Information Disclosure in Innovation Contests

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Abstract

In innovation contests, the progress of the competing firms in the innovation process is usually their private information. We analyze an innovation contest in which research firms have a stochastic technology to develop innovations at a fixed cost, but their progress is publicly announced. We make a comparison with the case of no information revelation: If the progress is disclosed, the expected profit of the firms is higher, but the expected profit of the sponsor is lower. Additionally, we show that firms may voluntarily reveal their information.

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1 Introduction

Contests have been used to enforce research in a variety of contexts: From refrigerators over computer programs to aerospace research. To win a contest, only the best innovation at the end of the research process matters. Nevertheless, intermediate stages of the research process can already indicate who is going to win the contest – if the progress of the participating firms is publicly known. Surprisingly, some contests force their participants to reveal their progress. This holds for example true for the Netflix Prize, an innovation race with a prize of $1,000,000 and build-in intermediate contests for progress prizes of $50,000 each. The goal of this race is to "[...] substantially improve the accuracy of predictions about how much someone is going to love a movie based on their movie preferences" (www.netflixprize.com). The submitted algorithms are measured by a score representing their performance – and the results of these score calculations are publicly accessible on the website.

Who benefits from disclosing the progress in a contest? On the one hand, the publication can serve as a kind of positive coordination device for the participants: A firm may

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recognize that she is far behind and thus abandon the contest. Furthermore, a firm may have a comfortable lead and decide to try to win the contest with her current innovation. These effects lead to lower research effort and thus lower costs, which can also go along with a less valuable winning innovation. On the other hand, there are opposing effects: The additional information can reveal that a firm is in fact behind in the contest, although without the information she would have believed to have a sufficiently good innovation to win it. This can encourage a firm to do more research, having a negative effect on the firm's research costs and a positive effect on the value of the winning innovation.

In this paper, we compare two settings in the framework of an innovation contest: In the main setting, the intermediate progress of the participants is made public, in the benchmark setting it is not. We show that firms prefer information disclosure, while the contest designer prefers to keep the information secret. However, if the contest design does not involve progress disclosure, we show that the firms may voluntarily reveal this information.

We use the analysis of research tournaments by Taylor (1995) as a benchmark. In his model, stochastic innovations can be made at a fixed cost. Firms have the possibility to conduct research several times, and their intermediate results are kept secret. He shows that there is a unique symmetric equilibrium in which firms continue to innovate if their best innovation value does not exceed a certain threshold. Building on this model, Fullerton et al. (2002) study auction-style research tournaments and Baye and Hoppe (2003) compare different contest types. Other models of research contests can also be found in the literature: Che and Gale (2003) find the optimal contest to be an auction given a deterministic research technology, Schoettner (2008) builds on the famous model by Lazear and Rosen (1981) to show that given a stochastic innovation technology, fixed-prize contests may in fact be superior to a first-price auction.

This paper also connects to the literature on multiple-round contests. In Konrad and Kovenock (2009), contestants have to win several component contests, modeled as all-pay auctions, to win the overall prize. Contrary to our model, the follower is not fully discouraged from continuing the contest even if he is far behind. Moldovanu and Sela (2006) investigate how to split contestants over sub-contests where only the winners continue to compete. In Yildirim (2005), building on work by Dixit (1987), heterogeneous participants can split their effort over two rounds with observable first-round effort. Playing a game ex ante where agents choose between non-observable effort (which equals one-shot play there) or two-round effort with intermediate revelation, they decide to reveal effort in equilibrium. In our model, we also get voluntary revelation – however, it is revelation of (stochastic) innovation values and not of effort. Furthermore, in our model a true multiple-round contest is played in case of secret intermediate results.

The idea of intermediate revelation of research results is also studied by Gill (2008) in the context of patent contests with exogenously given leader and follower. In his model, research is a two-stage process where both steps are necessary to develop a single innovation. The leader decides whether or not to disclose his performance after the first stage. Then, the follower may choose to drop out after the first stage. Whether or not the leader discloses depends on the research costs. By contrast, in our model leader and follower are endogenously determined, as multiple innovations can be developed. In Aoyagi (2007) all information on intermediate performance is controlled by the contest designer. Related to our model, performance is stochastic. Furthermore, it is additive over the two rounds, while we consider multiple independent innovations. The optimal feedback policy to the participants regarding this information is derived – it depends on the shape of the cost function whether a no-feedback or a full-feedback policy is optimal. In a related paper, Gershkov and Perry (2009) study the design of midterm reviews. Given a fixed prize, it is always optimal to have such a review, if the results of intermediate and final review are optimally aggregated.

An experimental study of information disclosure is provided by Ludwig and Lünser (2008). They compare two settings with and without intermediate information release, where equilibrium play is not affected by the information structure. Nevertheless, subjects in the experiments behave differently if they observe their opponent's effort.

We are interested in the effects of intermediate information disclosure when multiple independent innovations can be made. Building on the model by Taylor (1995), we add the revelation of the first-period innovation value to an innovation contest with two firms and two periods. Then, there is no longer a single cutoff for which firms below continue to innovate, but there are two cutoffs: If one firm has an innovation value in the high range, both stop innovating. If the highest innovation is in the intermediate range, only the follower continues to innovate – and if both innovations are below the lower cutoff, both firms continue. Thus, there is a coordination effect which is favorable for the firms: A contest with information disclosure is going along with lower research costs and a higher payoff for the firms. Contrary, the coordination effect regarding the value of the highest innovation is not large enough to make up for the lower number of innovations. Consequently, the prize sponsor gets a higher revenue in the setting without information disclosure. However, if firms are able to voluntarily reveal their first-period value, they do so. We model voluntary disclosure in two different ways: First, the firms decide in an ex ante-game whether they are going to reveal or not after the first period. Second, the decision to disclose is delayed until firms learn their first-period innovation value. In both cases, it turns out that there is essentially a unique equilibrium in which both firms disclose. Continuing this train of thought, the voluntary revelation has consequences for the contest designer: If he chooses the size of the prize optimally, he should choose it with respect to the setting where information is revealed – the firms will play it anyway.

The paper is organized as follows: The basic model and equilibrium behavior with information disclosure is presented in Section 2. We compare it to the benchmark case without disclosure in Section 3. In Section 4 we endogenize information revelation and conclude in Section 5. Proofs can be found in the Appendix.

2 The Model and Equilibrium Derivation

We consider two risk-neutral research firms, $i = 1, 2$. They compete in an innovation contest to win a fixed prize $p > 0$. Firms are assumed to know the price sponsor's utility function over research outcomes. Both firms have an innovation technology similar to Taylor (1995): Research is modeled as drawing an innovation x out of a probability distribution F with strictly positive density f. F is defined on $[0, b]$ with $b \leq \infty$. Each innovation draw is associated with a cost of $c > 0$ for each firm. Firms are not capital constrained. There are two periods $t = 1, 2$ in which firms may innovate. Innovation values x_i^t are independent across periods and firms. For each firm, only the best draw ($\max\{x_i^1, x_i^2\}$) is relevant for the contest. The firm with the highest draw wins the contest and the prize of p. Ties are randomly broken. We assume that innovations that do not win have a value of zero outside the contest, such that loosing innovations cannot be sold afterwards[1]. In the basic version of our model, in contrast to Taylor (1995), we assume in the spirit of Yildirim (2005) that first-period innovations become common knowledge after both firms have made their decision whether to conduct research or not and have taken their draw. We first analyze equilibrium behavior of the two firms. We look for subgame perfect Nash equilibria by backward induction and thus start with the second period. First note that for $p < c$ both firms would make a loss from conducting research. Thus, both do not conduct any research (neither in the first nor in the second period). Consequently, we focus on the case $c \leq p$ in the following.

2.1 Second Period

Suppose at least one firm has taken a draw in the first period, such that one of the two firms has taken the lead, $x_H^1 > x_L^2 \geq 0$. H stands for the firm with the higher first round innovation (the *higher* firm) and L for the firm with the lower innovation (the *lower* firm). We calculate best responses:

If the lower firm does not continue to innovate, it is a best response for the higher firm to stop innovating as well – she will win in any case.

So suppose now the higher firm does not draw again. Then, the lower firm wants to continue if the following condition holds:

$$P\left(x_L^2 > x_H^1\right) p - c \geq 0 \iff \left(1 - F\left(x_H^1\right)\right) p - c \geq 0 \iff F\left(x_H^1\right) \leq 1 - \frac{c}{p}$$

Let x^* solve $F(x^*) = 1 - \frac{c}{p}$. Then, if some $x > x^*$ is drawn by any of the two firms, the contest stops immediately and no new research will be conducted in the second round: The lower firm has no incentive to draw again if the opponent has already drawn such a high innovation. Then, the higher firm will obviously not draw again as well.

[1] This would give firms incentives to innovate that are outside the contest and thus outside of our model.

Now consider the case $x_H^1 \leq x^*$, such that the lower firm wants to draw again if the high firm does not. What is the best response of the high firm against the low firm drawing again? The high firm wants to draw again as well if the following condition holds:

$$\begin{aligned} & \left[P\left(x_H^2 > x_L^2 > x_H^1\right) + P\left(x_H^1 > x_L^2\right)\right]p - c \geq P\left(x_H^1 > x_L^2\right)p \\ \iff & \frac{1}{2}\left(1 - F\left(x_H^1\right)\right)^2 p - c \geq 0 \\ \iff & F\left(x_H^1\right) \leq 1 - \sqrt{2\frac{c}{p}} \end{aligned}$$

Let $\bar{x}$ solve $F(\bar{x}) = 1 - \sqrt{2\frac{c}{p}}$ and note that $\bar{x} < x^*$. What is the best response of the lower firm against a higher firm drawing again for $x_H^1 \leq \bar{x}$? Drawing again is a best response according to the following condition:

$$\begin{aligned} & P\left(x_L^2 > x_H^1, x_H^2\right)p - c \geq 0 \\ \iff & \left[\frac{1}{2}\left(1 - F\left(x_H^1\right)\right)^2 + \left(1 - F\left(x_H^1\right)\right)\left(F\left(x_H^1\right)\right)\right]p - c \geq 0 \end{aligned} \tag{1}$$

We know that

$$\frac{1}{2}\left(1 - F\left(x_H^1\right)\right)^2 p - c \geq 0$$

because $x_H^1 \leq \bar{x}$. Hence, (1) is fulfilled. Consequently, the lower firm wants to draw again in the second round as well. This is intuitive: The higher firm already has an advantage after the first round, so incentives for the lower firm to draw again are even higher.
We summarize our findings in the following proposition:

Proposition 1 *Given first-period innovations $x_H^1 > x_L^1$, we get the following second-period equilibrium strategies:*

- *If $x_H^1 > x^*$ both firms stop their research effort and the contest ends after the first period.*
- *If $x^* \geq x_H^1 > \bar{x}$ only the lower firm conducts research in the second period.*
- *If $\bar{x} \geq x_H^1$ both firms conduct research in the second period.*

Note that for small price values $p < 2c$ we get $\bar{x} < 0$, thus, the higher firm will never draw again in the second period.
Let us now consider the case that both firms have reached the same innovation level after the first period: Either because they both did not conduct any research in the first period, or because they both drew the same $x^1 := x_1^1 = x_2^1$. The following proposition holds:

Proposition 2 *Given first-period innovations $x_1^1 = x_2^1 = x^1$, we get the following second-period equilibria:*

- *If $p < 2c$, both firms do not conduct any research in the second period.*

- *If $p \geq 2c$, we have a unique equilibrium in case*
  1. $\frac{1}{2} - \frac{c}{p} \leq 1 - 2\frac{c}{p} \leq F(x^1)$*: Both firms do not draw again.*
  2. $F(x^1) \leq \frac{1}{2} - \frac{c}{p} \leq 1 - 2\frac{c}{p}$*: Both firms draw again.*
- *If $p \geq 2c$, we have two equilibria in case $\frac{1}{2} - \frac{c}{p} \leq F(x^1) \leq 1 - 2\frac{c}{p}$: Either both firms draw, or both firms do not draw.*

The proof is given in the Appendix. Note that equilibrium is unique in case firms did not conduct any research in the first period. If at least one firm takes a draw in the first period, a tie appears with zero probability, and thus second-period equilibrium is unique with probability 1.

2.2 First Period

In the same spirit the first-period pure-strategy equilibria can be derived, taking into account second-period equilibrium play. As the main focus of this paper is on information revelation after the first period, we are especially interested in the conditions under which both firms start innovating in the first period. If they do not innovate in the first period, information revelation is only of minor interest. It turns out that the size of the prize compared to the innovation costs is the crucial parameter for first-period innovation to take place. We make use of the following short notations: $r := \frac{c}{p}$ and $s := \sqrt{2r}$.

Proposition 3 *In the first period, we get the following pure-strategy equilibrium behavior with firms continuing in the second period as described in Proposition 1:*

- *For $r > \frac{1}{2}$ both firms do not conduct any research in the first period.*
- *For $0 < r < v^*$ both firms conduct research in the first period.*
- *For $\frac{1}{2} \geq r > v^*$ equilibrium behavior is asymmetric – one firm conducts research, the other does not.*

Proof See Appendix. □

The boundary value v^* is the solution of the following equation:

$$\frac{1}{6} - v^* + \frac{2}{3}v^*\sqrt{2v^*} - \frac{1}{2}(v^*)^2 - \frac{1}{2}(v^*)^3 = 0$$

Numerically, it is given by $v^* \approx 0.2428$. Intuitively, this value is reasonable: It represents $p \approx 4c$, which is in the size of total research costs for the two firms if all four possible innovations are developed.

The proposition shows that if the prize is too low compared to the costs, both firms will invest neither in the first nor in the second period. Additionally, there are two pure-strategy equilibria if r takes intermediate values. Consequently, a coordination problem arises with respect to pure-strategy equilibria. Furthermore, there is a symmetric mixed strategy equilibrium in this case as well, which we do not calculate here because we focus on $r < v^*$ in the following: We are interested in information revelation with firms in fact doing research in the first period.

3 Comparison with No Information Release

In this section, we compare the setting with information release after the first period, which we just analyzed, with the setting known from the literature (Taylor 1995) where information is kept secret after the first period. We want to find the preferred setting for both the firms and the contest designer. First, we compare the settings from the perspective of the firms, then we turn to the contest designer.

3.1 Firms' Perspective

To analyze the firms' perspective, we compare the expected number of innovation draws in the setting with information revelation to no information revelation after the first period – firms prefer the setting with lower research costs, which means less innovation draws in this context. The first step is to calculate the expected number of draws $d_R(r)$ in the equilibrium with information release, given that both firms do research in the first period.

Proposition 4 *Given $r < v^*$ and both firms doing research in the first period, the expected number of draws in equilibrium fulfills $d_R(r) = 4 - 2s + r^2$.*

Proof See Appendix. □

We now come back to the setting of Taylor (1995), where no information is released. He calculates a z representing a stop value for the firms: They take draws as long as they do not have an innovation that exceeds z. However, Taylor does not calculate the z explicitly. We thus take an approach to come closer to the exact value of z in our setting. According to Taylor (1995), z is the solution of the following equation:

$$\begin{aligned}
0 &= p\int_z^b \left[F^2(z) + \left(1 - F^2(z)\right)\frac{F(x)-F(z)}{1-F(z)} - F^2(z)\right] dF(x) - c \\
&= p\left(1+F(z)\right)\left[\int_z^b F(x)f(x)dx - F(z)\int_z^b f(x)dx\right] - c \\
&= p\left(1+F(z)\right)\left[\frac{1}{2}\left(1-F^2(z)\right) - F(z)\left(1-F(z)\right)\right] - c \\
&= p\frac{1}{2}\left(1+F(z)\right)\left(1-F(z)\right)^2 - c \\
\iff \quad 0 &= \left(1+F(z)\right)\left(1-F(z)\right)^2 - 2r \qquad (2)
\end{aligned}$$

Unfortunately, the explicit solution of this equation is quite messy. To anyway give a feeling of the size of z, one can see by plugging in $\bar{x}$ and x^* that $\bar{x} < z < x^*$ holds.

Nevertheless, we make a comparison between the setting of Taylor (1995) and our setting with the help of a numerical analysis. Our target value that we want to compare is the number of draws the firms take in each setting. For our case with information revelation we already calculated the expected number of draws ($d_R(r)$, Proposition 4). For the setting without information revelation, we perform a numerical analysis in the following way: For a given value of $F(z)$ we can calculate the corresponding r value such that (2) is fulfilled. Thus, by taking a reverse look at the resulting $(F(z), r)$ pairs we get solutions of (2) for these r values. The expected number of draws for such an r in the case without information disclosure is given by $d_{NR}(r) = 2(1 + F(z))$. The numerical analysis shows that the expected number of draws is higher if no information is revealed:

Proposition 5 *Considering $r < v^*$, the expected number of draws $d_{NR}(r)$ in case no information is revealed after the first period is larger then the expected number of draws $d_R(r)$ in case information is revealed.*

We immediately get the following corollary, as both players win in expectation $\frac{1}{2}p$ in equilibrium in both settings, but have lower costs in the setting with information disclosure because they take less draws:

Corollary 6 *For $r < v^*$, both research firms prefer the setting with information disclosure over the setting without information disclosure.*

Note that $r < v^*$ is exactly the range of r-values we are interested in, guaranteeing research draws by both firms in the first period.

3.2 Designer's Perspective

From the prize sponsor's perspective, a higher number of innovation draws is in principle favorable, as more draws suggest a higher expected final prize. However, it is not obvious that this relationship really holds in this context: Draws are taken conditional on already realized innovations. Thus, if a draw is not taken, a good innovation has already been made. But the equilibrium decision rules whether another draw is taken differ between the two settings. Thus, a higher number of draws is an indicator for a higher expected final innovation, but does not allow a sure conclusion.

The key to the comparison from the designer's perspective is the highest expected innovation generated by the two different settings. The designer prefers the setting yielding the higher one.

To calculate the highest expected innovation for the two settings, we need the respective distribution functions of the highest innovation. In the setting without information release, the two firms are innovating independently. Let Φ be the distribution of the highest innovation for a single firm. Then, the joint distribution is given by Φ^2. Using the result by Taylor (1995) regarding Φ, we get

$$\Phi^2(x) = \begin{cases} F^4(x) & \text{if } x \leq z \\ (F(x) - F(z) + F(z)F(x))^2 & \text{if } x > z \end{cases}$$

For the setting with information revelation, the two firms do not innovate independently. The distribution Ψ of the joint highest innovation has the following structure, given the equilibrium play of the two firms:

$$\Psi(x) = \begin{cases} F^4(x) & \text{if } x \leq \bar{x} \\ A & \text{if } \bar{x} < x \leq x^* \\ B & \text{if } x^* < x \end{cases}$$

Denote the highest innovation in period j by $x^j_{(1)}$. Then, A and B are given according to

$$\begin{aligned} A &= P\left(x^1_{(1)} < \bar{x}\right) P\left(x^2_{(1)} < x\right) + P\left(\bar{x} < x^1_{(1)} \leq x\right) P\left(x^2_{(1)} < x\right) \\ &= F^2(x)F^2(\bar{x}) + F(x)\left(F(x)^2 - F(\bar{x})^2\right) \\ B &= P\left(x^1_{(1)} < \bar{x}\right) P\left(x^2_{(1)} < x\right) + P\left(\bar{x} < x^1_{(1)} < x^*\right) P\left(x^2_{(1)} < x\right) + P\left(x^* < x^1_{(1)}\right) \\ &= F(x)^2F(\bar{x})^2 - F(x)F(\bar{x})^2 + F(x)F(x^*)^2 + F(x^2) - F(x^*)^2 \end{aligned}$$

Given these distribution functions, we can calculate which setting provides the higher expected innovation – no information revelation is preferred if the following condition holds:

$$\int_0^b 1 - \Phi^2(x)dx \geq \int_0^b 1 - \Psi(x)dx \iff \int_0^b \Phi^2(x) - \Psi(x)dx \leq 0 \tag{3}$$

Analyzing this equation yields the following theorem:

Theorem 7 *The expected value of the highest innovation is larger in the setting without information revelation if $r < v'$ holds. Then, this setting is preferred by the prize sponsor.*

The derivation of v' can be found in the Appendix, done by looking at a condition which yields a negative integrand of (3) on the whole interval $[0, b]$. This is the case for $v' = 0.1647$. In fact, for (3) to hold, the integrand would not need to be negative on the whole interval – the resulting condition is thus somewhat too strong and the result holds for some larger r-values as well.

4 Endogenous Information Release

We have seen in the previous section that firms prefer the setting with information disclosure after the first draw to the setting without information disclosure. However, the contest designer has opposite preferences, and he is the one to choose the setup. This raises the question whether firms can force the contest designer into the information revelation setting by voluntary revelation of their first-period innovation value. We take two approaches to model this: First, we extend our model by adding a stage zero in which firms can ex ante decide whether to disclose the level of their innovation after the first draw or not. This is an extension in the spirit of the analysis in Yildirim (2005) to our model. Second, we consider an intermediate decision, where the firms only decide whether they disclose the information after having observed the value of the first-period innovation.

4.1 Ex Ante Decision

We add an initial stage zero in which the firms simultaneously decide whether to reveal their information (action R) or whether they do not reveal (action N). It is our goal to identify equilibria of this simultaneous-move game to find out whether the analysis in the previous sections can be supported by endogenous information revelation. This would be the case if (R, R) is an equilibrium of this game. In case both firms play R, the contest following afterwards is the same as the one described in the previous sections. Hence, we already know the corresponding equilibrium strategies. The same holds true in case both firms play N. Then, we are back in the setting of Taylor (1995). To derive the best responses in this initial stage, we need to deduce the equilibrium strategies in the case of asymmetric information revelation. In the resulting contest, one firm reveals her first draw, the other one does not. We will analyze equilibria by backward induction. We focus on the main case $p > 2c$ in the following, and assume thus $r < 0.5$.
For the second-period equilibrium, we take the first draw as given. One firm has played R in the initial stage, we denote her draw by x_R^1 and call her firm R. The draw of the firm playing N (short: firm N) is denoted by x_N^1.

Proposition 8 *In the setting with asymmetric information release, given first-period innovations x_R^1 and x_N^1, we get the following second-period equilibrium strategies:*

- *Firm R takes a second draw iff $x_R^1 < z$.*
- *Firm N takes a second draw iff $x_N^1 < x_R^1 < x^*$ or $\bar{x} > x_N^1 > x_R^1$.*

In case firm N does not take a draw in the first period, in equilibrium with correct believes, firm R takes a second draw iff $\bar{x} > x_R^1$. Firm N takes a draw in case $x_R^1 < x^*$.

Proof See Appendix. □

Roughly speaking, firm R thus behaves as in the setting with no information release, while firm N plays the same strategy as with full information release. Note that the proposition ignores the case $x_R^1 = x_N^1$, which appears with zero probability – it is thus not payoff relevant and w.l.o.g. we omit it here.
The first-period equilibrium behavior can be summarized as follows:

Proposition 9 *In the first period of the contest with asymmetric information disclosure we get the following pure-strategy equilibrium behavior with firms continuing in the second period as described in proposition 8:*

- *For $r < \hat{v}$ there is an equilibrium where both firms draw in the first period.*
- *For $0.5 > r > \tilde{v}$ there is an equilibrium where firm R draws in the first period and firm N does not.*
- *For $0.5 > r > \hat{v}$ there is an equilibrium where firm N draws in the first period and firm R does not.*

The proof is given in the Appendix. It shows that $\hat{v}$ is given as the solution to the following equation:

$$-\frac{1}{24} - \frac{1}{3}\hat{v}^3 + \frac{1}{2}w - \frac{1}{4}w^2 - \frac{1}{6}w^3 + \frac{1}{8}w^4 - w\hat{v} = 0$$

Here, we use the short notation $w := F(z)$. w depends directly on the size of r (or $\hat{v}$ in this context), as given by (2). Consequently, we can approximate that $\hat{v} \approx 0.2623$.
Similarly, $\tilde{v}$ is the solution to

$$\frac{1}{6} - 2\tilde{v} - \frac{1}{2}\tilde{v}^2 - \frac{1}{6}\tilde{v}^3 + 2\sqrt{2\tilde{v}}\tilde{v} - \frac{1}{2}\sqrt{2\tilde{v}}\tilde{v}^2 = 0,$$

which yields $\tilde{v} \approx 0.1722$.
Note that firm R plays different strategies in the two equilibria involving a draw by firm R: As Proposition 8 shows, firm R will continue to innovate in less cases if firm N stops her innovation process. Consequently, the best reply of firm N is affected by the change in strategy, yielding two different equilibria involving a draw by firm R in the range $\tilde{v} < r < \hat{v}$.
With this characterization of pure-strategy equilibria we are ready to address the main question of this section: Are the two firms willing to ex ante commit to revealing their information after the first draw or not?
We focus our analysis on the first equilibrium identified in proposition 9. In this equilibrium, both firms take a draw in the first period and it is unique for $r < \tilde{v}$. Again, these r values are the interesting ones as it is common that the prize of the contest is substantially larger than the cost of a single research draw.
In the initial stage zero, we now have to identify the best responses of the two firms. What is the best response of a firm, if the other firm chooses to play R? If she plays R as well, they share the prize in expectation and $2 - s + \frac{1}{2}r^2$ research draws are taken by each of the firms. If a firm deviates to play N, the expected costs of drawing do not change (as she still gets the same information and plays the same strategy). However, it may happen that she receives in expectation less than half of the prize after the deviation, as follows by the following condition:

$$\left[\frac{17}{24} + \frac{1}{3}r^3 - \frac{1}{6}s^3 - \frac{1}{2}w + \frac{1}{4}w^2 + \frac{1}{6}w^3 - \frac{1}{8}w^4\right]p - \left(2 - s + \frac{1}{2}r^2\right)c = \frac{1}{2}p, \tag{4}$$

The left-hand side of (4) states the profit for the firm deviating to N, as derived in (15) in the Appendix. A r-value of $\bar{v} \approx 0.2325$ solves (4), and for $r < \bar{v}$ a firm playing N against

R receives in expectation less than half the share of the total prize. Combined with the fact that research costs do not change, it is the best response against a firm playing R to play R as well for these values.
What is the best response against a firm playing N? Playing N as well gives in expectation half of the prize while taking $1 + w$ draws. As we have just seen, a firm playing R receives in expectation more than half the prize against a firm playing N for $r < \bar{v}$. Additionally, she has to take the same number of draws in expectation. Hence, it is profitable to play R against a firm playing N.
This analysis results in the following theorem:

Theorem 10 *For $r < \bar{v}$ there is a subgame perfect Nash equilibrium in which both firms ex ante commit to revealing their information after the first period. For $r < \tilde{v}$ it is unique.*

Hence, we have shown that the disclosure of information can be endogenized – the firms are voluntarily agreeing to it ex ante.

4.2 Intermediate Decision

So far, we modeled the revelation decision as taking place before any research is done by the firms. In that setup, firms need to be able to commit to their decision. In the following, we drop the assumption that ex ante commitment is possible – the revelation decision is postponed after the first period, when firms are able to observe their first innovation. As the revelation decision works as a kind of signaling device, a firm holds a belief on the value of the other firm's innovation. We thus refine our equilibrium concept to Perfect Bayesian equilibrium. Nevertheless, firms reveal the information voluntarily, as the following theorem shows:

Theorem 11 *If firms make their revelation decision simultaneously after knowing their first-period innovation value, there is a unique perfect Bayesian equilibrium in which both firms reveal their value (up to indifference for value x^*). Off equilibrium path, in case one firm does not reveal, the other firm believes the deviating firm has value x^* with probability 1 and will react accordingly.*

The intuition for the proof is as follows: It is in fact an equilibrium as keeping the information secret is punished in the strongest possible way. To show the uniqueness, one has to consider the fact that a firm wants to clearly show that she has a high type (and discourage lower types from continuing to innovate) or a low type (and make intermediate types stop innovating). For intermediate types, one can show that if a firm keeps the information secret, she does so for an interval of values. However, for the lowest of these values a firm has an incentive to reveal – she does not want to pool with higher values against which the other firm would more often like to continue innovating.

5 Conclusion

We showed that although the contest designer prefers firms to keep intermediate information secret, they will voluntarily reveal their research progress. Especially, if the contest designer chooses the prize optimally, he has to take this into account: Using the optimal prize in the setting without information disclosure, which he prefers, can lead to a lower payoff than the optimal prize for the setting without disclosure, if the firms decide to disclose on their own. To show this effect quantitatively, it would be interesting to derive the optimal prize, which is still a topic for further research.
Extending the model to heterogeneous firms will have no qualitative effect on the results, although quantitatively different research costs or different research technologies will lead to different cutoffs for the two firms. Similarly, an extension to multiple firms results in more cutoff levels with respect to pure strategy equilibria. Furthermore, it gives rise to a mixed strategy equilibrium where firms who do not have the lead continue to innovate with a higher probability the lower the leading firm's innovation value is.

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A Appendix: Proofs

Proof of Proposition 2

Suppose firm i decides not to draw again. Then, not drawing again is a best response for firm $j \neq i$ in case

$$P\left(x_j^2 \geq x^1\right) p - c \leq \frac{1}{2}p \iff \left(1 - F\left(x^1\right)\right) p - c \leq \frac{1}{2}p \iff \frac{1}{2} - \frac{c}{p} \leq F\left(x^1\right) \qquad (5)$$

Thus, we get that both firms not drawing again is an equilibrium if (5) holds.
Here, we can directly see that both firms do not want to draw in the second period in case $p < 2c$. Even if both firms did not invest in the first period, and a firm could win for sure by conducting research, expected profit is higher if no research is done.
Let us get to the best response in case firm i decides to draw in the second period. Then, drawing is a best response for firm $j \neq i$ according to the following condition:

$$\frac{1}{2}p - c \geq P\left(x_i^2 \leq x^1\right) \frac{1}{2}p \iff \frac{1}{2}p - c \geq F\left(x^1\right) \frac{1}{2}p \iff 1 - 2\frac{c}{p} \geq F\left(x^1\right) \qquad (6)$$

Hence, both firms drawing again is an equilibrium if (6) is fulfilled. Again, we can see that a firm does not want to draw again in case $p < 2c$.
By noting that for $p \geq 2c$

$$1 - 2\frac{c}{p} = 2\left(\frac{1}{2} - \frac{c}{p}\right) \geq \frac{1}{2} - \frac{c}{p}$$

holds, the proposition follows. □

Proof of Proposition 3

To derive first-period equilibrium play, first consider the case $p < 2c$. As we have seen, both firms will not invest in the second period in case no research is done in the first period. If research is conducted by at least one firm, only the lower firm might invest again, because $\bar{x} < 0$ if $p < 2c$. By backward induction, we can conclude that both firms will not draw in the first period: We have seen in the analysis of the second period that a single draw is too expensive for a firm even when it wins for sure. In the first period, incentives for conducting research are even lower. An investing firm will not win for sure, as the other firm might decide to invest in the second period. Hence, both firms will not invest in the first period if the price is too low. This is no problem for the firms, as they make a positive expected profit of $\frac{1}{2}p$. It is a problem of the prize sponsor, who will get no research done but has to pay the prize anyway.

So let us consider the case $p \geq 2c$. What is the best response against an opponent not taking a draw in the first period? Note that we know the following:

$$P\left(x_i^2 > x^*\right) = 1 - F\left(x^*\right) = \frac{c}{p} = r$$
$$P\left(\bar{x} < x_i^2 \leq x^*\right) = F\left(x^*\right) - F\left(\bar{x}\right) = \sqrt{2\frac{c}{p}} - \frac{c}{p} = s - r$$
$$P\left(x_i^2 \leq \bar{x}\right) = F\left(\bar{x}\right) = 1 - \sqrt{2\frac{c}{p}} = 1 - s$$

We can thus write down the condition for player i taking a draw in the first round against a player $j \neq i$ not taking a draw in the first round, bearing in mind second-period equilibrium behavior:

$$\begin{aligned}
&\Bigg[P\left(x_i^1 > x^*\right) + P\left(\bar{x} < x_i^1 \leq x^*\right)\left(P\left(x_j^2 \leq \bar{x}\right) + \frac{1}{2}P\left(\bar{x} < x_j^2 \leq x^*\right)\right) \\
&\quad + P\left(x_i^1 \leq \bar{x}\right)\Bigg(P\left(x_i^2 > \bar{x}\right)\left(P\left(x_j^2 \leq \bar{x}\right) + \frac{1}{2}P\left(x_j^2 > \bar{x}\right)\right) \\
&\quad + \frac{2}{3}P\left(x_i^2 \leq \bar{x}\right)P\left(x_j^2 \leq \bar{x}\right)\Bigg)\Bigg] p - c - P\left(x_i^1 \leq \bar{x}\right)c \geq \frac{1}{2}p - c \\
\iff\quad &\Bigg[r + (s-r)\left((1-s) + \frac{1}{2}(s-r)\right) + (1-s)\Bigg(s\left((1-s) + \frac{1}{2}s\right) \\
&\quad + \frac{2}{3}(1-s)(1-s)\Bigg)\Bigg] p - (1-s)c \geq \frac{1}{2}p \\
\iff\quad &\left[r + (s-r)\left(1 - \frac{1}{2}(s+r)\right) + (1-s)\left((s-r) + \frac{2}{3}(1-2s+2r)\right)\right] p \\
&\quad - (1-s)c \geq \frac{1}{2}p \\
\iff\quad &\left[s - r + \frac{1}{2}r^2 + \frac{2}{3} - \frac{1}{3}s + \frac{1}{3}r - \frac{2}{3}s + \frac{2}{3}r - \frac{1}{3}rs\right] p - (1-s)c \geq \frac{1}{2}p \\
\iff\quad &\left[\frac{1}{6} - \frac{1}{3}rs + \frac{1}{2}r^2\right] - (1-s)r \geq 0 \\
\iff\quad &\frac{1}{6} - r + \frac{2}{3}rs + \frac{1}{2}r^2 \geq 0 \\
\iff\quad &\frac{1}{6} - r + \frac{2}{3}\sqrt{2}r^{\frac{3}{2}} + \frac{1}{2}r^2 \geq 0 \qquad (7)
\end{aligned}$$

We thus have to show now that (7) holds. To check this, we calculate the minimum of the left side in (7) with the help of the substitution $t := \sqrt{r}$. The FOC with respect to r is

$$\begin{aligned} & -1+\sqrt{2}r^{\frac{1}{2}}+r=0 \\ \Longleftrightarrow \quad & t^2+\sqrt{2}t-1=0 \\ \Longrightarrow \quad & t=-\sqrt{\frac{1}{2}}+\sqrt{\frac{3}{2}}=\frac{\sqrt{3}-1}{\sqrt{2}} \end{aligned}$$

Only the positive solution matters here, as $t = \sqrt{r}$ is restricted to be positive. Hence, we get $r = \frac{(\sqrt{3}-1)^2}{2} \approx 0.2679$, leading to an expected gain from drawing compared to not drawing of approximately $0.0654 > 0$, which is clearly a minimum on $[0; 0.5]$ ($r \leq 0.5$ holds as $p \geq 2c$). Hence, it is always a best response to draw in the first period if the opponent does not take a draw.

Finally, we get to the best response of firm i in case the other agent $j \neq i$ takes a draw in the first period. We compare the expected profit of drawing as well (and thus playing the same strategy and sharing the prize) with the expected profit of not drawing in the first period. Note that we just calculated above the expected share of the prize a firm gets when taking a draw in the first period against a firm not taking a draw in the first period. We can thus subtract this share from the whole prize to get the share of the firm not drawing against a drawing firm.

$$\begin{aligned} \frac{1}{2}p-c-\left[P\left(x_i^1\leq\bar{x}\right)P\left(x_j^1\leq x^*\right)+\frac{1}{2}P\left(\bar{x}<x_i^1\leq x^*\right)P\left(\bar{x}<x_j^1\leq x^*\right)\right]c \\ \geq\left[1-\left(\frac{2}{3}-\frac{1}{3}rs+\frac{1}{2}r^2\right)\right]p-P\left(x_j^1\leq x^*\right)c \qquad (8) \end{aligned}$$

Computing the probabilities yields

$$\begin{aligned} & \frac{1}{2}p-c-\left[(1-s)(1-r)+\frac{1}{2}(s-r)(s-r)\right]c\geq\left[\frac{1}{3}+\frac{1}{3}rs-\frac{1}{2}r^2\right]p-(1-r)c \\ \Longleftrightarrow \quad & \frac{1}{2}p-c-\left[1-s+\frac{1}{2}r^2\right]c\geq\left[\frac{1}{3}+\frac{1}{3}rs-\frac{1}{2}r^2\right]p-(1-r)c \\ \Longleftrightarrow \quad & \left[\frac{1}{6}-\frac{1}{3}rs+\frac{1}{2}r^2\right]-\left[1-s+r+\frac{1}{2}r^2\right]r\geq 0 \\ \Longleftrightarrow \quad & \frac{1}{6}-r+\frac{2}{3}rs-\frac{1}{2}r^2-\frac{1}{2}r^3\geq 0 \qquad (9) \end{aligned}$$

We can see that the left side of (9) is decreasing by checking the first derivative, bearing in mind that $r \in [0, 0.5]$:

$$-1+\sqrt{2r}-r-\frac{3}{2}r^2\leq -r-\frac{3}{2}r^2\leq 0$$

Numerically, we get that the left side of (9) equals zero for $r \approx 0.2428$ – we call this boundary value v^*. Hence, drawing as well is a best response for all $r < v^* = 0.2428$. For larger r values, firm i does not want to draw in the first period if firm j takes a draw. □

Proof of Proposition 4

Both firms take a draw in the first period. At least one additional draw is taken in case no innovation has a value above x^*:

$$P(x_i^1 \leq x^*)P(x_j^1 \leq x^*) = (1-r)^2$$

A second additional draw is taken in case both values are below $\bar{x}$:

$$P(x_i^1 \leq \bar{x})P(x_j^1 \leq \bar{x}) = (1-s)^2$$

This gives us a total number of

$$d_R(r) = 2 + (1-r)^2 + (1-s)^2 = 4 + r^2 - 2s$$

concluding the proof. □

Proof of Theorem 7

We derive a condition on r making $\int_0^b \Phi^2(x) - \Psi(x)dx < 0$ in a rather coarse way by looking for a non-positive integrand on the whole interval $[0, b]$. We proceed in several steps, cutting the interval into different parts:

i) $[0, \bar{x}]$

In this case, it is easy to see that $\int_0^{\bar{x}} \Phi^2(x) - \Psi(x)dx = \int_0^{\bar{x}} 0dx = 0$ holds.

ii) $(\bar{x}, z]$

Here, we get

$$\int_{\bar{x}}^{z} \Phi^2(x) - \Psi(x)dx = \int_{\bar{x}}^{z} \underbrace{\left(F(x)^2 - F(x)\right)}_{<0} \underbrace{\left(F(x)^2 - F(\bar{x})^2\right)}_{>0} dx < 0.$$

iii) $(z, x^*]$

First, we rewrite

$$\int_{z}^{x^*} \Phi^2(x) - \Psi(x)dx = \int_{z}^{x^*} \underbrace{F(z)^2 + F(x)\left(F(\bar{x})^2 - 2F(z) - 2F(z)^2\right) + F(x)^2\left(1 + 2F(z) + F(z)^2 - F(\bar{x})^2\right) - F(x)^3}_{=:h(x)} dx$$

We now show that the integrand $h(x)$ is negative by analyzing its first derivative, which is given as follows:

$$h'(x) = F(\bar{x})^2 - 2F(z) - 2F(z)^2 + 2F(x)\left(1 + 2F(z) + F(z)^2 - F(\bar{x})^2\right) - 3F(x)^2$$

At z, h' is positive:

$$h'(z) = \underbrace{\left(F(z)^2 - F(\bar{x})^2\right)}_{>0}\underbrace{(2F(z)-1)}_{>0 \text{ for } F(z)>\frac{1}{2}}$$

As $F(z)$ is implicitly given by (2) we get

$$F(z) > \frac{1}{2} \iff r < \frac{1}{2}\cdot\frac{3}{2}\cdot\left(\frac{1}{2}\right)^2 = 0.1875$$

and consequently $h'(z)$ is positive in this case. Additionally, a numerical check shows that $h'(x^*)$ is positive as well (for all $z \in [0,b]$). Furthermore, h' is a quadratic function which has a maximum (this follows from $h'''(x) = -6$). Taking these facts together, we get that h' is positive on $[z, x^*]$ given $r < 0.1875$. Hence, h is increasing on $[z, x^*]$. A numerical check shows that $h(x^*) < 0$ for $r < 0.1647$ – thus, for these r-values h is negative on the whole interval (as it is largest at x^*).

iv) $(x^*, b]$

In this case, we get the following:

$$\begin{aligned}&\int_{x^*}^{b} \Phi^2(x) - \Psi(x)dx \\ &\quad= \int_{x^*}^{b} \underbrace{F(z)^2 + F(x^*)^2 + F(x)\left(F(\bar{x})^2 - 2F(z) - 2F(z)^2 - F(x^*)^2\right) + F(x)^2\left(2F(z) + F(z)^2 - F(\bar{x})^2\right)}_{=:l(x)} dx\end{aligned}$$

As $l(x^*) = h(x^*)$, we know that $l(x^*)$ is negative for $r < 0.1647$. Furthermore, l is a quadratic function having a minimum (as $l''(x) = 2\left(2F(z) + F(z)^2 - F(\bar{x})^2\right) > 0$). Hence, as $l(b) = 0$, l is negative on $(x^*, b]$.

Thus, we can conclude that the integrand (and thus the whole integral) is negative if $r < 0.1647 = v'$ holds. □

Proof of Proposition 8

First, we know from proposition 1 that no firm will draw again in case she knows that an innovation larger than x^* has been drawn. The conclusion of this proposition applies to asymmetric information release as well: In the situation of proposition 1 a firm does not want to draw again even if she knows that she is behind. If a firm with such a high draw does not know the opponent's draw, incentives for drawing again are even lower.

Additionally, proposition 1 implies that firm N will not draw again if $x_N^1 > \bar{x}$ and $x_N^1 > x_R^1$.

We first consider the following case: Both firms have taken a draw in the first round. Firm R has a draw $\bar{x} < x_R^1 < x^*$ and faces the decision whether to draw again or not. For the moment we assume that firm N behaves according to proposition 1 and thus draws again if she is behind (the case of equality of draws can be ignored from firm R's perspective as it is a zero probability event). It is beneficial for firm R to draw again if the following condition holds:

$$\left[P\left(x_N^1 < x_R^1\right)\left(P\left(x_N^2 < x_R^1\right) + \frac{1}{2}P\left(x_N^2 > x_R^1\right)P\left(x_R^2 > x_R^1\right)\right)\right.$$
$$\left. + \frac{1}{2}P\left(x_N^1 > x_R^1\right)P\left(x_R^2 > x_R^1\right)\right] p - c \geq P\left(x_N^1 < x_R^1\right)P\left(x_N^2 < x_R^1\right)p$$

This yields the following probabilities:

$$\left[F\left(x_R^1\right)\left(F\left(x_R^1\right) + \frac{1}{2}\left(1 - F\left(x_R^1\right)\right)^2\right) + \frac{1}{2}\left(1 - F\left(x_R^1\right)\right)^2\right] p - c \geq F\left(x_R^1\right)^2 p$$
$$\Longleftrightarrow \quad \left(1 + F\left(x_R^1\right)\right)\left(1 - F\left(x_R^1\right)\right)^2 - 2\frac{c}{p} \geq 0 \qquad (10)$$

Note that (10) has the same structure as (2). Hence, firm R will draw again exactly in case her first draw is smaller than z, which solves both (2) and (10). We denote $F(z) =: w$ in the following.

For the calculation above, we assumed that firm N follows the strategy described in proposition 1, but it is not clear that this strategy is a best reply. Obviously, it is a best reply in case firm N is leading. Not drawing is then profitable even against an opponent who draws. However, it could be profitable for firm N to stop drawing in case she is behind and firm R has a draw $\bar{x} < x_R^1 < z$ with $x_R^1 > x_N^1$. In this case, firm R will draw again as well – she would not do so if she knew that she is in front. We check whether it is anyway profitable to draw again for firm N:

$$\begin{aligned}
&P\left(x_N^2 > x_R^1\right)\left(P\left(x_R^2 < x_R^1\right) + \frac{1}{2}P\left(x_R^2 > x_R^1\right)\right)p - c \\
=&\left(1 - F\left(x_R^1\right)\right)\left(F\left(x_R^1\right) + \frac{1}{2}\left(1 - F\left(x_R^1\right)\right)\right)p - c \\
>&\frac{1}{2}\left(1 + F\left(x_R^1\right)\right)\left(1 - F\left(x_R^1\right)\right)^2 p - c \\
\geq&0
\end{aligned}$$

The strict inequality holds by direct comparison (and $0 < F\left(x_R^1\right) < 1$). The last inequality holds as $x_R^1 < z$ in this case and (10) applies. Hence, it is in fact a best reply for firm N to follow the strategy derived in proposition 1.

If the draw of firm R fulfills $x_R^1 < \bar{x}$, the incentives to draw again are the same for firm N as in proposition 1. Hence, firm N behaves similarly here. For firm R, we consider an estimate of their profit from drawing again, looking only at the largest terms:

$$
\begin{aligned}
&\left[P\left(x_N^1 < x_R^1\right)\left(P\left(x_N^2 < x_R^1\right) + \frac{1}{2}P\left(x_N^2 > x_R^1\right)P\left(x_R^2 > x_R^1\right)\right)\right. \\
&\qquad\qquad\qquad\qquad \left.+\frac{1}{2}P\left(x_N^1 > \bar{x} > x_R^1\right)P\left(x_R^2 > \bar{x} > x_R^1\right)\right]p - c \\
&= \left[F\left(x_R^1\right)\left(F\left(x_R^1\right) + \frac{1}{2}\left(1 - F\left(x_R^1\right)\right)^2\right) + \frac{1}{2}\left(1 - F\left(\bar{x}\right)\right)^2\right]p - c \\
&> \left[F\left(x_R^1\right)F\left(x_R^1\right) + \frac{1}{2}\left(1 - \left(1 - \sqrt{2\frac{c}{p}}\right)\right)^2\right]p - c \\
&= F\left(x_R^1\right)F\left(x_R^1\right)p
\end{aligned}
$$

Hence, drawing again is beneficial for firm R.

What happens if one of the firms plays a strategy where she does not take a draw in the first period? If firm N faces a firm R taking no draw, the second period behavior is similar to playing against a firm with a draw of zero. For firm R, things change: If she faces a firm not drawing in the first period, her best reply is similar as in the situation of full information release. Thus, if she believes with probability one that she faces a not-drawing firm, she plays the same strategy as firm N in that case: She will only draw again if $x_R^1 < \bar{x}$. □

Proof of Proposition 9

In the first period, both firms have to compare the expected profits of taking a draw with the expected profits of waiting one period. Consider first the case of firm R not drawing in the first round. What is the best reply of firm N? This is basically the same exercise as deriving inequality (7), with one slight difference: Firm N is not able to discourage firm R from taking a draw in case $x_N^1 > x^*$. This slightly reduces the probability of winning the price for firm N compared to the setting of full revelation: It is now possible that firm R beats firm N with a draw $x_R^2 > x_N^1 > x^*$. This is the case with probability $\frac{1}{2}\left(1 - F\left(x^*\right)\right)^2 = \frac{1}{2}r^2$. We can include this probability change into (7) by subtracting $\frac{1}{2}r^2$, which gives us the following condition for a profitable draw in the first round:

$$
\frac{1}{6} - r + \frac{2}{3}\sqrt{2}r^{\frac{3}{2}} \geq 0 \qquad (11)
$$

The analysis of the first order condition shows that the left side of (11) has a minimum at $r = \frac{1}{2}$. For $r = \frac{1}{2}$, equality holds in (11). Hence, taking a draw is profitable for firm N in the first period in this case.

What is the best reply for firm R against this strategy of firm N? We first calculate the probability for firm R to win the prize if she is taking a draw in the first period (and following the equilibrium strategy of the second period afterwards).

$$
\begin{aligned}
& P\left(x_R^1 > x^*\right)\left[P\left(x_N^1 < x^*\right) + \frac{1}{2}P\left(x_N^1 > x^*\right)\right] \\
+ & P\left(z < x_R^1 < x^*\right)\left[\frac{1}{2}P\left(z < x_N^1 < x^*\right)\left(\frac{2}{3}P\left(z < x_N^2 < x^*\right) + P\left(x_N^2 < z\right)\right)\right. \\
& \left. + P\left(x_N^1 < z\right)\left(P\left(x_N^2 < z\right) + \frac{1}{2}P\left(z < x_N^2 < x^*\right)\right)\right] \\
+ & P\left(\bar{x} < x_R^1 < z\right)\left[\frac{1}{2}P\left(x_N^1 > x^*\right)P\left(x_R^2 > x^*\right)\right. \\
& + P\left(z < x_N^1 < x^*\right)\left(P\left(x_R^2 > x^*\right) + \frac{1}{2}P\left(z < x_R^2 < x^*\right)\right) \\
& + P\left(\bar{x} < x_N^1 < z\right)\left(\frac{1}{2}\left(P\left(x_R^2 > z\right) + \frac{1}{3}P\left(\bar{x} < x_R^2 < z\right)\right)\right. \\
& + \frac{1}{2}\left[P\left(x_R^2 > z\right)\left(\frac{1}{2}P\left(x_N^2 > z\right) + P\left(x_N^2 < z\right)\right)\right. \\
& + P\left(\bar{x} < x_R^2 < z\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{3}{4}P\left(\bar{x} < x_N^2 < z\right)\right) \\
& \left.\left. + P\left(x_R^2 < \bar{x}\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{2}{3}P\left(\bar{x} < x_N^2 < z\right)\right)\right]\right) \\
& + P\left(x_N^1 < \bar{x}\right)\left(P\left(x_R^2 > z\right)\left(P\left(x_N^2 < z\right) + \frac{1}{2}P\left(x_N^2 > z\right)\right)\right. \\
& + P\left(\bar{x} < x_R^2 < z\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{2}{3}P\left(\bar{x} < x_N^2 < z\right)\right) \\
& \left.\left. + P\left(x_R^2 < \bar{x}\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{1}{2}P\left(\bar{x} < x_N^2 < z\right)\right)\right)\right] \\
+ & P\left(x_R^1 < \bar{x}\right)\left[\frac{1}{2}P\left(x_N^1 > \bar{x}\right)P\left(x_R^2 > \bar{x}\right)\right. \\
& + P\left(x_N^1 < \bar{x}\right)\left(P\left(x_R^2 > \bar{x}\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{1}{2}P\left(x_N^2 > \bar{x}\right)\right)\right. \\
& \left.\left. + \frac{1}{2}P\left(x_R^2 < \bar{x}\right)P\left(x_N^2 < \bar{x}\right)\right)\right]
\end{aligned}
$$

Using the short notations we introduced ($P(x < \bar{x}) = s$, $P(x < z) = w$ and $P(x < x^* = r)$, simplifying and subtracting the costs, this reduces to the following expected profit:

$$\left(\frac{7}{24} - \frac{1}{3}r^3 + \frac{1}{6}s^3 + \frac{1}{2}w - \frac{1}{4}w^2 - \frac{1}{6}w^3 + \frac{1}{8}w^4\right)p - (1+w)\,c \tag{12}$$

Furthermore, we have to calculate the expected profit of firm R when she is waiting for the second period without taking a draw (and faces a drawing firm N):

$$
\begin{aligned}
& \left[\frac{1}{2}P\left(x_N^1 > \bar{x}\right)P\left(x_R^2 > \bar{x}\right) + P\left(x_N^1 < \bar{x}\right)\left(\frac{1}{3}P\left(x_R^2 < \bar{x}\right)P\left(x_N^2 < \bar{x}\right)\right.\right. \\
& \left.\left. + P\left(x_R^2 > \bar{x}\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{1}{2}P\left(x_N^2 > \bar{x}\right)\right)\right)\right] p - c
\end{aligned}
$$

$$= \left[\frac{1}{3} + \frac{1}{6}s^3\right] p - c \tag{13}$$

Drawing in the first period is thus profitable if the value of (12) is larger than the value of (13). Comparing these two terms, we get

$$\begin{aligned} & \left(\frac{7}{24} - \frac{1}{3}r^3 + \frac{1}{6}s^3 + \frac{1}{2}w - \frac{1}{4}w^2 - \frac{1}{6}w^3 + \frac{1}{8}w^4\right) p - (1+w)\, c \geq \left[\frac{1}{3} + \frac{1}{6}s^3\right] p - c \\ \Longleftrightarrow \quad & \left(-\frac{1}{24} - \frac{1}{3}r^3 + \frac{1}{2}w - \frac{1}{4}w^2 - \frac{1}{6}w^3 + \frac{1}{8}w^4\right) p - wc \geq 0 \\ \Longleftrightarrow \quad & -\frac{1}{24} - \frac{1}{3}r^3 + \frac{1}{2}w - \frac{1}{4}w^2 - \frac{1}{6}w^3 + \frac{1}{8}w^4 - wr \geq 0 \end{aligned}$$

A numerical analysis shows that the left hand side equals zero for $r \approx 0.2623$ – we call this critical value $\hat{v}$. For larger r values firm R prefers to wait for the second period to take her draw. In this case, we showed that there is an asymmetric equilibrium with firm N drawing in the first and firm R drawing in the second period. Firm N then follows her second-period equilibrium strategy.

For smaller r values, firm R takes a draw in the first period as well. To confirm that this constellation is consistent with an equilibrium behavior, we have to check the incentives of firm N to take a draw in this case. If she does not take a draw, her expected profit is

$$\begin{aligned} & \Bigg[P\left(z < x_R^1 < x^*\right)\left(P\left(x_N^2 > x^*\right) + \frac{1}{2}P\left(z < x_N^2 < x^*\right)\right) \\ & + P\left(x_R^1 < z\right)\left(\frac{1}{3}P\left(x_N^2 < z\right)P\left(x_R^2 < z\right)\right. \\ & \left.+\ P\left(x_N^2 > z\right)\left(P\left(x_R^2 < z\right) + \frac{1}{2}P\left(x_R^2 > z\right)\right)\right)\Bigg] p - P\left(x_R^1 < x^*\right) c \\ = & \left[\frac{1}{2} - \frac{1}{2}w + \frac{1}{2}w^2 - \frac{1}{6}w^3 - \frac{1}{2}r^2\right] p - (1-r)c \end{aligned} \tag{14}$$

We compare this with the expected profit of taking a draw. As part of (12), we already calculated the probability that firm R wins the contest in case both firms take a draw in the first round. Consequently, this number and the probability that firm N wins this contest add up to one. Hence, firm N makes an expected profit according to the following expression:

$$\left[\frac{17}{24} + \frac{1}{3}r^3 - \frac{1}{6}s^3 - \frac{1}{2}w + \frac{1}{4}w^2 + \frac{1}{6}w^3 - \frac{1}{8}w^4\right] p - \left(2 - s + \frac{1}{2}r^2\right) c \tag{15}$$

Comparing (15) with (14), we get the following condition for a profitable draw in the first period:

$$\begin{aligned} & \left[\frac{17}{24} + \frac{1}{3}r^3 - \frac{1}{6}s^3 - \frac{1}{2}w + \frac{1}{4}w^2 + \frac{1}{6}w^3 - \frac{1}{8}w^4\right] p - (2 - s + \frac{1}{2}r^2)c \\ & \geq \left[\frac{1}{2} - \frac{1}{2}w + \frac{1}{2}w^2 - \frac{1}{6}w^3 - \frac{1}{2}r^2\right] p - (1-r)c \\ \Longleftrightarrow\ & \frac{5}{24} - \frac{1}{6}r^3 + \frac{1}{3}w^3 - \frac{1}{4}w^2 - \frac{1}{8}w^4 - \frac{1}{6}s^3 - \frac{1}{2}r^2 - r + rs \geq 0 \end{aligned}$$

Again, a numerical analysis shows that the left hand side equals zero for $r \approx 0.2939$. For smaller r values, the inequality is fulfilled and drawing in the first period is profitable for firm N – we found an equilibrium in that case. For larger r values, firm N's best reply is not to draw in the first round. We thus have to check how firm R's best reply against a waiting firm N looks like (with respect to correct believes). Note that firm R will only draw again in the second-period equilibrium if $x_R^1 < \bar{x}$. Hence, incentives to draw are similar to the case of full information release and result in condition (7). The analysis of that condition showed that it is thus profitable for firm R to draw against a waiting firm N.

Finally, we have to analyze the incentives of the waiting firm N – is it profitable to draw against a drawing firm R who believes to face a firm N that does not draw? The expected profit of drawing can be calculated as follows:

$$
\begin{aligned}
&\Bigg[\frac{1}{2}P\left(x_R^1 > x^*\right)P\left(x_N^1 > x^*\right) + P\left(\bar{x} < x_R^1 < x^*\right)\Bigg(P\left(x_N^1 > x^*\right) \\
&+P\left(\bar{x} < x_N^1 < x^*\right)\left(\frac{1}{2} + \frac{1}{2}\left(P\left(x_N^2 > x^*\right) + \frac{1}{3}P\left(\bar{x} < x_N^2 < x^*\right)\right)\right)\Bigg) \\
&P\left(x_R^1 < \bar{x}\right)\left(\frac{1}{2}P\left(x_N^1 < \bar{x}\right) + P\left(x_N^1 > \bar{x}\right)\left(\frac{1}{2}P\left(x_R^2 > \bar{x}\right) + P\left(x_R^2 < \bar{x}\right)\right)\right)\Bigg] p \\
&-\left(2 - s + \frac{1}{2}r^2\right)c \\
=&\left[\frac{1}{2} - \frac{1}{2}s^2 + \frac{2}{3}s^3 + \frac{1}{3}r^3 - \frac{1}{2}r^2 s\right]p - \left(2 - s + \frac{1}{2}r^2\right)c
\end{aligned}
$$

If firm N does not draw in the first period, she is in the same situation as in the right hand side of (8). We compare the expected profits of drawing and not drawing:

$$
\left[\frac{1}{2} - \frac{1}{2}s^2 + \frac{2}{3}s^3 + \frac{1}{3}r^3 - \frac{1}{2}r^2 s\right]p - \left(2 - s + \frac{1}{2}r^2\right)c \geq \left[\frac{1}{3} + \frac{1}{3}rs - \frac{1}{2}r^2\right]p - (1 - r)\,c
$$

Simplifying and using $s = \sqrt{2r}$, we get that drawing is profitable in case

$$
\frac{1}{6} - 2r - \frac{1}{2}r^2 - \frac{1}{6}r^3 + 2sr - \frac{1}{2}sr^2 \geq 0
$$

This condition holds for $r < 0.1722$, as a numerical analysis shows. We call this critical value $\tilde{v}$. Given this condition, we are back in the situation where both want to draw (and our previous analysis showed that this is an equilibrium for this range of r-values). For $r > 0.1722$, firm N does not want to draw and we are hence in an equilibrium as well – the best reply for firm R against a firm N that does not draw is to draw. □

Proof of Theorem 11 We first show that it is in fact an equilibrium. Note that the point of revealing (or not revealing) is to make the other firm stop researching in as many cases as possible. Suppose firm i deviates and does not reveal her value. This deviation cannot be beneficial: If firm j has a value $x_j > x^*$, the reaction of this firm does not change – she always stops researching in this case. Additionally, if $x_j < x^*$, firm j will continue to do research, and thus goes on in the maximum number of cases. Revealing a value $x_i < x^*$ would have made a firm with value $x_j \in (x_i, x^*)$ stop researching, increasing the chances of firm i to win.

To show the uniqueness, suppose there is another equilibrium. Consider the strategy of firm i, and first assume that this firm always keeps the information secret in case $x_i \in X_1 \subset (x^*, b]$ (and reveals her value for $x_i \notin X_1$). Thus, in equilibrium, if firm j observes that firm i does not reveal any information, she correctly believes that $x_i > x^*$. Consequently, firm j stops innovating, no matter what value her first-period innovation has. This provides firm i with an incentive to always keep her information secret, as this will make firm j stop. Hence, in any equilibrium where information is kept secret for values in X_1, this has to be done also for some values $x_i \in X_2 \subset [0, x^*]$. Furthermore, X_2 has to be large enough such that firm j continues to innovate for some values x_j when receiving no information by firm i (and believing correctly that $x_i \in X_1 \cup X_2$). However, if $x_i \in X_1$, firm i has a profitable deviation by simply revealing her value and making firm j stop innovating in any case. Thus, there cannot be an equilibrium in which firm i with value $x_i \in (x^*, b]$ keeps this value secret.

A similar reasoning applies in case we assume that information is kept secret only for values $x_i \in X_3 \subset [0, \bar{x}]$ – firm j with any $x_j \in (\bar{x}, x^*)$ would stop innovating, and firm i with $x_i \in (\bar{x}, x^*)$ had an incentive to keep her information secret and make firm j stop for these x_j. Additionally, consider the case of a set $X_4 \subset (\bar{x}, x^*)$ for which values are kept secret on top of X_3 (making some $x_j \in (\bar{x}, x^*)$ continue to innovate): Then, it is profitable for $x_i \in X_3$ to reveal and make firm j stop innovating for all $x_j \in (\bar{x}, x^*)$. Thus, there cannot be an equilibrium in which firm i with value $x_i \in [0, \bar{x}]$ keeps this value secret.

Finally, consider the case where information is kept secret by firm i for values $x_i \in X_5 \subset (\bar{x}, x^*)$. Then, firm j will continue to innovate for all $x_j < \inf X_5$, if she does not observe any information by firm i. As a firm with any value in $x_i \in X_5$ decides to keep her information secret, firm i cannot be better off by revealing (and making firm j for all $x_j < x_i$ continuing). Thus, there can be no set X_6 with $x_j \in X_6$ continuing to innovate in equilibrium and $X_6 \cap (\inf X_5, x^*)$ having a positive mass. Otherwise, there would be some $x'_j \in X_6 \cap (\inf X_5, x^*)$ dividing this set in two parts with a positive mass. Consequently, some $x_i \in X_5 \cap (\inf X_5, x'_j)$ would exist for which firm i had a profitable deviation by revealing her type (and making firm j stop innovating in the part above x'_j). This shows that in equilibrium firm j does not continue to innovate for all $x_j \in (\inf X_5, x^*)$, if she does not observe information by firm i. Keeping this in mind, we can conclude that X_5 is in fact an interval of the form $(\inf X_5, x^*)$ (possibly including the end points, which we ignore for notational purpose). Suppose this were not the case. Then, there is some $x_i \in (\inf X_5, x^*)$ for which firm i would reveal her value. However, she could do strictly better for that value by keeping the information secret and making firm j stop innovating for all $x_j \in (\inf X_5, x_i)$.

To go on, let us assume first that both firms keep the information secret for such an interval, and this interval is the same for both, having the form (x', x^*). Then, both firms do not continue to innovate if they do not observe any information. Consider some $x_i'' \in (x', x^*)$. From the equilibrium derivation in case of full information revelation we know that any $x_j \in (x', x_i'')$ makes a positive profit against x_i'' by continuing to innovate. We denote the expected profit for firm j against values in (x_i'', x^*) by δ (it is independent of the size of x_j, as long as $x_j < x_i''$). Against all values in (x_j, x_i''), the expected profit is even larger than δ. Now consider some fixed $x_j < x_i''$ for which the probability that firm j is in the lead if she does not receive any information is less or equal to ε. If firm j would deviate for x_j and continue to innovate, this would have two effects: On the one hand, she would make an expected profit of at least δ against firm i having a higher valuation (up to x^*). On the other hand, firm i would continue to innovate for values in (x', x_j), which she would not have done otherwise – the relative loss in such a case is bounded by $p + c$. Thus, innovating is in fact profitable, if the following condition holds:

$$(1 - \varepsilon)\delta - \varepsilon(c + p) > 0$$

As this condition is fulfilled for ε small enough, firm j in fact has the profitable deviation to continue innovating. Thus, we cannot have a symmetric equilibrium with both firms keeping the information secret for values other than x^*.

Let us now consider the case of an asymmetric equilibrium, where (w.l.o.g.) firm i has a lower value for which her interval of keeping the information secret starts (this includes the case where firm j reveals for all values). This case differs from the above analysis in so far that firm j reveals her value for some values above x', such that firm i continues to innovate against firm j for some more values. However, this does not distort the profit for firm j against values above x_j if she continues to innovate, and the bound for the possible loss against lower values still holds. Hence, there is an interval of x_j-values for which it is a profitable deviation for firm j to continue innovating in any case. All in all, there cannot be an equilibrium in which a firm does not reveal her information after the first stage for values other than x^*.

Nevertheless, both firms are indifferent whether they reveal their information or not if their first-period innovation has the value x^* – but this event has zero probability. □