$	0	0	0	0	0	0	0	100	0	0
$\gamma^{c,x}$	35.17	1.22	1.26	1.4	1.25	7.59	45.89	0.17	5.98	0.07
$\gamma^{c,F}$	28.48	0.62	0.65	0.74	0.76	6.56	60.8	0.86	0.51	0.04
$\pi^{c,x}$	30.22	0.89	0.68	0.96	0.93	6.71	46.81	0.09	12.65	0.05
$mc^{c,x}$	23.35	0.41	0.32	0.44	0.5	5.56	64.88	0.02	4.49	0.03
γ^c	28.54	0.62	0.65	0.74	0.76	6.57	60.94	0.62	0.51	0.04
γ^{EA}	28.7	0.62	0.65	0.74	0.77	6.61	61.29	0.07	0.51	0.04
Exp	35.17	1.22	1.26	1.4	1.25	7.59	45.89	0.17	5.98	0.07
m	24.48	0.56	8.01	0.53	0.58	5.72	58.93	0.75	0.4	0.03

Table 4: Variance decomposition for the full EP DSGE model (in per cent)

States	Shocks																				
	u^a	u^g	u^β	u^x	u^l	u^w	u^{λ^p}	u^{risk}	$u^{\lambda^{c,F}}$	u^{λ^x}	u^q	$u^{q,EA}$	$u^{\lambda^{p,EA}}$	$u^{w,EA}$	$u^{r^{n,EA}}$	$u^{\beta,EA}$	$u^{a,EA}$	$u^{x,EA}$	$u^{l,EA}$	$u^{\pi,EA}$	$u^{g,EA}$
y	0.14	0.02	0	0.01	0.01	0.03	0.14	0	0	0.01	0	0.01	7.58	0.36	11.36	16.13	7.01	0.53	56.52	0.03	0.12
c	0.35	0.03	0.17	0.02	0.02	0.07	0.37	0.11	0	0	0	0	1.58	0.21	13.32	1	12.9	0.61	69.2	0.03	0.01
inv	1.11	0.1	0.12	0.85	0.07	0.19	0.59	0.16	0	0.01	0.06	0	1.86	0.39	12.02	1.32	10.16	2.7	68.25	0.03	0.01
r	0.1	0	0.01	0	0	0.02	0.19	0.1	0	0	0	0	2.55	0.22	14.26	1.44	13.79	0.91	66.35	0.02	0.02
r^n	0.12	0	0.01	0	0	0.03	0.21	0.27	0	0	0	0	1.53	0.09	11.82	5.27	29.64	1.26	49.7	0.01	0.02
π^D	0.11	0	0	0	0	0.03	0.38	0.01	0	0	0	0	3.35	0.45	13.5	10.18	5.94	0.66	65.31	0.04	0.02
π^c	0.09	0	0	0	0	0.02	0.3	0.01	0	0	0	0	4.99	0.7	14.16	9.54	4.39	0.54	65.19	0.05	0.02
k	1.71	0.22	0.21	0.89	0.13	0.28	0.68	0.24	0	0.02	0.05	0	1.73	0.56	11.76	1.84	8.22	5.45	65.96	0.03	0.03
l	0.1	0.02	0	0.01	0.01	0.04	0.11	0	0	0.01	0	0.01	8.16	0.36	11.16	17.21	6.88	0.53	55.21	0.03	0.14
q	0.18	0	0.02	0.01	0.01	0.04	0.27	0.1	0	0	1.24	0	1.8	0.2	13.4	1.02	13.14	0.59	67.94	0.02	0.01
mc	0.63	0	0	0	0	0.17	0.36	0.01	0	0	0	0.01	4.76	0.34	12.21	10.92	7.8	0.49	62.2	0.03	0.05
w	0.42	0	0.01	0	0.01	0.34	0.48	0.01	0	0	0	0	3.91	0.33	12.52	9.27	8.01	0.5	64.11	0.03	0.04
r^k	0.14	0.01	0	0.01	0.01	0.03	0.19	0.01	0	0	0	0.01	6.84	0.34	11.57	14.9	7.4	0.46	57.94	0.03	0.1
fa	0.27	0	0.03	0.01	0.01	0.06	0.49	0.11	0	0	0	0.02	4.29	0.28	10.9	21.7	6.49	0.89	54.29	0.03	0.13
$\pi^{c,F}$	0	0	0	0	0	0	0	0	0.05	0	0	0	12.2	2.09	15.21	6.33	1.37	0.04	62.65	0.07	0
$mc^{c,F}$	0	0	0	0	0	0	0	0	0.04	0	0	0	59.44	2.21	4.23	3.33	2.61	0.01	28.13	0.02	0
$\gamma^{x,c}$	0.06	0	0	0	0	0.01	0.08	0.01	0	0.01	0	0	13.86	0.28	11.7	8.62	15.68	2.33	47.27	0.03	0.03
$\gamma^{F,c}$	0.08	0	0	0	0	0.02	0.18	0.01	0	0	0	0	1.68	0.3	12.5	10.81	7.42	0.97	65.96	0.03	0.02
$\pi^{c,x}$	0.04	0	0	0	0	0.01	0.07	0.01	0	0.02	0	0	2.63	0.49	15.46	9.74	4.25	0.82	66.38	0.05	0.02
$mc^{c,x}$	0.15	0	0	0	0	0.04	0.42	0.01	0	0.03	0	0	3.97	0.4	12.04	10.62	7.21	0.52	64.52	0.03	0.03
γ^c	0.08	0	0	0	0	0.02	0.18	0.01	0	0	0	0	1.7	0.28	12.64	10.82	7.25	0.96	65.98	0.04	0.02
γ^{EA}	0.11	0	0	0	0	0.03	0.24	0.01	0	0	0	0	4.16	0.09	11.72	10.26	12.96	1.34	59	0.03	0.03
Exp	0.05	0	0	0	0	0.01	0.07	0.01	0	0.01	0	0.02	9.27	0.42	10.3	20.78	5.97	0.71	52.21	0.02	0.16
m	0.05	0	0.02	0	0	0.01	0.11	0.02	0	0	0	0	1.76	0.29	12.37	8.55	8.61	0.63	67.52	0.03	0.02
c^{EA}	0	0	0	0	0	0	0	0	0	0	0	0	0.39	0.06	5.59	48.13	7.23	8.2	30.36	0.01	0.03
inv^{EA}	0	0	0	0	0	0	0	0	0	0	0	0.05	1	0.12	9.93	2.33	9.03	31.12	46.33	0.03	0.05
q^{EA}	0	0	0	0	0	0	0	0	0	0	0	6.42	0.51	0.19	9.24	4.09	11.59	8.69	59.22	0.01	0.03
$r^{k,EA}$	0	0	0	0	0	0	0	0	0	0	0	0.01	1.85	5.21	9.04	9.87	7.68	2.18	64.12	0.02	0.03
$r^{n,EA}$	0	0	0	0	0	0	0	0	0	0	0	0.01	2.01	0.26	7.71	37.86	35.21	4.36	12.55	0.01	0.02
π^{EA}	0	0	0	0	0	0	0	0	0	0	0	0	53.04	1.59	6.96	3.55	1.59	0.02	33.22	0.03	0
k^{EA}	0	0	0	0	0	0	0	0	0	0	0	0.02	1.09	0.18	10.71	2.24	8.24	32.43	45	0.03	0.05
wp^{EA}	0	0	0	0	0	0	0	0	0	0	0	0	3.49	24.05	3.99	8.23	0.91	0.32	58.99	0.01	0
l^{EA}	0	0	0	0	0	0	0	0	0	0	0	0.01	0.88	1.71	9.63	15.11	29.38	3.19	39.93	0.02	0.13
y^{EA}	0	0	0	0	0	0	0	0	0	0	0	0.01	0.98	0.13	10.68	17.13	12.14	2.29	56.5	0.03	0.11
$y^{P,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.46	35.18	1.29	62.03	0	0.04
$c^{f,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6.48	34.52	6.6	52.35	0	0.05
$inv^{f,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	13.74	22.86	13.21	50.14	0	0.06
$q^{f,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	30.45	29.47	2.92	37.14	0	0.03
$r^{k,f,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.84	32.23	3.84	60.07	0	0.02
$r^{n,f,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.14	33.5	28.02	1.48	33.81	0.03	0.03
$\pi^{f,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10.48	33.85	25.7	0.54	29.39	0.03	0.02
$k^{f,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11.99	21.37	13.27	53.29	0	0.07
$w^{f,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.07	98.85	0.07	1.02	0	0
$l^{f,EA}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.96	19.29	1.76	76.95	0	0.04

7 Conclusion

This paper presents a detailed theoretical structure and the set of empirical results for the first version of EP DSGE model. The latter is a two area DSGE model specifically designed to match the essential characteristics of Estonian economy: the currency board regime, free capital mobility, and dependence on the outside economic environment via foreign trade. These are typical features of a small open economy in the vicinity of a much bigger economic zone. The EP DSGE model consists of two interlinked parts, the domestic economy part describing Estonian economy, and the euro area part acting as a large outside closed economy with monetary policy and trade links to the first part. The Estonian economy part has 24 state variables and 11 structural shocks, while the euro area part consists of 13 state variables and 10 structural shocks.

The first version of EP DSGE model focuses on the business cycle frequency fluctuations of the main Estonian macroeconomic aggregates, leaving their long run trends aside. The future developments of the model are likely to incorporate the long run dynamics as well, considering that Estonia is still subject to effects of real and nominal convergence stemming from its catch-up with the developed euro area economies. Other areas of the future theoretical developments of the model include incorporation of the financial sector together with relevant frictions, adding the housing sector combined with collateral-constrated type of households, and expanding the government sector part of the model.

Empirical part of this paper reports Bayesian estimation results for model's structural parameters, impulse response functions and variance decomposition of the state variables. Out of 52 structural parameters in EP DSGE, 34 are estimated using a data sample consisting of 14 macroeconomic series for Estonia and the euro area. Statistical estimates of the main structural parameters are largely in line with previous studies for Estonia, when a direct comparison can be made. It is also worth mentioning that the net foreign asset position of Estonia has been found an important and statistically significant factor in explaining the country risk premium in UIP equation for the interest rates, but the results suggest that other explanatory factors need to be considered as well.

The empirical relevance of structural shocks is assessed using the variance decomposition. It has been found that the most important domestic shocks in explaining the variability of Estonian macroeconomic series are the price mark up shock, that often dominates the other shocks contributing 50% or more of the variability of the state variables, and the technology shock. The euro area shocks also play a very significant role in driving the dynamics of Estonian macroeconomic aggregates. Among the most prominent euro area shocks that affect Estonia are the labor supply, the interest rate and the technology shock.

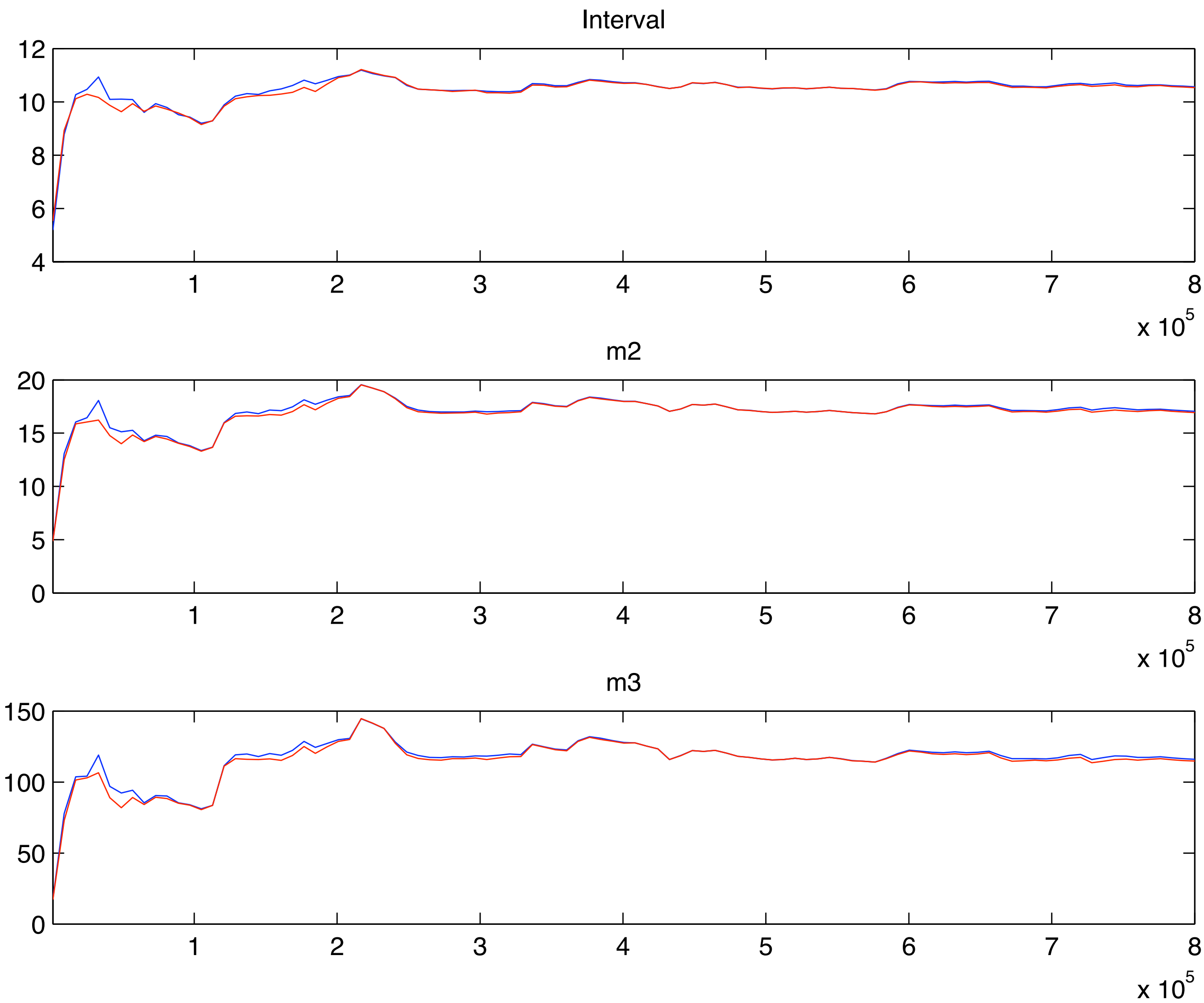

Figure 8: The red and blue lines represent specific measures of the parameter vector both within and between chains. For the results to be sensible, they should be relatively constant (although there will always be some variation) and should converge. Dynare reports three measures: "interval", being constructed from an 80% confidence interval around the parameter mean, "m2", being a measure of the variance and "m3" based on third moment. In each case, Dynare reports both within and between chains measures. The overall convergence picture presents results of the same nature, except that they reflect an aggregate measure based on the eigenvalues of the variance-covariance matrix of each parameter

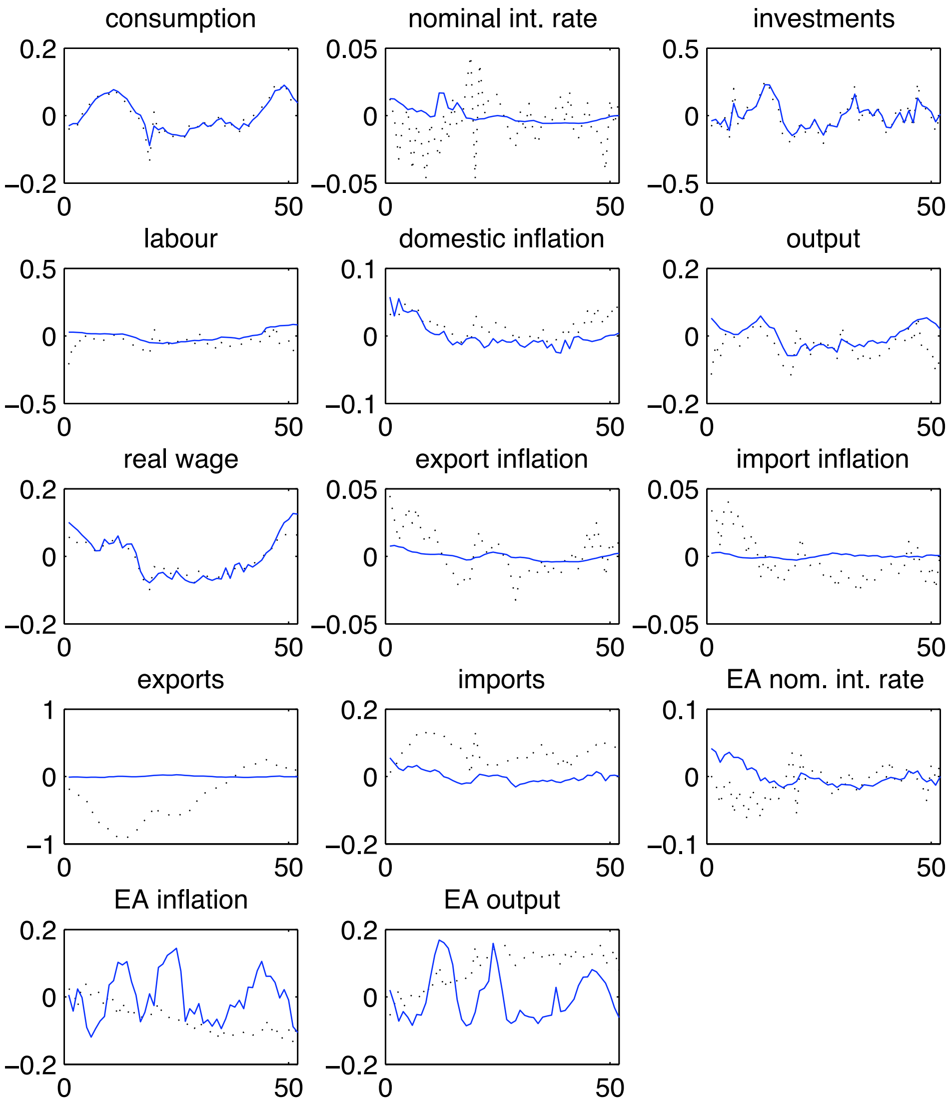

Figure 9: Sample data (blue solid line) and one-sided predicted values (black dotted line)

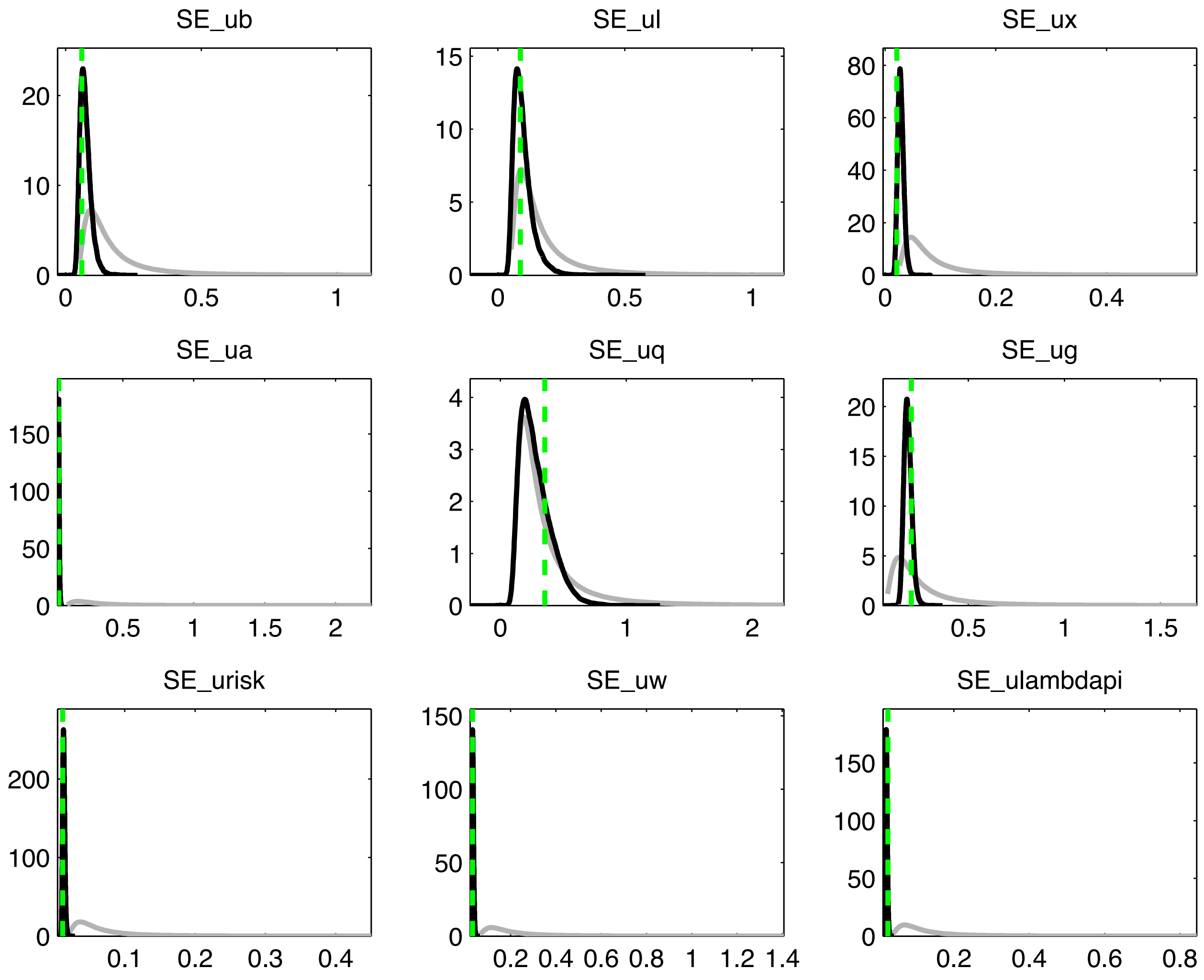

Figure 10: The prior (light colored line) and posterior (dark colored line) distributions; green vertical line is the posterior mode obtained by the posterior kernel maximization

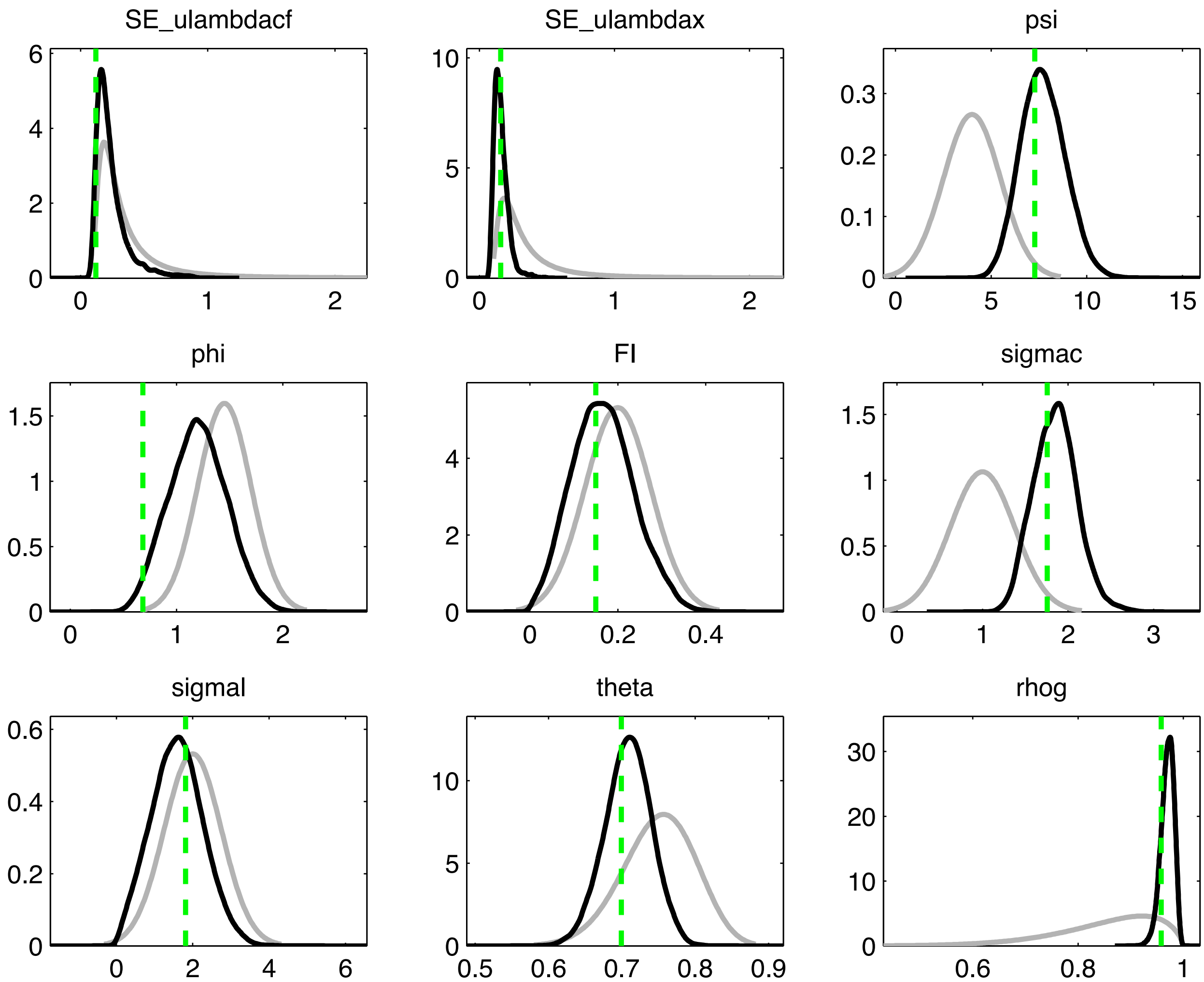

Figure 11: The prior (light colored line) and posterior (dark colored line) distributions; green vertical line is the posterior mode obtained by the posterior kernel maximization

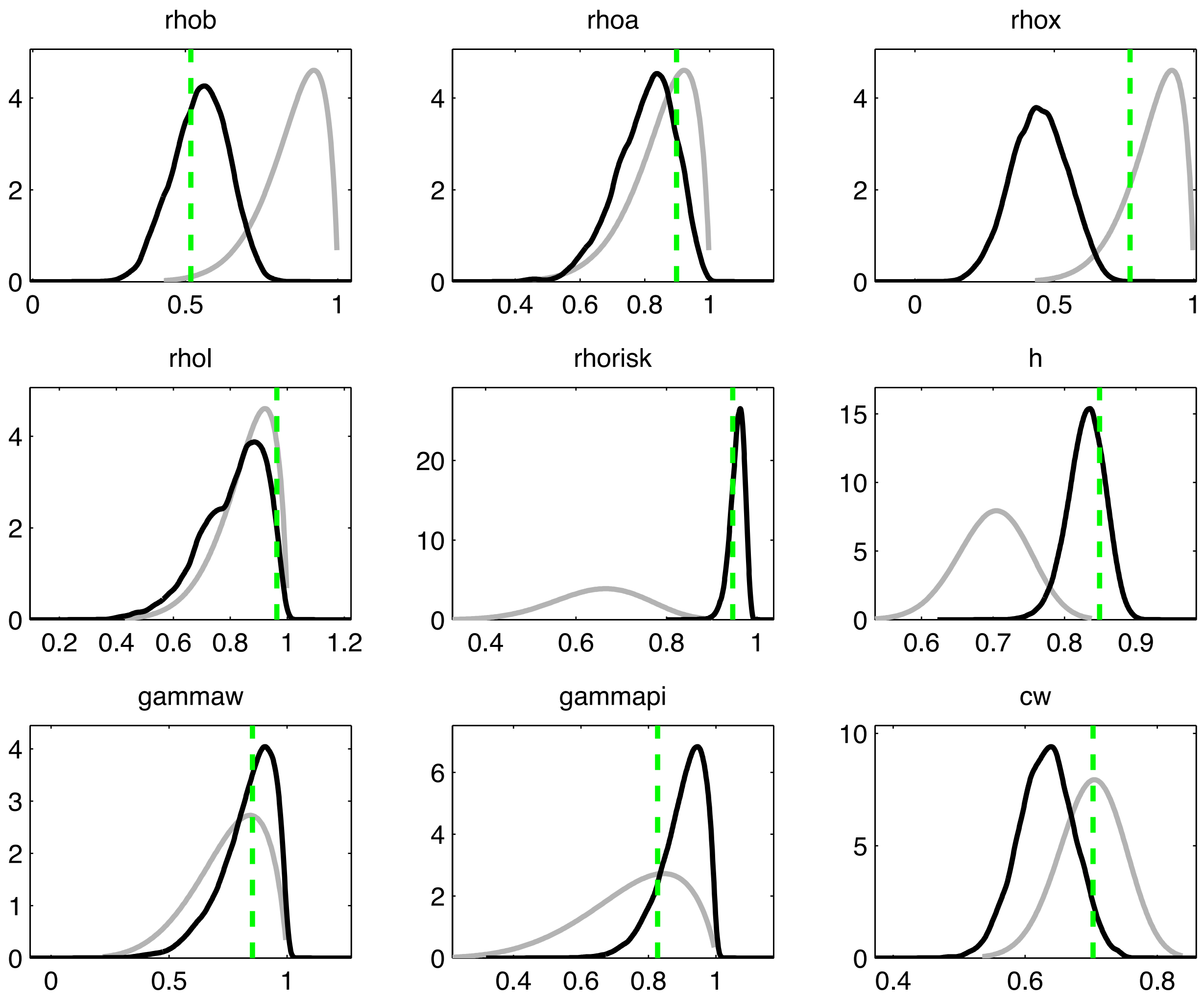

Figure 12: The prior (light colored line) and posterior (dark colored line) distributions; green vertical line is the posterior mode obtained by the posterior kernel maximization

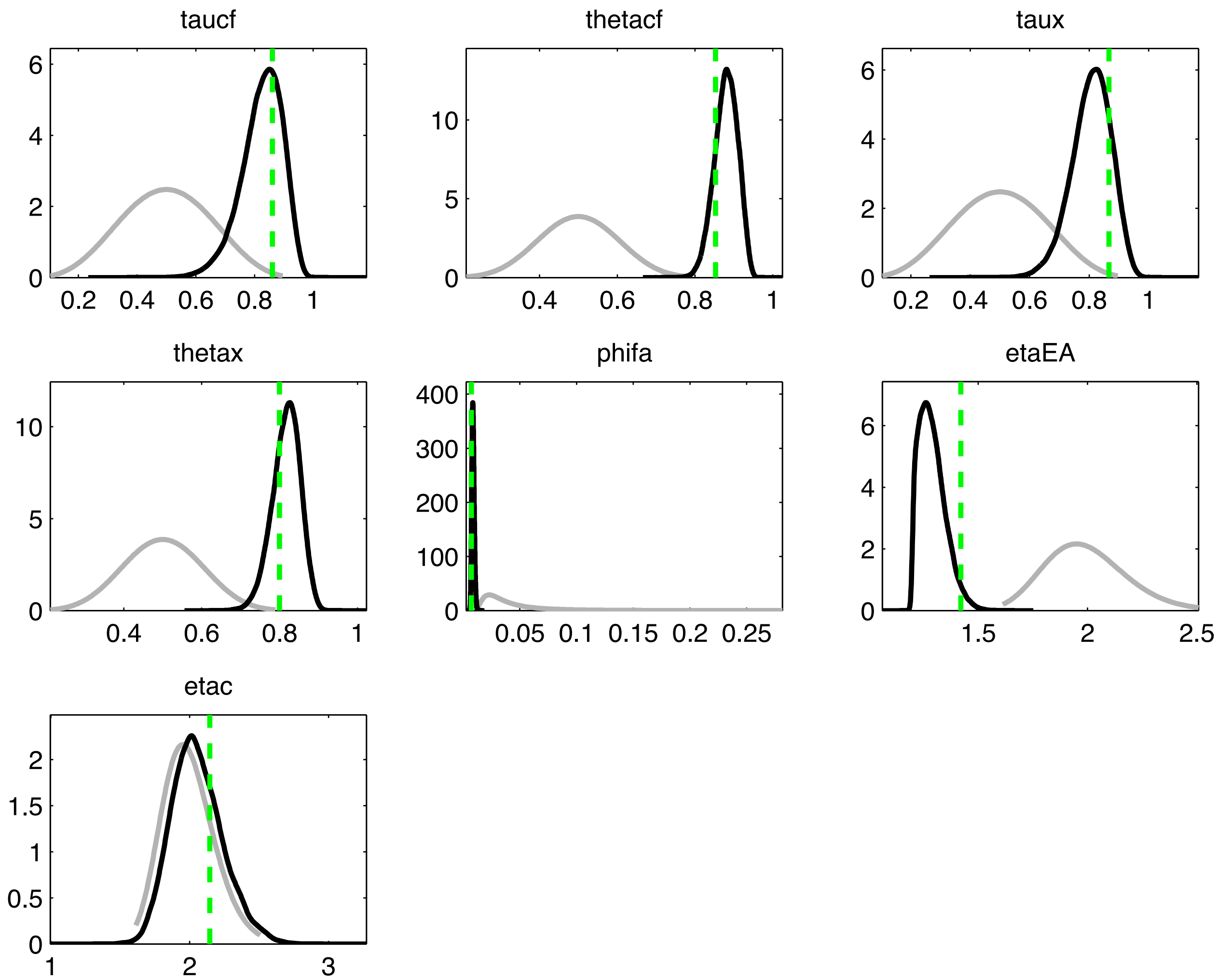

Figure 13: The prior (light colored line) and posterior (dark colored line) distributions; green vertical line is the posterior mode obtained by the posterior kernel maximization

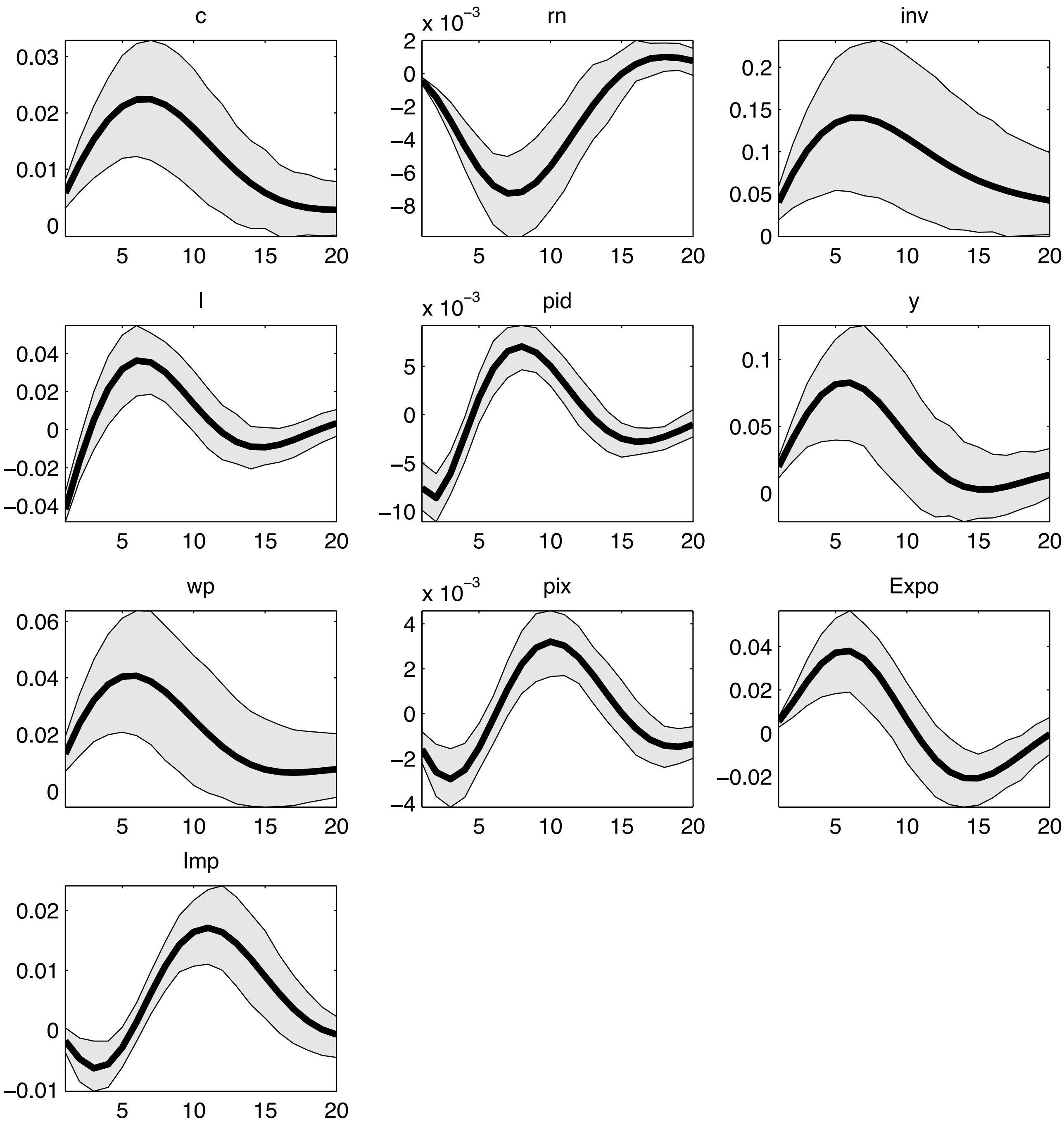

Figure 14: Model variables response to one standard deviation technology shock measured in percentage deviation from the steady state

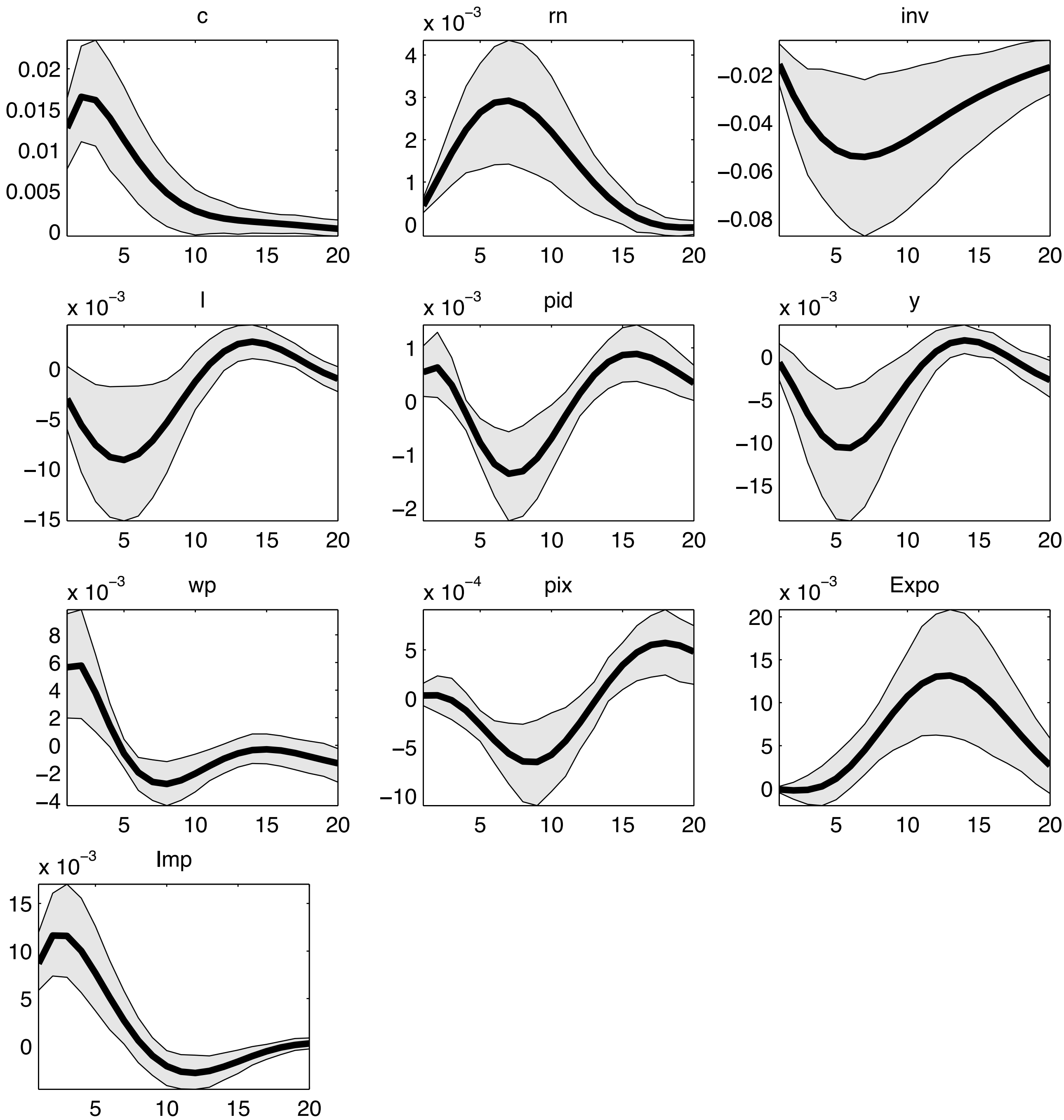

Figure 15: Model variables response to one standard deviation preference shock measured in percentage deviation from the steady state

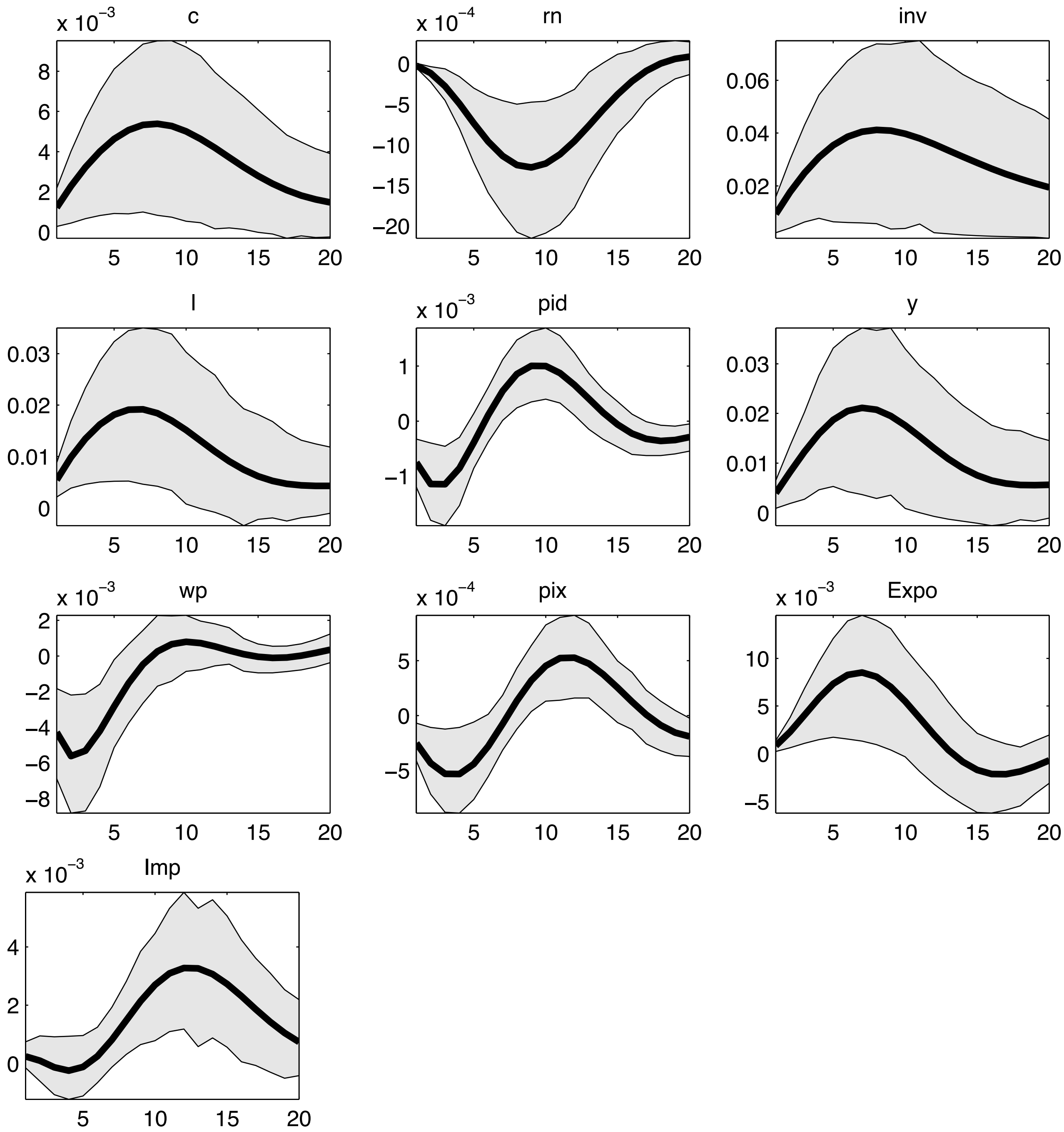

Figure 16: Model variables response to one standard deviation labour supply shock measured in percentage deviation from the steady state

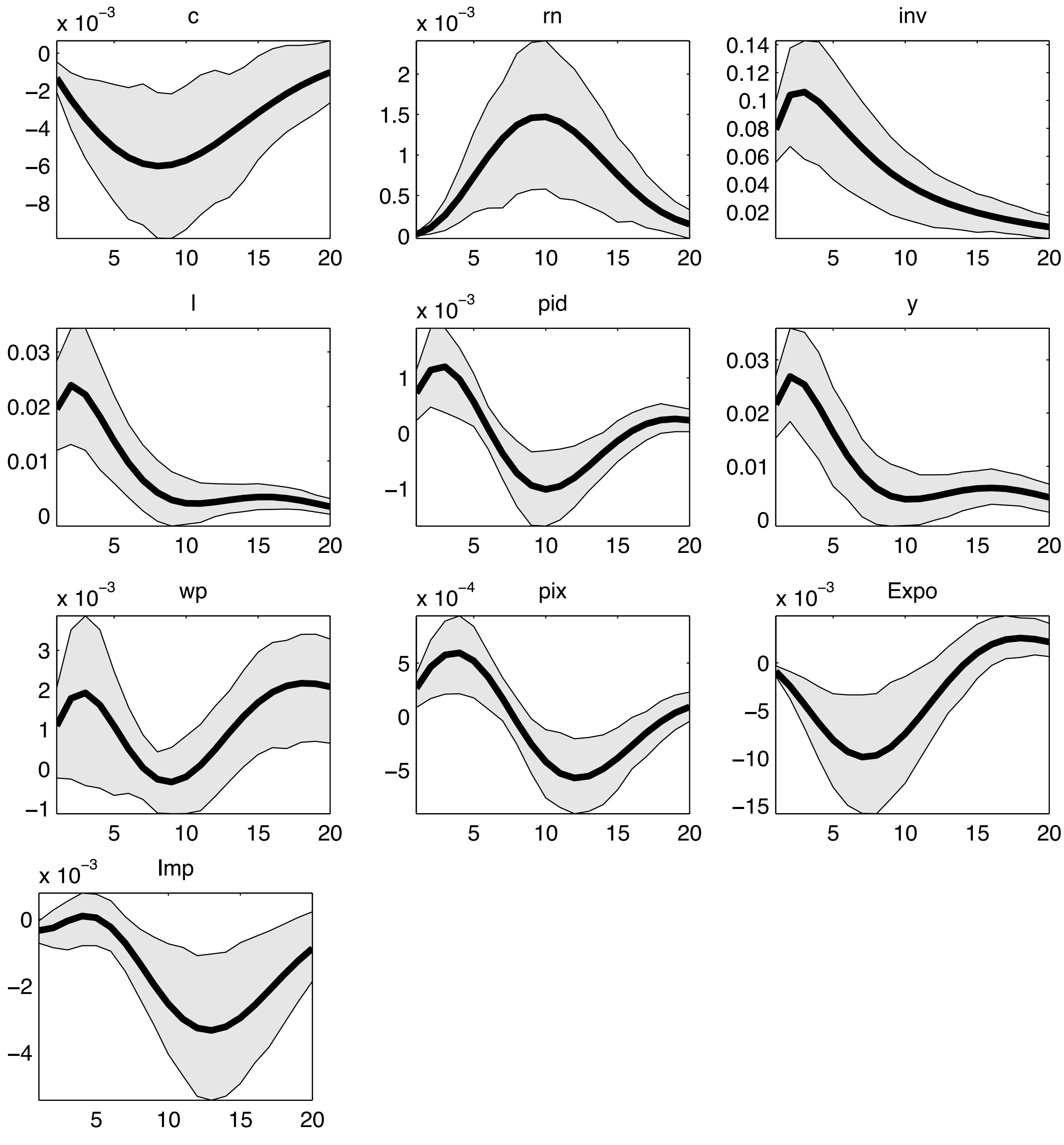

Figure 17: Model variables response to one standard deviation investment specific shock measured in percentage deviation from the steady state

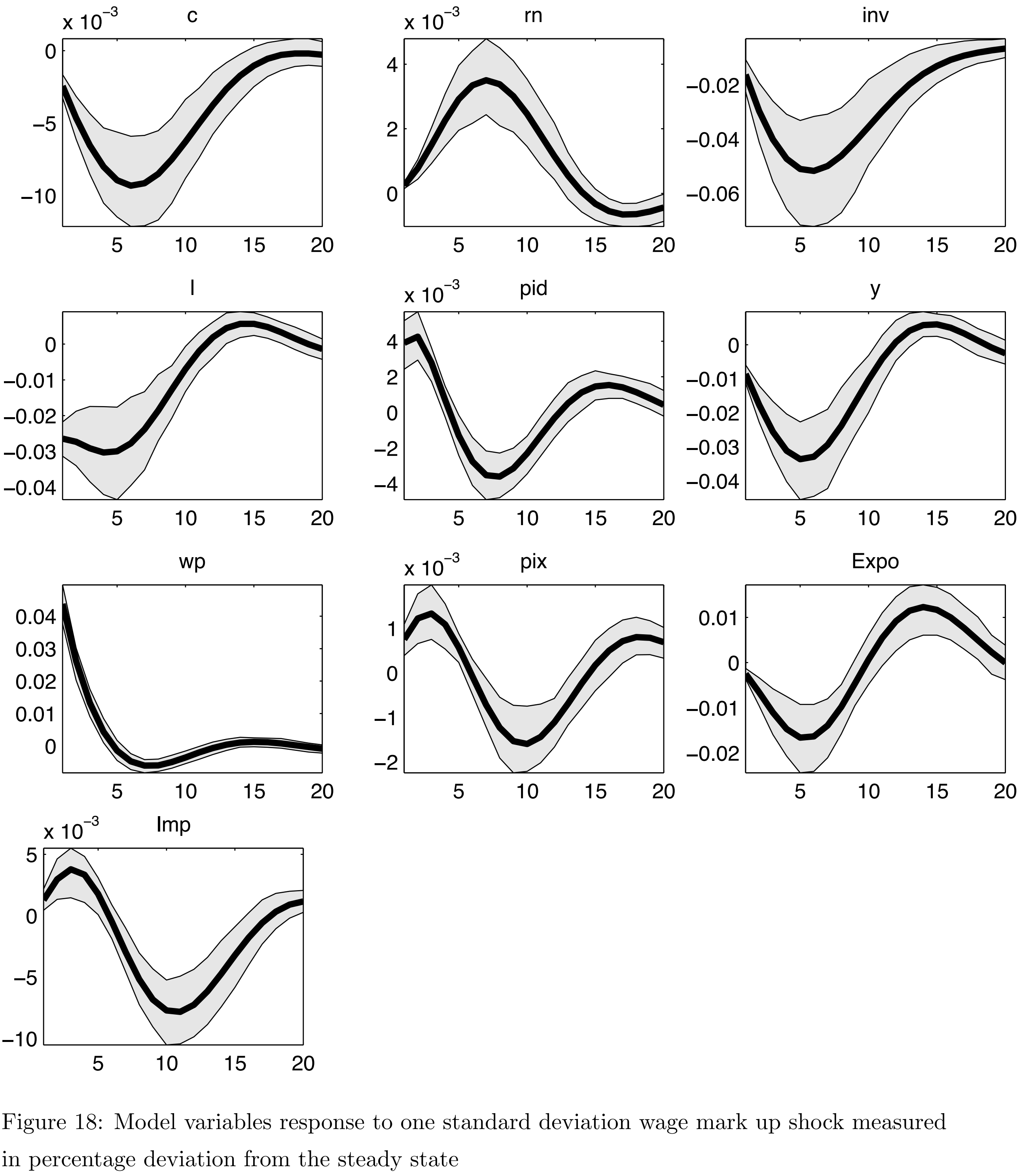

Figure 18: Model variables response to one standard deviation wage mark up shock measured in percentage deviation from the steady state

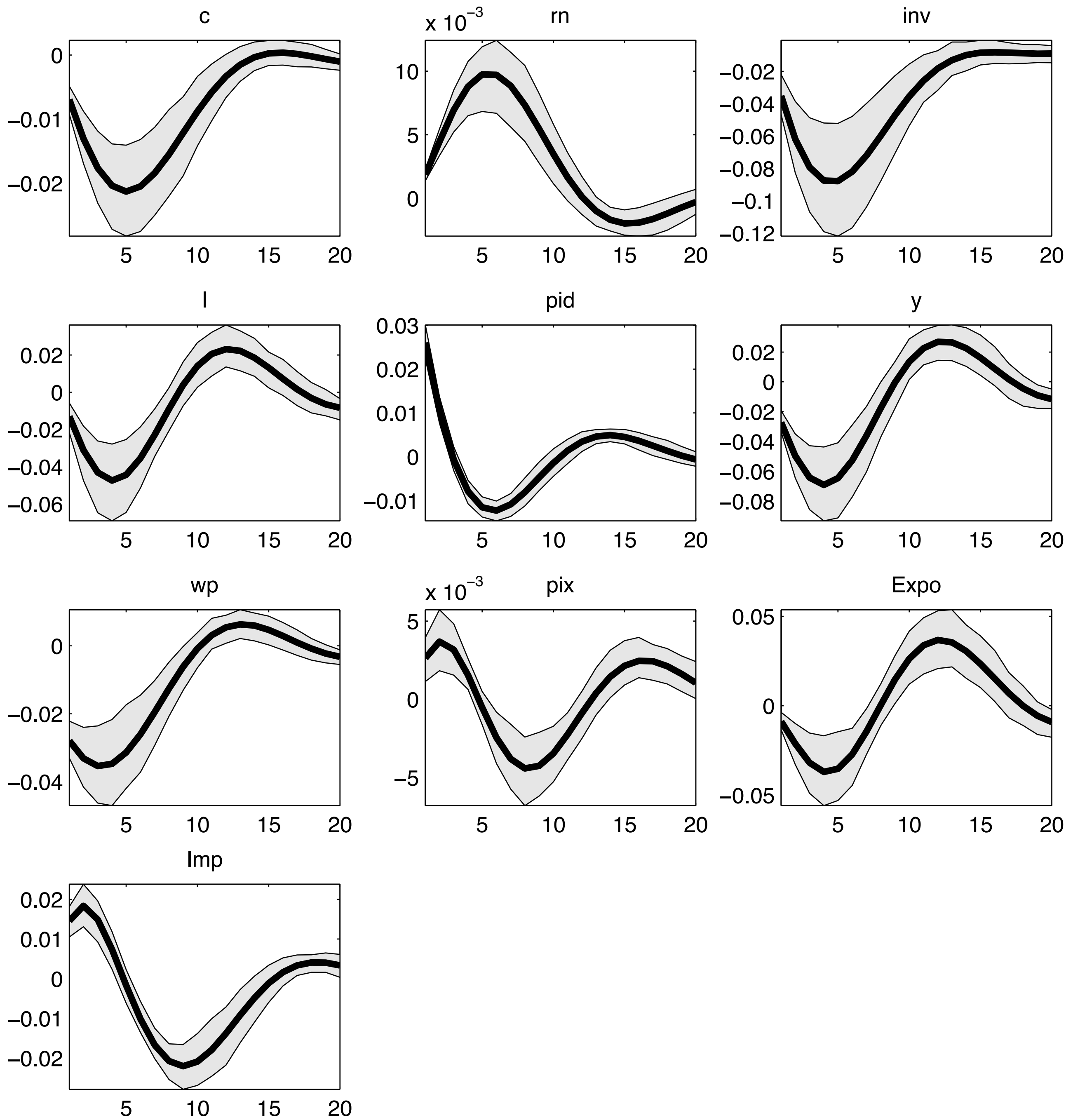

Figure 19: Model variables response to one standard deviation domestic mark up shock measured in percentage deviation from the steady state

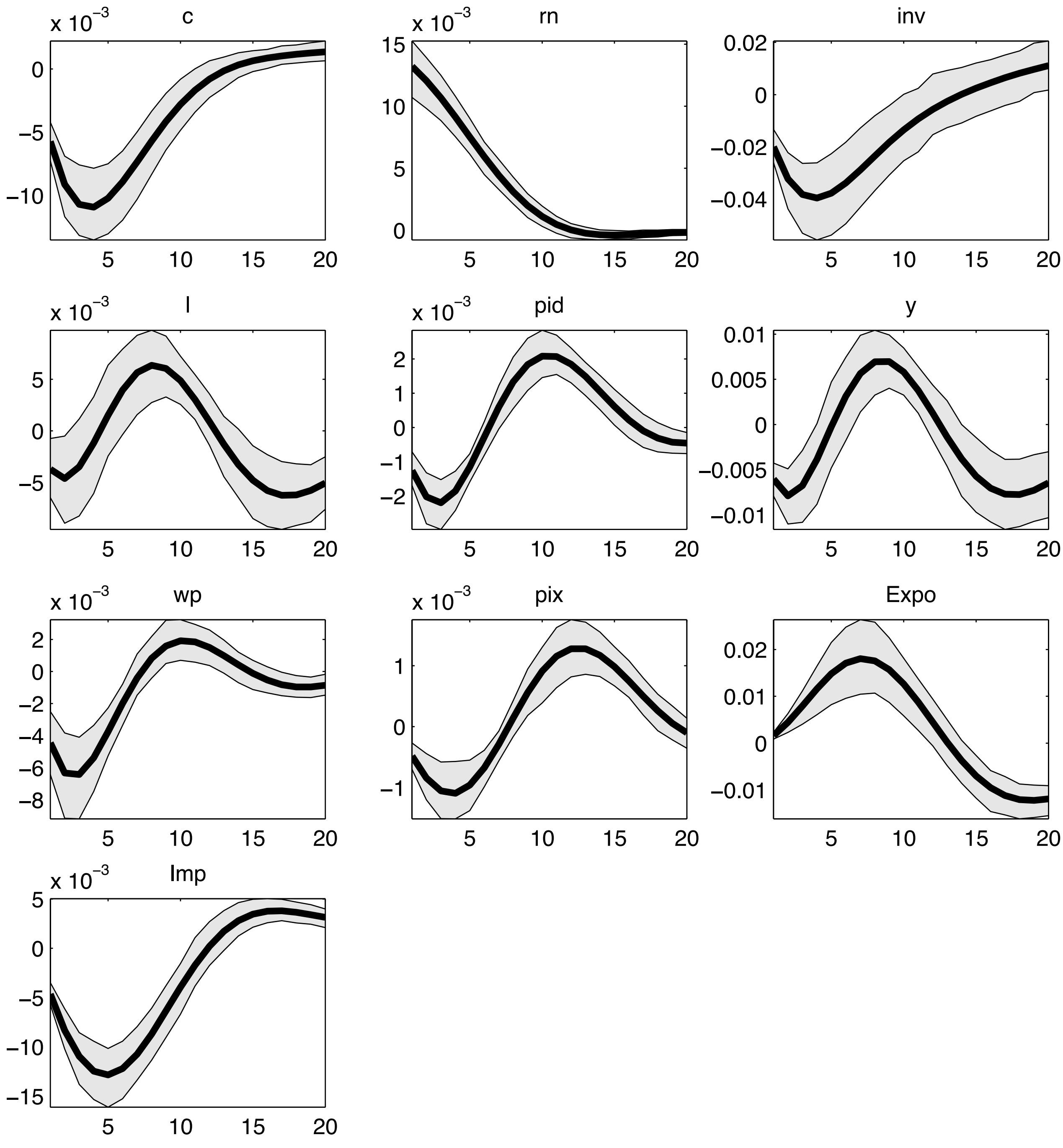

Figure 20: Model variables response to one standard deviation risk premium shock measured in percentage deviation from the steady state

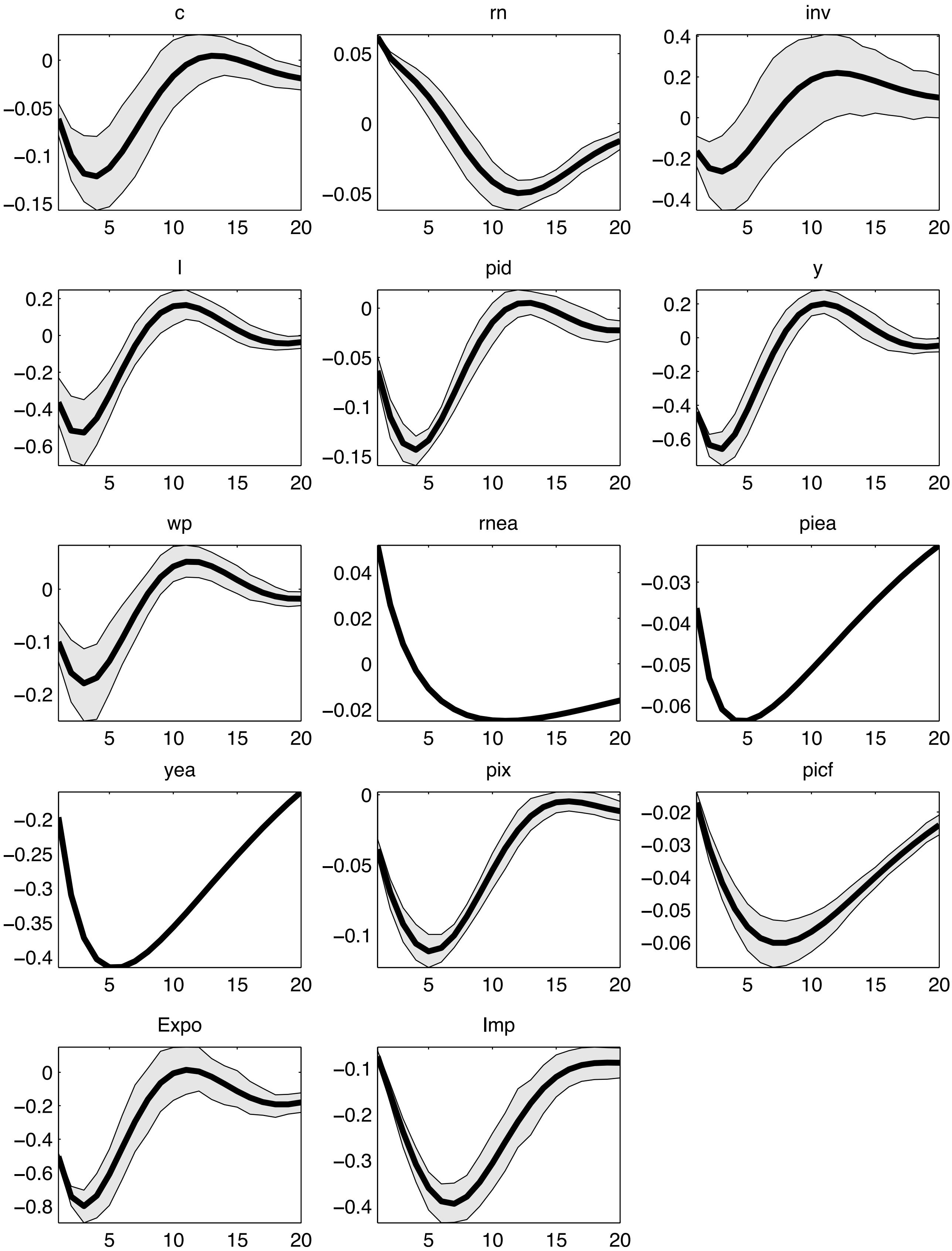

Figure 21: Model variables response to one standard deviation euro area monetary policy shock measured in percentage deviation from the steady state

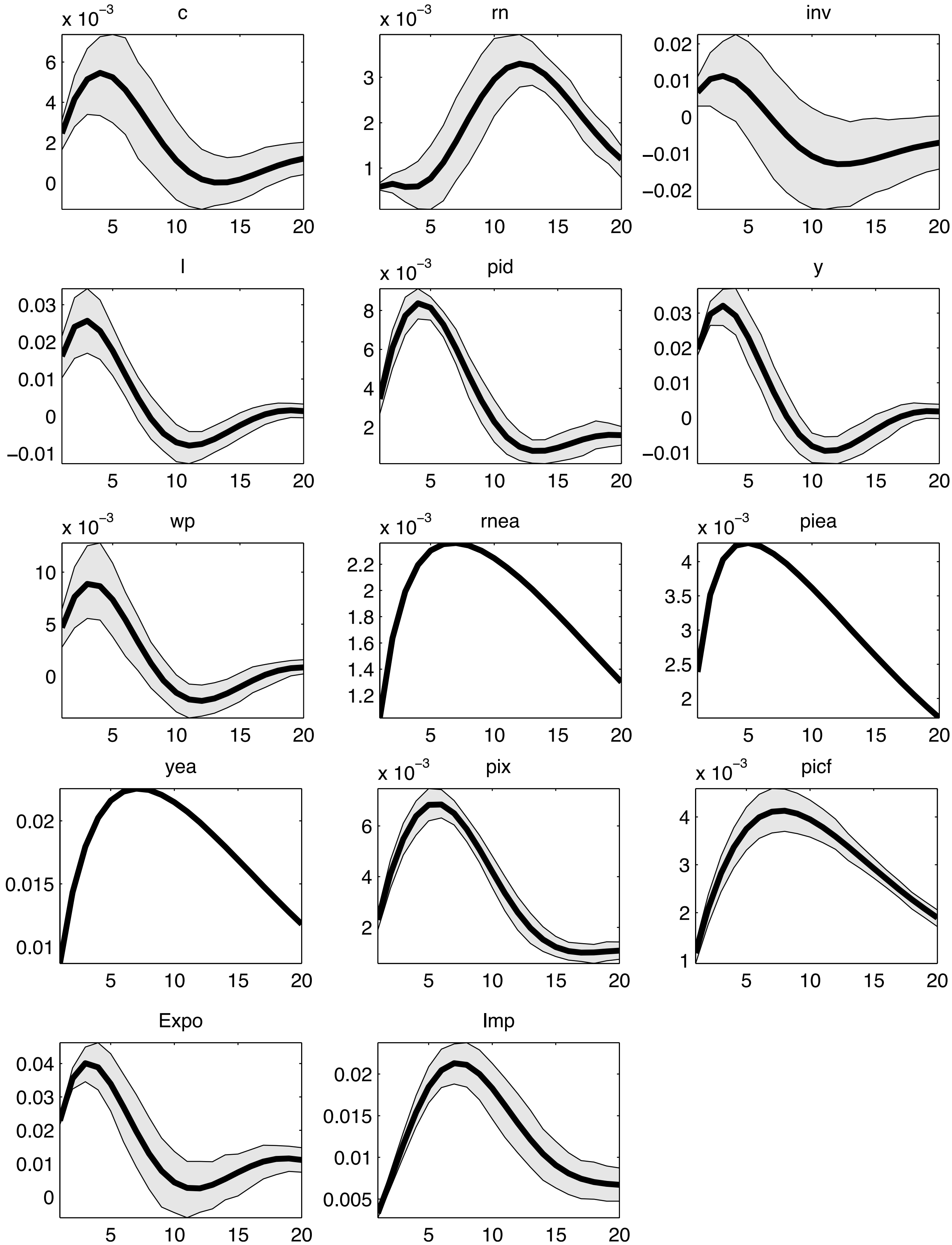

Figure 22: Model variables response to one standard deviation euro area inflation objective shock measured in percentage deviation from the steady state

8 Appendix

8.1 Relative prices, marginal costs and the real exchange rate

In this appendix some notions related to the relative prices, useful for the log-linearizarion of the EP DSGE model, are introduced. Their log-linearization and relationship with the marginal costs is highlighted.

Definition and log linearization of γ_t^c:

$$\gamma_t^c \equiv \frac{P_t^c}{P_t^D} ,$$

$$\widehat{\gamma}_t^c = \widehat{p}_t^c - \widehat{p}_t^D .$$

It is easy to show that by simply summing and subtracting $\widehat{p}_{t-1}^c$ and $\widehat{p}_{t-1}^D$ on the hand left side:

$$\widehat{\gamma}_t^c = \widehat{\pi}_t^c - \widehat{\pi}_t^D + \widehat{\gamma}_{t-1}^c ,$$

where, following the definition of P_t^c in equation (14):

$$\widehat{\pi}_t^c = \left[(1-\alpha_c) \left(\gamma^c\right)^{1-\eta_c} \right] \widehat{\pi}_t^D + \left[\alpha_c \left(\gamma^{F,c}\right)^{1-\eta_c} \right] \widehat{\pi}_t^{c,F} .$$

Definition and log linearization of $\gamma_t^{x,c}$:

$$\gamma_t^{x,c} \equiv \frac{P_t^{x,c}}{P_t^{EA}} ,$$

$$\widehat{\gamma}_t^{x,c} = \widehat{p}_t^{x,c} - \widehat{p}_t^{EA} ,$$

$$\widehat{\gamma}_t^{c,x} = \widehat{\pi}_t^{c,x} - \widehat{\pi}_t^{EA} + \widehat{\gamma}_{t-1}^{c,x} .$$

Definition and log linearization of γ_t^{EA}:

$$\gamma_t^{EA} \equiv \frac{P_t^D}{\bar{e} P_t^{EA}} = MC_t^{c,x} \gamma_t^{c,x} ,$$

$$\widehat{p}_t^c - \widehat{p}_t^{EA} = \widehat{mc}_t^{c,x} + \widehat{\gamma}_t^{c,x} . \tag{A.1}$$

Given that $\widehat{mc}_t^{c,x} = \widehat{p}_t^D - \widehat{p}_t^{x,c}$, one can write:

$$\widehat{mc}_t^{c,x} = \widehat{\pi}_t^D - \widehat{\pi}_t^{c,x} + \widehat{mc}_{t-1}^{c,x} .$$

Definition and log linearization of $\gamma_t^{F,c}$:

$$\gamma_t^{F,c} \equiv \frac{P_t^{F,c}}{P_t^D} ,$$

$$\widehat{\gamma}_t^{F,c} = \widehat{p}_t^{F,c} - \widehat{p}_t^D , \tag{A.2}$$

$$\widehat{\gamma}_t^{F,c} = \widehat{\pi}_t^{F,c} - \widehat{\pi}_t^D + \widehat{\gamma}_{t-1}^{F,c} .$$

These equations allow to re-write $\widehat{mc}_t^{c,F}$ in the following form:

$$\widehat{mc}_t^{c,F} = \widehat{p}_t^{EA} - \widehat{p}_t^{F,c} .$$

Using equation (A.1) to substitute out $\widehat{p}_t^{EA}$:

$$\widehat{mc}_t^{c,F} = \widehat{p}_t^c - \widehat{mc}_t^x - \widehat{\gamma}_t^x - \widehat{p}_t^{c,I} .$$

Then, using the definition of $\widehat{\gamma}_t^{F,c}$ in equation (A.2):

$$\widehat{mc}_t^{c,F} = -\widehat{mc}_t^x - \widehat{\gamma}_t^x + \widehat{\gamma}_t^{F,c} .$$

Finally, a definition of the real exchange rate re_t in terms of the relative prices is necessary. By definition, the real exchange rate is given by:

$$re_t \equiv \frac{\overline{e} P_t^{EA}}{P_t^c} .$$

Multiplying it by $\frac{P_t^D}{P_t^D}$:

$$re_t = \frac{\overline{e} P_t^{EA}}{P_t^c} \frac{P_t^D}{P_t^D} .$$

Using the definition of γ_t^{EA} and γ_t^c:

$$re_t = \frac{1}{\gamma_t^{EA} \gamma_t^c} . \tag{A.3}$$

An increase (decrease) in either γ_t^{EA} or γ_t^c or both implies a decrease (increase) of the real exchange rate, which means an appreciation (depreciation) given the definitions above.

8.2 Log-linearized model for Estonia

Aggregate resource constraint:

$$\widehat{y}_t = (1-\alpha_c)\left(\gamma^c\right)^{\eta_c} \frac{C}{Y}\left(\widehat{c}_t + \eta_c \widehat{\gamma}_t^c\right) + \frac{I}{Y}\widehat{i}_t + \frac{G}{Y}\widehat{g}_t + \frac{EXP}{Y}\widehat{x}_t ;$$

Exports dynamics:

$$\widehat{x}_t = \left(\gamma^{c,x}\right)^{-\eta_{EA}} \frac{Y^{EA}}{EXP}\left(\widehat{y}_t^{EA} - \eta_{EA}\widehat{\gamma}_t^{c,x}\right) ;$$

Imports dynamics:

$$\widehat{m}_t = \frac{C}{M}\left(-\eta^c(1-\alpha_c)\left(\gamma^c\right)^{-(1-\eta^c)}\widehat{\gamma}_t^{F,c} + \widehat{c}_t\right) ;$$

Net foreign assets low of motion:[45]

$$\widehat{fa}_t = Y^{EA}\left(\widehat{y}_t^{EA} - \widehat{mc}_t^{c,x} - \eta_{EA}\widehat{\gamma}_t^{c,x}\right) + M\,\widehat{\gamma}_t^{EA} - M\left(-\eta^c(1-\alpha_c)\left(\gamma^c\right)^{-(1-\eta^c)}\widehat{\gamma}_t^{F,c} + \widehat{c}_t\right) + R\,\widehat{fa}_{t-1} ;$$

Fisher equation:

$$\widehat{r}_t = \widehat{r}_t^n - \mathbb{E}_t\,\widehat{\pi}_{t+1}^c ;$$

[45]The steady–state assumptions under which this equation is derived are following: $FA = 0$, $\Omega(0,0) = 1$, $\gamma^{c,x} = \gamma^{EA} = MC^{c,x} = 1$, $R^n = R^{n,EA}$.

Consumption Euler equation:

$$\widehat{c}_t = \frac{h}{1+h}\widehat{c}_{t-1} + \frac{1}{1+h}\mathbb{E}_t\,\widehat{c}_{t+1} - \frac{1-h}{\sigma_c\,(1+h)}\widehat{r}_t + \frac{1-h}{\sigma_c\,(1+h)}\widehat{\varepsilon}^{\beta}_t\,;$$

Real wage dynamics:

$$\begin{aligned}\widehat{w}_t \;=\; & \frac{\beta}{1+\beta}\mathbb{E}_t\,\widehat{w}_{t+1} + \frac{1}{1+\beta}\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\,\widehat{\pi}^c_{t+1} - \frac{1+\beta\tau_w}{1+\beta}\widehat{\pi}^c_t + \frac{\tau_w}{1+\beta}\widehat{\pi}^c_{t-1} \\ & -\frac{1}{1+\beta}\frac{(1-\beta\theta_w)\,(1-\theta_w)}{[1+(1+\lambda_w)\sigma_l/\lambda_w]\,\theta_w}\left[\widehat{w}_t - \sigma_l\widehat{l}_t - \frac{\sigma_c}{1-h}\,(\widehat{c}_t - h\widehat{c}_{t-1}) + \widehat{\varepsilon}^L_t\right] + u^w_t\,;\end{aligned}$$

Households' investment decision equation, where φ is the inverse of the elasticity of the capital utilization cost function:

$$\widehat{i}_t = \frac{1}{1+\beta}\widehat{i}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\,\widehat{i}_{t+1} + \frac{\varphi}{1+\beta}\widehat{q}_t + \widehat{x}_t\,;$$

Price of capital dynamics:

$$\widehat{q}_t = -(\widehat{r}^n_t - \mathbb{E}_t\,\widehat{\pi}_{t+1}) + \frac{1-\delta}{1-\delta+R}\mathbb{E}_t\,\widehat{q}_{t+1} + \frac{R}{1-\delta+R}\mathbb{E}_t\,\widehat{r}^k_{t+1} + u^q_t\,;$$

Production function, where $\psi = \frac{\Psi'(1)}{\Psi''(1)}$ is the inverse elasticity of the capital utilization cost function Ψ and ϕ is one plus the share of fixed costs in production:

$$\widehat{y}_t = \phi(\widehat{a}_t + \alpha\widehat{k}_{t-1} + \alpha\psi\widehat{r}^k_{t+1} + (1-\alpha)\widehat{l}_t)\,;$$

Return on capital:[46]

$$(1+\psi)\widehat{r}^k_t = \widehat{l}_t + \widehat{w}_t - \widehat{k}_{t-1}\,;$$

Marginal costs:

$$\widehat{mc}_t = \alpha\widehat{r}^k_t + (1-\alpha)\widehat{w}_t - \widehat{a}_t\,;$$

Capital accumulation equation:

$$\widehat{k}_t = \delta\,\widehat{i}_t + (1-\delta)\widehat{k}_{t-1}\,;$$

Domestic inflation dynamics is given by New Keynesian Phillips Curve:

$$\widehat{\pi}^D_t = \frac{\beta}{1+\beta\tau_\pi}\mathbb{E}_t\,\widehat{\pi}^D_{t+1} + \frac{\tau_\pi}{1+\beta\tau_\pi}\,\widehat{\pi}^D_{t-1} + \frac{1}{1+\beta\tau_\pi}\frac{(1-\beta\theta)(1-\theta)}{\theta}\,\widehat{mc}_t + u^{\lambda^p}_t\,;$$

Inflation dynamics of imported consumption goods:

$$\widehat{\pi}^{c,F}_t = \frac{\beta}{1+\tau_{c,F}\beta}\mathbb{E}_t\,\widehat{\pi}^{c,F}_{t+1} + \frac{\tau_{c,F}}{1+\tau_{c,F}\beta}\,\widehat{\pi}^{c,F}_{t-1} + \frac{1}{1+\tau_{c,F}\beta}\frac{(1-\theta_{c,F})(1-\beta\theta_{c,F})}{\theta_{c,F}}(\widehat{mc}^{c,F}_t + u^{\lambda^{c,F}_t}_t)\,;$$

Inflation dynamics of exported consumption goods:

$$\widehat{\pi}^{c,x}_t = \frac{\beta}{1+\tau_{c,x}\beta}\mathbb{E}_t\,\widehat{\pi}^{c,x}_{t+1} + \frac{\tau_{c,x}}{1+\tau_{c,x}\beta}\,\widehat{\pi}^{c,x}_{t-1} + \frac{1}{1+\tau_{c,x}\beta}\frac{(1-\theta_{c,x})(1-\beta\theta_{c,x})}{\theta_{c,x}}(\widehat{mc}^{c,x}_t + u^{\lambda^{c,x}}_t)\,;$$

[46]See footnote 48 for derivations details.

Marginal costs for importers of consumption goods

$$\widehat{mc}_t^{c,F} = -\widehat{mc}_t^{c,x} - \widehat{\gamma}_t^{c,x} + \widehat{\gamma}_t^{F,c}\,;$$

Exporters marginal costs:

$$\widehat{mc}_t^{c,x} = \widehat{\pi}_t^{c} - \widehat{\pi}_t^{c,x} + \widehat{mc}_{t-1}^{c,x}\,;$$

Ratio of consumer prices to domestic prices:

$$\widehat{\gamma}_t^{c} = \widehat{\pi}_t^{c} - \widehat{\pi}_t^{D} + \widehat{\gamma}_{t-1}^{c}\,;$$

Ratio of consumer prices to imported consumption goods prices:

$$\widehat{\gamma}_t^{F,c} = \widehat{\pi}_t^{c} - \widehat{\pi}_t^{F,c} + \widehat{\gamma}_{t-1}^{F,c}\,;$$

Ration of exported consumption goods prices to euro area consumer prices:

$$\widehat{\gamma}_t^{c,x} = \widehat{\pi}_t^{c,x} - \widehat{\pi}_t^{EA} + \widehat{\gamma}_{t-1}^{c,x}\,;$$

Consumer price inflation:

$$\widehat{\pi}_t^{c} = (1-\alpha_c)\left(\gamma^c\right)^{1-\eta_c}\,\widehat{\pi}_t^{D} + \alpha_c\left(\gamma^{F,c}\right)^{1-\eta_c}\,\widehat{\pi}_t^{c,F}\,;$$

Monetary policy equation; modified UIP condition:

$$\widehat{r}_t^{n} = \widehat{r}_t^{n,EA} - \phi_{\text{fa}}\widehat{fa}_t + \widehat{\epsilon}_t^{\text{risk}}\,;$$

Fiscal policy:[47]

$$\widehat{g}_t = \rho_g\widehat{g}_{t-1} + u_t^g\,, \qquad u_t^g \sim \text{WN}(0,\sigma_g^2)\,;$$

The preference shock:

$$\widehat{\varepsilon}_t^{\beta} = \rho_\beta\widehat{\varepsilon}_{t-1}^{\beta} + u_t^\beta\,, \qquad u_t^\beta \sim \text{WN}(0,\sigma_\beta^2)\,;$$

The labour supply shock:

$$\widehat{\varepsilon}_t^{L} = \rho_L\widehat{\varepsilon}_{t-1}^{L} + u_t^L\,, \qquad u_t^L \sim \text{WN}(0,\sigma_L^2)\,;$$

[47]Since this process is give by:

$$\log G_t = \rho_g \log G_{t-1} + u_t^g\,,$$

substituting the variable with its definition in terms of percentage deviation from its steady state value, i.e. $G_t \equiv G\,(1+\widehat{g}_t)$ one has:

$$\log G\,(1+\widehat{g}_t) = \rho_g \log G\,(1+\widehat{g}_{t-1}) + u_t^g\,,$$

which can be approximated as:

$$\widehat{g}_t = \rho_g\widehat{g}_{t-1} + u_t^g\,.$$

The technology shock:

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + u_t^a \,, \qquad u_t^a \sim \mathrm{WN}(0, \sigma_a^2)\,;$$

The investment-specific technology shocks:

$$\widehat{x}_t = \rho_x \widehat{x}_{t-1} + u_t^x \,, \qquad u_t^x \sim \mathrm{WN}(0, \sigma_x^2)\,;$$

The risk premium shock:

$$\widehat{\epsilon}_t^{\text{risk}} = \rho_{\text{risk}} \widehat{\epsilon}_{t-1}^{\text{risk}} + u_t^{\text{risk}} \,, \qquad u_t^{\text{risk}} \sim \mathrm{WN}(0, \sigma_{\text{risk}}^2)\,.$$

The remaining shocks are assumed to be white noise processes:

$$u_t^w \sim \mathrm{WN}(0, \sigma_w^2)\,, \quad u_t^{\lambda^p} \sim \mathrm{WN}(0, \sigma_{\lambda^p}^2)\,, \quad u_t^{c,F} \sim \mathrm{WN}(0, \sigma_{\lambda^{c,F}}^2)\,, \quad u_t^{c,x} \sim \mathrm{WN}(0, \sigma_{\lambda^{c,x}}^2)\,, \quad u_t^q \sim \mathrm{WN}(0, \sigma_q^2)\,.$$

8.3 Log-linearized model for euro area

The resource constraint:

$$\widehat{y}_t^{EA} = \left(1 - \frac{\delta^{EA} K^{EA}}{INV^{EA}} - \frac{G^{EA}}{Y^{EA}}\right) \widehat{c}_t^{EA} + \frac{\delta^{EA} K^{EA}}{INV^{EA}} \widehat{Inv}_t^{EA} + \frac{G^{EA}}{Y^{EA}} \widehat{g}_t^{EA}$$

The Fisher equation:

$$\widehat{r}_t^{EA} = \widehat{r}_t^{n,EA} - E_t \left\{\widehat{\pi}_{t+1}^{EA}\right\}$$

The consumption Euler equation:

$$\widehat{c}_t^{EA} = \frac{h^{EA}}{1 + h^{EA}} \widehat{c}_{t-1}^{EA} + \frac{1}{1 + h^{EA}} E_t \left\{\widehat{c}_{t+1}^{EA}\right\} - \frac{1 - h^{EA}}{\sigma_c^{EA} \left(1 + h^{EA}\right)} \widehat{r}_t^{EA} + \frac{1 - h^{EA}}{\sigma_c^{EA} \left(1 + h^{EA}\right)} \widehat{\varepsilon}_t^{\beta,EA}$$

The household labour equation:

$$\begin{aligned}
\widehat{w}_t^{EA} \quad = \quad & \frac{\beta^{EA}}{1 + \beta^{EA}} E_t \left\{\widehat{w}_{t+1}^{EA}\right\} + \frac{1}{1 + \beta^{EA}} \widehat{w}_{t-1}^{EA} + \frac{\beta^{EA}}{1 + \beta^{EA}} E_t \left\{\widehat{\pi}_{t+1}^{EA}\right\} - \frac{1 + \beta^{EA} \tau_w^{EA}}{1 + \beta^{EA}} \widehat{\pi}_t^{EA} + \\
& + \frac{\tau_w^{EA}}{1 + \beta^{EA}} \widehat{\pi}_{t-1}^{EA} - \frac{1}{1 + \beta^{EA}} \frac{\left(1 - \beta^{EA} \theta_w^{EA}\right)\left(1 - \theta_w^{EA}\right)}{\left[1 + \frac{(1 + \lambda_w^{EA}) \sigma_l^{EA}}{\lambda_w^{EA}}\right] \theta_w^{EA}} \times \\
& \times \left[\widehat{w}_t^{EA} - \sigma_l^{EA} \widehat{l}_t^{EA} - \frac{\sigma_c^{EA}}{1 - h^{EA}} \left(\widehat{c}_t^{EA} - h^{EA} \widehat{c}_{t-1}^{EA}\right) + \widehat{\varepsilon}_t^{L,EA}\right] + u_t^{w,EA}
\end{aligned}$$

The household investment decision equation:

$$\widehat{Inv}_t^{EA} = \frac{1}{1 + \beta^{EA}} \widehat{Inv}_{t-1}^{EA} + \frac{\beta^{EA}}{1 + \beta^{EA}} E_t \left\{\widehat{Inv}_{t+1}^{EA}\right\} + \frac{\varphi^{EA}}{1 + \beta^{EA}} \widehat{q}_t^{EA} + \widehat{x}_t^{EA}$$

The condition on the price of capital:

$$\widehat{q}_t^{EA} = -\left(\widehat{r}_t^{n,EA} - E_t \left\{\widehat{\pi}_{t+1}^{EA}\right\}\right) + \frac{1 - \delta^{EA}}{1 - \delta^{EA} + R^{EA}} E_t \left\{\widehat{q}_{t+1}^{EA}\right\} + \frac{R^{EA}}{1 - \delta^{EA} + R^{EA}} E_t \left\{\widehat{r}_{t+1}^{k,EA}\right\} + u_t^{q,EA}$$

The production function:

$$\widehat{y}_t^{EA} = \phi^{EA} \widehat{a}_t^{EA} + \phi^{EA} \alpha^{EA} \widehat{k}_{t-1}^{EA} + \phi^{EA} \alpha^{EA} \psi^{EA} \widehat{r}_{t+1}^{k,EA} + \phi^{EA} \left(1 - \alpha^{EA}\right) \widehat{l}_t^{EA}$$

where $\psi^{EA} = \frac{\Psi'(1)}{\Psi''(1)}$ is the inverse of the elasticity of capital utilization cost function and ϕ^{EA} is one plus the share of fixed cost in production. The labour demand:[48]

$$\widehat{l}_t^{EA} = -\widehat{w}_t^{EA} + \left(1+\psi^{EA}\right)\widehat{r}_t^{k,EA} + \widehat{k}_{t-1}^{EA}$$

The marginal costs:

$$\widehat{mc}_t^{EA} = \alpha^{EA}\widehat{r}_t^{k,EA} + \left(1-\alpha^{EA}\right)\widehat{w}_t^{EA} - \widehat{a}_t^{EA}$$

The accumulation of capital:

$$\widehat{k}_t^{EA} = \delta^{EA}\widehat{Inv}_t^{EA} + \left(1-\delta^{EA}\right)\widehat{k}_{t-1}^{EA}$$

The New Keynesian Phillips Curve (NKPC) for the domestic inflation:

$$\widehat{\pi}_t^{EA} = \frac{\beta^{EA}}{1+\beta^{EA}\tau_\pi^{EA}}E_t\left\{\widehat{\pi}_{t+1}^{EA}\right\} + \frac{\tau_\pi^{EA}}{1+\beta^{EA}\tau_\pi^{EA}}\widehat{\pi}_{t-1}^{EA} + \frac{\left(1-\beta^{EA}\theta^{EA}\right)\left(1-\theta^{EA}\right)}{\left(1+\beta^{EA}\tau_\pi^{EA}\right)\theta^{EA}}\left(\widehat{mc}_t^{EA}\right) + u_t^{\lambda^p,EA}$$

Fiscal policy:

$$\widehat{g}_t^{EA} = \rho_g^{EA}\widehat{g}_{t-1}^{EA} + u_t^{g,EA}$$

Preference shock:[49]

$$\widehat{\varepsilon}_t^{\beta,EA} = \rho_\beta^{EA}\widehat{\varepsilon}_{t-1}^{\beta,EA} + u_t^{\beta,EA}$$

Labour supply shock:

$$\widehat{\varepsilon}_t^{L,EA} = \rho_L^{EA}\widehat{\varepsilon}_{t-1}^{L,EA} + u_t^{L,EA}$$

[48] The cost minimization problem of the intermediate goods producers leads to the following optimality condition:

$$W_t^{EA}L_t^{EA} = \frac{1-\alpha^{EA}}{\alpha^{EA}}R_t^{k,EA}z_t^{EA}K_{t-1}^{EA} \tag{A.4}$$

Moreover, from the consumer's f.o.c.-s we have that

$$R_t^{k,EA} = \Psi'\left(z_t^{EA}\right)$$

Log-linearizing, taking into account that in steady state $R^{k,EA} = \Psi'\left(z^{EA}\right)$ and that $z^{EA} = 1$

$$R^{k,EA}\left(1+\widehat{r}_t^{k,EA}\right) = \Psi'(1) + \Psi''(1)\,\widehat{z}_t^{EA}$$

$$\widehat{r}_t^{k,EA} = \frac{\Psi''(1)}{\Psi'(1)}\widehat{z}_t^{EA}$$

or

$$\widehat{z}_t^{EA} = \frac{\Psi'(1)}{\Psi''(1)}\widehat{r}_t^{k,EA}$$

Log linearizing equation (A.4)

$$\widehat{l}_t^{EA} = -\widehat{w}_t^{EA} + \widehat{r}_t^{k,EA} + \widehat{z}_t^{EA} + \widehat{k}_{t-1}^{EA}$$

Substituting for $\widehat{z}_t^{EA}$ and re-arranging one obtains the equation in the text.

[49] Other shocks are assumed to be white noise processes: $u_t^{w,EA}$, $u_t^{\lambda^p,EA}$, $u_t^{r^{n},EA}$, $u_t^{q,EA}$

Technology shock:

$$\widehat{a}_t^{EA} = \rho_a^{EA} \widehat{a}_{t-1}^{EA} + u_t^{a,EA}$$

Investment specific shock:

$$\widehat{x}_t^{EA} = \rho_x^{EA} \widehat{x}_{t-1}^{EA} + u_t^{x,EA}$$

Inflation objective shock:

$$\overline{\pi}_t = \rho_{\overline{\pi}} \overline{\pi}_{t-1} + u_t^{\overline{\pi}}$$

The monetary policy rule:

$$\begin{aligned} \widehat{r}_t^{n,EA} &= \phi_m \widehat{r}_{t-1}^{n,EA} + (1-\phi_m)\left[\overline{\pi}_t + r_{\pi^{EA}}\left(\widehat{\pi}_{t-1}^{EA} - \overline{\pi}_t\right) + r_{y^{EA}}\left(\widehat{y}_{t-1}^{EA} - \widehat{y}_{t-1}^{P,EA}\right)\right] + \\ & +r_{\Delta\pi}\left(\widehat{\pi}_t^{EA} - \widehat{\pi}_{t-1}^{EA}\right) + r_{\Delta y}\left[\widehat{y}_t^{EA} - \widehat{y}_t^{P,EA} - \left(\widehat{y}_{t-1}^{EA} - \widehat{y}_{t-1}^{P,EA}\right)\right] + u_t^{r^{n,EA}} \end{aligned}$$

8.4 The steady state

In this section we derive the steady state of the model. We start by assuming that in the steady state:[50]

$$R^n = R^{n,EA}$$

Then, given that assumption it comes from equation (13) that:

$$\mathbf{\Omega}\left(FA, risk\right) = 1 \tag{A.5}$$

which given that $\mathbf{\Omega}\left(FA, risk\right) = \exp\left(-\phi_{fa} FA + \log risk\right)$ equation (A.5) implies that $B^{EA} = FA = 0$.

The real marginal cost is the inverse of the mark-up:

$$MC = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \tag{A.6}$$

where ε is the elasticity of substitution among the domestic differentiated goods. The gross mark up steady state value is $1 + \lambda^p = \frac{\varepsilon}{\varepsilon - 1}$.

The steady state value of the return on capital R^k is

$$R^k = R^n + 1 - \delta$$

Remembering that $R^n = (1+i) = (1+r) = \frac{1}{\beta}$ (because of the zero inflation steady state), I can write R^k as

[50]Because of the zero inflation steady state

$$R^n = 1 + i = 1 + r = \frac{1}{\beta}$$

where r is the steady state of the real interest rate.

$$R^k = \frac{1}{\beta} - 1 + \delta \tag{A.7}$$

Q=1

Profits are

$$\Pi = \lambda_d Y - R^k K - WL - F$$

where λ_d is the price mark up. We know that in equilibrium $Y = R^k K + WL$, and that thanks to the fix cost profits are zero in steady state. Hence.

$$\Pi = \lambda_d Y - Y - F = 0$$

Solving for F

$$F = (\lambda_d - 1)\, Y$$

But Y still includes F. Hence an alternative way to write it is

$$F = (\lambda_d - 1)\left[\left(\frac{K}{L}\right)^{\alpha} L - F\right]$$

Solving again for F

$$F = \frac{(\lambda_d - 1)}{\lambda_d}\left(\frac{K}{L}\right)^{\alpha} L \tag{A.8}$$

The the steady state value of the fixed cost in production is

$$F = \frac{\lambda_d - 1}{\lambda_d}\left(\frac{K}{L}\right)^{\alpha} L \tag{A.9}$$

This implies that the steady state value of Y is

$$Y = \frac{1}{\lambda_d}\left(\frac{K}{L}\right)^{\alpha} L \tag{A.10}$$

The aggregate resource constraint is

$$Y = C^D + C^F + I + G - M + EXP \tag{A.11}$$

but we know from equation (31) that $M = C^F$ and hence

$$Y = C^D + I + G + EXP \tag{A.12}$$

Now, given the steady state value of the foreign assets position derived in the beginning of this section, equation (32) implies that imports and exports are equal in steady state:

$$EXP = M = C^F = \alpha_c \left(\frac{P^{F,c}}{P^c} \right)^{-\eta_c} C$$

where the last equality comes from equation (16). Moreover, from equation (15):

$$C^D = \alpha_c \left(\frac{P^{D,c}}{P^c} \right)^{-\eta_c} C$$

Multiplying the terms $\frac{P^{F,c}}{P^c}$ by $\frac{P^D}{P^D}$ we have and using the definition of $\frac{P^{D,c}}{P^c}$:

$$EXP = \alpha_c \left(\frac{\gamma^c}{\gamma^{F,c}} \right)^{\eta_c} C \tag{A.13}$$

$$C^D = \alpha_c \left(\gamma^c \right)^{\eta_c} C \tag{A.14}$$

Substituting equations (A.13) and (A.14) in equation (A.12) and bearing in mind that from equation 2, the steady state value of $I = \delta K$:

$$Y = \alpha_c \left(\gamma^c \right)^{\eta_c} C + \delta K + gY + \alpha_c \left(\frac{\gamma^c}{\gamma^{F,c}} \right)^{\eta_c} C$$

where g$\equiv \frac{G}{Y}$.

Re-arranging and solving for C

$$\left[\alpha_c \left(\gamma^c \right)^{\eta_c} + \alpha_c \left(\frac{\gamma^c}{\gamma^{F,c}} \right)^{\eta_c} \right] C = (1 - g) Y - \delta K$$

$$C = (1 - g) \frac{Y}{H} - \delta \frac{K}{H}$$

where $H \equiv \left[\alpha_c \left(\gamma^c \right)^{\eta_c} + \alpha_c \left(\frac{\gamma^c}{\gamma^{F,c}} \right)^{\eta_c} \right]$.

Using the production function

$$C = (1 - g) \frac{1}{H} \left[\left(\frac{K}{L} \right)^{\alpha} L - F \right] - \delta \frac{K}{H}$$

Substitute out F using equation A.9

$$C = (1 - g) \frac{1}{H} \left[\left(\frac{K}{L} \right)^{\alpha} L - \frac{\lambda_d - 1}{\lambda_d} \left(\frac{K}{L} \right)^{\alpha} L \right] - \delta \frac{K}{H}$$

$$C = \frac{1}{H} (1 - g) \frac{1}{\lambda_d} \left(\frac{K}{L} \right)^{\alpha} L - \delta \frac{K}{H}$$

Or equivalently

$$C = \left[(1-g) \frac{1}{\lambda_d} \left(\frac{K}{L} \right)^{\alpha} - \delta \frac{K}{L} \right] \frac{L}{H} \tag{A.15}$$

We need an expression for $\frac{K}{L}$
From the cost minimization problem we have

$$R^k = \alpha MC \frac{Y}{K}$$

$$\frac{W}{P^D} = (1-\alpha) MC \frac{Y}{L}$$

where $\frac{W}{P^D}$ is the steady state value of the real wage.
Combining the two

$$\frac{W}{P^D} = (1-\alpha) R^k \frac{1}{\alpha} \frac{K}{Y} \frac{Y}{L}$$

Re arranging and solving for $\frac{K}{L}$

$$\frac{K}{L} = \frac{\alpha}{(1-\alpha)} \frac{W}{P^D} \frac{1}{R^k} \tag{A.16}$$

We know that the marginal costs are

$$\frac{1}{\lambda_d} = \left(\frac{1}{1-\alpha} \right)^{(1-\alpha)} \left(\frac{1}{\alpha} \right)^{\alpha} \left(R^k \right)^{\alpha} \left(\frac{W}{P^D} \right)^{1-\alpha}$$

Solving for $\frac{W}{P^D}$

$$\frac{W}{P^D} = \left[\frac{1}{\lambda_d \left(\frac{1}{1-\alpha} \right)^{(1-\alpha)} \left(\frac{1}{\alpha} \right)^{\alpha} (R^k)^{\alpha}} \right]^{\frac{1}{1-\alpha}} \tag{A.17}$$

Using the expression for R^k (equation A.7)

$$\frac{W}{P^D} = \left[\frac{1}{\lambda_d \left(\frac{1}{1-\alpha} \right)^{(1-\alpha)} \left(\frac{1}{\alpha} \right)^{\alpha} \left(\frac{1}{\beta} - 1 + \delta \right)^{\alpha}} \right]^{\frac{1}{1-\alpha}}$$

Substituting in equation A.16

$$\frac{K}{L} = \left[\frac{1}{\lambda_d \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{\beta} - 1 + \delta\right)^{\alpha}} \right]^{\frac{1}{1-\alpha}} \frac{\alpha}{(1-\alpha) R^k}$$

$$\frac{K}{L} = \frac{\alpha}{(1-\alpha)\left(\frac{1}{\beta} - 1 + \delta\right)} \frac{1}{\lambda_d^{\frac{1}{1-\alpha}} \left(\frac{1}{1-\alpha}\right) \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{\beta} - 1 + \delta\right)^{\frac{\alpha}{1-\alpha}}}$$

$$\frac{K}{L} = \left[\frac{1}{\lambda_d \left(\frac{1}{\alpha}\right) \left(\frac{1}{\beta} - 1 + \delta\right)} \right]^{\frac{1}{1-\alpha}} \tag{A.18}$$

Substituting the previous in the consumption equation (equation A.15)

$$C = \left\{ (1-g) \frac{1}{\lambda_d} \left[\frac{1}{\lambda_d \left(\frac{1}{\alpha}\right) \left(\frac{1}{\beta} - 1 + \delta\right)} \right]^{\frac{\alpha}{1-\alpha}} - \delta \left[\frac{1}{\lambda_d \left(\frac{1}{\alpha}\right) \left(\frac{1}{\beta} - 1 + \delta\right)} \right]^{\frac{1}{1-\alpha}} \right\} \frac{L}{H} \tag{A.19}$$

From the labour supply condition we know that

$$\frac{W}{P^D} = \frac{L^{\sigma_l}}{[(1-h) C]^{\sigma_c}}$$

$$\frac{W}{P^D} = \frac{L^{\sigma_l}}{[(1-h) C]^{\sigma_c}}$$

Solving for L and substitute out for $\frac{W}{P^D}$ (using equation A.17)

$$L = \left\{ \left[\frac{1}{\lambda_d \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)} \left(\frac{1}{\alpha}\right)^{\alpha} (R^k)^{\alpha}} \right]^{\frac{1}{1-\alpha}} [(1-h) C]^{\sigma_c} \right\}^{\frac{1}{\sigma_l}} \tag{A.20}$$

Using equation A.19 to substitute C in equation A.20

$$L = \left\{ \frac{1}{\lambda_d^{\frac{1}{1-\alpha}} \left(\frac{1}{1-\alpha}\right) \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} (R^k)^{\frac{\alpha}{1-\alpha}}} \left[(1-h) \frac{L}{H} \left\{ \begin{array}{c} (1-g) \frac{1}{\lambda_d} \left[\frac{1}{\lambda_d \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{\alpha}{1-\alpha}} - \\ -\delta \left[\frac{1}{\lambda_d \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)} \right]^{\frac{1}{1-\alpha}} \end{array} \right\} \right]^{\sigma_c} \right\}^{\frac{1}{\sigma_l}}$$

Re arranging

$$L=\left[\frac{1}{\lambda_d^{\frac{1}{1-\alpha}}\left(\frac{1}{1-\alpha}\right)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\left(R^k\right)^{\frac{\alpha}{1-\alpha}}}\right]^{\frac{1}{\sigma_l}}(1-h)^{\frac{\sigma_c}{\sigma_l}}\left(\frac{L}{H}\right)^{\frac{\sigma_c}{\sigma_l}}\left[\begin{array}{c}(1-g)\frac{1}{\lambda_d}\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{\alpha}{1-\alpha}}-\\-\delta\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{1}{1-\alpha}}\end{array}\right]^{\frac{\sigma_c}{\sigma_l}}$$

The solution for L is

$$L=(H)^{-\frac{\sigma_c}{\sigma_c-\sigma_l}}\left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c-\sigma_l}}$$

where $A=\left[\frac{1}{\lambda_d^{\frac{1}{1-\alpha}}\left(\frac{1}{1-\alpha}\right)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\left(R^k\right)^{\frac{\alpha}{1-\alpha}}}\right]^{\frac{1}{\sigma_l}}(1-h)^{\frac{\sigma_c}{\sigma_l}}\left[\begin{array}{c}(1-g)\frac{1}{\lambda_d}\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{\alpha}{1-\alpha}}-\\-\delta\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{1}{1-\alpha}}\end{array}\right]^{\frac{\sigma_c}{\sigma_l}}$

I can then derive the steady state of C substituting out L in equation A.19

$$C=\left\{(1-g)\frac{1}{\lambda_d}\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{\alpha}{1-\alpha}}-\delta\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{1}{1-\alpha}}\right\}(H)^{-\frac{\sigma_c}{\sigma_c-\sigma_l}}\left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c-\sigma_l}}$$

The steady state value of the fix cost is (from equation A.8)

$$F=\frac{(\lambda_d-1)}{\lambda_d}\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{\alpha}{1-a}}(H)^{-\frac{\sigma_c}{\sigma_c-\sigma_l}}\left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c-\sigma_l}}$$

The steady state value for Y is (using equation A.10)

$$Y=\frac{1}{\lambda_d}\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{\alpha}{1-\alpha}}(H)^{-\frac{\sigma_c}{\sigma_c-\sigma_l}}\left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c-\sigma_l}}$$

Using equation A.18 we can campute the steady state value of K

$$K=\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{1}{1-\alpha}}L$$

$$K=\left[\frac{1}{\lambda_d\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}-1+\delta\right)}\right]^{\frac{1}{1-\alpha}}(H)^{-\frac{\sigma_c}{\sigma_c-\sigma_l}}\left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c-\sigma_l}}$$

In turns the investement steady state

$$I = \delta \left[\frac{1}{\lambda_d \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta} - 1 + \delta\right)} \right]^{\frac{1}{1-\alpha}} (H)^{-\frac{\sigma_c}{\sigma_c - \sigma_l}} \left(\frac{1}{A}\right)^{\frac{\sigma_l}{\sigma_c - \sigma_l}}$$

References

Adolfson, Malin, Stefan Laséen, Jesper Lindé and Mattias Villani (2007) Evaluating An Estimated New Keynesian Small Open Economy Model. *Working Paper Series*, No. 203, Sveriges Riksbank

Adolfson, Malin, Stefan Laséen, Jesper Lindé and Mattias Villani (2005) Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass–Through. *Working Paper Series*, No. 179, Sveriges Riksbank

An, Sungbae and Frank Schorfheide (2007) Bayesian Analysis of DSGE Models. *Econometric Reviews*, vol. 26, pp. 113–172

Bems R. and Hartelius K. Jönsson (2006) Trade Deficits in the Baltic States: How Long Will the Party Last? *Review of Economic Dynamics*, vol. 9, pp. 179–209

Benigno P. (2001) Price Stability with Imperfect Financial Integration. Mimeo

Brooks S.P. and A. Gelman (1998) General Methods for Monitoring Convergence of Iterative Simulations. *Journal of Computational and Graphical Statics*, vol. 7, pp. 434–455

Calvo, Guillermo (1983) Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics*, vol. 12, pp. 383–398

Canova, Fabio (2008) How much structure in empirical models? *CEPR Discussion Paper*, No. 6791

Canova, Fabio (2007) *Methods For Applied Macroeconomic Research.* Princeton University Press

Chari, V.V., Patrick J. Kehoe and Ellen R. McGrattan (2008) New Keynesian Models: Not Yet Useful for Policy Analysis. *NBER Working Paper*, No. 14313

Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans (2005) Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, vol. 113, pp. 1–45

Colantoni, Luca (2007) Monetary Transmission in Estonia: The Interest Rate Channel, Mimeo

Collard, F. and H. Dellas (2002) Exchange Rate Systems and Macroeconomic Stability, *Journal of Monetary Economics*, vol. 49, pp. 571–599

Dabu*inskas, Aurelijus and Dmitry Kulikov (2007) New Keynesian Phillips curve for Estonia, Latvia and Lithuania. *Working Papers Series*, No. 7/2007, Eesti Pank

Erceg, Christopher J., Dale W. Henderson and Andrew T. Levin (2000) Optimal Monetary Policy with Staggered Wage and Price Contacts. *Journal of Monetary Economics*, vol. 46, pp. 281–313

Fagan, Gabriel, Jérôme Henry and Ricardo Mestre (2005) An area-wide model (AWM) for the euro area. *Economic Modelling*, vol. 22, pp. 39–59

Fernandez-Villaverde, Jesús and Juan F. Rubio-Ramirez (2007) A Baseline DSGE Model, Mimeo

Galì, Jordi and Tommaso Monacelli (2005) Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies*, vol. 72, pp. 707–734

Galì, Jordi and Mark Gertler (1999) Inflation Dynamics: a Structural Econometric Analysis. *Journal of Monetary Economics*, vol. 44, pp. 195–222

Galì, Jordi (1999) Technlogy, Employment, and the Business Cycle: Do Techinolosy Shocks Explain Aggregate Fluctuations? *The American Economic Review*, vol. 89, pp. 249–271

Gelman A., J. B. Carlin, H. S. Stern and D. B. Rubin (2004) *Bayesian Data Analysis.* Chapman and Hall/CRC

Greenwood J., Z. Hercowitz and G. W. Huffman (1988) Investment, Capacity Utilization and the Real Business Cycles. *The American Economic Review*, vol. 78, pp. 402–417

Hamilton, James (1994) *Time series analysis*, Princeton

Iskrev, Nikolay (2008) How much do we learn from the estimation of DSGE models? A case study of identification issues in a New Keynesian business cycle model. Mimeo

Juillard, Michel (2004) *Dynare Manual.* CEPREMAP

Kattai, Rasmus (2005) EMMA: a Quarterly Model of the Estonian Economy. *Working Papers Series*, No. 12/2005, Eesti Pank

Kollmann, Robert (2001) The Exchange Rate in a Dynamic-Optimizing Current Account Model with Nominal Rigidities: a Quantitative Investigation. *Journal of International Economics*, vol. 55, pp. 243–262

Lendvai, Julia an Werner Roeger (2008) Estonia 1995 to 2007: Catching-up and Beyond. Mimeo

Lundvik P. (1992) *Foreign Demand and Domestic Business Cycles: Sweden 1891-1987.* In Business Cycles and Growth, Monograph Series No. 22, Institute for International Economic Studies, Stockholm University

Obstfeld, Maurice and Kenneth Rogoff (2000) The Six Major Puzzles in International Macroeconomics: Is There a Common Cause? *NBER Macroeconomics Annual*, MIT Press.

Poirer, Dale J. (1995) *Intermediate statistics and econometrics: a comparative approach*, MIT Press

Ratto, Marco, Werner Roeger and Jan in't Veld (2008) Quest III: An Estimated DSGE Model of the Euro Area with Fiscal and Monetary Policy. *DG ECFIN Economic Paper*, No. 335

Robert, Christian P. and George Casella (2004) *Monte Carlo statistical methods*, Springer

Schmitt-Grohé, Stephanie and Martín Uribe (2003) Closing Small Open Economy Models. *Journal of International Economics*, vol. 61, pp. 163–185

Smets, Frank and Raf Wouters (2005) Comparing Shocks and Frictions in US and Euro Area Business Cycles: a Bayesian DSGE Approach. *Journal of Applied Econometrics*, vol. 20, pp. 161–183

Smets, Frank and Raf Wouters (2003) An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, vol. 1, pp. 1123–1175

Smets, Frank and Raf Wouters (2002) Openness, Imperfect Exchange Rate Pass-Through and Monetary Policy. *Journal of Monetary Economics*, vol. 49, pp. 947–981

Staehr, Karsten (2008) Estimates of Employment and Welfare Effects of Personal Labour Income Taxation in a Flat-Tax Country: The Case of Estonia. *Working Papers Series*, No. 3/2008, Eesti Pank

Tovar, Camilo E. (2008) DSGE models and central banks. *BIS Working Papers*, No. 258

Uhlig, Harald (1999) A toolkit for analyzing nonlinear dynamic stochastic models easily. In R. Marimón and A. Scott (eds.) *Computational Methods for the Study of Dynamic Economies*, Oxford University Press