Pricing in a Market with Imitative and Habit Forming Consumers*

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We present results about the strategic behavior of firms in markets where consumers follow simple behavioral decision rules based on imitation and habit as suggested in consumer research, social learning, and related fields. On this basis, we investigate monopoly and competition between firms, described via an open-loop differential game. We explain for the monopoly case that a reduction of the space of all price paths in time to the space of time-constant prices is sensible since the latter in general contains Nash equilibria. We also show that the equilibrium price of the weakest active firm tends to marginal cost as the number of (non-identical) firms grows. Our model is consistent with observed market behavior such as product life cycles.

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1 Introduction

In many situations, strategic pricing represents a difficult task for firms, especially if the consumers are not known to strictly follow a given demand function. In reality, consumers behave boundedly rationally as observed in numerous psychological and experimental investigations (cf. Conlisk 1996) and appreciated in some areas of industrial organization (cf. Ellison 2006). In the present work, we examine a model which describes how firms shall optimally, i. e. strategically, set their prices or advertising levels when confronted with habitual imitative consumers. Such consumers imitate popular product choices and form a habit to repeatedly purchase the same product. This rule of thumb behavior is psychologically supported (Assael 1984) and acknowledged in the economic literature (e. g. Stigler and Becker 1977, Schlag 1998).

The demand side of a market with habitual imitative consumers has been examined in Matzke and Wirth (2008a). The corresponding model has been stated in form

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of a population game as defined in Sandholm (2005), using the fact that for a large population size the stochastic process generated by the evolutionary process can be approximated by solutions to ordinary differential equations (Benaïm and Weibull 2003). This demand side is complemented with a supply side model in Matzke and Wirth (2008b). The purpose of this paper is to give a brief overview over the results.

The demand side consists of the continuous-time consumer population game from Matzke and Wirth (2008a). The game of the supply side describes the strategic behavior of firms, which anticipate the consumers' behavior and thus the demand dynamics. With such a description at hand, we examine how advertising might influence the results, and show that the model is consistent with observed market patterns such as product life cycles.

The consumer model is in the spirit of Smallwood and Conlisk (1979) as well as von Thadden (1992) in that the consumers are unable or not willing to act strategically and thus act adaptively. Our rational supply side approach differs in methodology. We employ a normal form game in which firms choose a price path at the beginning of the game and gain an according profit. In fact, we use a differential game, i. e. a time-continuous game where the state variables (here the consumer subpopulation sizes) follow first-order ordinary differential equations. More specifically, the chosen product prices (control variables) determine the rate of change in consumer subpopulation sizes (state variables), where consumers are grouped into subpopulations according to the product they own. Technically, there is no effect in the opposite direction, which renders the dynamics an open-loop system. However, in a deterministic setting we have equivalence of open and closed loops. This is convenient since closed loops are considered—in contrast to static open loops—as being genuinely strategic since they comprise a feedback in which the control variables are affected by the state variables.

In this context, the following result will turn out to be very interesting: The firms' action space generally contains all possible price paths and thus is very complex. However, for a monopoly, a Nash equilibrium often lies in the reduced space of time-constant prices. This justifies that most of the time we return to time-constant price paths and steady state analysis. Moreover, we will see that markets with imitative and habitual consumers behave naturally in that e. g. an increasing number of firms enhances competition and reduces prices. However, perfect competition is generally only achieved in symmetric markets. Finally, advertising is shown to be an effective method to sustain demand.

1.1 Further motivation and related literature

Boundedly rational consumer behavior—as advocated by Ellison (2006), Conlisk (1996), and many others—is often observed in consumer research, in particular imitation of group behavior or habitual purchase (cf. Assael 1984, p. 371ff, 53). Just to mention some exemplary laboratory experiments, Venkatesan (1966) shows that consumers generally conform to group norms and Corriveau, Fusaro, and Harris (2009) find a sensibility to group consensus for young children. Pingle and Day (1996) summarize experiments which show that boundedly rational behavior such as imitation and habit (which they call "economic choices in reality") represents means in order to get well-performing economic choices in presence of decision costs. Our focus here lies on markets with boundedly rational consumers that follow habitual imitative decision rules as introduced in Matzke and Wirth (2008a).

Closely related to the demand dynamic employed here is the model by Smallwood and Conlisk (1979). They consider consumers who buy the same product each period until a breakdown occurs. Then, they choose another product depending on its market share. It is examined how strongly the consumers should rely on product popularities. Despite having been published in 1979 already, there is still social learning literature building on this model, for example Ellison and Fudenberg (1995).

Imitative behavior in general constitutes a well-known and frequently used concept in evolutionary game theory and social learning, compare for instance Schlag (1998), Ellison and Fudenberg (1993), and Banerjee (1992), just to name a few notable papers. Habit, on the other hand, occurs in the habit formation literature (Heaton 1993) as well as implicitly in some industrial organization models (for instance in Smallwood and Conlisk 1979, where habit is implicitly formed as long as no breakdown occurs). Habit may also be interpreted as a special case of learning, since agents learn from past experience (Sobel 2000, p. 257) and positive experience with a good may cause habitual purchase behavior.

Firms are usually more rational than consumers can or aim to be. The reason lies in the large number of agents and equipment that are employed in order to avoid costly wrong decisions. The approximation of rational firms seems reasonable, even though some early work in the field of bounded rationality assumes the opposite, i. e. boundedly rational firms (e.g. Rothschild 1947, Cyert and March 1956). However, in line with most of the recent literature, we restrict bounded rationality to the consumers and to assume fully rational firms (Ellison 2006, p. 4). We will stick to this convention, i. e. our firms aim at maximizing their profits given the demand side and the pricing strategies of the competitors which is modeled via a differential game as introduced by Isaacs (1954). In combination with the two previously mentioned simple rule of thumb ingredients, imitation and habit by consumers, the model will be able to generate typical patterns observed in consumer markets such as product life cycles (de Kluyver 1977, Brockhoff 1967, Polli and Cook 1969).

The outline of this paper is as follows. Section 2 recapitulates the consumer model, defining the consumers' behavioral rules and deducing the resulting demand dynamics. Subsequently, section 3 introduces and analyzes the competition game played by the supply side. In section 4, a possible generation of product life cycles is described. Additionally, a model extension by advertising is suggested. Finally, we conclude in section 5.

2 Model for boundedly rational demand side

In this section we shall present the model for boundedly rational consumers and the resulting demand dynamics from Matzke and Wirth (2008a), which presents the demand side of the market.

2.1 Methodology

The methodology applied builds upon the work of Sandholm (2006) and consists mainly of a population game with a particular choice of a conditional switch rate.

Each consumer owns at most one unit of n possible products. A consumer owning good i is equipped with an independent Poisson alarm clock of rate R_i, i. e. an alarm clock which rings after an exponentially distributed time with expected value R_i^{-1}. Each time the alarm rings (which is associated with a broken product), the consumer switches to product j with switching probability $p_{ij} = \frac{\rho_{ij}}{R_i}$. If the consumer does not own a product yet, the alarm clock signalizes an arising interest in buying a good. Typically, the frequency R_0 of revisions without any good is larger than the rate R_i of possible replacements of good i, since the goods usually survive longer than the consumers without any good are satisfied.

Let us denote $\rho_{ij}(x,t)$ the conditional switch rate from product i to product j at time t and state $x = (x_1, \dots, x_n)$, where x_i denotes the market share of consumers owning product i. Obviously, $R_i = \sum_{j=0}^{n} \rho_{ij}(x,t)$ (where subscript 0 stands for consumers without any good).

The switching rates and probabilities of course depend on the product prices, and exactly this dependence will later form the instrument via which firms exert an influence on consumer behavior during their competition. However, the price dependence is not necessary to understand the dynamics of the demand side. Hence, the reader may implicitly understand all parameters to depend on the product prices, but this dependence will not be introduced explicitly until the treatment of the supply side.

The previous definitions characterize a population game with all potential consumers as players. This game uniquely determines a mean dynamic which describes the temporal change of market shares,

$$\dot{x}_i = \sum_{j=0}^{n} x_j \rho_{ji}(x,t) - x_i \sum_{j=0}^{n} \rho_{ij}(x,t), \quad x_i(0) = x_i^0, \quad i = 0, \dots, n. \tag{1}$$

We define the sales of product i, $S_i(t)$, as the number of units of product $i \in \{1, \dots, n\}$ sold at time t,

$$S_i(t) \quad = \quad N \sum_{j=0}^{n} x_j(t) \rho_{ji}(x(t),t), \quad i = 0, \dots, n, \tag{2}$$

where N denotes the number of possible consumers. Using mean dynamic (1), we obtain a relation between sales and consumer subpopulations,

$$\dot{x}_i + x_i R_i = \sum_{j=0}^{n} x_j \rho_{ji} \quad \Leftrightarrow \quad \frac{S_i}{N} = \dot{x}_i + x_i R_i, \quad i = 0, \ldots, n. \tag{3}$$

2.2 Consumer dynamics

Now only the switching probabilities $p_{ij}(x,t) = \frac{\rho_{ij}(x,t)}{R_i}$ remain to be specified. Of those people who do not own any product, the fraction of consumers deciding to buy product i is described by p_{0i}. Consumers' choices are sensitive to market shares or popularities of the products (Smallwood and Conlisk 1979): When consumers passively encounter a product, its level of familiarity rises, thus increasing the possibility for this product to be bought. Consumers may also actively imitate others in buying the same good since the popularity of a product might give information about the product's past performance (Ellison and Fudenberg 1993). As discussed in Matzke and Wirth (2008a) a linear relation

$$p_{0i} = \varphi_i x_i, \quad i \neq 0, \tag{4}$$

seems to be a good modeling approach. $\varphi_i \in [0,1]$ generally differs from product to product (Assael 1984, p. 432, 414) and can even be time dependent. It constitutes the accumulated influence of product frequency on the consumers' purchase decision via different mechanisms and can be interpreted as an anticipated product quality. Of course, φ_i depends on the good's properties as there are the price, the (expected) quality, the strength of networking and fashion effects for that product etc.

Let us now turn to those people owning product i. Someone who is content with that good tends to buy a new unit of the same good, even though a better product might exist. Assael (1984, p. 53) summarizes several studies on the topic and comes to the conclusion that a form of habit evolves, leading to repeat purchases of a product without further information search or evaluating brand alternatives. Hence we assume a fixed, product-specific percentage of consumers to develop a buying habit so that

$$p_{ii} = s_i \in [0,1], \quad i \neq 0. \tag{5}$$

The fraction of switching consumers $(1 - p_{ii})$ divides up into the fractions p_{ij} of people switching to product $j \neq i$. They behave just like those consumers not yet owning any good, i. e.

$$p_{ij} \quad = \quad (1 - p_{ii}) p_{0j} = (1 - s_i) \varphi_j x_j, \quad i \neq 0 \wedge j \neq 0, i. \tag{6}$$

The switching probabilities p_{i0} and p_{00} are now uniquely determined by the constraints $\sum_{j=0}^{n} p_{ij} = 1$ and $\sum_{j=0}^{n} p_{0j} = 1$.

For $i = 1, \ldots, n$, the mean dynamic (1) eventually takes the form

$$\begin{aligned} \dot{x}_i &= x_i \Bigg(\varphi_i R_0 - (1 - s_i) R_i - \varphi_i \sum_{\substack{j=1 \\ j \neq i}}^{n} [R_0 - (1 - s_j) R_j] \, x_j - \varphi_i R_0 x_i \Bigg) \\ &= \varphi_i R_0 x_i \Bigg(\Psi_i - x_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \Phi_j x_j \Bigg), \end{aligned} \tag{7}$$

where $\Psi_i := 1 - \frac{R_i}{R_0} \frac{1 - s_i}{\varphi_i}$ and $\Phi_i := 1 - \frac{R_i}{R_0}(1 - s_i)$ stand for "quality" and "habit induction" of product i.

All constants R_i, φ_i and s_i may in principle (and will later) be time-dependent so that product modifications or fashion trends can be modeled.

Population games with the switching probabilities as defined above will represent the demand side of our market model. Let us hence define:

Definition 2.1 (Habitual imitative consumers)**.** *Agents who behave according to the above model with switching probabilities* (4) *to* (6) *are called* habitual imitative consumers*. A population game with such agents is called the demand side of a* market with habitual imitative consumers*.*

3 Strategic pricing in a monopoly & oligopoly

After having repeated the framework of the demand side model, let us now examine markets in which firms anticipate the consumers' actions (indeed, companies do try to predict consumer behavior) and set their prices accordingly. For details and proofs we refer to Matzke and Wirth (2008b).

Naturally, the imitation factor φ_i and habit coefficient s_i depend on the good prices, i. e. $\varphi_i = \varphi_i(\xi_1(t), \ldots, \xi_n(t))$, $s_i = s_i(\xi_1(t), \ldots, \xi_n(t))$, $i = 1, \ldots, n$, where the price of good j at time t is denoted $\xi_j(t)$. To keep things simple while staying sufficiently realistic, we shall assume φ_i and s_i to depend on ξ_i only. The consumers see the prices of all goods, and the probability to buy product i (encoded by φ_i and s_i) rises with falling price ξ_i. They behave like many small iron particles which are attracted by different magnets, representing the products. The strength of a magnet relative to its competitors determines the eventual amount of trapped particles, which illustrates the mechanism of competition. Competing firms will seek a compromise between large margins and sufficiently low prices to attract consumers more strongly than their competitors (via high φ_i and s_i).

Recall that the imitation function has the following interpretation: A consumer owning no good or switching product imitates the population of consumers owning good i with probability $\varphi_i(\xi_i)$. Equivalently, the fraction $\varphi_i(\xi_i)$ of the whole population would purchase good i at a price of ξ_i. Obviously, $\varphi_i(\xi_i)$ represents the normalized demand function of product i, or in probabilistic terms, $\varphi_i(\xi_i)$ is the demand distribution of product i. Hence, let us agree upon the following

Condition 3.1. *In a market with habitual imitative consumers, let $\varphi_i(\vec{\xi})$ and $s_i(\vec{\xi})$ denote the imitation and habit coefficient for product i, depending on the vector $\vec{\xi}=(\xi_1,\ldots,\xi_n)$ of product prices. Then φ_i and s_i are monotonously decreasing in ξ_i.*

In the following we will abbreviate vectors of scalars according to $(\sigma_i)_{i=1,\ldots,m} = \vec{\sigma}$. We are now able to describe a normal form competition game of the firms.

Definition 3.2 (Normal form competition game)**.** *The* normal form competition game *in a market with habitual imitative consumers is a normal form game $G = (n, \mathfrak{S}, \Pi)$ with*

- *the number of agents n being the number of firms, where each firm produces one product and the products are understood to be characterized by the functions $s_i(\xi_1,\ldots,\xi_n)$ and $\varphi_i(\xi_1,\ldots,\xi_n)$,*
- *the set $\mathfrak{S}$ of all possible strategy combinations with $\mathfrak{S} \subseteq [\mathbb{R}_+^{\mathbb{R}_+}]^n = \mathbb{R}_+^{\mathbb{R}_+}\times\cdots\times\mathbb{R}_+^{\mathbb{R}_+}$ (a subset of the space of n-tuples over maps $\xi_i : \mathbb{R}_+ \to \mathbb{R}_+$, $t \mapsto \xi_i(t)$, where $\xi_i(t)$ denotes the price of good i at time t),*
- *the utility function $\Pi : \mathfrak{S} \to \mathbb{R}_+^n$, $\Pi_i(\xi_1,\ldots,\xi_n) = F\left[(\xi_i(t) - c_i)S_i\left(\vec{s}(\vec{\xi}(t)), \vec{\varphi}(\vec{\xi}(t)), \vec{x}(t)\right)\right]$, being the firms' profit, where there are no fixed costs, c_i denotes the (time-independent) marginal cost of production for good i, and $\vec{S}(\vec{s}(\vec{\xi}(t)), \vec{\varphi}(\vec{\xi}(t)), \vec{x}(t)) \equiv \vec{S}(t, \vec{x}(0))$ is the sales vector (cf. (2)) belonging to the population game (7) with ρ defined by (4) to (6). The operator $F : \mathbb{R}^{\mathbb{R}_+} \to \mathbb{R}_+$, which assigns a non-negative real number to each map $f : \mathbb{R}_+ \to \mathbb{R}$, may assume different forms.*

The imitation and habit function $\vec{\varphi}(\vec{\xi})$ and $\vec{s}(\vec{\xi})$ may in general be time dependent. The operator F can e. g. constitute the *cumulated discounted profit* over a certain time period $[0, T]$,

$$F_T\left[\pi(t)\right] = \int_0^T \exp[-rt]\pi(t)\mathrm{d}t,$$

where $\pi(t)$ represents the firm's profit at time t. For an infinite time horizon this is extended to

$$F_\infty\left[\pi(t)\right] = \int_0^\infty \exp[-rt]\pi(t)\mathrm{d}t.$$

For a zero discount rate r, the latter definition is not well-defined. In this case we resort to the *long-term profit rate*,

$$F_\partial\left[\pi(t)\right] = \lim_{T\to\infty}\frac{1}{T}\int_0^T \pi(t)\mathrm{d}t = \lim_{t\to\infty}\pi(t),$$

where the last expression only holds for time-invariant prices in the steady state. In this case (which is for example of interest when the firms would like to validate their prices in an equilibrated market), we will also denote F_∂ as the *steady state profit.*

Note that 3.2 in conjunction with (7) defines a differential game as in Isaacs (1954).

Definition 3.2 allows for a time dependent price. The firms set their price paths initially and then strictly follow these. The lack of opportunities for price path revisions implies no disadvantage for the firms, since the consumer model is deterministic and consumer behavior thus predictable. Therefore, even if the firms would be able to change their prices during the game, they would not do so unless the market conditions were changed by an external event.

Obviously, firms do not at each time maximize their current profit, but choose their price paths in order to obtain an optimal overall profit in the long run.

3.1 Monopoly

A product is feasible if it persists in the steady state and the firm does not make any losses when selling the good. The feasibility of a monopoly product can be expressed in terms of its imitation and habit functions (Matzke and Wirth 2008b):

Proposition 3.1. *The single product on the market with habitual imitative consumers is feasible if and only if* $\Psi_1[\xi_1 = c_1] > 0$.

A widely observed pricing strategy consists in charging an elevated price most of the time with (more or less regular) intermittent special offers. This strategy probably aims at making people buy the product during the low-price period and thereby inducing a habit for the high-price period. However, for periodic price changes we can show that in a market with habitual imitative consumers a Nash equilibrium is found to lie in the space of time-constant prices, which in many cases justifies to a priori confine ourselves to steady states and constant pricing.

Under fairly mild conditions on the functions φ and Ψ at the critical point ξ^* we obtain the following optimality result (Matzke and Wirth 2008b).

Proposition 3.2. *Let us consider a monopoly market with habitual imitative consumers in which the firm has a periodic price path, i. e. in each time period* $[kT, kT+T], k \in \mathbb{N}$, *the same price path* $\xi_1(t) = \xi_1(t+T) = \xi_1(t+2T) = \cdots$ *is pursued. The appropriate normal form competition game reads*

$$G = \left(1, \mathbb{R}_+^{[0,T]}, \frac{1}{T}\int_0^T (\xi_1(t) - c_1) S_1(s_1(\xi_1(t)), \varphi_1(\xi_1(t)), t)\mathrm{d}t\right),$$

assuming that a periodic state, i. e. a state with $x_1(t) = x_1(t+T) = \cdots$, *has been reached. Moreover, assume* $R(\varphi\Psi)' \geq \Psi\Psi'\varphi^2$ *at the constant critical price* ξ^*. *Then, for a feasible good, if* $2(\Psi'(\xi^*))^2 > \Psi''(\xi^*)\Psi(\xi^*)$, *a constant price is a (local) optimum for the monopoly.*

The first condition holds for R small enough, i. e. at least for long-lasting products, the latter condition holds for instance for affine Ψ. Hence it indeed makes sense for

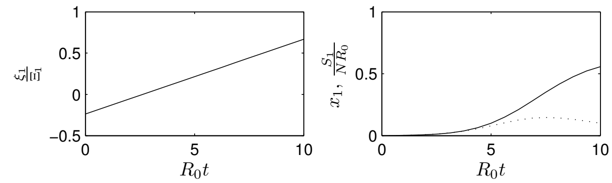

Figure 1: Optimal affine price evolution (left) as well as subpopulation (solid line) and sales (dotted line) evolution (right) for parameter values $R_1 = 0.1R_0$, $T = 10/R_0$, $r = 0$, $c_1 = 0.1/\Xi_1$, and $x_1(0) = 0.001$ (cf. example 3.1).

some cases to reduce the complex price space $\mathbb{R}_+^{[0,T]}$ by focussing on time-constant prices.

For the issues dealt with so far, the general mathematical system was analytically treatable. For other questions, we have to resort to specific examples in order to obtain a qualitative insight into the characteristics of a market with habitual imitative consumers. Clearly, in such cases it is instructive to consider only very simple forms of s, φ, and especially ξ that just capture the necessary features for the discussed problem at hand. In particular, affine functions (the simplest case possible) are well-suited to study trends (e. g. price trends). The following example is meant to examine the optimal price trend for a good that is sold during a finite time period. It illustrates that proposition 3.2 does not hold for bounded time intervals.

Example 3.1 (Cumulated discounted profit in a monopoly setting)**.** *For simplicity, let us assume* $\varphi_1 = s_1 = 1 - \frac{\xi_1}{\Xi_1}$*, and let us only allow for affine price functions* $\xi_1(\cdot) \in \mathfrak{L}([0,T]) := \{f : [0,T] \to \mathbb{R} \,|\, \exists\, a, b : f(t) = a + bt\}$*. Consider the normal form competition game*

$$G = \left(1, \mathfrak{L}([0,T]), \int_0^T \exp[-rt](\xi_1(t) - c_1) S_1\left(s_1(\xi_1(t)), \varphi_1(\xi_1(t)), t\right) \mathrm{d}t\right).$$

For given parameters $R_1, T, r, c_1, x_1(0)$*, the optimal price path* $\xi_1(t)$ *can be found numerically. As a result, for a whole range of realistic parameters we obtain that the product is initially sold below marginal cost, and then the price rises. One example calculation is depicted in figure 1.*

Of course, when r is chosen extremely large, this trend is reversed. However, this only happens for values of $r \sim R_0$ to $2R_0$. This would correspond to an interest rate of above 100 % within the time R_0^{-1}, i. e. if on average a consumer thinks of the good only once a year, the interest rate would have to be above 100 % per annum!

From this example, we may conclude that on a market with habitual imitative consumers a beneficial pricing strategy consists in starting at a low price and then increasing the price steadily. It might even be advantageous to initially give away products for free. The underlying idea is to initially strongly increase the market share in order to exploit habitual behavior.

3.2 Oligopoly and polygopoly

The feasibility of a product in an oligopoly can be described similarly to the monopoly (Matzke and Wirth 2008b).

Proposition 3.3. *Consider an n-product market with habitual imitative consumers on which the products i, $i = 1, \ldots, n-1$, coexist with $0 < \Phi_i < 1$. Then product n is feasible if and only if*

$$\Psi_n[\xi_n = c_n] > \vec{\tilde{x}} \cdot \Phi(\vec{\tilde{\xi}}) = \sum_{i=1}^{n-1} \Phi_i(\tilde{\xi}_i)\tilde{x}_i$$

where $\vec{\tilde{x}}$ is the vector of market shares on the $(n-1)$-goods market (i. e. without product n) in the steady state and $\vec{\tilde{\xi}}$ the corresponding price vector.

$\Psi_n[\xi_n = c_n]$ represents the hypothetic monopoly market share when the price equals the marginal costs. Hence, intuitively, the above proposition implies that this hypothetic monopoly market share has to be larger than the weighted sum of market shares of products 1 to $n-1$, where the weights $\Phi_i \leq 1$ are the larger the stronger the corresponding goods induce habit.

Next, we shall study market implications from rising numbers of competitors: The prices of the weakest products on the market converge against their marginal costs as the number of competitors rises to infinity (Matzke and Wirth 2008b):

Proposition 3.4. *Consider a polygopoly with n firms and habitual imitative consumers, where all n products coexist in the steady state Nash equilibrium. For given n, let i_n denote the index of the "weakest" good, i. e. the one with lowest market share $x_{i_n} = \min_{j=1,\ldots,n}\{x_j\}$. Let $\Phi_{i_n}(\xi_{i_n})$ and $\Psi_{i_n}(\xi_{i_n})$ be differentiable. If there is $\nu > 0$ such that $\frac{\partial \Psi_{i_n}}{\partial \xi_{i_n}}\big|_{\xi_{i_n}=\xi^{*,n}_{i_n}} < -\varepsilon < 0$ for all $n > \nu$, then as the number of firms tends to infinity, the price of good i_n converges to marginal cost, i. e. $(\xi^{*,n}_{i_n} - c_{i_n}) \to 0$.* □

The proof of this result (given in Matzke and Wirth (2008b)) can inductively be repeated for the second "weakest" good, the third "weakest" one, and so on up to the mth "weakest" good, where m is any positive integer.

Corollary 3.5. *Consider a polygopoly with n firms and let the assumptions from proposition 3.4 hold. As the number of firms tends to infinity, the m "weakest" products' prices converge to their marginal costs, where m is any positive integer.* □

As a consequence, since on a symmetric market any good is the "weakest" one, the prices of all goods converge to marginal costs, resulting in perfect competition. In general, however, there may be products so superior to the rest of the market that their prices stay away from marginal costs while all other prices converge against marginal costs.

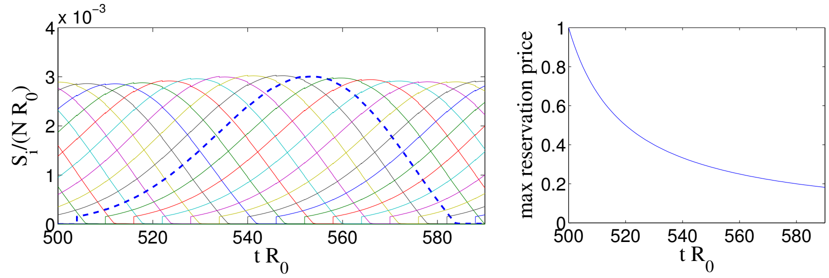

Figure 2: Product life cycles (left) when many products enter the market successively and the maximum reservation price Ξ_i of each product decreases in time (right). In this simulation we chose $\varphi_i(\xi_i) = s_i(\xi_i) = 1 - \frac{\xi_i}{\Xi_i}$ with maximum reservation price $\Xi_i = \frac{\Xi}{1+\alpha R_0(t-t_i)}$, $\alpha = 5 \cdot 10^{-2}$. Furthermore, we have a time interval $T = \frac{6}{R_0}$ between the introduction of products, an alarm clock rate $R_i = 0.025 R_0$ for all goods, an initial subpopulation $x_i(0) = 10^{-3}$, and constant product prices $\xi_i = 0.1\Xi$.

4 Product life cycles and advertising

To point into possible directions of further research we will briefly present two extensions to our model, the generation of product life cycles and advertising.

4.1 The generation of product life cycles

In this paragraph, we briefly illustrate how a realistic product life cycle may emerge from our model. As a simple example, consider a consecutive introduction of many products, all competing with each other. Think for instance of the mobile phone market, where new (innovative) mobile phones frequently enter the market. The maximum reservation price for a product may be assumed highest at its introduction on the market when it still represents the state of the art, and then it decreases in time, as innovation goes on. Hence, also habit and imitation function are highest at the time of product introduction.

The most simple setting is to assume fixed prices ξ_i, simple imitation and habit functions $\varphi_i(\xi_i) = s_i(\xi_i) = 1 - \frac{\xi_i}{\Xi_i}$, new product introductions equally distributed over time, and a simple evolution of the maximum reservation price Ξ_i in time, e. g. $\Xi_i = \frac{\Xi}{1+\alpha R_0(t-t_i)}$, where t_i is the time of introduction of product i. Figure 2 shows the resulting product life cycles of the successively introduced products, obtained from an exemplary simulation.

Notice the classical pattern with a gentle increase of the sales right after product launch, a broad maturity period and a quite steep decline until the product vanishes (cf. for example de Kluyver (1977), Polli and Cook (1969) and others).

4.2 Marketing strategies: Advertisement

In section 2.2, in order to employ specific switching probabilities, we used the mechanism of imitation (4), that is, the probability of buying good i is proportional to the amount x_i of people who already own it. x_i may here be interpreted as the probability that the consumer gets to know the product from other consumers. The multiplicative imitation factor φ_i represents how strongly the consumer is convinced to buy the good when she knows it. However, consumers can also get to know the good via advertisements, which constitute an effective tool for firms to influence the consumers' buying behavior. The probability to see the product's commercial is given by $a_i \in [0, 1]$, where a_i depends positively on the advertising budget. The overall probability to become aware of product i (via commercials or other consumers) hence is $a_i + x_i - a_i x_i$ so that (4) and (6) change to

$$p_{0i} = \varphi_i(x_i + a_i - x_i a_i), \quad i \neq 0, \tag{8}$$

$$p_{ij} = (1 - s_i)\varphi_j(x_j + a_j - x_j a_j), \quad i \neq 0 \wedge j \neq 0, i. \tag{9}$$

For the same motivation as in section 2.2, equation (5) remains unchanged,

$$p_{ii} = s_i \in [0, 1], \quad i \neq 0. \tag{10}$$

We shall in the following always assume $\varphi_i, s_i > 0$. An interesting question would be whether a non-feasible product can be made feasible by advertising. The following proposition from Matzke and Wirth (2008b) provides an answer (where we disregard advertising costs and only examine whether a demand for that good exists).

Proposition 4.1. *On an n-product market with habitual imitative consumers, product i is always feasible if it is advertised, i. e.* $a_i > 0$ *(unless there is a good j with* $\Phi_j = 1$*).*

Apparently, commercials help the good to survive on the market. This statement is illustrated in figure 3 where the steady state market share is shown for different advertising levels in a monopoly. For a positive level, the market share is always positive and hence the product feasible.

Note that in proposition 4.1 we only consider the demand side of the market, i. e. we examine whether the product is demanded by consumers in the steady state. We ignored that the firm might not be able to operate in the black because of immense advertising costs. Let us therefore introduce advertising costs c_i^a for good i. Obviously, the derivative $\frac{\partial a_i}{\partial c_i^a}$ has to be non-negative. With this altered model at hand, various simulations can be performed for specific functions $\varphi_i(\xi_i)$, $s_i(\xi_i)$ and $a_i(c_i^a)$. One could for instance examine the product feasibility including advertising costs, whether advertising is profitable at all, how large a_i should optimally be, or whether there is a threshold value for x_i above which advertising is no longer beneficial. For illustration, we pick up example 3.1 and add advertising. We will compute the optimal affine pricing and advertising strategy.

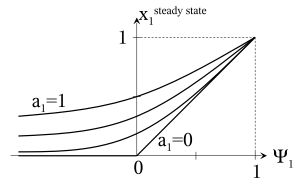

Figure 3: Stable steady state value of the market share x_1 for different advertising levels a_1.

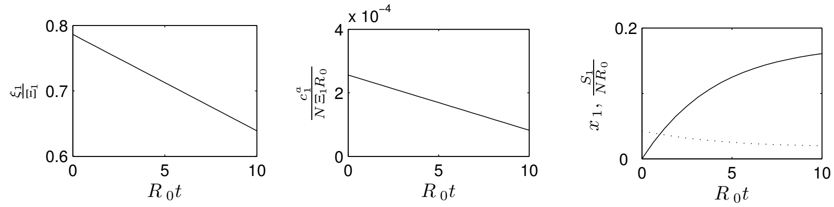

Figure 4: Optimal affine price evolution (left) and optimal affine advertising expenses (middle), as well as subpopulation (solid line) and sales (dotted line) evolution (right) for parameter values $R_1 = 0.1R_0$, $T = 10/R_0$, $r = 0$, $c_1 = 0.1/\Xi_1$, $K = 2 \cdot 10^{-6}$, $x_1(0) = 0$ (cf. example 4.1).

Example 4.1 (Cumulated discounted profit in a monopoly setting with advertising). *For simplicity, let us assume* $\varphi_1 = s_1 = 1 - \frac{\xi_1}{\Xi_1}$ *and* $a_1 = \frac{1}{1+K/c_1^a}$, *and let us only allow for affine price and advertising cost functions* $\xi_1(\cdot), c_1^a(\cdot) \in \mathfrak{L}([0,T]) := \{f : [0,T] \to \mathbb{R} \,|\, \exists\, a, b : f(t) = a + bt\}$. *Consider the normal form competition game*

$$G = \left(1, \mathfrak{L}^2([0,T]), \int_0^T \exp[-rt]\left[(\xi_1(t) - c_1)S_1\left(s_1(\xi_1(t)), \varphi_1(\xi_1(t)), a_1(c_1^a(t)), t\right) - c_1^a(t)\right] \mathrm{d}t\right).$$

For given parameters $R_1, T, r, c_1, x_1(0)$, *the optimal price paths* $\xi_1(t)$ *and* $c_1^a(t)$ *can be found numerically. As a result, for a whole range of realistic parameters we obtain the reverse of example 3.1: The price decreases with time. One example calculation is depicted in figure 4.*

Apparently, a firm is recommended to start an advertising campaign in parallel to the product launch and steadily decrease the product price as well as the advertising expenses during the lifespan of the product. Due to the initial advertising, the market share is rapidly increased with brute force. Via the subsequent price decrease, habit purchases can be kept on a high level, and reluctant customers are attracted. Thereby, the market is optimally exploited by initially letting customers with a high reservation price pay high prices and only later reducing the price to make people with low reservation prices buy the product (similarly to the concept of price discrimination). Advertising becomes less crucial when the market share has already reached a certain level (the product sells itself) and is therefore reduced.

5 Conclusion

We examined the optimal strategic pricing for firms when the demand evolution is generated by the behavior of boundedly rational consumers who follow a rule of thumb and base their decisions on imitation and habit. The demand dynamic is described within the framework of a population game with associated switching probabilities, and it serves as a basis for strategic pricing of a monopoly or oligopoly in a differential game. The optimal price paths correspond to Nash equilibria of a normal form competition game.

The modeling approach is supported by psychological and experimental studies, and the introduced methodology allows for broad applications and qualitative theoretical analysis.

We investigated product feasibility (i. e. the conditions under which firms operate profitably in the long-term) and expressed it with the help of the hypothetical popularity of the product if it was sold for a price equal to the marginal cost. Furthermore, we showed that markets with habitual imitative consumers are in a sense well-behaved: For a rising number of firms, the prices decrease, the prices of the weakest products (but not necessarily of all products) converge against marginal costs. Such results (despite the boundedly rational consumer behavior) prove once more the existence of some kind of efficiency in not totally rational markets.

We also saw for the monopoly that under certain conditions, Nash equilibria are found in the strategy space of all time-constant price paths so that a reduction of the (quite complex and untractable) strategy space of all possible price paths is at least sometimes sensible.

Finally, the assumed boundedly rational consumer behavior was shown to lead to observed market patterns such as product life cycles, and as an extension, we explored optimal advertising strategies.

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