International Portfolios with Nominal Rigidities and Capital Accumulation

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PRELIMINARY AND INCOMPLETE

Abstract

We analyse bond and equity portfolios in models with nominal price rigidities, capital accumulation, trade in bonds and equities and endogenous monetary policy and find that the implied bond and equity portfolios are usually excessive and unstable. Equities and bonds are used to hedge fluctuations in the real exchange rate and human capital returns and robust portfolios obtain if equities are significantly better hedges for human capital than bonds, while the opposite is true for the real exchange rate. Here, bond returns are closely related to the nominal, but not necessarily the real exchange rates, while capital accumulation implies that equity returns are often highly correlated with the real exchange rate. The empirical analysis finds some support for a hedging explanation of equity home bias and suggests that the failure of the model may stem from its inability to produce realistic behaviour of equity returns and real exchange rates.

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1 Introduction

Recent decades have seen substantial increases in international gross asset positions and capital flows (see e.g. Lane and Milesi Feretti (2001, 2007)). Given the large size of the outstanding positions, changes in valuations of these assets can potentially have a large effect on macroeconomic outcomes. However, until recently, economists were not able to shed much light on the determinants of these asset positions, as standard open economy models contained very simple portfolio decisions, if any at all. What is more, stylised facts on the nature of international asset positions were thin on the ground. It is therefore not surprising that the literature has mostly focussed on trying to explain "home equity bias", the well documented fact that a large fraction of domestic equity is held by domestic owners (see French and Poterba (1991)). While home equity bias has somewhat decreased over the past few decades, it is still sizable and very common. This fact has been called a bias, as the most basic optimising model of portfolio choice would imply full diversification (Lucas (1982)), suggesting that all agents hold the same risky portfolio with the share of domestic equity held equal to the share of the country in the world market portfolio. Many explanations have since been put forward to account for the observed bias, including frictions in the trading of goods or assets or informational frictions, with varying degrees of success (see Lewis (1999), Karolyi and Stulz (2003) and Sercu and Vanpee (2007) for surveys of the literature). Another strand of the literature, however, has argued that once we go beyond the basic models that have been used to study international asset positions and introduce features which have been shown to be important in macroeconomic analyses of closed economies, equity home bias can be explained in the context of standard macroeconomic models, without introducing additional frictions that pertain only to an international dimension. In most of these models, equity positions can be understood as positions derived from an explicit diversification motive, and positions driven by motives to hedge against consumption expenditure risk and risk arising from nontradable income, most notably human capital. [1]. Thus, Heathcote and

[1] In many settings, consumption expenditure and the real exchange are perfectly correlated and in those cases we will use the latter expression as a synonym for the former.

Perri (2008) show that simply introducing capital accumulation is enough to generate home bias in equity positions, as the response of investment implies that equity holdings are good hedges against consumption expenditure and human capital risk. However, in their model, portfolios are also very sensitive to changes in parameters and portfolios generally exhibit excessive home bias for realistic parameter values.

In all of the models above, the portfolio decision of agents essentially only consists of choosing the share of domestic equity held. But after recent methodological advances that allowed the use of standard solution methods to solve models with more sophisticated portfolio choice, a growing literature has developed that analyses optimal portfolios in an environment with multiple assets. [2] Thus, Coeurdacier et al. (2008) and Engel and Matsumoto (2008) develop models which are able to generate relatively realistic degrees of equity home bias and robust equity and bond portfolios. In the former paper, the key innovation is to allow for trade in real bonds in addition to capital accumulation, while the latter paper introduces sticky prices and trade in foreign exchange forwards, while abstracting from capital accumulation. In both papers, the basic mechanism that generates equity home bias and robust portfolios is similar and consists of two distinct features. Firstly, equities are good at hedging human capital risk, due to the possibility of capital accumulation in Coeuracier et al (2009) and due to sticky prices in Engel and Matsumoto (2009). Secondly, bonds are used to hedge real exchange rate risk. This is because the real bonds traded in Coeurdacier et al (2009) are perfectly correlated with real exchange rate risk, while in Engel and Matsumoto (2008), equities are a poor hedge for real exchange rate risk.

We analyse models that feature capital accumulation, nominal rigidities, endogenous monetary policy and trade in nominal bonds as well as equities. Models which allow for capital accumulation are, of course, standard in closed economy business cycle analysis, going back at least to the seminal contributions of Kydland and Prescott (1982) and Long and

[2] Devereux and Sutherland (2007a, 2008a) and Tille and van Wincoop (2008) present an essentially identical solution method that allows a general class of open economy model with multiple assets to be solved using standard algorithms. Devereux and Saito (2005), Evans and Hnatkovska (2006) and Judd et al. (2002) describe alternative solution approaches.

Plosser (1983), owing to the fact that the volatility of investment accounts for a significant fraction of the volatility of output. For open economies, there are at least two additional reasons that call for the inclusion of investment. Firstly, one of the key functions of the international capital market is to finance international investment. Secondly, net exports are to found to be strongly countercyclical and investment is key in generating this correlation, both theoretically and in the data (Backus et al (1992, 1994)). Nominal rigidities and Taylor type monetary policy rules are by now the standard way to introduce real effects of monetary policy into macroeconomic models (see Gali (2008) and Woodford (2003) for introductions). Finally, the bulk of bond issuance is in nominal bonds.

We find that once we allow for capital accumulation and some nominal risk in bond trading, the portfolio implications of standard macroeconomic models are no longer as convincing. In particular, we find that equity home bias is often excessively large or negative and that bond positions can be vary from large positive to large negative positions in domestic bonds. More generally, portfolios are often unstable and sensitive to changes in parameters, as equity and bond returns are often highly correlated. The reason is twofold: Firstly, equities are often a good hedge for consumption expenditure risk in the general model: As in Heathcote and Perri (2008) and Coeurdacier et al (2008), equity returns tend to be high when the real exchange rate appreciates, because of the response of investment. Secondly, nominal bonds are usually not much better at hedging real exchange rate risk, and often worse, due to nominal risk. Equity returns and bond returns are frequently highly correlated, implying that both equity and bond positions are heavily affected by the desires to hedge consumption expenditure risk and generating relatively extreme positions. Our benchmark model features productivity and interest rate shocks, trade in equities and one period nominal bonds and a central bank that is moderately aggressive in targeting CPI inflation. However, the result is remarkably robust, spanning most of the plausible parameter space, as well as allowing for trade in long term bonds, investment shocks or different monetary policy reaction functions.

In Engel and Matsumoto (2008a), equity returns are procyclical. In a boom, output is demand determined in the short term, as the nominal rigidities imply that firms cannot lower prices to stimulate demand to the same extent as with flexible prices. As a consequence, labour demand falls and, with a sufficient degree of price rigidity, so do human capital returns. Firm profits rise due to higher productivity and so do relative equity returns, implying that domestic equity is a good hedge for human capital risk. We will call this the "sticky price channel". On the contrary, we find that, as in Heathcote and Perri (2008) and Coeurdacier et al (2008), equity returns are countercyclical in the model, as higher investment in a boom and nominal depreciation imply a persistent relative fall in dividends. Nevertheless, domestic equity continues to be a good hedge for human capital risk, as both labour demand and wages rise due to the increase in demand. We will call this the "investment channel" and it is the investment channel that dominates in this respect in our model.

In most models considered below, we will have two shocks and two assets per country. It can then be shown that markets are effectively complete, up to a linear approximation. In those cases, linearised consumption expenditures are perfectly correlated with the real exchange rate. Real exchange rates are found to be procyclical, implying that the Home real exchange rate depreciations during a boom[3]. Since equity returns also fall during a boom, domestic equity is then a good hedge for consumption expenditure risk. In our benchmark model, the Home currency initially depreciates during a boom, but appreciates in the long term. Since returns of one period bonds are equal to the unexpected appreciation during the following period, while returns of a long term bond are equal to the presented discounted value of all future appreciations, short term bond returns are countercyclical, while long term bond returns are procyclical. Thus, while a long position in short term bonds would be a good hedge for real exchange rate risk, a long position in long term bonds would not be, indicating that long term bonds are not necessarily better at hedging long term exchange rate risk, despite depending on all future nominal exchange rates rather than just the next

[3]Note that real exchange rates are defined as the Foreign price level times the nominal exchange rate divided by the Home price level, implying that a rise in the real exchange rate implies a Home real *depreciation*.

period appreciation.

More generally, we find that equities are surprisingly often as good at hedging real exchange rate risk as nominal bonds. Firstly, this is because equity returns are often highly correlated with real exchange rate risk, due to the dynamic nature of the model and the role of capital accumulation. Secondly, nominal bond returns are often not very closely related to real exchange rates, again for two reasons. Firstly, the presence of nominal risk puts an upper bound on the degree to which nominal and real exchange rates are related here. Bonds do become better hedges for real exchange rates, as we increase the degree of price rigidity, but we often require substantial (and sometimes unrealistic) degrees of price rigidity to achieve that bond returns are significantly more highly correlated with real exchange rate risk than equity returns. What is more, the model with capital accumulation and nominal rigidities exhibits real indeterminacy for high degrees of price rigidity, limiting the degree to which we can arbitrarily increase the extent to which bonds can hedge real exchange rate risk through this route without simultaneously changing other parts of the model (see Carlstrom and Fuerst (2005) for a discussion in a closed economy context). Secondly, there is an issue with the time dimension. In the long run, all prices are flexible. This implies that, as we look further into the future, nominal and real exchange rates are less closely related and even long lived bonds will be less good at hedging real exchange rates far in the future. In the case of short term bonds, there is also "maturity mismatch" in the sense that the maturity of the risk is infinite, while the maturity of the asset is only one period (and the maturity of the alternative asset, equity, is infinite as well). If current and future real exchange rate changes are not too closely correlated, short term bonds will then not be very good at hedging real exchange rate risk.

As in Devereux and Sutherland (2007a, 2008b), monetary policy does affect asset returns here. In particular, monetary policy affects the dynamics of the nominal exchange rates and thereby the hedging properties of bond returns, but only in very extreme cases can it increase the correlation between real exchange rate risk and bond returns enough to produce robust portfolios. In the wide majority of cases, equities will be as good as bonds at hedging real exchange rate risk. Since real exchange rate risk is large in this model and correlations between the two asset returns are high, this implies that asset positions are often extreme, ranging from excessive home bias to excessive foreign bias and large positive to large negative positions in domestic bonds. While we can in most cases allocate the asset positions to parts driven by real exchange rate risk and human capital risk, respectively, rationalising the positions is often difficult beyond the fact that high correlations between the assets imply large and offsetting positions.

We experiment with a variety of model specifications, in order to test the robustness of our results. In particular, we also investigate a version of the model with investment efficiency shocks. Investment shocks have recently been shown to be important for explaining output fluctuations in closed economies (see Fisher (2002,2006), Justiciano and Primiceri (2006), Justiciano et al (2007)). However, in our setting, while they often change portfolios substantially, they usually do not help in bringing them closer to the data, as they lower the correlation with real exchange rate risk for both equities and bonds.

In our empirical analysis, we use data on G7 countries to estimate the relevant covariances of equity and bond returns with human capital and real exchange rate risk. We find that these hedging demands can in fact explain equity home bias in the US, Japan, and to some extent the UK, but not in other countries. What is more, equity home bias is in all countries driven by the motive to hedge human capital risk and not real exchange rate risk. This empirical exercise also sheds also highlights two areas, where the model and data are quite far apart: Firstly, equity return innovations are not volatile enough and too correlated with the sources of risk in our model. Secondly, real exchange rate risk in the model is too large.

2 Model

There are two symmetric countries, Home (H) and Foreign (F), indexed by i. Each country is specialised in the production of a composite good using a continuum of country specific intermediate goods. Intermediate goods are produced using labour and capital. The factors of production and the intermediate goods which immobile between countries, but composite goods are traded. In addition to trade in composite goods, countries trade in nominal bonds and equities.

2.1 Households

Country i is inhabited by a representative consumer with a utility function that is separable in consumption and labour:

$$U_i = \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{i,t+j}^{1-\sigma}}{1-\sigma} - \iota \frac{L_{i,t+j}^{1+\omega}}{1+\omega} \right), \tag{1}$$

where $C_{i,t}$ is the consumption aggregator of country i, and $P^i_{C,t}$ is the consumption price index. The discount rate β, the intertemporal elasticity of substitution σ, the Frisch labour supply elasticity ω and the parameter ι governing labour supply in the steady state are common across countries. The consumption aggregator for country i is defined as:

$$C_{i,t} = \left[a_C^{1/\phi} \left(C^i_{i,t} \right)^{\frac{\phi-1}{\phi}} + (1-a_C)^{1/\phi} \left(C^i_{j,t} \right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \tag{2}$$

where $a_C > 1/2$ is the consumption home bias parameter and ϕ is the elasticity of substitution between Home and Foreign goods. These preferences imply the following consumption price indices for Home and Foreign:

$$P^i_{C,t} = \left[a_C \left(P^i_{i,t} \right)^{1-\phi} + (1-a_C) \left(P^i_{j,t} \right)^{1-\phi} \right]^{\frac{1}{1-\phi}}, \tag{3}$$

where $P^i_{j,t}$ is the price in country i of the composite good produced in country j. All prices are quoted in terms of the local currency. The optimal allocation across consumption goods is then given by:

$$C^i_{i,t} = a_C \left(\frac{P^i_{i,t}}{P^i_{C,t}} \right)^{-\phi} C_{i,t} \qquad\qquad C^i_{j,t} = (1 - a_C) \left(\frac{P^i_{j,t}}{P^i_{C,t}} \right)^{-\phi} C_{i,t} \tag{4}$$

where $C^i_{j,t}$ denotes consumption of good j by agent i. The first order conditions for labour supply are given by:

$$\iota l^{\omega}_{i,t} = \left(\frac{W_{i,t}}{P^i_{C,t}} \right) C^{-\sigma}_{i,t}, \tag{5}$$

which describe the standard condition that the marginal disutility of labour today has to equal the marginal utility of consumption times the real wage.

Consumption of the Home and Foreign aggregate consumption goods are given by

$$C^i_{j,t} = \left(\int_0^1 \left(C^i_{j,t}(k) \right)^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{6}$$

where $C^i_{j,t}(k)$ is consumption of the kth intermediate good in country j by the agent in country i and ε is the elasticity of substitution between varieties in consumption. Optimal consumption of Home and Foreign intermediate goods then implies:

$$C^i_{j,t}(k) = \left(\frac{P^i_{j,t}}{P^i_{j,t}(k)} \right)^{\varepsilon} C^i_{j,t} \tag{7}$$

where $P^i_{j,t}$ is the price in country i of the good produced in country j. The price indices for Home and Foreign composite goods are then:

$$P^i_{j,t} = \left(\int_0^1 \left(P^i_{j,t}(k) \right)^{1-\varepsilon} dk \right)^{\frac{1}{1-\varepsilon}} \tag{8}$$

and the real exchange rate Q is defined as:

$$Q_t = \frac{P^H_{C,t}}{S_t P^F_{C,t}} \tag{9}$$

2.2 Capital Accumulation

At time t, each country possesses a capital stock $K_{i,t}$. Country specific capital stocks depreciate at rate δ and are augmented by country specific investment $I_{i,t}$:

$$K_{i,t+1} = (1-\delta) K_{i,t+1} + I_{i,t}, \tag{10}$$

where investment in country $i's$ capital stock is a CES aggregate of the Home and Foreign composite good:

$$I_{i,t} = \left[a_I^{1/\phi} \left(I_{i,t}^i \right)^{\frac{\phi-1}{\phi}} + (1-a_I)^{1/\phi} \left(I_{j,t}^i \right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}. \tag{11}$$

ϕ is the elasticity of substitution between the Home and Foreign good in investment, and $a_I > 1/2$ signifies local bias in investment. The Home and Foreign composite investment goods are produced using by a CES composite of Home and Foreign varieties:

$$I_{j,t}^i = \left(\int_0^1 \left(I_{j,t}^i (k) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{12}$$

where ε is the elasticity of substitution between intermediate goods of the same country in investment. We do not distinguish between the elasticities of substitution in consumption and investment to save on notation. Demand for investment is then given by:

$$I_{j,t}^i (k) = \left(\frac{P_{j,t}^i}{P_{j,t}^i (k)} \right)^{\varepsilon} I_{j,t}^i, \tag{13}$$

where the price indices for Home and Foreign goods are as described above.

At time t, the stock of capital in each country is rented out in a spot market for a rental rate $R_{i,t}^K$. Since optimal investment implies that the expected discounted payoff from investment is equal to its marginal cost at time t, it is determined by:

$$1 = \beta E_t \left[\varpi_{t,t+1}^i \frac{1}{P_{I,t}^i} \left(R_{i,t}^K + (1-\delta) P_{I,t+1}^i \right) \right], \tag{14}$$

where $\varpi^i_{t,t+1}$ is the stochastic discount factors applied by the owners of the country i capital stock at time t to discount date $t+1$ profits.

2.3 Firms

Each country contains a continuum of firms, each producing a differentiated intermediate good, indexed by k, using capital and labour. The production function is given by:

$$Y_{i,t}(k) = A_{i,t}(L_{i,t}(k))^{\alpha}(K_{i,t}(k))^{1-\alpha}, \tag{15}$$

where $A_{i,t}$ is the exogenous level of productivity in country i and $alpha$ is the elasticity of production with respect to labour. In each country, capital and labour are traded on an aggregate spot market. Firms maximise the present discounted value of profits and choose labour and capital to minimise the cost of production. The cost function of a producer is then given by the solution to the following problem

$$\begin{aligned}
&\min_{K_{i,t}(k),L_{i,t}(k)} R^K_{i,t}K_{i,t}(k) + W_{i,t}L_{i,t}(k) \\
&s.t. \\
&Y_{i,t}(k) = A_{i,t}(L_{i,t}(k))^{\alpha}(K_{i,t}(k))^{1-\alpha}
\end{aligned}$$

which implies

$$\frac{L_{H,t}(k)}{K_{i,t}(k)} = \frac{\alpha}{1-\alpha}\frac{R^K_{i,t}}{W_{i,t}}. \tag{16}$$

Thus, all producers produce using the same ratio of labour to capital. The total and marginal cost functions, $\Xi(Y)$ and $\varrho(Y)$ are then given by:

$$\Xi_{i,t}(Y_{i,t}(k)) = R^K_{i,t}K_{i,t}(k) + W_{i,t}L_{i,t}(k) = \frac{Y_{i,t}(k)}{A_{i,t}}\left(R^K_{i,t}\right)^{1-\alpha}W^{\alpha}_{i,t}(1-\alpha)^{-(1-\alpha)}\alpha^{-\alpha} \tag{17}$$

$$\varrho_{i,t}(Y_{i,t}(k)) = \frac{\partial TC_{i,t}}{\partial Y_{i,t}} = \frac{1}{A_{i,t}}\left(R^K_{i,t}\right)^{1-\alpha}W^{\alpha}_{i,t}(1-\alpha)^{-(1-\alpha)}\alpha^{-\alpha}. \tag{18}$$

Total demand for each intermediate good is composed of domestic demand for consumption and investment and foreign demand for consumption and investment. Equating production with total demand then implies:

$$Y_{i,t}(k) = C_{i,t}^{i}(k) + C_{i,t}^{i}(k) + I_{i,t}^{j}(k) + I_{i,t}^{j}(k), \tag{19}$$

where the demand functions are as given above. Firms take these demand functions as given and choose prices in local currency in Home and Foreign to maximise profits. Per period profits of Home firms are given by:

$$\Pi_{H,t}(k) = P_{H,t}^{H}(k) Y_{H,t}^{H}(k) + P_{H,t}^{F}(k) S_t Y_{H,t}^{F}(k) - \Theta_H(Y_{H,t}(k)), \tag{20}$$

where $Y_{H,t}^{H}(k) = C_{H,t}^{H}(k) + I_{H,t}^{H}(k), Y_{H,t}^{F}(k) = C_{H,t}^{F}(k) + I_{H,t}^{F}(k)$ and $\Theta_H(\cdot)$ is the total cost function of a firm in country i. S_t is the nominal exchange rate defined as number of units of Home currency per unit of Foreign currency. While firms rent capital and labour on spot markets, they can only reset prices with a probability of $1-\theta$ every period. A firm reoptimising in period t will choose a price $\widetilde{P}_{i,t}^{j}$ that maximises the current market value of profits generated while the price remains in effective, taking all other prices as given. Thus, it solves:

$$\max_{\widetilde{P}_{i,t}^{j}, \widetilde{P}_{i,t}^{i}} \sum_{l=0}^{\infty} \theta^l E_t \left[\varpi_{t,t+l}^{i} \left(\Pi_{i,t} \left(k, \widetilde{P}_{i,t}^{j}, \widetilde{P}_{i,t}^{i} \right) \right) \right], \tag{21}$$

where $\varpi^i_{t,t+l}$ is the stochastic discount factor used to discount profits of a firm in country i at time $t+l$ back to time t. Optimal prices are then given by:

$$\widetilde{P}^H_{H,t}(k) = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi^H_{t,t+l}\varrho_{H,t}\left(P^H_{H,t+l}\right)^{\varepsilon}Y^H_{H,t+l}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi^H_{t,t+l}\left(P^H_{H,t+l}\right)^{\varepsilon}Y^H_{H,t+l}} \tag{22}$$

$$\widetilde{P}^F_{H,t}(k) = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi^H_{t,t+l}\left(P^F_{H,t+l}\right)^{\varepsilon}\varrho_{H,t}Y^F_{H,t+l}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi^H_{t,t+l}S_{t+l}\left(P^F_{H,t+l}\right)^{\varepsilon}Y^F_{H,t+l}} \tag{23}$$

$$\widetilde{P}^H_{F,t}(k) = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi^F_{t,t+l}\left(P^H_{F,t+l}\right)^{\varepsilon}\varrho_{F,t}Y^H_{F,t+l}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi^F_{t,t+l}\frac{1}{S_{t+l}}\left(P^H_{F,t+l}\right)^{\varepsilon}Y^H_{F,t+l}} \tag{24}$$

$$\widetilde{P}^F_{F,t}(k) = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi^F_{t,t+l}\left(P^F_{F,t+l}\right)^{\varepsilon}\varrho_{F,t}Y^F_{F,t+l}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi^F_{t,t+l}\left(P^F_{F,t+l}\right)^{\varepsilon}Y^F_{F,t+l}}. \tag{25}$$

As usual, optimal prices thus depend on discounted marginal costs over the expected lifetime of the price set. The nature of price rigidities implies that Home and Foreign goods prices evolve according to:

$$P^i_{j,t} = \left(\theta\left(P^i_{j,t-1}\right)^{1-\varepsilon} + (1-\theta)\left(\widetilde{P}^i_{j,t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{26}$$

2.4 Financial Markets

There is trade in Home and Foreign one period nominal bonds and Home and Foreign equity. Nominal bonds pay one unit of the domestic currency in each period and are in net zero supply. Owners of equity of country i receive a claim to country i dividends $D_{i,t}$ defined below. The total supply of equity in each country is normalised to unity. It is assumed that in period 0 each household owns the stock of domestic equity and that bond positions are zero. The gross nominal returns in domestic currency for bonds and equity are:

$$R^S_{i,t+1} = \frac{D_{i,t} + P^S_{i,t+1}}{P^S_{i,t}} \qquad R^B_{i,t+1} = \frac{1}{P^B_{i,t}} \tag{27}$$

where $R^S_{i,t}$ $\left(R^B_{i,t}\right)$ is the return on holdings of country i equity (bonds) and $P^S_{i,t}$ $\left(P^B_{i,t}\right)$ are the prices of country i equity (bonds). Dividends are now composed of profits earned by the firm plus the rent earned for using capital minus investment spending:

$$D_{i,t} = \Pi_{i,t} + R^{K}_{i,t} K_{i,t} - P^{i}_{I,t} I_{i,t}$$

The budget constraint for Home in period t is then given by:

$$\begin{aligned} & S^{H}_{H,t} P^{S}_{H,t} + S^{H}_{F,t} P^{S}_{F,t} S_t + B^{H}_{H,t} P^{B}_{H,t} + B^{H}_{F,t} P^{B}_{H,t} S_t \\ = \; & W_{H,t} L_{H,t} + S^{H}_{H,t-1} \left(D_{H,t} + P^{S}_{H,t} \right) + S^{H}_{F,t-1} \left(D_{F,t} + P^{S}_{F,t} \right) S_t \\ & + B^{H}_{H,t-1} + B^{H}_{F,t-1} S_t - P^{H}_{C,t} C_{H,t}, \end{aligned} \tag{28}$$

where $S^{i}_{j,t}$ $\left(B^{i}_{j,t}\right)$ are holdings of country j equity (bonds) by country i in period t. The asset Euler equations are then given by:

$$\beta E_t \left[\left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\sigma} \frac{P^{H}_{C,t}}{P^{H}_{C,t+1}} \frac{D_{H,t+1} + P^{S}_{H,t+1}}{P^{S}_{H,t}} \right] = 1 \tag{29}$$

$$\beta E_t \left[\left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\sigma} \frac{P^{H}_{C,t}}{P^{H}_{C,t+1}} \frac{D_{F,t+1} + P^{S}_{F,t+1}}{P^{S}_{F,t}} \frac{S_{t+1}}{S_t} \right] = 1 \tag{30}$$

$$\beta E_t \left[\left(\frac{C_{F,t+1}}{C_{F,t}} \right)^{-\sigma} \frac{P^{F}_{C,t}}{P^{F}_{C,t+1}} \frac{D_{H,t+1} + P^{S}_{H,t+1}}{P^{S}_{H,t}} \frac{S_t}{S_{t+1}} \right] = 1 \tag{31}$$

$$\beta E_t \left[\left(\frac{C_{F,t+1}}{C_{F,t}} \right)^{-\sigma} \frac{P^{F}_{C,t}}{P^{F}_{C,t+1}} \frac{D_{F,t+1} + P^{S}_{F,t+1}}{P^{S}_{F,t}} \right] = 1 \tag{32}$$

$$\beta E_t \left[\left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\sigma} \frac{P^H_{C,t}}{P^H_{C,t+1}} \frac{1}{P^B_{H,t}} \right] = 1 \tag{33}$$

$$\beta E_t \left[\left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\sigma} \frac{P^H_{C,t}}{P^H_{C,t+1}} \frac{1}{P^B_{F,t}} \frac{S_t}{S_{t+1}} \right] = 1 \tag{34}$$

$$\beta E_t \left[\left(\frac{C_{F,t+1}}{C_{F,t}} \right)^{-\sigma} \frac{P^F_{C,t}}{P^F_{C,t+1}} \frac{1}{P^B_{H,t}} \frac{S_{t+1}}{S_t} \right] = 1 \tag{35}$$

$$\beta E_t \left[\left(\frac{C_{F,t+1}}{C_{F,t}} \right)^{-\sigma} \frac{P^F_{C,t}}{P^F_{C,t+1}} \frac{1}{P^B_{F,t}} \right] = 1 \tag{36}$$

2.5 Market Clearing

The market clearing condition for goods are given by:

$$C^H_{H,t} + C^F_{H,t} + I^H_{H,t} + I^F_{H,t} = Y_{H,t} \tag{37}$$

$$C^H_{F,t} + C^F_{F,t} + I^H_{F,t} + I^F_{F,t} = Y_{F,t} \tag{38}$$

For assets, we have:

$$S^H_{H,t} + S^F_{H,t} = 1 \qquad S^H_{F,t} + S^F_{F,t} = 1 \tag{39}$$

$$B^H_{H,t} + B^F_{H,t} = 0 \qquad B^H_{F,t} + B^F_{F,t} = 0 \tag{40}$$

The real exchange rate Q is:

$$Q_t = \frac{P^H_{C,t}}{S_t P^F_{C,t}} \tag{41}$$

2.6 Monetary Authorities

We assume that the monetary authority sets the nominal interest rate according to a simple rule that is, however, subject to shocks. In Devereux and Sutherland (2008), these shocks are interpreted as financial market shocks, but as in their paper, the role of these shocks more generally here is to introduce shocks to domestic inflation rates which are not related to productivity. Devereux and Sutherland (2007) also provide a more extensive discussion of the role of the monetary policy rule. In our benchmark case, monetary policy is set according to:

$$R_{i,t}^{N} = \frac{1}{\beta} \left(\frac{P_{C,t}^{i}}{P_{C,t-1}^{i}} \right)^{\gamma} \exp\left(m_{i,t}\right), \tag{42}$$

where γ determines the monetary policy responsiveness to inflation and m_t is a mean zero shock to interest rates. It is worth pointing out that, while in Devereux and Sutherland (2008), firms set prices according to producer currency pricing and the central bank stabilises PPI inflation rates, we assume that prices are set according to local currency pricing and the central bank stabilises CPI inflation.

2.7 Exogenous Processes

In order to close the model, we need to specify the nature of the stochastic processes. There are two sources of uncertainty per country. Productivity shocks in each country evolve according to:

$$\log\left(A_{i,t+1}\right) = \rho_A \log\left(A_{i,t}\right) + \varepsilon_{A,i,t+1}, \tag{43}$$

where $\varepsilon_{A,i,t+1}$ is a mean zero shock to productivity, and ρ_A governs the persistence of productivity. The interest rate shock is represented by:

$$m_{i,t+1} = \rho_M m_{i,t} + \varepsilon_{M,i,t+1} \tag{44}$$

where $\varepsilon_{M,i,t+1}$ is a mean zero shock to interest rates, and ρ_M governs the persistence of interest rates.

2.8 Equilibrium and Solution Method

An equilibrium in this economy is a set of quantities $NFA_{H,t}, K_{H,t+1}, K_{F,t+1}, Y_{H,t}, Y_{F,t}, C_{H,t}, C_{F,t}, I_{H,t}, I_{F,t}, C_{H,t}^{H}, C_{F,t}^{H}, C_{F,t}^{F}, C_{H,t}^{F}, I_{H,t}^{H}, I_{F,t}^{H}, I_{F,t}^{H}, I_{H,t}^{F}, D_{H,t}, D_{F,t}, W_{H,t}, W_{F,t}, S_{H,t}^{H}, S_{F,t}^{H}, S_{H,t}^{F}, S_{F,t}^{F}, B_{H,t}^{H}, B_{F,t}^{H}, B_{H,t}^{F}, B_{F,t}^{F},$ prices $P_{H,t}^{H}, P_{F,t}^{H}, P_{F,t}^{F}, P_{H,t}^{F}, P_{C,t}^{H}, P_{C,t}^{F}, P_{I,t}^{H}, P_{I,t}^{F}, Q_t, S_t$ and shocks $A_{H,t}, A_{F,t}, M_{H,t}, M_{F,t}$ which satisfy the following conditions:

1. the consumption allocations (equations 4 & 7)
2. the first order conditions for labour supply (equations 5)
3. the investment allocations (equations 13)
4. the first order conditions for capital accumulation (equations 14)
5. the Home household's budget constraint (equation 28)
6. the household's first order conditions for asset purchases (equations 29 - 36)
7. the aggregate capital labour ratios (equations 16)
8. the firms' pricing decisions (equations 22 - 25)
9. the market clearing conditions for goods (equations 37 - 38)
10. the market clearing conditions for assets (equations 39 - 40)
11. the laws of motion for the exogenous processes (equations 43 - 44)
12. the monetary policy rule (equations 42)

However, we do not solve for this equilibrium. Instead, we solve for a linear approximation to this equilibrium around the nonstochastic steady state, applying the methods developed by Devereux and Sutherland (2008). The solution method consists of three steps. First, we solve for a first order approximate solution to the above equations around the nonstochastic steady state, conditional on the steady state portfolio. Secondly, we solve for the steady state portfolio as a function of the provisional first order approximate solution. Finally, we combine the provisional first order accurate solution and steady state portfolios to arrive at the full solution of the model. The appendix gives some more detail on the solution method.

3 General Results

As mentioned above, we focus on a first order accurate solution of the equilium. In the appendix, we present all the linear model equations. Here, we comment on several important relationships that hold in the linearised model. Firstly, all assets are equivalent to the first order:

$$E_t\left[\widehat{R}^H_{S,t+j}\right] = E_t\left[\widehat{R}^H_{B,t+j}\right] = E_t\left[\widehat{R}^F_{S,t+j}\right] = E_t\left[\widehat{R}^F_{B,t+j}\right] = \frac{1}{\beta}, j \geq 1$$

implying zero linearised return differentials in expectation. Note that our solution approach will use a second order approximation of returns in order to be able to solve for portfolios. Secondly, *realised* nominal returns in terms of Home currency for Home and Foreign one period nominal bonds are, in linearised form:

$$\widehat{R}^B_{H,t} = -E_{t-1}\left[-\sigma\left(\widehat{C}_{H,t} - \widehat{C}_{H,t-1}\right) + \left(\widehat{P}^H_{C,t-1} - \widehat{P}^H_{C,t+j-1}\right)\right] \tag{45}$$

$$\widehat{\widetilde{R}}^B_{F,t} = \widehat{R}^B_{F,t} + \widehat{Z}_t - \widehat{Z}_{t-1} \tag{46}$$

$$= -E_{t-1}\left[-\sigma\left(\widehat{C}_{H,t} - \widehat{C}_{H,t-1}\right) + \left(\widehat{P}^H_{C,t-1} - \widehat{P}^H_{C,t+j-1}\right) + \widehat{Z}_t\right], \tag{47}$$

which implies that the linearised return differential between Home and Foreign bonds is equal to unexpected nominal depreciation in the following period:

$$\widehat{\widetilde{R}}^B_{H,t} - \widehat{\widetilde{R}}^B_{F,t} = \widetilde{E}_t\left[-\widehat{Z}_t\right], \tag{48}$$

where $\widetilde{E}_t = E_t - E_{t-1}$. Thus, Home bonds will have a positive relative payoff when the Home currency unexpectedly appreciates, while they will offer a negative relative payoff if the Home currency unexpectedly depreciates. Relative equity returns are, to a first order approximation, given by:

$$\widehat{\widetilde{R}}^S_{F,t} - \widehat{\widetilde{R}}^S_{H,t} = (1-\beta)\,\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j} - \widehat{D}_{H,t+j} + \widehat{Z}_{t+j}\right)\right], \tag{49}$$

which implies that the relative return of Home equities is given by the present discounted sum of unexpected changes to all future dividends. Thus, bond returns only depend on an expectation of values in the following period, while equity returns depend on the infinite future.

The stochastic discount factors for Home and Foreign households are $\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma}\left(\frac{P^H_{C,t}}{P^H_{C,t+1}}\right)$ and $\left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\sigma}\left(\frac{P^F_{C,t}}{P^F_{C,t+1}}\right)$, respectively, or, linearising, $-\sigma\left(\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right)-\left(\widehat{P}^H_{C,t+1}-\widehat{P}^H_{C,t}\right)$ and $-\sigma\left(\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right)-\left(\widehat{P}^F_{C,t+1}-\widehat{P}^F_{C,t}\right)$. The discount factors enter the Euler equations for investment, (14), and the pricing equations for financial assets, (22) - (25). When markets are complete, the stochastic discount factors are equalised between Home and Foreign agents in all states of nature. With incomplete markets, however, the stochastic discount factors will in general differ between different investors. However, as is shown in the appendix, the stochastic discount factor of the Home and the Foreign agent are identical at this level of approximation, once we express them in the same units:

$$\begin{aligned} & E_t\left[-\sigma\left(\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right)-\left(\widehat{P}^H_{C,t+1}-\widehat{P}^H_{C,t}\right)\right] && (50)\\ = \; & E_t\left[-\sigma\left(\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right)-\left(\widehat{P}^F_{C,t+1}-\widehat{P}^F_{C,t}\right)\right]+\widehat{Z}_t-\widehat{Z}_{t+1}. && (51)\end{aligned}$$

In our benchmark model, there are two sources of uncertainty and two assets per country. With zero initial net foreign asset positions, it can then be shown that markets are complete to a first order approximation and we have perfect risk sharing. This means that, to a first order approximation, the decentralised equilibrium could be found as a solution to a planner's problem with the appropriate welfare weights. This property has the following implications. Firstly, it implies that the Backus-Smith-Kollmann (BSK) condition holds in linearised form:

$$-\sigma\left(\widehat{C}_{H,t}-\widehat{C}_{F,t}\right)=\widehat{P}^H_{C,t}-\widehat{S}_t-\widehat{P}^F_{C,t}=-\widehat{Q}_t. \tag{52a}$$

Thus, relative consumption is perfectly correlated with the real exchange rate. Note that unlike in closed economies, complete markets do not imply that the marginal utility of consumption is perfectly correlated, as it is now efficient for agents to consume less when prices are high. Secondly, the portfolios derived are independent of the covariance matrix of innovations. It also implies that all real quantities in this economy are stationary. Note that the previous result, that stochastic discount factors are equalised between the two countries once they are expressed in the same units, is not due to the fact that markets are complete to a first order approximation, but simply as a result of the order of approximation. Thus, in the case where the number of assets is smaller than the number of shocks, the result will still hold, while risk sharing will be imperfect.

The appendix also shows that the budget constraint of the Home agent can be written as:

$$NFA_{H,t} = NFA_{H,t-1}R^S_{H,t} + \xi_{H,t} + NX_{H,t}, \tag{53}$$

where the net foreign asset position of the Home agent, $NFA_{H,t}$, is given by Foreign assets held by the Home agent minus Home assets held by the Foreign agent:

$$NFA_{H,t} = S^H_{F,t}P^S_{F,t}Z_t + B^H_{F,t}Z_t - S^F_{H,t}P^S_{H,t} - B^F_{H,t}P^B_{H,t}. \tag{54}$$

$\xi_{H,t}$ is the excess return on the Home portfolio defined as the difference between actual net foreign assets at the beginning of period t and net foreign assets at period t had all wealth been invested in Home equity:

$$\begin{aligned} \xi_{H,t} &= \left(S^H_{H,t-1} - 1\right)\left(P^S_{H,t} + D_{H,t}\right) + S^H_{F,t-1}\left(P^S_{F,t} + D_{F,t}\right)S_t \\ &\quad + B^H_{H,t-1} + B^H_{F,t-1}S_t - NFA_{H,t-1}R^S_{H,t}. \end{aligned} \tag{55}$$

$NX_{H,t}$ are Home net exports defined as:

$$NX_{H,t} = Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}S_t - I_{H,t}P^H_{I,t} - P^H_{C,t}C_{H,t}. \tag{56}$$

Linearising (54) gives:

$$\widehat{NFA}_{H,t} = \widehat{NFA}_{H,t-1}\frac{1}{\beta} + \widehat{\xi}_{H,t} + \widehat{NX}_{H,t}, \tag{57}$$

where $\widehat{NFA}_{H,t} = \frac{NFA_{H,t}}{Y}$ and $\widehat{NX}_{H,t} = \frac{NX_{H,t}}{Y}$ and Y is steady state output.

Using (52a) and (57), we can then derive a partial equilibrium expression for equity holdings as a function of hedging motives (for details, see the appendix):

$$S = \frac{1}{2}\left(1 + \frac{(1-\beta)\left(Ccov_{R_t^B}\left(\widehat{R}_t^q, \widehat{R}_t^S\right) + cov_{R_t^B}\left(\widehat{\widetilde{R}}_t^Q, \widehat{R}_t^S\right)\right) - WLcov_{R_t^B}\left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{Dvar_{R_t^B}\left(\widehat{R}_t^S\right)}\right), \tag{58}$$

where $cov_{R_t^B}\left(\widehat{R}_t^q, \widehat{R}_t^S\right) = \widetilde{E}_t\left[\left(1-\frac{1}{\sigma}\right)\left(\widehat{Q}_t - P\left[\widehat{Q}_t|\widehat{R}_t^B\right]\right)\left(\widehat{R}_t^S - P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)\right]$, $cov_{R_t^B}\left(\widehat{R}_t^{\widetilde{Q}}, \widehat{R}_t^S\right) = \widetilde{E}_t\left[\left(1-\frac{1}{\sigma}\right)\left(\sum_{j=1}^{T}\beta^j\left(\widehat{Q}_{t+j} - P\left[\widehat{Q}_{t+j}|\widehat{R}_t^B\right]\right)\right)\left(\widehat{R}_t^S - P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)\right]$, $cov_{R_t^B}\left(\widehat{R}_t^W, \widehat{R}_t^S\right) = \widetilde{E}_t\left[\left(\widehat{R}_t^W - P\left[\widehat{R}_t^W|\widehat{R}_t^B\right]\right)\left(\widehat{R}_t^S - P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)\right]$, $var_{R_t^B}\left(\widehat{R}_t^S\right) = \widetilde{E}_t\left(\widehat{R}_t^S - P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)^2$ and D, W, L, S are steady state dividends, wages, labour, and equity holdings, respectively. $\widehat{R}_t^W$ is the (linearised) relative return to human capital defined as: $\widehat{R}_{H,t}^W - \widehat{R}_{F,t}^W - \widehat{Z}_t = (1-\beta)\,\widetilde{E}_t\sum_{j=0}^{\infty}\beta^j\left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} - \widehat{W}_{F,t+j} - \widehat{L}_{F,t+j} - \widehat{Z}_{t+j}\right)$.

While condition (58) is not structural, it provides some useful intuition about the drivers of equity positions in this model. It is also worth noting that this condition holds in a fairly general class of models, i.e. all dynamic models with complete markets, trade in equities and one period nominal bonds and labour income as the only nontradable source of income.

According to (58), the optimal equity position depends on four terms. The first term shows that this model would generate perfectly diversified portfolios $\left(S^S = \frac{1}{2}\right)$ in the absence of any concern for the real exchange rate or labour income, i.e. for $\sigma = 1$ and $WL = 0$. The second and third term arise from a motive to hedge movements in personal consumption expenditures which are here perfectly correlated with the real exchange rate[4]. The BSK condition, (52a) indicates that optimal risk sharing implies that consumption falls when the real exchange rate appreciates and the fall in consumption is larger the larger is the intertemporal elasticity of substitution. For high values of the intertemporal elasticity of substitution $(\sigma < 1)$, relative consumption falls so much in response to a rise in the real exchange rate that relative consumption expenditure also falls. Since optimal consump-

[4]Note that a rise in Q here denotes a real exchange rate *depreciation*

tion spending would be lower in the case when the real exchange rate appreciates, agents would, ceteris paribus, prefer to hold an asset that has low payoffs when the real exchange rate appreciates. For the more realistic case of low intertemporal elasticity of substitution ($\sigma > 1$), agents would prefer to hold an asset that pays more when the real exchange rate appreciates. Finally, the last term arises from a motive to hedge movements in the return to human capital and it implies that agents would prefer to hold an asset that has high payoffs when the returns to human capital are low, and vice versa. Note, however, that nominal bonds can also be used to hedge some or all of these sources of risk. What is important for equity positions is thus not the unconditional comovement of equity returns with these sources of risk, but their comovement conditional on the returns to the nominal bond.

It is instructive at this point to compare this expression with equivalent ones implied by other papers in the literature. In Heathcote and Perri (2008), there are no nominal rigidities and equities are the only asset traded. Since their benchmark model also only features one shock, markets are effectively complete. This implies that the reduced form expression analogous to (58) would be very similar, with the main difference being that the equity position only depends on the the unconditional comovement between relative equity returns and the sources of risk. In Coeurdacier et al. (2008), prices are flexible and infinitely lived real bonds are traded instead of nominal bonds. This implies that while the relevant covariances are conditional on bond returns, it is the real bond returns that we have to condition on. Since the payoff of the real bonds is perfectly correlated with current and future real exchange rates, movements in the real exchange rate are completely hedged using real bonds and the equity positions are determined by the conditional covariance between relative equity returns and returns to human capital. Coeurdacier and Gourinchas (2008) provide an analysis of a very general class of models in a static setting. Their expressions would thus often look similar to (58), with the key difference being that the terms would only involve contemporaneous covariances. Finally, Engel and Matsumoto (2008) analyse a model with nominal rigidities and complete markets in a dynamic setting which would imply the same partial equilibrium expression that we present here.

We can derive a similar expression for bond holdings which is:

$$B = \frac{1}{2}\left(\frac{Ccov_{R_t^S}\left(\widehat{R}_t^q, \widehat{R}_t^B\right) + Ccov_{R_t^S}\left(\widehat{\widetilde{R}}_t^Q, \widehat{R}_t^B\right) + \frac{WL}{1-\beta}cov_{R_t^S}\left(\widehat{R}_t^W, \widehat{R}_t^B\right)}{Dvar_{R_t^S}\left(\widehat{R}_t^B\right)}\right), \tag{59}$$

where $cov_{R_t^S}\left(\widehat{\widetilde{R}}_t^Q, \widehat{R}_t^B\right) = \widetilde{E}_t\left[\left(\left(1-\frac{1}{\sigma}\right)\sum_{j=1}^{T}\beta^j\left(\widehat{Q}_{t+j} - P\left[\widehat{Q}_{t+j}|\widehat{R}_t^S\right]\right)\right)\left(\widehat{R}_t^B - P\left[\widehat{R}_t^B|\widehat{R}_t^S\right]\right)\right]$, $cov_{R_t^S}\left(\widehat{R}\right.$ $\widetilde{E}_t\left[\left(\left(1-\frac{1}{\sigma}\right)\left(\widehat{Q}_t - P\left[\widehat{Q}_t|\widehat{R}_t^S\right]\right)\right)\left(\widehat{R}_t^B - P\left[\widehat{R}_t^B|\widehat{R}_t^S\right]\right)\right]$, $cov_{R_t^S}\left(\widehat{R}_t^W, \widehat{R}_t^B\right) = \widetilde{E}_t\left[\left(\widehat{R}_t^W - P\left[\widehat{R}_t^W|\widehat{R}_t^S\right]\right)\left(\widehat{R}\right.\right.$ and $var_{R_t^S}\left(\widehat{R}_t^B\right) = \widetilde{E}_t\left(\widehat{R}_t^S - P\left[\widehat{R}_t^B|\widehat{R}_t^S\right]\right)^2$. The bond position is determined by three terms, which are very similar to the terms that determine the equity portfolio. The first and second terms are again related to the desire to hedge movements in the real exchange rate and imply that, for $\sigma > 1$, agent go long in domestic bonds, if bond returns are negatively correlated with the real exchange rate, while they hold positive positions in domestic bonds if $\sigma < 1$. The motive to hedge movements in the return to human capital imply that domestic bonds are held if their returns are negatively correlated with the returns to human capital. Again, as in the case of equity, the relevant covariances are conditional, in this case conditional on relative equity returns.

The expressions in (58) and (59) are very useful, because they consist of the regression coefficients of the following regressions:

$$\widehat{R}_t^W = \beta_{w,s}\widehat{R}_t^S + \beta_{w,b}\widehat{R}_t^B + \varepsilon_{w,t} \tag{60}$$

$$\widehat{R}_t^q = \beta_{q,s}\widehat{R}_t^S + \beta_{q,b}\widehat{R}_t^B + \varepsilon_{q,t} \tag{61}$$

$$\widehat{R}_t^{\widetilde{Q}} = \beta_{\widetilde{q},s}\widehat{R}_t^S + \beta_{\widetilde{q},b}\widehat{R}_t^B + \varepsilon_{\widetilde{q},t}. \tag{62}$$

Below, we thus use (58) and (59) as well as the regression coeffients obtained from (60) to (62) extensively in order to gain intuition about the drivers of portfolios.

3.1 Calibration

The parameters that need to be calibrated in our benchmark model are $\beta, \rho_A, \rho_M, \sigma, \phi, a_C, a_I, \delta, \alpha, \iota, \theta, \varepsilon, \omega,$ and the elements of the covariance matrix Σ. We adopt a benchmark calibration that closely follows the literature on international business cycles and portfolios, whenever possible (e.g. Backus et al. (1994), Chari et al. (2002), Coeurdacier et al. (2007, 2008), Heathcote and Perri (2008)). The model is assumed to run at quarterly frequency and we thus set the discount factor β to 0.99 which implies an annual steady state real interest rate of 4.1%. The rate of depreciation is set to $\delta = 0.025$ implying an annual rate of depreciation of almost 10%. The degrees of consumption and investment home bias are set to $a_C = a_I = 0.85$, implying a steady state import/GDP ratio of 15%.The elasticity of output with respect to labour, α, is set to 0.66. The elasticity of substitution between individual varieties, ε, is set to 10, implying a steady state markup of 11%. The elasticity of substitution between Home and Foreign goods, ϕ, is set to 1.5 and the risk aversion coefficient is set to be $\sigma = 2$. This implies that the share of consumption in total output is 0.78. The parameter governing the disutility of labour is set to $\iota = 9.7$. We set $\theta = 0.75$, implying that one quarter of firms can change prices during every quarter and an average life of prices of one year. The elasticity of labour supply, ω, is set to one, which, together with i, ε, and α implies a labour share of 0.59. The persistence of the productivity and interest rate shocks are set to $\rho_A = 0.91$ and $\rho_M = 0.9$, respectively. As mentioned above, markets are complete to a first order which implies that portfolios are not sensitive to changes in the covariance matrix. However, the covariance matrix does, of course, matter for the volatility and the comovement of many real variables in the system. In our benchmark, the covariance matrix is given by:

$$\Sigma = \begin{bmatrix} 0.012 & 0.0054 & 0 & 0 \\ 0.0054 & 0.012 & 0 & 0 \\ 0 & 0 & 0.012 & 0 \\ 0 & 0 & 0 & 0.012 \end{bmatrix}.$$

4 Portfolios

In this section, we compute optimal portfolios in a variety of settings. First, we consider a model without nominal rigidities in order to illustrate the main drivers arising from the possibility of capital accumulation. In such a model, optimal portfolios involve zero positions in nominal bonds, while domestic equity is a good hedge for both human capital and real exchange rate risk, leading to equity home bias. We then analyse portfolios in a model without capital accumulation, but with price rigidities. In such a model, equities are a good hedge for human capital risk, while nominal bonds are a good hedge for real exchange rate risk, for sufficient degrees of price rigidity. In the general model with both capital accumulation and nominal rigidities, the investment channel dominates. This again implies that equities are a good hedge for real exchange rate risk, and in many cases a better hedge for real exchange rate risk than nominal bonds, even for high degrees of nominal rigidity or monetary policy responsiveness. This result is quite robust in a variety of settings, including other values for the intertemporal elasticity of substitution, the elasticity of substitution between Home and Foreign goods, with long term bonds, different monetary policy functions or pricing regimes, consumption and labour shares, shock persistence, or investment shocks. Shocks to equity prices make portfolios more robust by shutting down equity trade completely.

4.1 Model without Nominal Rigidities

First, we consider a version of the model where all firms are free to set prices in all periods. This model is very similar to the model in Heathcote and Perri (2008) and generates the same equity portfolio for their parameterisation ($\varepsilon \to \infty, \sigma = 1, \phi = 1$). There are four main differences to the model in Heathcote and Perri (2008): i) we allow for monopolistic competition, ii) we introduce a monetary authority that sets interest rates, iii) we introduce a second shock, namely shocks to the nominal interest rate, and iv) we allow for the trading of nominal bonds, but it turns out that the only relevant changes for portfolios are our different parameterisations, including the degree of monopolistic competition. Even these effects, however, are only quantitative and of limited size. Trading of nominal bonds, interest rate shocks and the monetary policy rule all have no effect on equity portfolios, as will be clear below, and optimal bond portfolios are always zero.

In order to build intuition for the structure of optimal portfolios and the mechanics of models with capital accumulation, it is useful to focus on the effects of productivity shocks. Figures 1 and 2 present the impulse responses to a positive relative shock to Home productivity for a number of Home variables. A positive relative shock to Home productivity implies that Home productivity rises, but Foreign productivity falls by the same amount to leave average world productivity unchanged. Due to the symmetry of the two countries in the model, the responses of Foreign variables will be the exact mirror image of the responses of Home variables and we can ignore them without loss of information. In response to a positive relative Home productivity shock Home output rises. This rise in Home output is initially driven by the increase in Home productivity and a rise in Home labour, but high investment in the initial periods also implies an increase in the capital stock which makes the rise in Home output more persistent. Home labour output rises in the initial periods, as wages rise and the substitution effect outweighs the income effect. The rise in wages is driven by the increase in productivity and an increase in the demand for consumption and investment. The increase in investment is driven by two forces: a desire to save in order to smooth consumption and a desire to build up capital, as it is now more productive. The positive relative shock to Home productivity also implies a real depreciation of Home currency due to the increase in the supply of the Home good which is more prominent in the Home consumption basket.

The behaviour of nominal variables is strongly affected by monetary policy. With our parameterisation, monetary policy responds moderately strongly to CPI inflation. Initially, the Home CPI rises, as demand for foreign goods is high. The nominal exchange rate depreciates to help bring about the real depreciation. Over time, import demand falls and the price of the Home good continues to fall. However, as the supply of the Home good returns to the steady state level, a real *appreciation* is required and this requires a nominal appreciation. A different way of looking at exchange rate dynamics is that the fall in the Home price level induces a fall in Home nominal interest rates. In order for expected bond returns to be equalised, a nominal appreciation of the Home currency is then necessary. It is worth noting here that the monetary authority stabilises CPI *inflation* and not the price level and that we therefore do not observe that prices return to the steady state level.

Now that we have analysed the basic responses in this economy, let us look at the hedging demands identified in (58)[5]. Above, we identified real exchange rate movements and movements in the returns to human capital as the drivers of portfolio decisions. As mentioned above, the real exchange rate depreciates initially and only slowly converges from above to its steady state value. Thus, for $\sigma > 1$, in order to hedge movements in the real exchange rate, agents would like to hold an asset that has low payoffs in response to a positive productivity shock. We also noted that Home wages and labour hours increased in response to a productivity shock. Relative returns to human capital are also affected by the nominal exchange rate which first depreciates and then appreciates, but it can be shown that relative returns to Home human capital increase in response to a productivity shock. Thus, again, agents would prefer to hold an asset that has a low payoff in response to a positive productivity shock. So how do asset returns respond? (49) shows that relative equity returns depend on the expected discounted value of future dividends. Dividends in turn are the sum of profits and capital income, minus investment. Home profits and capital income increase, despite a rise in the Home wage, but this effect is dominated by the increase in investment which is also more important than the movement in nominal exchange rates. The strong response of investment implies a persistent negative response of relative dividends, implying that relative equity returns fall in response to a productivity shock. This means that Home equity is in principle a good hedge for both real exchange rate risk and human capital risk. Relative nominal bond returns are equal to unexpected nominal exchange rate appreciation. Since the nominal exchange rate initially depreciates, nominal bonds can thus also be

[5]The correct partial equilibrium expression for portfolios is slightly different in this case. See the appendix for details.

a good hedge for both real exchange rate and human capital risk in response to productivity shocks.

What about responses to monetary shocks? In this model - with perfect price flexibility -, monetary shocks only affect nominal variables in equilibrium - all prices and the nominal exchange rate. Thus, all real variables are unaffected by monetary shocks, as long as the net foreign asset position is not affected. Consequently, relative equity returns and relative returns to human capital and the real exchange rate are also unaffected. The nominal exchange rate, however, does move in response to monetary shocks and so do relative bond returns. Thus, Home equity is again a good hedge for real exchange rate and human capital risk, while, here, nominal bonds are not. In fact, the only way for agents to insulate themselves from the monetary shock is by holding zero bond positions. We can then think of portfolios in two ways. Structurally, domestic equity is used to hedge against productivity shocks, while bonds are used to hedge nominal shocks. Another way to think about it is that Home equity is used to hedge both human capital and real exchange rate risk, while bonds do not contribute to insure against these sources of uncertainty. Note that since markets are complete to a first order, these portfolios achieve perfect risk sharing at this level of approximation. It is worth remembering that the main mechanism that drives portfolios here is capital accumulation: The response of investment implies that returns to human capital and relative equity returns are negatively correlated. The response of investment also implies that relative equity returns and the real exchange rate are negatively correlated. Finally, the possibility of investment increases the persistence of the model potentially increasing asset positions.

Tables A1 and A2 in the appendix give portfolios for a variety of parameter combinations and decompose asset positions into the parts driven by real exchange rate and human capital risk. Here, we only comment on a few major characteristics. As already noted by Heathcote and Perri (2008), this model can generate significant home bias in equity portfolios. The home bias in equity portfolios is driven both by the motive to hedge movements in real exchange rates and by the motive to hedge labour income risk. It is worth noting that, quantitatively, the motive to hedge movements in real exchange rates is more important for equity portfolios than the desire to smooth human capital risk. In general, the model generates too much home bias, often involving short positions in foreign equity. Portfolios are discontinuous in some parameters. For example, for high enough values of the elasticity of substitution between Home and Foreign goods, relative equity returns respond positively to productivity shocks, making them poor hedges for human capital and real exchange rate risk and implying short positions, often large in magnitude, in domestic equity. Finally, as already mentioned, optimal international asset portfolios involve zero bond positions which are clearly counterfactual. Monetary policy, here simply parameterised by γ, only affects the response of nominal variables and thus has no effect on portfolios.

4.2 Model without Investment

We now consider a model with nominal price rigidities, but without investment. To that purpose, we set the rate of depreciation, δ, to zero and assume investment to be zero at all times, implying that the capital stock in each country is fixed. Otherwise, we keep all other elements of the model unchanged. This model is very similar in spirit to the models presented in Engel and Matsumoto (2008a,b). The major differences are the type of price stickiness and the nature of monetary policy. Engel and Matsumoto (2008a,b) assume that a fraction of firms chooses prices before shocks are observed, while the remainder of firms is free to choose prices after observing the shocks. They further assume that agents derive utility from holding real money balances and make the supply of nominal money balances exogenous and stochastic.

Both here and in Engel and Matsumoto (2008a,b), money has real effects due to the presence of nominal rigidities: Since not all firms can change prices, changes in the nominal interest rate also imply changes in the real interest rate which in turn affect other real variables. Figure 3 and 4 present impulse responses to productivity shocks in this model with our benchmark calibration, while figures 5 and 6 present the responses to monetary shocks.

Following a relative Home productivity shock, Home output increases, but by less than the increase in productivity, as labour demand falls. As productivity rises, marginal costs fall. With perfect price flexibility, optimal prices would also fall, as they are a markup over marginal costs. In this model, optimal prices also fall, as firms would like to reduce prices in order to increase the quantity produced and sold. Due to the price rigidities, however, not all firms can reduce prices, so demand cannot expand as strongly as with flexible prices. Thus, with sufficient degrees of price flexibility, firms cut labour demand, as they can now produce more output with the same amount of labour input. Since those firms that are allowed to change prices, do cut them, the Home price level falls. As in the model without nominal rigidities, the real exchange rate depreciates due to the increase in the supply of the Home good, while the nominal exchange rate first depreciates, but appreciates in the long run. Firm profits and dividends rise, as productivity rises and labour demand falls and this effect outweighs the fall in prices. Due to the rise in dividends, relative Home equity returns increase, while the relative returns to labour income are negative, as labour demand falls. A long position in domestic equity is then a good hedge for human capital risk, but a poor hedge for real exchange rate risk. Nominal bond returns are positive, as the nominal exchange rate initially depreciates. Thus, nominal bonds are a good hedge for real exchange rates, but a poor hedge for human capital.

Following a monetary shock that increases nominal interest rates, real interest rates increase. The increase in real interest rates reduces domestic demand, leading to a fall in output and labour demand. The fall in demand induces a fall in prices further increasing real interest rates. Since output of the Home good has fallen, its relative price should increase which, due to local bias in consumption implies a real appreciation of the Home currency. Since Home prices have fallen, this implies that the nominal exchange rate has to appreciate by more than the appreciation of the real exchange rate. Despite the positive shock to nominal interest rates, nominal interest rates end up falling initially, as the central bank can react contemporaneously to the fall in inflation. Relative Home dividends increase, as the fall in Home dividends is more than compensated by the increase in the nominal exchange rate. The persistent rise in relative Home dividends implies a positive response of relative Home equity returns. For relative Home labour income, however, the appreciation of the nominal exchange rate is not sufficient to compensate for the fall in nominal Home labour income and returns to human capital are negative. Thus, domestic equity is again a good hedge for human capital. Since the real exchange rate appreciates, domestic equity is also a good hedge for real exchange rate fluctuations. Since the nominal exchange rate initially appreciates, relative nominal bond returns are positive and bonds are also good hedges for human capital and real exchange rate risk.

We thus find that, for sufficiently sticky nominal goods prices, domestic equity is a good hedge for human capital risk under both shocks, while nominal bonds are good hedges for real exchange rate risk. This intuition is confirmed when looking at the optimal portfolios in this economy. Again, we defer a more detailed discussion of portfolios to the appendix, but comment only on the most salient features here. Firstly, we can see that for sufficiently sticky nominal goods prices, equity portfolios exhibit realistic home bias, as the share of domestic equities held is between one half and one. Equity portfolios are mainly determined by a motive to hedge movements in the returns to human capital, while mainly the nominal bonds are used to hedge movements in the real exchange rate. A relatively high degree of price stickiness is crucial for this result: As the degree of price stickiness falls, equity portfolios become less home biased, for two reasons. Firstly, as prices become less sticky, returns to equity and human capital become more correlated, until at some point, the correlation becomes positive and a short position in domestic equity is required to hedge human capital risk. Secondly, as price stickiness falls, nominal bond returns become less closely related to movements in the real exchange rate and equity positions correspondingly take one a bigger role in hedging real exchange rate risk also. Since equity returns and the real exchange rate are positively correlated, this implies again that agents would be pushed towards holding a negative position in domestic equity to hedge real exchange rate risk.

4.3 Model with Nominal Rigidities and Investment

We now consider a model that features both nominal rigidities and investment. Both nominal rigidities and investment are widely assumed to be important for business cycle fluctuations though many models continue to feature only one of the two properties. In open economy settings, models similar to ours have been used to generate realistic real exchange rate dynamics (Chari et al. (2002)) and cross country correlations in output and asset returns (Kollmann (2001)). Our contribution is to introduce nontrivial portfolio choice with trade in nominal bonds and equities into these models.

Figures 5 to 6 present the impulse responses of various variables to a relative Home productivity shock. As we can see, this model shares many qualitative features with the model without nominal rigidities. However, the combination of nominal rigidities and capital accumulation endogenously creates substantial persistence in the response to shocks. Home output rises and the rise in output is initially magnified by an increase in labour hours and driven both by an increase in consumption and investment. The resulting increase in the capital stock makes the rise in Home output both stronger and more persistent. The increase in investment and consumption demand induce an increase in the demand for labour in the initial periods after the shock, while the increase in labour demand and the rise in productivity induce an increase in the Home wage. The rise in consumption and investment drives an initial negative response of net exports. The rise in imports also drives an increase in the Home price of the Foreign good which in turn induces an increase in the Home price level. The real exchange rate depreciates due to the increase in the supply of the Home good and the local bias in consumption and investment. Since the Home CPI initially rises, a real depreciation necessitates a nominal depreciation. Over time, however, as Home demand and therefore prices fall, the nominal exchange rate appreciates. As in the model without nominal rigidities, relative Home dividends fall, mainly due to the rise in investment. The persistent fall in relative dividends implies a negative relative equity return. Due to an increase in both labour demand and wages, Home labour income and the returns to human capital respond positively, implying that, again, Home equity is a good hedge for domestic labour income risk. Since the real exchange rate depreciates, Home equity is again also a good hedge for real exchange rate risk. Relative returns of Home nominal bonds are negative, as the nominal exchange rate initially depreciates, which implies that Home nominal bonds are in principle also good hedges for both human capital and real exchange rate risk.

Now consider the monetary shock. Due to nominal price rigidities, this monetary shock now has real effects. In particular, a positive relative shock to the Home nominal interest rate implies a rise in the Home real interest rate also. This rise in the real interest rate chokes off domestic demand for both consumption and investment. The fall in demand induces a fall in prices. Since the central bank here can react even to a contemporaneous fall in inflation, it lowers the nominal interest rate, and to an extent that the nominal interest rate in fact falls despite the positive monetary shock. Note, however that prices still fall more so that real interest rates increase. The fall in demand induces a fall in output which, with unchanged productivity, translates initially into a fall in labour demand and a decrease in the demand for investment. The fall in Home output induces a relative increase in the scarcity of the Home good and thus a real Home appreciation. Since Home prices fall, this implies an even stronger nominal Home appreciation. Both the Home wage and labour demand fall, implying a fall in Home labour income and the returns to human capital. Home dividends increase, as profits increase due to lower labour costs and the nominal appreciation, while investment falls. The persistent rise in dividends implies positive relative returns to Home equity. This implies that Home equity is again a good hedge for human capital risk and real exchange rate risk. Similarly, Home nominal bonds are also a good hedge for real exchange rate and human capital risk.

Thus the model with both nominal rigidities and investment behaves more like the model without nominal rigidities when considering the productivity shock, and more like the model without investment when considering the monetary shock. Overall, however, capital accumulation remains an important driver of the behaviour of many variables, including portfolio decisions. In particular, we saw that relative equity returns fell in response to a productivity shock, as in the model without nominal rigidities, while they rose in response to a monetary shock, unlike in the model without investment. The returns to human fall increase in response to a productivity shock and fall in response to a monetary shock, just as in the model without price rigidities and unlike in the case of the model without investment. The behaviour of nominal and real exchange rates is qualitatively much like in the case without investment. But remember that for virtually all variables, the persistence of the responses to shocks is magnified by the combination of nominal rigidity and capital accumulation. So what does that imply for portfolios? As noted before, both equities and nominal bonds are in principle good hedges for both real exchange rate and human capital risk in this model. What is more, neither asset is perfectly correlated with one of the sources of risk so that, both assets are in general affected by both sources of risk and asset returns are highly correlated. What is more, for our benchmark parameterisation, equity returns and nominal bond returns are highly correlated. This implies that in order to span the space of consumption allocations, agents may ideally hold large and offsetting positions. What is more, the fact that assets are highly correlated implies that asset positions can be quite sensitive to the value of certain parameters and are not easy to understand. Finally, the high endogenous persistence of the model will also tend to produce large asset positions.

With our benchmark calibration, agents take short positions in domestic equity and large long positions in domestic bonds. Both bonds and equity are highly correlated with real exchange rate risk, but the correlation is marginally higher for bonds which explains their long position. The equity position is negative due to offsetting the effect of nominal bonds when it comes to real exchange rate hedging, while human capital risk actually makes a positive contribution to equity home bias. Thus, unlike Coeurdacier et al. (2008), we do not find that portfolios are robust, because different sources of risk are allocated to different assets. So why are our results so different? Firstly, in Coeurdacier et al. (2008), infinitely lived real bonds are traded. With infinitely lived real bonds, relative bond returns are perfectly correlated with the real exchange rate. It can then be shown that real exchange rate risk is entirely hedged using bonds and does not affect equity positions. Equity positions then solely depend on the conditional covariance with human capital returns. More generally, in order to generate robust portfolios, we need to reduce the correlation between the two assets, and achieve that the relative correlation of equity with respect to real exchange rate risk is substantially different from the relative correlation with respect to human capital risk. So why are portfolios so unstable in this model? There are a number of reasons. Firstly, the real exchange rate motive is very prominent here and it is this motive that drives extreme portfolio positions. The real exchange rate motive is large, because the size of consumption spending in this model is relatively large (more on this below). Secondly, equity is a good hedge for real exchange rate risk. This property is quite closely linked to the possibility of capital accumulation. Capital accumulation implies that relative equity returns are negatively correlated with the real exchange rate and the correlation is very strong here, as there is no strong force to separate movements in investment from changes in output. Thirdly, while nominal bonds are highly correlated with real exchange rate risk, they are not necessarily better hedges for this source of risk than equities. The association between nominal bond returns and real exchange rate risk is weakened by two forces. Firstly, there is the presence of nominal risk. Secondly, bonds only have a maturity of one period and are thus unlikely to be perfectly correlated with real exchange rate risk of infinite maturity. Note, however, that longer maturity of bonds does not necessarily increase its usefulness as an asset to hedge long run risk. Below, we discuss the effects of several features of the model and present portfolios for many parameter configurations. However, we find that it is surprisingly difficult to come up a model that generates robust portfolios along the lines of Coeurdacier et al. (2008).

4.3.1 Intertemporal Elasticity of Substitution/ Risk Aversion

As is well known, standard macroeconomic models have great difficulties to explain many asset pricing facts. In particular, some authors (see e.g. Mehra and Prescott (1985)) find that very high degrees of risk aversion are necessary to explain equity risk premia. We thus investigate the robustness of our results with respect to different values for the coefficient of relative risk aversion σ. As can be seen in table 1a, higher values of σ cannot generate more realistic asset positions. On the contrary, we find that higher degrees of risk aversion increase asset position. This implies that positions in Foreign equity become even larger meaning that agents take larger short positions in Home equity. Home bias in equities is negative even for low values of σ. The large foreign bias in equity positions is mainly due to real exchange rate hedging, while the human capital motive makes a positive contribution to equity positions for sufficiently high degrees of risk aversion. Optimal bond positions are always positive and also increase with σ. The increase is again mainly driven by the real exchange rate motive and bond positions quickly become very large, as agents have a stronger motive to hedge risk. Asset positions are mainly affected by an increased desire to hedge risk, but asset correlations also change somewhat. In general, we find that the correlation between bond returns and current and future exchange rates is more strongly negative than for equity returns. Thus, for $\sigma > 1$, long positions in bonds are used to hedge consumption expenditure. As σ rises, investment reacts more strongly to shocks, and equity thus becomes a better hedge for human capital risk.

4.3.2 Elasticity of Substitution between Home and Foreign Goods

Cole and Obstfeld (1991) recognised that in models with more than one goods, diversification using assets may not be necessary in order to hedge the international dimension of risk. This is because changes in relative prices can provide some insurance without explicit diversification of portfolios. The strength of the response of relative prices is strongly affected by the elasticity of substitution between Home and Foreign goods, ϕ. Correspondingly, this parameter has a relatively large influence on portfolios in many settings, including Engel and and Matsumoto (2008a,b) and Heathcote and Perri (2008). This is also true here, as can be seen in table 1b. However, the table also shows that there is only a small range for ϕ that can produce a realistic degree of home bias in equity positions. Equity positions become more negative, as we increase ϕ, turning from excessive home bias to large negative positions in domestic equity. This is mainly due to the effect of human capital hedging. As ϕ increases, the response of relative prices becomes smaller. This implies that relative equity returns become less negatively correlated with human capital returns, and the correlation in fact ultimately turns positive. Thus, human capital hedging increase home bias for low values of ϕ, while it increases foreign bias for high values of ϕ. The real exchange rate motive induces foreign bias in equity positions, but its effect becomes progressively smaller, as ϕ increases, as the effect of changes in relative prices becomes smaller. Only for ϕ close to one, are equity positions of a realistic magnitude. However, even in that case, realistic equity positions arise as the result of the fact that the contributions of the two hedging motives offset each other to a large extent, implying that the resulting portfolio is not very robust. Positions in Home bonds rise strongly, as ϕ increases, moving from large negative to large positive positions. This is mainly driven by human capital hedging. For low values of ϕ, bonds and equities have similar hedging properties for human capital, implying opposite effects of human capital on equity and debt. For higher values of ϕ, bonds become better at hedging human capital risk, implying long positions in bonds. Again, as ϕ rises, the effects of the real exchange rate motive become smaller. It is worth noting that while ϕ has a large effect on asset positions, portfolios are not discontinuous in ϕ, unlike in Heathcote and Perri (2008), Coeurdacier (2008) and the model without nominal rigidities presented above. This is due to the fact that, as long bond positions become less good at hedging real exchange rate positions, progressively more hedging of human capital risk is done using the bonds, as in the model without investment above or Engel and Matsumoto (2008a,b).

4.3.3 The Degree of Nominal Rigidity

As shown above, with perfect price flexibility, optimal bond positions are always zero. As noted above, this is the result of two effects. Firstly, monetary shocks are then pure noise and only affect prices and nominal exchange rates. Zero bond positions then allow agents to avoid letting the noise affect net foreign asset positions. Secondly, trade in equities is enough to allow perfect risk sharing With nominal rigidities, however, money is no longer neutral. This implies that shocks to the interest rate have an effect on real variables. What is more, trade in equities is no longer sufficient to complete the market. Both of these effects imply that optimal bond positions are no longer zero for $\theta > 0$. In table 2a, we present portfolios for different degrees of price rigidity. An increase in θ has several effects that are relevant for portfolios. For low levels of θ, portfolios are, as expected, very similar to the ones in the model without nominal rigidities. Relative equity returns are strongly correlated with both human capital and real exchange rate risk and home equity bias is too strong as a result. Bond positions are small and negative, as nominal bond returns are very noisy. As θ increases, the contributions of all hedging motives become larger and home bias in equity and positions in foreign bonds become larger as a result. This is because equity and bond returns become more correlated, implying that agents take larger offsetting positions in order to span the space of allocations. The correlation between the two asset returns at some point becomes so high that asset positions become very large. As mentioned before, initially equity returns are more highly (negatively) correlated with real exchange rate risk than bond returns. However, as θ increases, the correlation between equity returns and real exchange rate risk falls, while the correlation between bond returns and real exchange rate risk is only moderately negative for low values of θ, but increases quickly. We can see that for a certain value of θ, θ^*, the correlation between bond returns and real exchange rates becomes larger than for equity returns and we observe a discontinuity in portfolio positions. For values of θ larger than θ^*, the nominal bonds are better hedges for real exchange rates than equity and drive positive positions in domestic bonds. For $\theta > \theta^*$, initially optimal positions in Home equity are negative, as the correlation between equity and bond returns remains high and implies offsetting positions in the two assets. As θ further increase, the difference between the correlation of bond returns with real exchange rate risk and of equity returns becomes larger, implying that equity returns are then mainly used to hedge human capital risk, while bonds are used hedge real exchange rate risk. There are thus some values of θ for which we observe realistic degrees of home bias, though the appropriate degrees of price rigidity are somewhat outside the range used in the literature. It is also worth noting that in this model, the contribution of human capital hedging to Home equity bias is always positive, as the correlation between relative equity returns and human capital returns is negative even at low levels of price rigidity, due to the response of investment.

However, we cannot arbitrarily increase the degree of nominal rigidity in this model, for the following reason. As shown by Carlstrom and Fuerst (2000), compared to standard New Keynesian models, the addition of investment spending makes models with simple monetary rules more likely to exhibit real indeterminacy. By real indeterminacy, it is meant that the behaviour one of more real variables is not pinned down by the model. In particular, Carlstrom and Fuerst (2000) show that the Taylor principle does no longer hold, i.e. that it is not sufficient for real determinacy that monetary policy responds at least one-to-one to increases in expected inflation. We find that our model with the benchmark calibration exhibits real indeterminacy for values of θ larger than around 0.8, whereby the precise cutoff value for $\theta, \widetilde{\theta}$, above which the model displays real indeterminacy, depends on a number of other parameters. In particular, as expected, it depends on the response of monetary policy. We thus find that the model is determinate for $\theta = 0.85$ for $\gamma > 3$ and for $\theta = 0.95$ for $\gamma > 25$, which are the values used in table 2a and throughout the rest of the paper whenever we consider values degrees of nominal rigidity above 0.8. It is worth noting that while the model generates realistic and robust levels of equity home bias and bond portfolios for these parameters, they lie somewhat outside the range of values for these parameters that the theoretical and empirical literature regards as plausible.

4.3.4 Monetary Policy

As in all New Keynesian models, monetary policy has important welfare effects due to inefficiencies present in these models. Above, we also commented on the role of monetary in bringing about real determinacy. In addition, as already emphasised by Devereux and Sutherland (2007b, 2008b), monetary policy also affects portfolios by affecting asset returns. In particular, monetary policy strongly affects the dynamics of nominal exchange rates and thus the behaviour of bond returns. In Devereux and Sutherland (2007b, 2008b), monetary policy was irrelevant, as long as markets were complete to a first order approximation. Here, however, monetary policy has an effect even with first order complete markets. Of particular interest here is whether a strongly responsive monetary policy, by making prices less responsive to shocks, can increase the association between nominal and real exchange rates and thus increase the extent to which nominal bonds can hedge against real exchange rate fluctuations. However, as can be seen in table 3, the effects of changes in γ on portfolios are both qualitatively and quantitatively insignificant. This is because due the short maturity of nominal bonds, monetary policy only has a limited effect on the properties of bond returns. Thus, the stance of monetary policy, ceteris paribus, does not allow the model to produce more realistic portfolios.

4.3.5 Long Term Bonds

In our benchmark model, agents trade in equities and one period nominal bonds. Bond returns are then given by the unexpected contemporaneous appreciation of the nominal exchange rate. rates. It could then be conjectured that the reason why bonds are not more closely related with real exchange rate risk is the short maturity of the bonds and that by increasing the maturity of the bonds, realistic and robust a la Coeuracier et al. (2008) could be generated. We therefore now allow agents to trade equities and infinitely lived nominal bonds. These bonds pay one unit of domestic currency in each period. Bond prices are then, in domestic currency:

$$\begin{aligned} P_{i,t}^{B,long} &= \beta E_t \left[\left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-\sigma} \left(\frac{P_{C,t}^i}{P_{C,t+1}^i} \right) \left(1 + P_{i,t+1}^{B,long} \right) \right] \\ &= \beta E_t \left[\sum_{j=1}^{\infty} \left(\frac{C_{i,t+j}}{C_{i,t}} \right)^{-\sigma} \left(\frac{P_{C,t}^i}{P_{C,t+j}^i} \right) \right] \end{aligned} \tag{63}$$

Relative bond returns are then, to a first order approximation, given by the discounted sum of future changes in the nominal exchange rate:

$$\widehat{\widetilde{R}}_{F,t}^{B,long} - \widehat{\widetilde{R}}_{H,t}^{B,long} = (1-\beta)\, \widetilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(-\widehat{S}_{t+j} \right) \right].$$

Markets are again complete, to a first order approximation. However, we need to slightly amend our partial expressions for portfolios to accomodate the fact that bonds are now long lived. We have:

$$S = \frac{1}{2}\left(1 + \frac{(1-\beta)\,C\left(cov_{\widehat{R}_t^{B,long}}\left(\widehat{R}_t^{q}, \widehat{R}_t^{S}\right) + cov_{\widehat{R}_t^{B,long}}\left(\widehat{R}_t^{\widetilde{Q}}, \widehat{R}_t^{S}\right)\right) - WLcov_{\widehat{R}_t^{B,long}}\left(\widehat{R}_t^{W}, \widehat{R}_t^{S}\right)}{Dvar_{\widehat{R}_t^{B,long}}\left(\widehat{R}_t^{S}\right)}\right),$$

where $cov_{R_t^B}\left(\widehat{R}_t^{q}, \widehat{R}_t^{S}\right) = \widetilde{E}_t\left[\left(\widehat{Q}_t - P\left[\widehat{Q}_t|\widehat{R}_t^{B,long}\right]\right)\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B,long}\right]\right)\right]$, $cov_{R_t^B}\left(\widehat{R}_t^{\widetilde{Q}}, \widehat{R}_t^{S}\right) = \left(1-\frac{1}{\sigma}\right)\widetilde{E}_t\left[\left(\sum_{j=1}^{T}\beta^j\left(\widehat{Q}_{t+j} - P\left[\widehat{Q}_{t+j}|\widehat{R}_t^{B,long}\right]\right)\right)\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B,long}\right]\right)\right]$, $cov_{R_t^{B,long}}\left(\widehat{R}_t^{W}, \widehat{R}_t^{S}\right) = \left(\widehat{R}_t^{W} - P\left[\widehat{R}_t^{W}|\widehat{R}_t^{B,long}\right]\right)\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B,long}\right]\right)$, and $var_{R_t^{B.long}}\left(\widehat{R}_t^{S}\right) = \left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B,long}\right]\right)^2$.
Bond positions are now given by:

$$B = \frac{1}{2}\left(\frac{(1-\beta)\,C\left(cov_{R_t^S}\left(\widehat{R}_t^{q}, \widehat{R}_t^{B,long}\right) + cov_{R_t^S}\left(\widehat{R}_t^{\widetilde{Q}}, \widehat{R}_t^{B,long}\right)\right) - WLcov_{R_t^S}\left(\widehat{R}_t^{W}, \widehat{R}_t^{B,long}\right)}{Dvar_{R_t^S}\left(\widehat{R}_t^{B,long}\right)}\right), \tag{64}$$

where $cov_{R_t^S}\left(\widetilde{R}_t^{q}, \widehat{R}_t^{B,long}\right) = \widetilde{E}_t\left[\left(1-\frac{1}{\sigma}\right)\left(\widehat{Q}_t - P\left[\widehat{Q}_t|\widehat{R}_t^{S}\right]\right)\left(\widehat{R}_t^{B,long} - P\left[\widehat{R}_t^{B,long}|\widehat{R}_t^{S}\right]\right)\right]$, $cov_{R_t^S}\left(\widetilde{R}_t^{\widetilde{Q}}, \widehat{R}_t^{B,long}\right) = \widetilde{E}_t\left[\sum_{j=1}^{T}\beta^j\left[\left(1-\frac{1}{\sigma}\right)\left(\widehat{Q}_{t+j} - P\left[\widehat{Q}_{t+j}|\widehat{R}_t^{S}\right]\right)\right]\left(\widehat{R}_t^{B,long} - P\left[\widehat{R}_t^{B,long}|\widehat{R}_t^{S}\right]\right)\right]$, $cov_{R_t^S}\left(\widehat{R}_t^{W}, \widehat{R}_t^{B,long}\right) = \widetilde{E}_t\left[\left(\widehat{R}_t^{W} - P\left[\widehat{R}_t^{W}|\widehat{R}_t^{S}\right]\right)\left(\widehat{R}_t^{B,long} - P\left[\widehat{R}_t^{B,long}|\widehat{R}_t^{S}\right]\right)\right]$, and $var_{R_t^S}\left(\widehat{R}_t^{B,long}\right) = \left(\widehat{R}_t^{B,long} - \left[\widehat{R}_t^{B,long}|\widehat{R}_t^{S}\right]\right)^2$.

However, and perhaps counterintuitively, it can be shown that longer lived bonds are not necessarily better assets to hedge real exchange rate risk. This is true in our setup, for two reasons. The first reason is apparent from a look at figure 11 which reproduce the impulse responses of real and nominal exchange rates for our benchmark calibration. The first row shows the impulse responses to a relative Home productivity shock. The real exchange rate always depreciates, but the nominal exchange rate initially depreciates, but it appreciates in the long run. Since nominal bond returns now depend on the discounted sum of all current and future changes in nominal exchange rates, nominal bond returns respond positively, implying that taking a short position in nominal bonds would now provide some insurance against real exchange rate risk. However, as the second row in figure 11 shows, in response to an interest rate shock, both the nominal and the real exchange rate appreciate, implying that a long position in domestic bonds would hedge against real exchange rate exposure. Thus, the dynamic response of nominal exchange rates implies that we would expect the correlation between long term nominal bond returns and real exchange rate risk to be *lower* than with short term bonds. There is in fact a second reason to suspect that long term bonds are less strongly correlated with real exchange rate risk than short term bonds. This is because in this setup the association between nominal and real exchange rates is determined mainly by the degree of price rigidity. In our benchmark calibration, only one quarter of firms can reset prices in every quarter. However, in the long run, prices are flexible and that implies that, while the current nominal and real exchange rates will move closely in line with each other, nominal and real exchange rates far in the future will do less so. Thus the expected value of future discounted changes in nominal and real exchange rates will be less closely related than contemporaneous ones. This, again, will tend to reduce the correlation between long term nominal bond returns and real exchange rate risk. In tables 4 and 5, we present portfolios and correlations for different values of nominal rigidity and monetary policy responsiveness when agents trade long term bonds.

There are several interesting findings. Firstly, for none of the parameter combinations are nominal bonds better hedges for future real exchange rates than domestic equities. Correspondingly, agents take long positions in equity in order to hedge their real exchange rate exposure and take short positions in nominal bonds, for virtually all parameter values. Only for very high levels of nominal rigidity do agents take long positions in domestic bonds to hedge real exchange rate risk. A look at the correlations indicates that correlations between short term bonds and real exchange rate risk are, in general, more strongly negative than for long term bonds, especially for low values of θ. Only at very high degrees of nominal rigidity, are long term bonds more highly correlated with real exchange rate risk. As before, equity positions generally exhibit too much home bias, and the home bias is driven both by hedging against human capital and real exchange rate risk. Equity positions are qualitatively and, for the most part, quantitatively little affected by changes in the degree of nominal rigidity or monetary policy responsiveness. As expected, however, bond positions are strongly affected by the degree of nominal rigidity. As θ increases, the correlation between bond returns and real exchange rate risk quickly increases from very low levels and is the main driver behind the negative bond positions. Increases in monetary policy responsiveness have a relatively strong effect on correlations between asset returns and sources of risk. However, while monetary policy now has a quantitatively larger effect on portfolios, it again does not lead to qualitatively different asset positions. We thus conclude that allowing agents to trade long term bonds does not generate robust and realistic portfolios, if anything, it does the opposite here.

4.3.6 Consumption and Labour Share

In virtually all our exercises, portfolios are more affected by the motive to hedge movements in personal consumption expenditures/real exchange rates than human. The big size of the real exchange rate motive also appears to be the prime culprit for the instability of portfolios in many cases. In this section, we examine whether the instability can be ameliorated by reducing the relative importance of the exchange rate motive. The strength of the two hedging motives is affected by many aspects of the model. Two of these are the level of consumption spending and labour income in the steady state. In table 6, we thus present portfolios with parameter configurations that imply a lower consumption share or higher labour income.

In our benchmark calibration, the elasticity of output with respect to labour is $\alpha = 0.66$. However, due to imperfect competition and the presence of profits in the model, this value is not equal to the labour share. With an elasticity of substitution between individual varieties of $\varepsilon = 10$, the labour share is then 0.59 of output. As we increase α to 0.7, the total labour share increases to 0.63. The real exchange rate motive then indeed becomes less important and home bias becomes less negative, though agents still take negative positions in Home equity. Further increasing α to 0.75 implies a labour share of 0.68 and Home agents now take a positive position in Home equity, but foreign bias remains. An increase in the elasticity of substitution between individual varieties, ε, has the effect of reducing the steady state markup and thus profits. This also has the effect of increasing the labour share in the steady state. However, as we increase ε to 20 and thus the labour share to 0.63, the real exchange rate motive does not become less important and the foreign equity bias in fact becomes even larger.

In our benchmark calibration, the consumption share is 0.78 of output. Two parameters that affect the steady state consumption share are the discount factor β and the rate of depreciation δ. A higher discount factor β implies lower steady state consumption, as agents are more happy to defer consumption to the future. A higher rate of depreciation implies that investment needs to be raised, on average, in order to replenish the capital stock. Note that changes in the intertemporal elasticity of substitution σ does not affect the consumption share, as agents increase the capital stock in a way that increases both consumption and output equally, leaving the consumption share unchanged. For $\beta = 0.995$, the consumption share c/y falls to 0.75. However, perhaps somewhat surprisingly, asset positions become more extreme and more affected by real exchange rates. This is because, as the discount rate increases, agents care more about the future and therefore increase their asset positions. What is more, the more important the future is, the less will agents like to hold nominal bonds, as nominal bond returns become increasingly less correlated with real exchange rates as we look further into the future. An increase of the rate of depreciation, δ, to 0.05 reduces the steady state consumption share to 0.75 also. We now see a smaller effect of the real exchange rate motive on portfolios and that agents now take substantially positive positions in Home equity though home bias is still negative. However, it is worth noting that $\delta = 0.05$ implies an annual depreciation rate of 20% which is far outside the range generally used in business cycle studies.

The right value for the consumption share is also not entirely uncontroversial. OECD data that includes government consumption shows that most countries have consumption shares, including durables, of $0.7 - 0.8$, but the share becomes smaller when durables consumption is included in investment (add some discussion and show some data here). It is also worth noting that, as noted above, our benchmark calibration is in line with other work using this type of models. Thus, Chari et al. (2002) use $\alpha = 0.66$, $\beta = 0.99$, $\varepsilon = 20$ and $\delta = 0.021$. Kollmann (2005) uses somewhat different parameter values that imply a steady state labour share of 0.66. In particular, he sets $\varepsilon = 6$ and $\alpha = 0.79$. In that case, the importance of the exchange rate motive in portolios is somewhat smaller, but foreign bias remains.

4.3.7 Shock Persistence

In the absence of empirical data on monetary shocks, we chose the shock persistence to be similar to the persistence of productivity shocks. Here, we investigate whether higher or lower values of the shock persistence can bring about qualitative changes in the portfolios. In table 6, we can see that, as we lower ρ_M from 0.95 to 0.5 to 0, equity foreign bias increases and domestic bond positions become more strongly positive. Again, this is largely due to an increase in the effect of real exchange rate hedging, as human capital hedging actually works towards increaing home bias in equity portfolios.

4.3.8 Investment Shocks

One reason to find that in our setup bonds are relatively poor and equities relative good at hedging real exchange rate risk is that the presence of the monetary shock introduces some noise in bond returns, while there is no such noise in equity returns. We therefore replace the monetary shock with an investment shock. Several recent papers on business cycles and portfolio choice have recently stressed the importance of investment shocks for explaining business cycle fluctuations or international asset positions (see Fisher (2002,2006), Justiciano and Primiceri (2006), Justiciano et al (2007) and Coeurdacier et al (2008)). As in these papers, we introduce investment shocks as shocks to investment efficiency . Investment efficiency shocks then enter the Euler equation for output and the law of motion for capital which become:

$$
\begin{aligned}
& E_t\left[-\sigma\left(\widehat{C}_{i,t+1}-\widehat{C}_{i,t}\right)+\left(P^i_{C,t}-P^i_{C,t+1}\right)+\widehat{\chi}_{i,t}-\widehat{P}^i_{I,t}\right] \\
& +E_t\left[\widehat{R}^K_{i,t+1}\beta R^K+\beta\left(1-\delta\right)\left(\widehat{P}^i_{I,t+1}-\widehat{\chi}_{i,t+1}\right)\right] \\
= \; & 0 \qquad (65)
\end{aligned}
$$

$$
\widehat{K}_{i,t+1}=\left(1-\delta\right)\widehat{K}_{i,t+1}+\delta\left(\widehat{I}_{i,t}+\widehat{\chi}_{i,t}\right)
$$

where $\chi_{i,t}$ is a shock to investment efficiency that evolves according to:

$$
\widehat{\chi}_{i,t+1}=\rho_{\chi}\widehat{\chi}_{i,t}+\widehat{\varepsilon}_{i,\chi,t+1}.
$$

The removal of the interest rate shock implies that nominal interest rates, and by association nominal bond returns, should become less volatile. Investment shocks also increase the volatility of investment thereby raising the volatility of dividend and equity returns. Figures 12 and 13 present the impulse responses to a positive relative shock to Home investment efficiency. The investment efficiency shock reduces the effective cost of investment which induces a large rise in Home investment. Consumption falls, but the rise in investment, dominates and implies a persistent rise in Home output and, with unchanged productivity, a rise in Home labour demand and Home wages. Over time the increase in the capital stock also contributes to the rise in output, allowing Home labour to drop below the steady state level. The rise in investment demand drives a rise in the price of the Home good and therefore a rise in the Home CPI and IPI. The real exchange rate initially appreciates due to the rise in demand, but over time it depreciates, as Home output expands. Home nominal exchange rates also initially appreciate, but depreciate in the long run, as Home prices increase in the long term. The rise in investment implies a persistent fall in dividends which implies that relative Home equity returns respond negatively to an investment shock. The increase in Home investment demand, coupled with local bias in investment, implies that labour income rises persistently and so do Home human capital returns. Thus, Home equity would be a good hedge for Home human capital risk. As mentioned above, the real exchange rate first appreciates, but then depreciates. The real depreciation, however, dominates, implying that, as with monetary shocks, Home equity is a good hedge for real exchange rate fluctuations also. However, unlike with monetary shocks, the nominal exchange rate initially appreciates, while the long run exchange rate depreciates. The return to one period bonds is thus positive and a short position in Home bonds would hedge real exchange rate and human capital risk. Since the responses to productivity shocks in this economy are unchanged, optimal hedging of productivity shocks would require a long position in Home nominal bonds for both human capital and real exchange rate risk.

The fact that a long position in Home equity could be a good hedge for both real exchange rate and human capital risk, while any position in nominal bonds can only provide a good hedge for both sources of risk for one of the two shocks is reason to believe that a model with investment shocks and short term bonds will not be able to produce robust and realistic portfolios. This intuition is confirmed in tables 7 and 8 which depict portfolios and correlations for a variety of values for the degree of price rigidity and monetary policy responsiveness in this model. As before, the model displays real indeterminacy for high values of θ necessitating a rise in monetary policy responsiveness to make the model determinate again[6]. Several features are worth commenting on. Firstly, the introduction of investment shocks does make equity returns more and bond returns relatively less volatile than before. Secondly, the degree of price rigidity in this model is not nearly as significant as in the model with monetary shocks. In fact, in this model, price rigidities are not necessary to make nominal bonds useful assets. In the appendix, we show that agents hold positive amounts of nominal bonds in the case where prices are completely flexible. As expected, the introduction of investment shocks greatly lowers the correlation between equities and bonds, as they move in opposite directions in response to an investment shock. In fact, in most cases, the correlation between equity and bond returns is now negative. Most other correlations are also somewhat lower than in the presence of monetary shocks. For all values

[6]For $\theta = 0.85$, we have $\gamma = 3$ and for $\theta = 0.95$, we have $\gamma = 25$.

of θ and γ, equities are more highly correlated with future real exchange rates than bonds are, and correlations do not change much with the degree of price rigidity or monetary policy responsiveness, at least for all but very high degrees of nominal price rigidity. Equity positions are almost equally affected by human capital risk and real exchange rate hedging and both sources of risk work towards increasing home bias, which is too large as a result. Bond positions are mainly affected by real exchange rate hedging and are positive for all values of γ and θ.

As we can see in figure , the nominal exchange rate initially appreciates, but depreciates in the long run, implying that the returns to long run nominal bonds in fact respond negatively to investment shocks. Nominal bonds could then in principle also be good hedges for both human capital and real exchange rate risk in response to investment shocks. This is also the case closest to Coeurdacier et al. (2008) with the big exception being that we continue to let agents trade nominal and not real bonds. When we allow long term nominal bonds to be traded instead of short term bonds, correlations between bond and equity returns do indeed become significantly positive. Portfolios are indeed sometimes substantially different now, as a result. However, as it turns out, the correlation between equity returns and real exchange rates remains more strongly negative than for bond returns for realistic values of γ such that domestic equities are in virtually all cases still used to hedge real exchange rate risk. In fact, the real exchange rate motive tends to be more important for equity portfolios than human capital risk. Both hedging motives work towards producing home bias and home bias is again too strong for all but very high values of γ. Positions in domestic bonds are then usually negative. For very high values of monetary policy responsiveness, nominal bonds become more closely related to real exchange rates than domestic equities and bond positions change substantially. Agents take large long positions in nominal bonds to hedge real exchange rate risk. In those cases, the equity position is mainly affected by hedging human capital risk which takes on a realistic magnitude as a result, but it is worth noting that these cases imply an unreasonably strong response of monetary policy. For example, $\gamma = 100$ implies that the central bank raises the nominal interest rates by 100%(!) in response to a 1% increase in CPI inflation.

4.4 Risk Premium Shocks

Above we showed that it is very difficult to write models with nominal bonds and capital accumulation in which portfolios are robust and of a realistic size. The main reason is that due to the response of investment, equity returns are often highly correlated with the real exchange rate. What is more, equity returns are in general less volatile than bond returns. In addition to the fact that the high correlation between equity returns and the real exchange rate makes portfolio choice unstable, there are also strong empirical grounds to be skeptical of such high correlations and low volatility. For example, Van Wincoop and Warnock (2008) show that the correlation between equity returns and the contemporaneous real exchange rate is very low, both unconditionally and conditioning on many macroeconomic variables. While the measure they analyse is not the exact empirical counterpart to the relevant correlations here, it is highly unlikely that the correlations are as high as found in the models above. What is more, the volatility of equity returns in the data is quite significantly higher than the volatility of bond returns. Note that what matters here is the correlation and volatility of equity returns relative to the correlation and volatility of bond returns. Thus, introducing investment shocks lowers correlations and raises volatilities of equity returns somewhat, but affects bond returns in the same direction and therefore does not help in producing robust portfolios. In this section we therefore take a more direct approach and introduce shocks to equity prices in order to reduce the association between equity return and the real exchange rate and increase their volatility. Equity prices are now given by:

$$P_{i,t}^{S} = \beta\left(\kappa_{S,i,t}\right) E_t\left[\left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{-\sigma}\left(\frac{P_{C,t}^{i}}{P_{C,t+1}^{i}}\right)\left(D_{i,t+1} + P_{i,t+1}^{S}\right)\right], \tag{66}$$

where $\kappa_{i,t}$ is a shock to country $i's$ equity price which evolves according to

$$\ln \kappa_{i,t+1} = \ln \rho_\kappa \kappa_{i,t+1} + \varepsilon_{\kappa,i,t+1} \tag{67}$$

.The equity price shock is specific to each asset and thus affects both investors and is pure noise. In linear form, equity prices are now given by:

$$\begin{aligned} \widehat{P}^S_{H,t} &= E_t\left[-\sigma\left(\widehat{C}_{H,t+1} - \widehat{C}_{H,t}\right) + \left(\widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+1}\right) + (1-\beta)\,\widehat{D}_{H,t+1} + \beta\widehat{P}^S_{H,t+1}\right] + \widehat{\kappa}_{S,H,t} \\ &= (1-\beta)\,E_t\left[\sum_{j=0}^{\infty}\beta^j\left(-\sigma\left(\widehat{C}_{H,t+j+1} - \widehat{C}_{H,t}\right) + \left(\widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+j+1}\right) + \widehat{D}_{H,t+j+1}\right)\right] \\ &\quad + E_t\sum_{j=0}^{\infty}\beta^j\widehat{\kappa}_{S,H,t+j} \end{aligned} \tag{68}$$

and the equity price shock evolves as $\widehat{\kappa}_{H,t+1} = \rho_\kappa\widehat{\kappa}_{H,t} + \widehat{\varepsilon}_{\kappa,H,t+1}$. Note that this shock is pure noise in the sense that it does not directly affect the value of any real variables. In particular, it does not enter the Euler equations for investment. This shock is very similar to the risk premium shock in Smets and Wouters (2007), with the main difference being that in their model, the shock affects the price of bonds which are the only financial asset traded.

Relative equity returns in Home currency are now given by:

$$\begin{aligned} \left(\widehat{\widetilde{R}}^S_{F,t} - \widehat{\widetilde{R}}^S_{H,t}\right) &= (1-\beta)\,\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j} - \widehat{D}_{H,t+j} + \widehat{S}_{t+j}\right)\right] \\ &\quad + \beta\widetilde{E}_t\sum_{j=0}^{\infty}\beta^j\left(\widehat{\kappa}_{S,F,t+j} - \widehat{\kappa}_{S,H,t+j}\right) - \left(\widehat{\kappa}_{S,F,t-1} - \widehat{\kappa}_{S,H,t-1}\right), \end{aligned} \tag{69}$$

where the final term clearly signifies that relative returns are now affected by the equity price shock.

In addition to equity price shocks, the only other source of uncertainty in the model are now productivity shocks. Our partial equilibrium expression for stocks and bonds changes somewhat due to an additional term, arising from the presence of the equity price shock. We now have:

$$
\begin{aligned}
S &= \frac{1}{2}+\frac{1}{2}\frac{(1-\beta)\,C\left(cov_{R_t^B}\left(\widehat{R}_t^q,\widehat{R}_t^S\right)+cov_{R_t^B}\left(\widetilde{R}_t^{\widetilde{Q}},\widehat{R}_t^S\right)\right)}{Dvar_{R_t^B}\left(\widehat{R}_t^S\right)} \\
&\quad -\frac{1}{2}\frac{WLcov_{R_t^B}\left(\widehat{R}_t^W,\widehat{R}_t^S\right)-Dcov_{R_t^B}\left(\widehat{R}_t^\kappa,\widehat{R}_t^S\right)}{Dvar_{R_t^B}\left(\widehat{R}_t^S\right)} \\
B &= \frac{1}{2}\frac{Ccov_{R_t^S}\left(\widehat{R}_t^q,\widehat{R}_t^S\right)+Ccov_{R_t^S}\left(\widehat{R}_t^{\widetilde{Q}},\widehat{R}_t^S\right)}{Dvar_{R_t^B}\left(\widehat{R}_t^B\right)} \\
&\quad +\frac{Ccov_{R_t^S}\left(\widehat{Q}_t,\widehat{R}_t^S\right)+Ccov_{R_t^S}\left(\widetilde{Q}_t,\widehat{R}_t^S\right)}{Dvar_{R_t^B}\left(\widehat{R}_t^B\right)},
\end{aligned}
$$

where $\widehat{R}_t^\kappa=\beta\widetilde{E}_t\sum_{j=0}^{\infty}\beta^j\left(\widehat{\kappa}_{S,F,t+j}-\widehat{\kappa}_{S,H,t+j}\right)-\left(\widehat{\kappa}_{S,F,t-1}-\widehat{\kappa}_{S,H,t-1}\right).$

While we have two sources of uncertainty and two assets per country, the equity premium shock is pure noise. It thus does not present a real source of risk that needs to be hedged by the agents. Thus, just trade in nominal bonds is enough to achieve perfect risk sharing, for any equity portfolio. In particular, full equity home bias can be rationalised as an optimal portfolio with the appropriate bond positions. Equity positions are not pinned down, because while a shock to the equity price changes the nominal value of wealth, agents are aware that this shock is simply noise and does not reflect fundamental changes and thus do not want to trade. Tables 11 and 12 present portfolios when either short or long term bonds are traded in addition to equity, for different degrees of price rigidity and monetary policy responsiveness. We present portfolios for full equity home bias, as this is the most intuitive benchmark in an environment where agents prefer not to trade in equities. In terms of our partial equilibrium expressions, human capital and real exchange rate hedging play no role in determining equity positions, as trade in bonds is enough to hedge both real exchange rate and human capital risk. Thus, real exchange rate risk and human capital risk affect bond positinos, but so does the equity price shock. Domestic bond positions are generally positive and large when short term bonds are traded. The positions get somewhat smaller, as θ increases and somewhat larger, as the degree of monetary policy responsiveness rises. Both human capital and real exchange rate hedging make a positive contribution to domestic bond positions, while the equity price shock has a negative effect on bond positions. When long term bonds are traded, bond positions are huge. Agents generally take short positions in domestic bonds driven by human capital and real exchange rate hedging, while the equity price shock has a positive effect. As the degree of monetary policy responsiveness rises, bond positions change sign and become positive, as the response of bond returns to shocks changes sign. Since correlations are generally low, agents need to take large positions in order to hedge risk.

The equity price shock has the additional effect of making equity returns more volatile and less correlated with the other variables. Thus, the introduction of equity price shocks is partly successful: It makes equity prices behave more realistically and equity portfolios more robust in the sense that full equity home bias is consistent with perfect risk sharing. However, this comes at the expense of an ad hoc assumption that would also imply that any equity portfolio is consistent with perfect risk sharing, given the appropriate bond position. Also, bond positions are very large and unstable. What is more, it is still not true that equities hedge human capital risk, while bonds hedge real exchange rate risk.

5 Empirical Part (very incomplete)

The models discussed above predict that equity portfolios should be determined by the covariance between relative equity return innovations and human capital and real exchange rate risk, conditional on bond returns, while bond portfolios should depend on the covariance between relative bond return innovations and human capital and real exchange rate risk, conditional on relative equity return innovations. In this section, we use data on G 7 countries to calculate the empirical counterparts to the relevant covariances in order to compute the implied portfolios. Relative labour income, reak exchange rates and relative

Table 1

Labour and Consumption Shares

	Canada	France	Germany	Italy	Japan	UK	US	EU	Average
Labour Share	0.69	0.71	0.73	0.61	0.63	0.68	0.73	0.69	0.68
Naive Labour Share	0.60	0.61	0.59	0.49	0.57	0.63	0.62	0.57	0.59
Consumption Share	0.71	0.66	0.63	0.65	0.60	0.70	0.71	0.65	0.67
Share of World Output	0.04	0.08	0.11	0.08	0.16	0.08	0.45	0.26	1

equity and bond returns are constructed using national accounts and financial market data, as in Coeurdacier and Gourinchas (2009). The data series are quarterly and runs from 1970 to 2008 and are taken from Coeurdacier and Gourinchas (2009) to which we refer for further details.

Compared to Coeurdacier and Gourinchas (2009), there are three key differences, due to the dynamic nature of our approach. Firstly, our measure of real exchange rate risk comprise both current and expected future real exchange rate fluctuations. Similarly, we use innovations to equity and bond returns rather than bond and equity returns themselves. Finally, our reduced form expressions for portfolios are slightly different. We find that, based on the estimated moments, equity home bias can be rationalised in the US, Japan, and the UK. In all countries, equity positions are mainly affected by the human capital hedging motive and not by the real exchange rate motive. Our results also suggest two empirical failures of the models. Firstly, equity return innovations are not as volatile as in the data and too highly correlated with both sources of risk. Secondly, real exchange rate risk is too large in our models.

In Table 1, we summarise the labour shares for the G7 countries. The table shows both a measure of the labour share that includes a fraction of mixed surplus and a measure which only includes compensation of employees. It is worth noting that the difference between the two measures is quite sizable, a point emphasized by Gollin (2002).

5.1 Innovations and Risk Loadings

In order to to estimate the relevant covariances, we need to construct innovations to bond, equity and human capital returns, as well as a measure of real exchange rate risk.

Denote by $r^W_{i,t+1}$ the log of the gross simple return on human capital in country i between t and $t+1$. Following Campbell (1996), under the assumption that the dividend price ratio on human wealth is stationary, we can write:

$$r^W_{i,t+1} \equiv \log\left(LI_{i,t+1} + V^W_{i,t+1}\right) - \log\left(V^W_{i,t}\right) = k + \zeta_{i,t} - \rho\zeta_{i,t+1} + \Delta \log LI_{i,t+1},$$

where $V^W_{n,t}$ measures nonfinancial wealth, $\zeta_{i,t} = \log\left(LI_{i,t}/V^W_{i,t}\right)$ is the log-dividend price ratio, $\rho^{-1} = 1 + \exp(\zeta) = 1 + \left(LI_i/V^W_i\right) = \left(LI_i + V^W_i\right)/V^W_i$. As in Coeurdacier and Gourinchas, we will use $\rho = 0.98$. k is an unimportant constant. Solving this equation forward and imposing that $\lim_{t\to\infty} \rho^t\left(r^W_{i,t} - \Delta \log LI_{i,t}\right) = 0$, we obtain (up to a constant):

$$\zeta_{i,t} = \sum_{j=0}^{\infty} \rho^j \left(r^W_{i,t+j} - \Delta \log LI_{i,t+1+j}\right)$$

This expression states that the ratio of labour income to the value of human capital (the equivalent of a dividend-price ratio for human capital) is high today either when future human capital returns are high, or when future nonfinancial income growth is low. We do not estimate future expected returns to human capital. Instead, following Coeurdacier and Gourinchas (2009), we assume that the conditional expected return on human capital equals the conditional expected return on equities $\left(E_t r^W_{i,t+j} = E_t r^E_{i,t+j}\right)$. We then obtain the following expression:

$$r^W_{i,t+1} - E_t\left[r^W_{i,t+1}\right] = \left(E_{t+1} - E_t\right)\sum_{j=0}^{\infty} \rho^j \left(\Delta \log LI_{i,t+1+j}\right) - \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty} \rho^j r^W_{i,t+j+1}$$

This expression states that the innovation to the return on human capital depends upon innovations to the path of future expected labour income growth , as well as innovations to the path of future expected equity returns proxying for future expected human capital returns. Human capital return innovations today are high, if current and future innovations to expected labour income growth are high, or if innovations to future expected human capital returns are low. Finally, we obtain innovations to the relative expected human capital return by subtracting this expression for country i from the equivalent expression for the rest of the world, assuming that ρ is the same for all countries:

$$r_{i,t+1}^{W} - E_t\left[r_{i,t+1}^{W}\right] = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j\left(\Delta \log li_{i,t+1+j}\right) - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{i,t+j+1}^{E}.$$

We also need to obtain a measure for real exchange rate risk. Coeurdacier and Gourinchas (2009) use a static model and correspondingly their measure of real exchange rate risk is simply related to contemporaneous real exchange rate fluctuations. By contrast, due to the dynamic nature of our approach, our measure of real exchange rate risk implies that innovations to current and future expected real exchange rate changes need to be taken into ccount. Using a similar procedure as above, we then arrive at the following equation:

$$r_{i,t+1}^{Q} - E_t\left[r_{i,t+1}^{Q}\right] = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j\left(\Delta \log Q_{i,t+1+j}\right),$$

where $r_{i,t+1}^{Q} - E_t\left[r_{i,t+1}^{Q}\right]$ is the relevant measure of real exchange rate risk.

We then estimate the following vector autoregression:

$$g_{t+1} = Ag_t + \varepsilon_{t+1},$$

where $z_t' = \begin{bmatrix} r_t^E & \Delta li_t & r_t^B & \Delta q & x_t \end{bmatrix}$. x_t represents other controls that helps to predict the (relative) growth rate of labour income, bond and equity returns, and the real exchange rate. Here, we follow Coeurdacier and Gourinchas (2009) and use relative log consumption expenditure and the nxa variable from Gourinchas and Rey (2007). We then take the estimates from the VAR, $\widehat{A}$ and $\widehat{\varepsilon}$, to construct the innovations to human capital returns and real exchange rate risk. Human capital and real exchange rate risk are obtained as:

$$\begin{aligned} r^{W,1}_{i,t+1} - E_t\left[r^{W,1}_{i,t+1}\right] &= \left(e_2^{'} - \rho e_1^{'}\widehat{A}\right)\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1} \\ r^{Q}_{i,t+1} - E_t\left[r^{Q}_{i,t+1}\right] &= e_4^{'}\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1} \end{aligned}$$

while equity and bond return innovations are:

$$\begin{aligned} r^{E}_{i,t+1} - E_t\left[r^{E}_{i,t+1}\right] &= e_1^{'}\widehat{\varepsilon}_{t+1} \\ r^{B}_{i,t+1} - E_t\left[r^{B}_{i,t+1}\right] &= e_3^{'}\widehat{\varepsilon}_{t+1}, \end{aligned}$$

where e_i is a unit vector whose ith element is equal to one, while all other elements are equal to zero.

In Table 2, we present simple measures of volatility and comovement for asset returns and measures of risk for the US. As noted above, it is the *conditional* measures which are of importance for portfolios. However, these measures still contain some interesting information. Firstly, equity return innovationas are more volatile than bond return innovations by a factor of more than two, and the volatility of equity returns is of the same magnitude as the volatility of human capital and real exchange rate risk. Equity return innovations are also quite highly correlated with bond return innovations and real exchange rate risk, while the correlation with human capital is somewhat lower. Bond return innovations in turn are quite highly correlated with both human capital and real exchange rate risk.

5.1.1 Estimating the Loading on Human Capital Risk

We are now in a position to estimate the relevant covariance ratios. In order to do that, we follow Warnock and van Wincoop (2008) and Coeurdacier and Gourinchas (2009) and use a regression based approach. To that end, we run the following regression:

$$\widetilde{r}^{W}_{i,t+1} = k + \beta^{i}_{w,b}\widetilde{r}^{B}_{i,t+1} + \beta^{i}_{w,e}\widetilde{r}^{E}_{i,t+1} + \varepsilon^{w}_{i,t},$$

Table 2

Volatility and Correlation of Returns and Measures of Risk for the US

	Equity Returns	Human Capital Risk	Bond Returns	Real ER Risk
Variance	0.0048	0.0052	0.0019	0.0054
Correlation with:				
Equity Returns	1	0.12	0.46	0.45
Human Capital Risk	0.12	1	0.69	0.15
Bond Returns	0.46	0.69	1	0.66
Real ER Risk	0.45	0.15	0.66	1

where $\widetilde{r}^{j}_{i,t+1} = r^{j}_{i,t+1} - E_t r^{j}_{i,t+1}$ is he return innovation and $\varepsilon^{j}_{i,t}$ is attributed both to the measurement error in the construction of real exchange rate risk and to the fluctuations in real exchange rate risk not spanned by relative bond and equity returns (which is zero in our models due to effectively complete markets). Note that the difference to Coeurdacier and Gourinchas (2009) is that we use relative equity and bond return innovations rather than simple bond and equity returns in this regression.

The results are presented in table 3. We see that the coefficient on bond returns is always significantly positive and quite large. In fact, in most countries in the sample, the coefficient is larger than one in a statistical sense. For equities, the picture is more nuanced. In Japan, the UK and the US, equity returns are significantly negatively correlated with human capital returns, indicating that equities could hedge human capital risk. In Canada, Italy and the EU the coefficient is insignificantly different from zero, while it is significantly positive in a statistical sense in France and Germany, but still quite low. These results imply that long positions in domestic equity can in fact hedge human capital risk.

Finally, Benigno and Ristico (2009) argue in favour of computing covariance and variance ratios directly from the estimated residual covariance matrix of the VAR. We have done this and results are virtually identical to those of the above regression.

Table 3

Loadings of human capital risk on bond and equity return innovations

	Canada	France	Germany	Italy	Japan	UK	US	EU
Bonds	1.08	0.89	1.28	1.14	1.10	1.22	1.35	1.29
	0.08	0.06	0.07	0.08	0.16	0.08	0.11	0.07
Equity	−0.01	0.13	0.09	−0.02	−0.34	−0.09	−0.26	0.03
	0.04	0.02	0.03	0.03	0.08	0.03	0.04	0.04
R2	0.66	0.75	0.78	0.64	0.26	0.68	0.53	0.78
Obs.	136	136	136	136	136	136	136	136

5.1.2 Estimating the Loading on Real Exchange Rate Risk

In order to estimate the loadings on real exchange rate risk, we run:

$$\widetilde{r}^{Q}_{i,t+1} = k + \beta^{i}_{q,b}\widetilde{r}^{B}_{i,t+1} + \beta^{i}_{q,e}\widetilde{r}^{E}_{i,t+1} + \varepsilon^{q}_{i,t},$$

where $\varepsilon^{q}_{i,t}$ is attributed both to the measurement error in the construction of real exchange rate risk and to the fluctuations in real exchange rate risk not spanned by relative bond and equity returns (which is zero in our models due to effectively complete markets). Note that there are now two differences between our regression and Coeurdacier and Gourinchas (2009). Firstly, we use a measure of real exchange rate risk that takes into account current and future real exchange rates. Secondly, as above, we use return innovations rather than returns themselves.

The results are presented in table 4. The loadings of real exchange rate risk on bonds are always significant and quite large. What is more, they are statistically insignificantly different from one in France, the US and the EU, while they are always close to one in an economic sense. The loadings of equity are vary quite a bit, but they are generally fairly small and sometimes insignificantly different from zero. However, in the US, the coefficient is positive and modestly large. Again, the estimates from VAR based calculations are virtually identical.

Table 4

Loadings of real exchange rate risk on bond and equity return innovations

	Canada	France	Germany	Italy	Japan	UK	US	EU
Bonds	1.14	1.12	1.03	0.86	1.16	1.09	0.97	1.08
	0.06	0.11	0.07	0.09	0.06	0.05	0.12	0.10
Equity	−0.04	−0.07	0.14	0.01	0.07	−0.03	0.20	0.10
	0.03	0.04	0.04	0.03	0.03	0.02	0.08	0.06
R2	0.79	0.47	0.68	0.42	0.84	0.81	0.46	0.54
Obs.	136	136	136	136	136	136	136	136

Table 5

Estimate Equity Portfolios and Home Bias

	Canada	France	Germany	Italy	Japan	UK	US	EU
Output Share	0.04	0.08	0.11	0.08	0.16	0.08	0.45	0.26
Home Equity Share	0.06	−0.20	−0.11	0.10	0.65	0.25	0.84	0.21
Home Bias	0.02	−0.28	−0.22	0.02	0.48	0.17	0.38	−0.05
due to Human Capital	0.02	−0.28	−0.22	0.02	0.48	0.17	0.38	−0.05
due to Real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

5.2 Implied Bond and Equity Positions

In order to compare our empirical results with the model predictions, we need to adapt our reduced form expression for bond and equity portfolios to allow for countries of different size. We then have:

$$S^* = \widetilde{\lambda} + \left(1 - \widetilde{\lambda}\right) \left(\frac{-\left(1-\beta\right) C^* cov_{\widehat{R}_t^B}\left(\widehat{R}_t^Q, \widehat{R}_t^S\right) - W^* L_H cov_{\widehat{R}_t^B}\left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{D^* var_{\widehat{R}_t^B}\left(\widehat{R}_t^S\right)} \right),$$

where $\widetilde{\lambda} = \frac{C_H}{C_H + C_F} = \frac{D_H}{D_H + D_F} = \frac{Y_H}{Y_H + Y_F} \frac{\lambda}{1+\lambda}$ is the steady state ratio of consumption, dividends and output between the Home country and the rest of the world. Table 6 presents the implied equity portfolios:

The estimated covariances imply significant home bias in the UK, Japan, and the US. In all cases, the effect of real exchange rate fluctuations on equity positions is very small, implying that home bias is virtually entirely determined by a motive to hedge movements in real exchange rates.

6 Conclusion

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7 Appendix

7.1 Solution Method

Conventional linear solution methods cannot be applied in models with portfolio choice for two reasons. Firstly, in the steady state, portfolio choice is not well defined due to the absence of uncertainty. Secondly, all assets are identical in a first order approximation, as all components of order higher than one are neglected and first order expected returns need to be equal. Portfolio choice is then again indeterminate. The method presented in Devereux and Sutherland (2008) relies on a number of insights: Firstly, they show that a first order accurate solution to the model in general only requires a zero order accurate solution for portfolios. Secondly, they show that the zero order accurate portfolio can be derived by looking at the stochastic neighbourhood of the steady state and letting the noise go to zero. Thirdly, they show that the steady state portfolio can be derived using a second order approximation of the portfolio Euler equations and a first order approximation of all other equations of the model.

7.2 Relative Returns and Stochastic Discount Factors

Realised returns in domestic currency are given by:

$$R^B_{H,t} = \frac{1}{P^B_{H,t-1}} \qquad R^B_{F,t} = \frac{1}{P^B_{F,t-1}}$$

Relative returns in Home currency are then given by:

$$\frac{R^B_{H,t}}{\widetilde{R}^B_{H,t}} = \frac{P^B_{F,t-1}}{P^B_{H,t-1}} \frac{Z_{t-1}}{Z_t}$$

or in linearised form

$$\widehat{R}^B_{H,t} - \widehat{\widetilde{R}}^B_{F,t} = \widehat{P}^B_{F,t-1} - \widehat{P}^B_{H,t-1} + \widehat{Z}_{t-1} - \widehat{Z}_t.$$

From $\widehat{P}^B_{H,t} = E_t\left[-\sigma\left(\widehat{C}_{H,t+1} - \widehat{C}_{H,t}\right) + \widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+1}\right]$ and $\widehat{P}^B_{F,t} = E_t\left[-\sigma\left(\widehat{C}_{H,t+1} - \widehat{C}_{H,t}\right) + \widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+1}\right.$ -

we then have:

$$
\begin{aligned}
\widehat{R}^B_{H,t} - \widehat{\widetilde{R}}^B_{F,t} &= E_{t-1}\left[\widehat{Z}_t - \widehat{Z}_{t-1}\right] + \widehat{Z}_{t-1} - \widehat{Z}_t \\
&= -\left(E_t - E_{t-1}\right)\left[\widehat{Z}_t\right] = -\widetilde{E}_t\left[\widehat{Z}_t\right]
\end{aligned}
$$

The stochastic discount factors for Home and Foreign households are $\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma}\left(\frac{P^H_{C,t}}{P^H_{C,t+1}}\right)$ and $\left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\sigma}\left(\frac{P^F_{C,t}}{P^F_{C,t+1}}\right)$, respectively, or, linearising: $-\sigma\left(\widehat{C}_{H,t+1} - \widehat{C}_{H,t}\right) - \left(\widehat{P}^H_{C,t+1} - \widehat{P}^H_{C,t}\right)$ and $-\sigma\left(\widehat{C}_{F,t+1} - \widehat{C}_{F,t}\right) - \left(\widehat{P}^F_{C,t+1} - \widehat{P}^F_{C,t}\right)$. The Euler equations for investment in Home equity are:

$$
\beta E_t\left[\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma}\left(\frac{P^H_{C,t}}{P^H_{C,t+1}}\right)\left(\frac{D_{H,t+1} + P^S_{H,t+1}}{P^S_{H,t}}\right)\right] = 1 \tag{70}
$$

$$
\beta E_t\left[\left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\sigma}\left(\frac{P^F_{C,t}}{P^F_{C,t+1}}\right)\left(\frac{D_{H,t+1} + P^S_{H,t+1}}{P^S_{H,t}}\frac{Z_t}{Z_{t+1}}\right)\right] = 1 \tag{71}
$$

Linearising:

$$
\begin{aligned}
0 &= E_t\left[-\sigma\left(\widehat{C}_{H,t+1} - \widehat{C}_{H,t}\right) + \left(\widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+1}\right) + (1-\beta)\,\widehat{D}_{H,t+1} + \beta\widehat{P}^S_{H,t+1} - \widehat{P}^S_{H,t}\right] \\
0 &= E_t\left[-\sigma\left(\widehat{C}_{F,t+1} - \widehat{C}_{F,t}\right) + \left(\widehat{P}^F_{C,t} - \widehat{P}^F_{C,t+1}\right) + (1-\beta)\,\widehat{D}_{H,t+1} + \beta\widehat{P}^S_{H,t+1} - \widehat{P}^S_{H,t} + \widehat{Z}_t - \widehat{Z}_{t+1}\right]
\end{aligned}
$$

which imply:

$$
E_t\left[-\sigma\left(\widehat{C}_{H,t+1} - \widehat{C}_{H,t}\right) - \left(\widehat{P}^H_{C,t+1} - \widehat{P}^H_{C,t}\right)\right] \tag{72}
$$

$$
= E_t\left[-\sigma\left(\widehat{C}_{F,t+1} - \widehat{C}_{F,t}\right) - \left(\widehat{P}^F_{C,t+1} - \widehat{P}^F_{C,t}\right)\right] + \widehat{Z}_t - \widehat{Z}_{t+1} \tag{73}
$$

Thus, in a first order approximation, the Home and Foreign stochastic discount factors are the same, once they are expressed in the same units.

7.3 Budget Constraint

The budget constraint for the Home agent is:

$$\begin{aligned} & S^H_{H,t}P^S_{H,t} + S^H_{F,t}P^S_{F,t}Z_t + B^H_{H,t}P^B_{H,t} + B^H_{F,t}P^B_{F,t}Z_t \\ = \; & S^H_{H,t-1}\left(P^S_{H,t} + D_{H,t}\right) + S^H_{F,t-1}\left(P^S_{F,t} + D_{F,t}\right)Z_t + B^H_{H,t-1} \\ & + B^H_{F,t-1}Z_t + L_{H,t}W_{H,t} - P^H_{C,t}C_{H,t} \end{aligned} \quad (74)$$

The net foreign asset position is then given by assets held abroad minus domestic assets held by foreign agents:

$$NFA_{H,t} = S^H_{F,t}P^S_{F,t}Z_t + B^H_{F,t}Z_t - S^F_{H,t}P^S_{H,t} - B^F_{H,t}P^B_{H,t} \quad (75)$$

From the asset market clearing conditions:

$$S^F_{H,t} = 1 - S^H_{H,t} \qquad B^F_{H,t} = -B^H_{H,t} \quad (76)$$

which gives:

$$\begin{aligned} NFA_{H,t} &= S^H_{F,t}Z_t - \left(1 - S^H_{H,t}\right)P^S_{H,t} + B^H_{F,t}Z_t + B^H_{H,t}P^B_{H,t} \\ &= \left(S^H_{H,t} - 1\right)P^S_{H,t} + S^H_{F,t}Z_tP^S_{F,t} + B^H_{F,t}Z_t + B^H_{H,t}P^B_{H,t} \end{aligned} \quad (77)$$

Define the portfolio excess return as:

$$\begin{aligned} \xi_{H,t} &= \left(S^H_{H,t-1} - 1\right)\left(P^S_{H,t} + D_{H,t}\right) + S^H_{F,t-1}\left(P^S_{F,t} + D_{F,t}\right)Z_t \\ & + B^H_{H,t-1} + B^H_{F,t-1}Z_t - NFA_{H,t-1}R^S_{H,t}, \end{aligned} \quad (78)$$

i.e. the difference between actual net foreign assets at the beginning of period t and net foreign assets at period t had all wealth been invested in the home equity. Note that

$\widetilde{R}_H^S = R_H^S = \frac{P_{H,t}^S + D_{H,t}}{P_{H,t-1}^S}$, $\widetilde{R}_F^S = R_F^S \frac{Z_t}{Z_{t-1}} = \frac{P_{F,t}^S + D_{F,t}}{P_{F,t-1}^S} \frac{Z_t}{Z_{t-1}}$. Then,

$$\begin{aligned} \xi_{H,t} &= \left(S_{H,t-1}^H - 1\right) P_{H,t-1}^S R_{H,t}^S + S_{F,t-1}^H P_{F,t-1}^S Z_{t-1} R_{F,t}^S \frac{Z_t}{Z_{t-1}} \\ &\quad + B_{H,t-1}^H P_{H,t-1}^B R_{H,t}^B + B_{F,t-1}^H Z_{t-1} P_{F,t-1}^B R_{F,t}^B \frac{Z_t}{Z_{t-1}} - NFA_{H,t-1} R_{H,t}^S. \end{aligned} \tag{79}$$

From $NFA_{H,t} = \left(S_{H,t}^H - 1\right) P_{H,t}^S + S_{F,t}^H Z_t + B_{F,t}^H P_{F,t}^B + B_{H,t}^H P_{H,t}^B$

$$\begin{aligned} \xi_{H,t} &= S_{F,t-1}^H P_{F,t-1}^S Z_{t-1} \left(R_{F,t}^S \frac{Z_t}{Z_{t-1}} - R_{H,t}^S\right) + B_{H,t-1}^H P_{H,t-1}^B \left(R_{H,t}^B - R_{H,t}^S\right) \\ &\quad + B_{F,t-1}^H Z_{t-1} P_{F,t-1}^B \left(R_{F,t}^B \frac{Z_t}{Z_{t-1}} - R_{H,t}^S\right) \end{aligned} \tag{80}$$

Now rewrite the original budget constraint:

$$\begin{aligned} NFA_{H,t} + P_{H,t}^S &= S_{H,t-1}^H \left(P_{H,t}^S + D_{H,t}\right) + S_{F,t-1}^H Z_t \left(P_{F,t}^S + D_{F,t}\right) \\ &\quad + B_{H,t-1}^H \left(P_{C,t}^H + P_{H,t}^B\right) + B_{F,t-1}^H Z_t \left(P_{F,t}^B + P_{C,t}^F\right) + L_{H,t} W_{H,t} - P_{C,t}^H C_{H,t} \\ &= S_{H,t-1}^H P_{H,t-1}^S R_{H,t}^S + S_{F,t-1}^H Z_{t-1} P_{F,t-1}^S R_{F,t}^S \frac{Z_t}{Z_{t-1}} \\ &\quad + B_{H,t-1}^H P_{H,t-1}^B R_{H,t}^B + B_{F,t-1}^H Z_{t-1} P_{F,t-1}^B R_{F,t}^B \frac{Z_t}{Z_{t-1}} + L_{H,t} W_{H,t} - P_{C,t}^H C_{H,t} \end{aligned} \tag{81}$$

From $\xi_{H,t} = S_{F,t-1}^H P_{F,t-1}^S Z_{t-1} \left(R_{F,t}^S \frac{Z_t}{Z_{t-1}} - R_{H,t}^S\right) + B_{H,t-1}^H P_{H,t-1}^B \left(R_{H,t}^B - R_{H,t}^S\right) + B_{F,t-1}^H P_{F,t-1}^B Z_{t-1} \left(R_{F,t}^B \frac{Z_t}{Z_{t-1}} -\right.$

$$\begin{aligned} NFA_{H,t} &= S_{H,t-1}^H P_{H,t-1}^S R_{H,t}^S + \xi_{H,t} + S_{F,t-1}^H P_{F,t-1}^S Z_{t-1} R_{H,t}^S \\ &\quad + B_{H,t-1}^H P_{H,t-1}^B R_{H,t}^S + B_{F,t-1}^H P_{F,t-1}^B Z_{t-1} R_{H,t}^S + W_{H,t} - P_{C,t}^H C_{H,t} - P_{H,t}^S \end{aligned} \tag{82}$$

From $NFA_{H,t} = \left(S_{H,t}^H - 1\right) P_{H,t}^S + S_{F,t}^H P_{F,t}^S Z_t + B_{F,t}^H P_{F,t}^B Z_t + B_{H,t}^H P_{H,t}^B$:

$$NFA_{H,t} = NFA_{H,t-1} R_{H,t}^S + P_{H,t-1}^S R_{H,t}^S + \xi_{H,t} + L_{H,t} W_{H,t} - P_{C,t}^H C_{H,t} - P_{H,t}^S \tag{83}$$

From $R^S_{H,t} = \frac{D_{H,t}+P^S_{H,t}}{P^S_{H,t-1}}$, $D_{H,t} = \Pi_{H,t} + R^K_{H,t}K_{H,t} - I_{H,t}P^H_{I,t}$ and $\Pi_{H,t} = Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t - R^K_{H,t}K_{H,t} - W_{H,t}L_{H,t}$

$$\begin{aligned} NFA_{H,t} &= NFA_{H,t-1}R^S_{H,t} + D_{H,t} + P^S_{H,t} + \xi_{H,t} - P^H_{C,t}C_{H,t} - P^S_{H,t} + W_{H,t}L_{H,t} \\ &= NFA_{H,t-1}R^S_{H,t} + Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t + \xi_{H,t} - P^H_{C,t}C_{H,t} - I_{H,t}P^H_{I,t} \end{aligned} \tag{84}$$

Remembering that next exports were defined as: $NX_{H,t} = Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t - I_{H,t}P^H_{I,t} - P^H_{C,t}C_{H,t}$, we have:

$$NFA_{H,t} = NFA_{H,t-1}R^S_{H,t} + \xi_{H,t} + NX_{H,t} \tag{85}$$

or in linear form:

$$\widehat{NFA}_{H,t} = \widehat{NFA}_{H,t-1}\frac{1}{\beta} + \widehat{\xi}_{H,t} + \widehat{NX}_{H,t}, \tag{86}$$

Rewrite the portfolio excess return as:

$$\begin{aligned} \xi_{H,t} &= \left(S^H_{H,t-1} - 1\right)P^S_{H,t-1}R^S_{H,t} + S^H_{F,t-1}P^S_{F,t-1}R^S_{F,t}Z_t \\ &\quad + B^H_{H,t-1}P^B_{H,t}R^B_{H,t} + B^H_{F,t-1}P^B_{F,t}R^B_{F,t}Z_t - NFA_{H,t-1}R^S_{H,t}, \end{aligned} \tag{87}$$

Linearising:

$$\begin{aligned} \widehat{\xi}_{H,t} &= \left(S^H_{H,t-1} - 1\right)\frac{P^S}{\beta}\left(\widehat{P}^S_{H,t-1} + \widehat{R}^S_{H,t}\right) + \frac{P^S}{\beta}\left(\widehat{S}^H_{H,t-1}S\right) + \frac{P^S}{\beta}S^H_F\left(\widehat{S}^H_{F,t-1} + \widehat{P}^S_{F,t-1} + \widehat{R}^S_{F,t} + \widehat{Z}_t\right) \\ &\quad + B\left(\widehat{B}^H_{H,t-1} + \widehat{P}^B_{H,t} + \widehat{R}^B_{H,t}\right) - B\left(\widehat{B}^H_{F,t-1} + \widehat{P}^B_{F,t} + \widehat{R}^B_{F,t} + \widehat{Z}_t\right) - \widehat{NFA}_{H,t-1}\frac{1}{\beta}, \end{aligned} \tag{88}$$

Substituting for $\widehat{NFA}_{H,t}$, we have:

$$\widehat{\xi}_{H,t} = \left(S^H_H - 1\right)\frac{P^S}{\beta}\left(\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right) + B\left(\widehat{R}^B_{H,t} - \widehat{R}^B_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right), \tag{89}$$

We can then write the budget constraint in linearised form as:

$$\begin{aligned}\widehat{NFA}_{H,t} &= \widehat{NFA}_{H,t-1}\frac{1}{\beta} + (S-1)\frac{P^S}{\beta}\left(\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right) \\ &\quad + B\left(\widehat{R}^B_{H,t} - \widehat{R}^B_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right) + \widehat{NX}_{H,t} \qquad (90)\end{aligned}$$

Taking expectations at time t and rewriting:

$$\begin{aligned}\frac{1}{\beta}\widehat{NFA}_{H,t-1} &= E_t\left[\widehat{NFA}_{H,t} - \widehat{NX}_{H,t} - \frac{P^S}{\beta}(S-1)\left(\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right)\right] \\ &\quad - BE_t\left[\widehat{R}^B_{F,t} - \widehat{R}^B_{H,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \qquad (91)\end{aligned}$$

Now subtitute for $NFA_t = \beta E_{t+1}\left[NFA_{H,t+1} - \frac{P^S}{\beta}(S-1)\left(\widehat{R}^S_{H,t+1} - \widehat{R}^S_{F,t+1} - \widehat{Z}_{t+1} + \widehat{Z}_t\right)\right] - \beta BE_{t+1}\left(\widehat{R}^B_{F,t+1} - \widehat{R}^B_{H,t+1} - \widehat{Z}_{t+1} + \widehat{Z}_t\right)$. Continuing in this manner, we get:

$$\begin{aligned}\frac{1}{\beta}NFA_{H,t-1} &= \sum_{j=0}^{T} -\beta^j E_t\left[\widehat{NX}_{H,t+j}\right] + \frac{P^S}{\beta}(S-1)E_t\left[\widehat{R}^S_{H,t+j} - \widehat{R}^S_{F,t+j} - \widehat{Z}_{t+j} + \widehat{Z}_{t+j-1}\right] \\ &\quad - BE_t\left[\widehat{R}^B_{F,t+j} - \widehat{R}^B_{H,t+j} - \widehat{Z}_{t+j} + \widehat{Z}_{t+j-1}\right] + E_t\left[E_{t+T}NFA_{t+T}\right] \qquad (92)\end{aligned}$$

Imposing $T \to \infty$,$lim_{T\to\infty}E_t\left[NFA_{t+T}\right] = 0$ and using $E_t\left[\widehat{R}^S_{H,t+\tau} - \widehat{R}^S_{F,t+\tau} + \widehat{Z}_{t+\tau} - \widehat{Z}_{t+\tau-1}\right] = 0, \tau > 1$, we get:

$$\begin{aligned}\frac{1}{\beta}\widehat{NFA}_{H,t-1} &= \sum_{j=0}^{T} -\beta^j E_t\left[\widehat{NX}_{H,t+j}\right] - \frac{P^S}{\beta}(S-1)E_t\left[\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \\ &\quad - BE_t\left[\widehat{R}^B_{F,t} - \widehat{R}^B_{H,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \qquad (93)\end{aligned}$$

or, rearranging:

$$\begin{aligned}\sum_{j=0}^{T} -\beta^j E_t\left[NX_{H,t+j}\right] &= \frac{1}{\beta}NFA_{H,t-1} + \frac{P^S}{\beta}(S-1)E_t\left[\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \\ &\quad + BE_t\left[\widehat{R}^B_{F,t} - \widehat{R}^B_{H,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \qquad (94)\end{aligned}$$

From $\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1} = (1-\beta)\, \widetilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\widehat{D}_{H,t+j} - \widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right]$ and $\widehat{R}^B_{F,t} - \widehat{R}^B_{H,t} - \widehat{Z}_t + \widehat{Z}_{t-1} = \widetilde{E}_t \left[-\widehat{Z}_t\right]$, where $\widetilde{E}_t[X_t] = \widetilde{E}_t[X_t] - \widetilde{E}_{t-1}[X_t]$

$$\sum_{j=0}^{T} -\beta^j E_t \left[NX_{H,t+j}\right] = \frac{1}{\beta} NFA_{H,t-1} + D(S-1)\, \widetilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\widehat{D}_{H,t+j} - \widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right] + B\widetilde{E}_t \left[-\widehat{Z}_t\right] \quad (95)$$

This budget constraint holds if and only if:

$$\sum_{j=0}^{T} -\beta^j \widetilde{E}_t \left[NX_{H,t+j}\right] = D(S-1)\, \widetilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\widehat{D}_{H,t+j} - \widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right] + B\widetilde{E}_t \left[-\widehat{Z}_t\right] \quad (96)$$

and

$$\sum_{j=0}^{T} -\beta^j E_{t-1} \left[NX_{H,t+j}\right] = \frac{1}{\beta} NFA_{H,t-1} \quad (97)$$

Now remember that $NX_{H,t} = Y^H_{H,t} P^H_{H,t} + Y^F_{H,t} P^F_{H,t} Z_t - I_{H,t} P^H_{I,t} - P^H_{C,t} C_{H,t} = D_{H,t} + W_{H,t} L_{H,t} - P^H_{C,t} C_{H,t}$. In linear form, we have:

$$\widehat{NX}_{H,t} = \widehat{D}_{H,t} D + \left(\widehat{W}_{H,t} + \widehat{L}_{H,t}\right) WL - C \left(\widehat{P}^H_{C,t} + \widehat{C}_{H,t}\right) \quad (98)$$

Thus, we have:

$$\begin{aligned} & \sum_{j=0}^{T} -\beta^j \widetilde{E}_t \left[\widehat{D}_{H,t+j} D + \left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j}\right) WL - C \left(\widehat{P}^H_{C,t+j} + \widehat{C}_{H,t+j}\right)\right] \\ = \; & D(S-1)\, \widetilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\widehat{D}_{H,t+j} - \widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right] + B\widetilde{E}_t \left[-\widehat{Z}_t\right] \end{aligned} \quad (99)$$

or, rewriting:

$$
\begin{aligned}
& C\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{P}_{C,t}^{H}+\widehat{C}_{H,t}\right] \\
= \; & WL\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}\right]+SD\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\widehat{D}_{H,t+j}\right] \\
& +\left(1-S\right)D\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\left(\widehat{D}_{F,t+j}-\widehat{Z}_{t+j}\right)\right]+B\widetilde{E}_{t}\left[-\widehat{Z}_{t}\right] \qquad (100)
\end{aligned}
$$

The analogous expression for the foreign country is, in terms of Home currency:

$$
\begin{aligned}
& C\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{P}_{C,t+j}^{F}+\widehat{C}_{F,t+j}+\widehat{Z}_{t+j}\right] \\
= \; & WL\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}+\widehat{Z}_{t+j}\right]+SD\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\left(\widehat{D}_{F,t+j}+\widehat{Z}_{t+j}\right)\right] \\
& +\left(1-S\right)D\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\widehat{D}_{H,t+j}\right]-B\widetilde{E}_{t}\left[-\widehat{Z}_{t}\right] \qquad (101)
\end{aligned}
$$

Deducting the foreign budget constraint from the Home one, we obtain:

$$
\begin{aligned}
& C\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\left(\widehat{P}_{C,t}^{H}+\widehat{C}_{H,t}\right)-\left(\widehat{P}_{C,t+j}^{F}+\widehat{C}_{F,t+j}+\widehat{Z}_{t+j}\right)\right] \\
= \; & WL\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}-\left(\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}+\widehat{Z}_{t+j}\right)\right] \\
& +\left(2S-1\right)D\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\left(\widehat{D}_{H,t+j}-\left(\widehat{D}_{F,t+j}-\widehat{Z}_{t+j}\right)\right)\right] \\
& +2B\widetilde{E}_{t}\left[-\widehat{Z}_{t}\right] \qquad (102)
\end{aligned}
$$

As mentioned above, with two assets and two sources of uncertainty, markets are complete to a first order approximation. It can then be shown that the Backus Smith Kollmann condition holds in linearised form:

$$-\sigma\left(\widehat{C}_{H,t}-\widehat{C}_{F,t}\right)=\widehat{P}^{H}_{C,t}-\widehat{Z}_t-\widehat{P}^{F}_{C,t}=-\widehat{Q}_t \tag{103}$$

Consumption expenditure can then be expressed solely as a function of the real exchange rate:

$$\begin{aligned}&\left(\widehat{P}^{H}_{C,t}+\widehat{C}_{H,t}\right)-\left(\widehat{P}^{F}_{C,t+j}+\widehat{C}_{F,t+j}+\widehat{Z}_{t+j}\right)\\ =\ &\left(1-\frac{1}{\sigma}\right)\left(\widehat{P}^{H}_{C,t}-\widehat{Z}_t-\widehat{P}^{F}_{C,t}\right)=-\left(1-\frac{1}{\sigma}\right)\widehat{Q}_t\end{aligned} \tag{104}$$

Now define the return on human capital in country i as

$$R^{W}_{H,t}=\frac{W_{H,t}L_{H,t}+P^{W}_{H,t}}{P^{W}_{H,t-1},}, \tag{105}$$

where $P^{W}_{H,t}=E_t\sum_{j=1}^{\infty}\beta^j\left(\frac{C_{H,t+j}}{C_{H,t}}\right)^{-\sigma}\frac{P^{H}_{C,t}}{P^{H}_{C,t+j}}W_{H,t+j}L_{H,t+j}$ is the present value of labour income in Home. Linearising, the expression for returns, we have:

$$R^{W}_{H,t}=(1-\beta)\sum_{j=0}^{\infty}\beta^j\left(-\sigma\left(\widehat{C}_{H,t+j}-\widehat{C}_{H,t}\right)+\widehat{P}^{H}_{C,t}-\widehat{P}^{H}_{C,t+j}+\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}\right) \tag{106}$$

Deducting the analogous foreign expression in Home currency terms, we have:

$$\widehat{R}^{W}_{H,t}-\widehat{\widetilde{R}}^{W}_{F,t}=(1-\beta)\,\widetilde{E}_t\sum_{j=0}^{\infty}\beta^j\left(\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}-\widehat{W}_{F,t+j}-\widehat{L}_{F,t+j}-\widehat{Z}_{t+j}\right) \tag{107}$$

Now we can write the budget constraint as:

$$C\widehat{R}^{Q}_t=\frac{WL}{1-\beta}\widehat{R}^{W}_t+(2S-1)\frac{D}{1-\beta}\widehat{R}^{S}_t+2B\widehat{R}^{B}_t, \tag{108}$$

where $R^{W}_t=\widehat{R}^{W}_{H,t}-\widehat{\widetilde{R}}^{W}_{F,t}, R^{S}_t=\widehat{R}^{S}_{H,t}-\widehat{\widetilde{R}}^{S}_{F,t},\ \widehat{R}^{B}_t=\widehat{R}^{B}_{H,t}-\widehat{\widetilde{R}}^{B}_{F,t}=\widetilde{E}_t\left[-\widehat{Z}_t\right]$ and $\widehat{R}^{Q}_t=-\left(1-\frac{1}{\sigma}\right)\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\widehat{Q}_{t+j}\right]$

Now project this equation on relative bond returns $\widehat{R}_t^B$:

$$CP\left[\widehat{R}_t^Q|\widehat{R}_t^B\right] = \frac{WL}{1-\beta}P\left[\widehat{R}_t^W|\widehat{R}_t^B\right] + \frac{D}{1-\beta}(2S-1)\,P\left[\widehat{R}_t^S|\widehat{R}_t^B\right] + 2B\widehat{R}_t^B, \tag{109}$$

where $P\left[\widehat{X}_t|\widehat{Y}_t\right]$ is the projection of X_t on Y_t. Subtracting this equation from the one before, we have:

$$\begin{aligned} & C\left(\widehat{R}_t^Q - P\left[\widehat{R}_t^Q|\widehat{R}_t^B\right]\right) \\ = \; & \frac{WL}{1-\beta}\left(\widehat{R}_t^W - P\left[\widehat{R}_t^W|\widehat{R}_t^B\right]\right) + \frac{D}{1-\beta}(2S-1)\left(\widehat{R}_t^S - P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right) \end{aligned} \tag{110}$$

Rewriting:

$$\begin{aligned} & \frac{D}{1-\beta}(2S-1)\left(\widehat{R}_t^S - \left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right) \\ = \; & C\widetilde{E}_t\left(\widehat{R}_t^Q - P\left[\widehat{R}_t^Q|\widehat{R}_t^B\right]\right) - \frac{WL}{1-\beta}\left(\widehat{R}_t^W - P\left[\widehat{R}_t^W|\widehat{R}_t^B\right]\right) \end{aligned} \tag{111}$$

Rearranging, we have:

$$S = \frac{1}{2}\left(1 + \frac{(1-\beta)\,C\widetilde{E}_t\left(\widehat{R}_t^Q - P\left[\widehat{R}_t^Q|\widehat{R}_t^B\right]\right) - WL\left(\widehat{R}_t^W - P\left[\widehat{R}_t^W|\widehat{R}_t^B\right]\right)}{D\left(\widehat{R}_t^S - P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)}\right) \tag{112}$$

In order to compare this expression with the related literature, it is useful to distinguish between contemporaneous movements in the real exchange rate and future movements. Doing that, we obtain:

$$\begin{aligned} S \; = \; & \frac{1}{2} + \frac{1}{2}\frac{-(1-\beta)\left(1-\frac{1}{\sigma}\right)C\widetilde{E}_t\left(\widehat{R}_t^q - P\left[\widehat{R}_t^q|\widehat{R}_t^B\right]\right)}{D\left(\widehat{R}_t^S - P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)} \\ & + \frac{1}{2}\frac{(1-\beta)\,C\widetilde{E}_t\left(\widehat{R}_t^{\widetilde{Q}} - P\left[\widehat{R}_t^{\widetilde{Q}}|\widehat{R}_t^B\right]\right) - WL\left(\widehat{R}_t^W - P\left[\widehat{R}_t^W|\widehat{R}_t^B\right]\right)}{D\left(\widehat{R}_t^S - P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)}, \end{aligned} \tag{113}$$

where $R_t^{\widetilde{Q}} = -\left(1-\frac{1}{\sigma}\right)\sum_{j=1}^{T}\beta^j\widetilde{E}_t\left[\widehat{Q}_{t+j}\right]$ and $R_t^q = -\left(1-\frac{1}{\sigma}\right)\widetilde{E}_t\left[\widehat{Q}_t\right]$. Multiplying the numerator and the denominator by $\left(\widehat{R}_t^S - \left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)$, we have:

$$
\begin{aligned}
S &= \frac{1}{2}+\frac{1}{2}\frac{(1-\beta)\,C\widetilde{E}_t\left(\widehat{R}_t^q - P\left[\widehat{R}_t^q|\widehat{R}_t^B\right]\right)\left(\widehat{R}_t^S-\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)}{D\left(\widehat{R}_t^S-P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)^2} \\
&\quad +\frac{1}{2}\frac{\left((1-\beta)\,C\widetilde{E}_t\left(\widehat{R}_t^{\widetilde{Q}}-P\left[\widehat{R}_t^{\widetilde{Q}}|\widehat{R}_t^B\right]\right)-WL\left(\widehat{R}_t^W-P\left[\widehat{R}_t^W|\widehat{R}_t^B\right]\right)\right)\left(\widehat{R}_t^S-\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)}{D\left(\widehat{R}_t^S-P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)^2} \\
&= \frac{1}{2}\left(1+\frac{(1-\beta)\,C\left(cov_{R_t^B}\left(\widehat{R}_t^q,\widehat{R}_t^S\right)+cov_{R_t^B}\left(\widetilde{Q}_t,\widehat{R}_t^S\right)\right)-WLcov_{R_t^B}\left(\widehat{R}_t^W,\widehat{R}_t^S\right)}{Dvar_{R_t^B}\left(\widehat{R}_t^S\right)}\right) \quad (114)
\end{aligned}
$$

where $cov_{R_t^B}\left(\widehat{R}_t^q,\widehat{R}_t^S\right)=\widetilde{E}_t\left[\left(1-\frac{1}{\sigma}\right)\left(\widehat{Q}_t-P\left[\widehat{Q}_t|\widehat{R}_t^B\right]\right)\left(\widehat{R}_t^S-P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)\right]$, $cov_{R_t^B}\left(\widetilde{Q}_t,\widehat{R}_t^S\right)=$ $\widetilde{E}_t\left(\left(1-\frac{1}{\sigma}\right)\sum_{j=1}^{T}\beta^j\left(\widehat{Q}_{t+j}-P\left[\widehat{Q}_{t+j}|\widehat{R}_t^B\right]\right)\right)\left(\widehat{R}_t^S-P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)$, $cov_{R_t^B}\left(\widehat{R}_t^W,\widehat{R}_t^S\right)=\widetilde{E}_t\left(\widehat{R}_t^W-P\left[\widehat{R}_t^W|\right.\right.$ and $var_{R_t^B}\left(\widehat{R}_t^S\right)=\widetilde{E}_t\left(\widehat{R}_t^S-P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)^2$.

According to (114), the optimal equity position depends on four terms. The first term shows that this model would generate perfectly diversified portfolios $\left(S^S=\frac{1}{2}\right)$ in the absence of any concern for the real exchange rate or labour income, i.e. for $\sigma=1$ and $WL=0$. The second and third term arise from a motive to hedge movements in personal consumption expenditures which are here perfectly correlated with the real exchange rate[7]. The BSK condition, (52a) indicates that optimal risk sharing implies that consumption falls when the real exchange rate appreciates and the fall in consumption is larger the larger is the intertemporal elasticity of substitution. For high values of the intertemporal elasticity of substitution $(\sigma<1)$, relative consumption falls so much in response to a rise in the real exchange rate that relative consumption expenditure also falls. Since optimal consumption spending would be lower in the case when the real exchange rate appreciates, agents would, ceteris paribus, prefer to hold an asset that has low payoffs when the real exchange rate appreciates. For the more realistic case of low intertemporal elasticity of substitution

[7]Note that a rise in Q here denotes a real exchange rate *depreciation*

($\sigma > 1$), agents would prefer to hold an asset that pays more when the real exchange rate appreciates. Finally, the last term arises from a motive to hedge movements in the return to human capital and it implies that agents would prefer to hold an asset that has high payoffs when the returns to human capital are low, and vice versa. Note, however, that nominal bonds can also be used to hedge some or all of these sources of risk. What is important for equity positions is thus not the unconditional comovement of equity returns with these sources of risk, but their comovement conditional on the returns to the nominal bond.

We can also derive an analogous expression for bonds. Remember that the budget constraint could be written as:

$$C\widehat{R}_t^Q = \frac{WL}{1-\beta}\widehat{R}_t^W + \frac{D}{1-\beta}(2S-1)\widehat{R}_t^S + 2B\widehat{R}_t^B$$

Now project this equation on relative bond returns $\widehat{R}_t^S$:

$$CP\left[\widehat{R}_t^Q|\widehat{R}_t^S\right] = \frac{WL}{1-\beta}P\left[\widehat{R}_t^W|\widehat{R}_t^S\right] + \frac{D}{1-\beta}(2S-1)\widehat{R}_t^S + 2BP\left[\widehat{R}_t^B|\widehat{R}_t^S\right] \tag{115}$$

Subtracting this equation from the one before, we have:

$$-C\left(\widehat{R}_t^Q - P\left[\widehat{R}_t^Q|\widehat{R}_t^S\right]\right) = \frac{WL}{1-\beta}\left(\widehat{R}_t^W - P\left[\widehat{R}_t^W|\widehat{R}_t^S\right]\right) + 2B\left(\widehat{R}_t^B - \left[\widehat{R}_t^B|\widehat{R}_t^S\right]\right) \tag{116}$$

Rewriting:

$$\begin{aligned} B &= \frac{1}{2}\frac{C\left(\widehat{R}_t^Q - P\left[\widehat{R}_t^Q|\widehat{R}_t^S\right]\right) - \frac{WL}{1-\beta}\left(\widehat{R}_t^W - P\left[\widehat{R}_t^W|\widehat{R}_t^S\right]\right)}{\widehat{R}_t^B - P\left[\widehat{R}_t^B|\widehat{R}_t^S\right]} & (117)\\ &= \frac{1}{2}\left(\frac{Ccov_{R_t^S}\left(R_t^q, \widehat{R}_t^B\right) + Ccov_{R_t^S}\left(\widehat{R}_t^{\widetilde{Q}}, \widehat{R}_t^B\right) - \frac{WL}{1-\beta}cov_{R_t^S}\left(\widehat{R}_t^W, \widehat{R}_t^B\right)}{Dvar_{R_t^S}\left(\widehat{R}_t^B\right)}\right), & (118)\end{aligned}$$

where $cov_{R_t^S}\left(\widetilde{R}_t^q, \widehat{R}_t^B\right) = \left(1-\frac{1}{\sigma}\right)\widetilde{E}_t\left(\widehat{Q}_t - P\left[\widehat{Q}_t|\widehat{R}_t^S\right]\right)\left(\widehat{R}_t^B - P\left[\widehat{R}_t^B|\widehat{R}_t^S\right]\right), cov_{R_t^S}\left(\widetilde{R}_t^{\widetilde{Q}}, \widehat{R}_t^B\right) =$

$\widetilde{E}_t\left[\left(\left(1-\frac{1}{\sigma}\right)\sum_{j=1}^{T}\beta^j\left(\widehat{Q}_t - P\left[\widehat{Q}_{t+j}|\widehat{R}_t^S\right]\right)\right)\left(\widehat{R}_t^B - P\left[\widehat{R}_t^B|\widehat{R}_t^S\right]\right)\right]$, $cov_{R_t^S}\left(\widehat{R}_t^W,\widehat{R}_t^B\right) = \Big(\widehat{R}_t^W - P\Big[\widehat{R}_t^W|\widehat{R}_t^S$
and $var_{R_t^S}\left(\widehat{R}_t^B\right) = \left(\widehat{R}_t^S - P\left[\widehat{R}_t^B|\widehat{R}_t^S\right]\right)^2$.

Thus, we see that bond positions depend again on three terms. The first term shows the extent to which nominal bonds can hedge against contemporaneous changes in the real exchange rate. The second term shows the extent to which nominal bond returns can hedge against expected future changes in the real exchange rate. Finally, the last term is given by the extent to which nominal bond returns covary with labour income. All covariances are conditional on stock returns.

7.3.1 Polar cases

In order to understand this expression somewhat better, it is useful to consider some polar cases. We therefore consider the case where prices are completely fixed and when prices are completely flexible. Each time we dinstinguish the case where one period bonds are traded and when infinitely lived bonds are traded. We also consider real bonds and the static case.

Perfectly Fixed Prices When prices are completely fixed in both countries, $\widehat{P}_{H,t}^H = \widehat{P}_{F,t}^H = \widehat{P}_{H,t}^F = \widehat{P}_{F,t}^F = \widehat{P}_{C,t}^H = \widehat{P}_{C,t}^F = 0$, the nominal and the real exchange rate are identical, as any changes in the real exchange rates have to be transmitted through the nominal exchange rate:

$$\begin{aligned} \widehat{Q}_t &= \widehat{P}_{C,t}^F + \widehat{Z}_t - \widehat{P}_{C,t}^H; \\ &= \widehat{Z}_t \end{aligned} \tag{119}$$

As derived above, relative returns of one period nominal bonds are equal to the unexpected change in the next period nominal exchange rate. With the equivalence between nominal and real exchange rates, nominal bond returns are then perfectly correlated with the contemporaneous real exchange rate innovation. This has two effects for the expression for equity portfolios. Firstly, it implies that the relevant covariances are now conditional on the contemporaneous real exchange rate. Secondly, since nominal bond returns are perfectly correlated with the real exchange rate innovation in the same period, the second term is cancelled and we are left with:

$$S = \frac{1}{2}\left(1 + \frac{(1-\beta)\,Ccov_{Q_t}\left(\widehat{\widetilde{Q}}_t, \widehat{R}_t^S\right) - WLcov_{Q_t}\left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{Dvar_{Q_t}\left(\widehat{R}_t^S\right)}\right). \tag{120}$$

Note that since bonds only have a maturity of one period, they are not a perfect hedge for the total real exchange rate risk. Thus, future real exchange rate changes still enter in this expression. In the case of infinitely lived bonds, the relative return of the nominal bond is given by the expected discounted current and future changes in the nominal exchange rate. With full price rigidity, this implies that relative bond returns are perfectly correlated with real exchange rate risk. In that case, all movements in consumption expenditures are hedged using the bond position and the optimal equity position would only depend on the covariance of labour income returns with equity returns, conditional on the real exchange rate, as in Coeurdacier et al. (2008):

$$S = \frac{1}{2}\left(1 - \frac{WLcov_{\widehat{R}_{H,t}^B}\left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{Dvar_{\widehat{R}_{Ht}^B}\left(\widehat{R}_t^S\right)}\right). \tag{121}$$

The expression for short term bonds is unchanged:

$$B = \frac{1}{2}\left(\frac{Ccov_{R_t^S}\left(R_t^q, \widehat{R}_t^B\right) + Ccov_{R_t^S}\left(\widehat{R}_t^{\widetilde{Q}}, \widehat{R}_t^B\right) - \frac{WL}{1-\beta}cov_{R_t^S}\left(\widehat{R}_t^W, \widehat{R}_t^B\right)}{Dvar_{R_t^S}\left(\widehat{R}_t^B\right)}\right), \tag{122}$$

7.3.2 Full Price Flexibility

When prices are completely flexible and markets are complete, the correlation of the nominal and real exchange rate, conditional on equity returns is zero, as relative equity returns are perfectly correlated with real exchange rate risk (and also human capital risk). We are then back to the case of Heathcote and Perri (2008) and the optimal equity position will depend on the unconditional correlation of equity returns with the real exchange rate and on the covariance of equity returns and returns to human capital:

$$S = \frac{1}{2}\left(1 - \frac{(1-\beta)\, C\left(cov\left(R_t^q, \widehat{R}_t^S\right) + cov\left(R_t^{\widetilde{Q}}, \widehat{R}_t^S\right)\right) + WLcov\left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{Dvar\left(\widehat{R}_t^S\right)}\right)$$

Bond positions are now zero, both if short term bonds or long term bonds are traded:

$$B = 0.$$

7.3.3 Real Bonds

In the case of real bonds, relative bond returns are perfectly correlated with the real exchange rate. We then have the same expressions, as in the case of full price rigidity. With one period real bonds, the equity position is given by:

$$S = \frac{1}{2}\left(1 + \frac{(1-\beta)\, Ccov_{Q_t}\left(\widehat{\widetilde{Q}}_t, \widehat{R}_t^S\right) - WLcov_{Q_t}\left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{Dvar_{Q_t}\left(\widehat{R}_t^S\right)}\right). \tag{123}$$

With bonds with infinite maturity, we have:

$$S = \frac{1}{2}\left(1 - \frac{WLcov_{\widehat{R}_{H,t}^B}\left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{Dvar_{\widehat{R}_{Ht}^B}\left(\widehat{R}_t^S\right)}\right). \tag{124}$$

Short term bond positions are given by:

$$B = \frac{1}{2}\left(\frac{Ccov_{R_t^S}\left(R_t^q, \widehat{R}_t^B\right) + Ccov_{R_t^S}\left(\widehat{R}_t^{\widetilde{Q}}, \widehat{R}_t^B\right) - \frac{WL}{1-\beta} cov_{R_t^S}\left(\widehat{R}_t^W, \widehat{R}_t^B\right)}{Dvar_{R_t^S}\left(\widehat{R}_t^B\right)}\right), \tag{125}$$

7.4 Linear System

As in Devereux and Sutherland (2008), we first solve for the equilibrium conditional on portfolio choice before solving for the full equilibrium. For the first step, the equations are:

1. $-\sigma\left(\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right)+\widehat{P}^{H}_{C,t}-\widehat{P}^{H}_{C,t+1}+(1-\beta)\,\widehat{D}_{H,t}+\beta\widehat{P}^{S}_{H,t+1}-\widehat{P}^{S}_{H,t}$

2. $-\sigma\left(\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right)+\widehat{P}^{F}_{C,t}-\widehat{P}^{F}_{C,t+1}+(1-\beta)\,D_{F,t}+\beta\widehat{P}^{S}_{F,t+1}-\widehat{P}^{S}_{F,t}$

3. $E_t\left[\sigma\left(\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right)\right]=\sigma\left(\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right)+\widehat{Q}_{t+1}-\widehat{Q}_t$

4. $\widehat{i}^{H}_{H,t}=\frac{1-\theta}{\theta}(1-\theta\beta)\left((1-\alpha)\,\widehat{R}^{K}_{H,t}+\alpha\widehat{W}_{H,t}-\widehat{A}_{H,t}-\widehat{P}^{H}_{H,t}\right)+\beta E_t\left[\widehat{i}^{H}_{H,t+1}\right]$

5. $\widehat{\widehat{i}}^{F}_{F,t}=\frac{1-\theta}{\theta}(1-\theta\beta)\left((1-\alpha)\,\widehat{R}^{K}_{F,t}+\alpha\widehat{W}_{F,t}-\widehat{A}_{F,t}-\widehat{P}^{F}_{F,t}\right)+\beta E_t\left[\widehat{i}^{F}_{F,t+1}\right]$

6. $\widehat{i}^{F}_{H,t}=\frac{1-\theta}{\theta}(1-\theta\beta)\left(-\widehat{Z}_t+(1-\alpha)\,\widehat{R}^{K}_{H,t}+\alpha\widehat{W}_{H,t}-\widehat{A}_{H,t}-\widehat{P}^{H}_{H,t}\right)+\beta E_t\left[\widehat{i}^{F}_{H,t+1}\right]$

7. $\widehat{i}^{H}_{F,t}=\frac{1-\theta}{\theta}(1-\theta\beta)\left(\widehat{Z}_t+(1-\alpha)\,\widehat{R}^{K}_{F,t}+\alpha\widehat{W}_{F,t}-\widehat{A}_{F,t}-\widehat{P}^{H}_{F,t}\right)+\beta E_t\left[\widehat{i}^{H}_{F,t+1}\right]$

8. $E_t\left[\sigma\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right]+\widehat{P}^{H}_{C,t+1}-\widehat{P}^{H}_{C,t}-\widehat{M}_{H,t}-\gamma\widehat{i}^{H}_{C,t}=0$

9. $E_t\left[\sigma\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right]+\widehat{P}^{F}_{C,t+1}-\widehat{P}^{F}_{C,t}-\widehat{M}_{F,t}-\gamma\widehat{i}^{F}_{C,t}=0$

10. $0=E_t\left[-\sigma\left(\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right)+\left(P^{H}_{C,t}-P^{H}_{C,t+1}\right)-\widehat{P}^{H}_{I,t}+\left(\widehat{R}^{K}_{H,t+1}\beta R^{K}+\beta(1-\delta)\,\widehat{P}^{H}_{I,t+1}\right)\right]$

11. $0=E_t\left[-\sigma\left(\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right)+\left(P^{F}_{C,t}-P^{F}_{C,t+1}\right)-\widehat{P}^{F}_{I,t}+\left(\widehat{R}^{K}_{F,t+1}\beta R^{K}+\beta(1-\delta)\,\widehat{P}^{F}_{I,t+1}\right)\right]$

12. $\widehat{K}_{H,t+1}=(1-\delta)\,\widehat{K}_{H,t+1}+\delta\widehat{I}_{H,t}$

13. $\widehat{K}_{F,t+1}=(1-\delta)\,\widehat{K}_{F,t+1}+\delta\widehat{I}_{F,t}$

14. $\widehat{NFA}_{t+1}=\widehat{\xi}_t+\frac{1}{\beta}\widehat{NFA}_{t+1}+\left(\widehat{C}^{H}_{H,t}a_c\Lambda+\widehat{I}^{H}_{H,t}a_I(1-\Lambda)\right)+\left(a_c\Lambda+a_I(1-\Lambda)\right)\widehat{P}^{H}_{H,t}$
$+\left(\widehat{C}^{F}_{H,t}(1-a_C)\Lambda+(1-a_I)(1-\Lambda)\,\widehat{I}^{F}_{H,t}\right)+\left((1-a_C)\Lambda+(1-a_I)(1-\Lambda)\right)\left(\widehat{P}^{F}_{H,t}+\widehat{Z}_t\right)$
$-\left(\Lambda\left(\widehat{C}_{H,t}+\widehat{P}^{H}_{C,t}\right)+(1-\Lambda)\left(\widehat{I}_{H,t}+\widehat{P}^{H}_{I,t}\right)\right)$

15. $\widehat{Y}_{H,t}=\widehat{C}^{H}_{H,t}a_C\Lambda+\widehat{C}^{F}_{H,t}(1-a_C)\Lambda+I^{H}_{H,t}a_I(1-\Lambda)+I^{F}_{H,t}(1-a_I)(1-\Lambda)$

16. $\widehat{Y}_{F,t}=\widehat{C}^{F}_{F,t}a_C\Lambda+\widehat{C}^{H}_{F,t}(1-a_C)\Lambda+I^{F}_{F,t}a_I(1-\Lambda)+I^{H}_{F,t}(1-a_I)(1-\Lambda)$

17. $\widehat{P}^H_{C,t} = a_C \widehat{P}^H_{H,t} + (1 - a_C) \left(\widehat{P}^H_{F,t} \right)$

18. $\widehat{P}^F_{C,t} = a_C \widehat{P}^F_{F,t} + (1 - a_C) \left(\widehat{P}^F_{H,t} \right)$

19. $\widehat{P}^H_{I,t} = a_I \widehat{P}^H_{H,t} + (1 - a_I) \left(\widehat{P}^H_{F,t} \right)$

20. $\widehat{P}^F_{I,t} = a_I \widehat{P}^F_{F,t} + (1 - a_I) \left(\widehat{P}^F_{H,t} \right)$

21. $\widehat{Q}_t = \widehat{Z}_t + \widehat{P}^F_{C,t} - \widehat{P}^H_{C,t}$

22. $\widehat{\Pi}_{H,t} = \varepsilon(\left(\widehat{C}^H_{H,t} a_C \Lambda + \widehat{I}^H_{H,t} a_I (1 - \Lambda) \right) + (a_C \Lambda + a_I (1 - \Lambda)) \widehat{P}^H_{H,t} + \left(\widehat{C}^F_{H,t} (1 - a_C) \Lambda + \widehat{I}^F_{H,t} (1 - a_I) (1 -$
$+ ((1 - a_C) \Lambda + (1 - a_I) (1 - \Lambda)) \left(\widehat{P}^F_{H,t} + \widehat{Z}_t \right)) - (\varepsilon - 1) \left((1 - \alpha) \widehat{R}^K_{H,t} + \alpha \widehat{W}_{H,t} - \widehat{A}_{H,t} + \widehat{Y}_{H,t} \right)$

23. $\widehat{\Pi}_{F,t} = \varepsilon(\left(\widehat{C}^F_{F,t} a_C \Lambda + \widehat{I}^F_{F,t} a_I (1 - \Lambda) \right) + (a_C \Lambda + a_I (1 - \Lambda)) \widehat{P}^F_{F,t} + \left(\widehat{C}^H_{F,t} (1 - a_C) \Lambda + \widehat{I}^H_{F,t} (1 - a_I) (1 - \Lambda$
$+ ((1 - a_C) \Lambda + (1 - a_I) (1 - \Lambda)) \left(\widehat{P}^H_{F,t} - \widehat{Z}_t \right)) - (\varepsilon - 1) \left((1 - \alpha) \widehat{R}^K_{F,t} + \alpha \widehat{W}_{F,t} - \widehat{A}_{F,t} + \widehat{Y}_{F,t} \right)$

24. $\widehat{D}_{H,t} D = \widehat{\Pi}_{H,t} \Pi + \widehat{R}^K_{H,t} R^K + \widehat{K}_{H,t} K - \left(\widehat{I}_{H,t} I + I \widehat{P}^H_{I,t} \right)$

25. $\widehat{D}_{F,t} D = \widehat{\Pi}_{F,t} \Pi + \widehat{R}^K_{F,t} R^K + \widehat{K}_{F,t} K - I \left(\widehat{I}_{F,t} + \widehat{P}^F_{I,t} \right)$

26. $\widehat{i}^H_{H,t} = \left(\widehat{P}^H_{H,t} - \widehat{P}^H_{H,t-1} \right)$

27. $\widehat{i}^F_{H,t} = \left(\widehat{P}^F_{H,t} - \widehat{P}^F_{H,t-1} \right)$

28. $\widehat{i}^F_{F,t} = \left(\widehat{P}^F_{F,t} - \widehat{P}^F_{F,t-1} \right)$

29. $\widehat{i}^H_{F,t} = \left(\widehat{P}^H_{F,t} - \widehat{P}^H_{F,t-1} \right)$

30. $\widehat{Y}_{H,t} = \widehat{A}_{H,t} + \alpha \widehat{L}_{H,t} + (1 - \alpha) \widehat{K}_{H,t}$

31. $\widehat{Y}_{F,t} = \widehat{A}_{F,t} + \alpha \widehat{L}_{F,t} + (1 - \alpha) \widehat{K}_{F,t}$

32. $0 = \widehat{W}_{H,t} - \widehat{P}^H_{C,t} - \sigma \widehat{C}_{H,t} - \omega \widehat{L}_{H,t}$

33. $0 = \widehat{W}_{F,t} - \widehat{P}^F_{C,t} - \sigma \widehat{C}_{F,t} - \omega \widehat{L}_{F,t}$

34. $\sigma (C_{H,t+1} - C_{H,t}) + P^H_{C,t} - P^H_{C,t+1} - P^B_{H,t}$

35. $\sigma\left(C_{F,t+1}-C_{F,t}\right)+P^{F}_{C,t}-P^{F}_{C,t+1}-P^{B}_{F,t}$

36. $R^{B}_{H,t}=-P^{B}_{H,t}$

37. $R^{B}_{F,t}=-P^{B}_{F,t}$

38. $\widehat{C}^{H}_{H,t}=-\phi\left(\widehat{P}^{H}_{H,t}-\widehat{P}^{H}_{C,t}\right)+\widehat{C}_{H,t}$

39. $\widehat{C}^{F}_{H,t}=-\phi\left(\widehat{P}^{F}_{H,t}-\widehat{P}^{F}_{C,t}\right)+\widehat{C}_{F,t}$

40. $\widehat{C}^{F}_{F,t}=-\phi\left(\widehat{P}^{F}_{F,t}-\widehat{P}^{F}_{C,t}\right)+\widehat{C}_{F,t}$

41. $\widehat{C}^{H}_{F,t}=-\phi\left(\widehat{P}^{H}_{F,t}-\widehat{P}^{H}_{C,t}\right)+\widehat{C}_{H,t}$

42. $I^{H}_{H,t}=-\phi\left(\widehat{P}^{H}_{H,t}-\widehat{P}^{H}_{C,t}\right)+\widehat{I}_{H,t}$

43. $I^{F}_{H,t}=-\phi\left(\widehat{P}^{F}_{H,t}-\widehat{P}^{F}_{C,t}\right)+\widehat{I}_{F,t}$

44. $I^{F}_{F,t}=-\phi\left(\widehat{P}^{F}_{F,t}-\widehat{P}^{F}_{C,t}\right)+\widehat{I}_{F,t}$

45. $I^{H}_{F,t}=-\phi\left(\widehat{P}^{H}_{F,t}-\widehat{P}^{H}_{C,t}\right)+\widehat{I}_{H,t}$

46. $\widehat{\widehat{P}}^{H}_{H,t}=\frac{1}{(1-\theta)}\left(\widehat{P}^{H}_{H,t}-\theta\widehat{P}^{H}_{H,t-1}\right)$

47. $\widehat{\widehat{P}}^{F}_{H,t}=\frac{1}{(1-\theta)}\left(\widehat{P}^{F}_{H,t}-\theta\widehat{P}^{F}_{H,t-1}\right)$

48. $\widehat{\widehat{P}}^{F}_{F,t}=\frac{1}{(1-\theta)}\left(\widehat{P}^{F}_{F,t}-\theta\widehat{P}^{F}_{F,t-1}\right)$

49. $\widehat{\widehat{P}}^{H}_{F,t}=\frac{1}{(1-\theta)}\left(\widehat{P}^{H}_{F,t}-\theta\widehat{P}^{H}_{F,t-1}\right)$

50. $\widehat{L}_{H,t}-\widehat{K}_{H,t}=\widehat{R}^{K}_{H,t}-\widehat{W}_{H,t}$

51. $\widehat{L}_{F,t}-\widehat{K}_{F,t}=\widehat{R}^{K}_{F,t}-\widehat{W}_{F,t}$

52. $\widehat{i}^{H}_{C,t}=\left(\widehat{P}^{H}_{C,t}-\widehat{P}^{H}_{C,t-1}\right)$

53. $\widehat{i}^{F}_{C,t}=\left(\widehat{P}^{F}_{C,t}-\widehat{P}^{F}_{C,t-1}\right)$

54. $\widehat{A}_{H,t+1} = \rho_A \widehat{A}_{H,t} + \varepsilon_{H,t}$

55. $\widehat{A}_{F,t+1} = \rho_A \widehat{A}_{F,t} + \varepsilon_{F,t}$

56. $\widehat{M}_{H,t+1} = \rho_M \widehat{M}_{H,t} + \varepsilon_{H,M,t}$

57. $\widehat{M}_{F,t+1} = \rho_M \widehat{M}_{F,t} + \varepsilon_{F,M,t}$

58. ξ

7.5 Model without Nominal Rigidities

Tables A.1 and A.2 present portfolios for a variety of parameters for intertemporal elasticity of substitution, σ, and the elasticity of substitution between Home and Foreign goods, ϕ. Table A.1 focusses on the cases stressed in the benchmark calibration in Heatcote and Perri (2008), namely for $\sigma = \phi = 1$, while Table A.2 considers our benchmark calibration, with $\sigma = 2$ and $\phi = 1.5$. As mentioned in the main text, optimal bond positions are always zero, as the covariance of bond returns with the sources of risk is zero, once we condition on equity returns. Our discussion can thus focus on equity portfolios without loss of generality. We find that, in general, the real exchange rate depreciates, following a productivity shock. For low enough values of ϕ, relative equity returns are also negative, for two reasons. Firstly, investment rises which implies a fall in dividends. Secondly, the terms of trade depreciate. Both of these effects are persistent. For $\sigma < 1$, agents prefer to hold an asset that has high payoffs when the real exchange rate depreciates. Thus, the real exchange rate hedging motive pushes the Home equity position to be negative for $\sigma < 1$. For $\sigma > 1$, equity is a good hedge for real exchange rates. Thus, we find that the hedging motive for real exchange rates increases home bias in equities for $\sigma > 1$, while it decreases home bias for $\sigma < 1$. For $\sigma = 1$, the agent does not care about real exchange rate fluctuations and this motive correspondingly does not affect real exchange rates. The returns to human capital increase for low values of σ, while they are negative for higher values of σ. This implies that for high enough values of the intertemporal elasticity of substitution, domestic equity is a good hedge for human capital. As an increase in σ implies that agents are more risk averse and less ready to substitute intertemporally, asset positions in general increase. As we increase the substitutability between Home and Foreign goods, keeping σ constant at $\sigma = 1$, we find that Home equity positions initially increase and become very high for a range of values. This is because higher ϕ implies a smaller fall in relative prices, as Home output increases. The smaller fall in relative prices implies that relative Home equity returns respond less negatively to a productivity shock which induces agents to hold more of the asset in order to achieve the same degree of insurance. As in Heathcote and Perri (2008) and Coeurdacier (2008), portfolios are discontinuous in ϕ. Above a certain level of ϕ, relative equity returns become positively correlated with the returns to human capital which induces agents to go short in domestic equity. As ϕ further increases, the positive correlation between equity returns and the returns to human capital becomes stronger and thus allows positions to become smaller, but equity positions always exhibit excessive foreign bias. Since we kept $\sigma = 1$, the real exchange rate motive does not affect portfolio positions.

Table 2 gives analogous findings for our preferred parameter values, namely $\phi = 1.5$ and $\sigma = 2$. Portfolios and their main drivers are for the most part very similar, at least qualitatively, but there are two major differences. Firstly, we find that for low enough values of ϕ, human capital returns respond negatively to productivity shocks implying that the hedging motive for human capital actually decreases home bias in equities. Secondly, since $\sigma \neq 1$, the hedging motive for real exchange rates is again important. Since relative equity returns are negative only for low values of ϕ, while the real exchange rate always depreciates, we find that this motive increases home bias for low values of ϕ, while it decreases home bias for high values of ϕ.

In general, while the model can produce substantial home bias, their are a number of potential weaknesses: i) equity positions are very sensitive to changes in parameter values and exhibit discontinuities 2) for realistic parameter values($\sigma > 1, \phi > 1$), the model produces too much home bias, 3) optimal bond holdings are zero and there are no real effects of monetary policy, 4) the motive to hedge real exchange rate risk is usually as, and often more, important for equity positions as the motive to hedge returns to human capital. This may raise concerns about the plausibility of the model in light of the evidence that contemporaneous real exchange rates and equity returns are not significantly correlated in the data.

Table A.1

without nominal rigidities ($\phi = 1$)

Statistic		$\sigma = 0.8$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 3$	$\sigma = 5$	$\sigma = 10$
Share of Home Equity		**0.57**	**0.79**	**1.01**	**1.09**	**1.16**	**1.22**	**1.25**
	due to:							
	human capital	0.60	0.29	−0.01	−0.13	−0.23	−0.30	−0.35
	real ER	−0.02	0	0.02	0.02	0.03	0.03	0.04
	future real ER	−0.52	0	0.50	0.69	0.86	0.98	1.06
No. of Home Bonds		**0**	**0**	**0**	**0**	**0**	**0**	**0**
	due to:							
	human capital	0	0	0	0	0	0	0
	real ER	0	0	0	0	0	0	0
	future real ER	0	0	0	0	0	0	0

without nominal rigidities ($\sigma = 1$)

Statistic		$\phi = 0.8$	$\phi = 1$	$\phi = 1.5$	$\phi = 2$	$\phi = 3$	$\phi = 5$	$\phi = 10$
Share of Home Equity		**0.54**	**0.79**	**1.91**	**5.98**	**−5.67**	**−2.32**	**−1.69**
	due to:							
	human capital	0.04	0.29	1.41	5.48	−6.17	−2.82	−2.19
	real ER	0	0	0	0	0	0	0
	future real ER	0	0	0	0	0	0	0
No. of Home Bonds		**0**	**0**	**0**	**0**	**0**	**0**	**0**
	due to:							
	human capital	0	0	0	0	0	0	0
	real ER	0	0	0	0	0	0	0
	future real ER	0	0	0	0	0	0	0

Table A.2

without nominal rigidities ($\phi = 1.5$)

Statistic		$\sigma = 0.8$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 3$	$\sigma = 5$	$\sigma = 10$
Share of Home Equity		**1.83**	**1.91**	**1.97**	**1.99**	**2.01**	**2.02**	**2.03**
	due to:							
	human capital	2.25	1.41	0.72	0.48	0.29	0.16	0.07
	real ER	−0.02	0	0.02	0.03	0.04	0.04	0.07
	future real ER	−0.90	0	0.73	0.98	1.18	1.32	1.41
No. of Home Bonds		**0**	**0**	**0**	**0**	**0**	**0**	**0**
	due to:							
	human capital	0	0	0	0	0	0	0
	real ER	0	0	0	0	0	0	0
	future real ER	0	0	0	0	0	0	0

without nominal rigidities ($\sigma = 2$)

Statistic		$\phi = 0.8$	$\phi = 1$	$\phi = 1.5$	$\phi = 2$	$\phi = 3$	$\phi = 5$	$\phi = 10$
Share of Home Equity		**0.86**	**1.09**	**1.99**	**4.10**	**−17.81**	**−3.07**	**−1.90**
	due to:							
	human capital	−0.28	−0.13	0.48	1.92	−13.01	−2.99	−2.21
	real ER	0.02	0.23	0.03	0.05	−0.11	−0.01	0.00
	future real ER	0.62	0.69	0.98	1.64	−5.18	−2.99	−0.19
No. of Home Bonds		**0**	**0**	**0**	**0**	**0**	**0**	**0**
	due to:							
	human capital	0	0	0	0	0	0	0
	real ER	0	0	0	0	0	0	0
	future real ER	0	0	0	0	0	0	0

7.6 Model without Investment

The model without investment generates realistic degrees of home bias with our benchmark calibrtation whereby the equity position is largely determined by a desire to hedge movements in the return to human capital and only marginally by the motive to hedge real exchange rate risk. This is because Home equity is a good hedge for human capital risk under both shocks, while nominal bonds are a good hedge for real exchange rate risk. In general, however, both asset positions are affected by both the desire to hedge human capital risk and the desire to hedge real exchange rate risk which implies that finding an intuition for portfolios is sometimes complex. Tables 3 and 4 depict portfolios for various values of the intertemporal elasticity of substitution/ risk aversion σ, the elasticity of substitution ϕ, the degree of price rigidity θ and the responsiveness of monetary policy γ.

As in the case of the model without price rigidities, increasing σ has the effect of increasing asset positions, as agents' risk aversion increases. Since human capital returns and relative equity returns are always negatively correlated, the contribution of human capital risk to equity positions is always positive for Home equity positions. This contribution does not change much, however, as we change σ. The contribution of real exchange rate risk is positive for $\sigma < 1$ and negative for $\sigma > 1$ and is largely determined by the residual risk left after nominal bonds have hedged the other portions of real exchange rate risk. Nominal bond positions are positive for $\sigma > 1$, as they are used to hedge real exchange rate risk.

As we change ϕ, the effects of changes in the relative supply of goods on relative prices becomes smaller which implies that labour demand does not fall (rise) by as much in response to an increase (decrease) in supply. This means that relative equity returns become more positively correlated with returns to human capital, and at some point, the correlation in fact turns positive. As in the case of the model without price rigidities, this implies that hedging human capital risk reduces home equity bias. Unlike in the case of the model without price rigidities, however, this effect is not strong enough to induce agents to take short positions in domestic equity though the negative effect on equity portfolios actually increases monotonically, as ϕ rises. Domestic bond positions are always positive and monotonically increase, as ϕ rises. Initially, bond positions are mainly driven by a desire to change movements in the real exchange rate, but as ϕ rises, real exchange rate risk falls and so does their contribution to bond positions. The increase in bond positions is then mainly driven by a positive and increasing contribution of hedging human capital risk, as equities become less good at hedging this source of risk.

Table 4a highlights the importance of the degree of price rigidity. The share of domestic equity held increases monotonically, as we increase the degree of price rigidity, while the size of bond positions exhibits a hump shape, increasing initially, but falling for high levels of θ. Equity positions are strongly negative for low values of θ, but increase for three reasons. For low values of price rigidity, labour demand does not fall much in response to increases in the supply of the Home good implying that equity is not a good hedge for human capital. What is more, nominal bonds are a poor hedge for real exchange rate risk leaving much of the real exchange rate risk to be hedged using bonds. As θ increases, labour demand falls more strongly, leading to a larger positive contribution of human capital hedging to real exchange rate risk. What is more, the correlation between equity returns and the real exchange rate becomes smaller, as we increase price stickiness and in fact turns negative for higher values of θ. Thus, we observe that for lower values of θ, real exchange rate hedging leads to less home bias, while for high values of θ, it increases home bias though the effect is small. Moreover, nominal bonds become more closely related to real exchange rate fluctuations and reduce the extent to which equity positions are affected by the real exchange rate motive. These effects also play out in bond positions. The contribution of human capital risk becomes more negative, as θ increases. Bond positions nevertheless initially increase, as this effect is dominated by the increasing positive contribution of real exchange rate hedging which results from the higher correlation of nominal and real exchange rates, as the degree of price rigidity rises.

The degree of monetary policy responsiveness mainly has an effect on the size and dynamics of fluctuations in nominal prices, including the nominal exchange rate. The behaviour of the nominal exchange rate, in turn, determines the behaviour of bond returns. However, with one period bonds, this effect is rather limited and so are changes in portfolios with respect to changes in γ. As γ increases, the negative effect of real exchange rate risk on equity positions becomes smaller and this appears to be a result of the fact that bond positions become smaller also and equity returns become less (negatively) correlated with the real exchange rate. However, both for equity and bond positions, the effect is rather limited. Bond positions are nonmonotonic with respect to γ, and so are the individual hedging demands. Bond returns become more positively correlated with human capital returns, moving from highly negatively correlated to highly positively correlated, while they are always highly negatively correlated with real exchange rates. Since human capital concerns appear to be more important for equity portfolios, these effects are rather small though. It is also worth noting that equity returns and bond returns become less correlated as γ increases, with the correlations turning from highly positively to negatively correlated.

In summary, the model without investment can generate equity home bias for sufficient degrees of price rigidity and sufficiently low values of the substitution elasticity between home and foreign goods. In those cases, equity portfolios appear to be driven more by concerns to hedge human capital than movements in the real exchange rate, as in Coeurdacier et al (2008). However, due to the absence of investment in the model, it generates counterfactual procyclial net exports.

Table 3

without investment ($\phi = 1.5$)

Statistic		$\sigma = 0.8$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 3$	$\sigma = 5$	$\sigma = 10$
Share of Home Equity		**0.77**	**0.76**	**0.74**	**0.73**	**0.72**	**0.72**	**0.72**
	due to:							
	human capital	0.22	0.26	0.30	0.32	0.34	0.35	0.36
	real ER	0.01	0	−0.02	−0.03	−0.04	−0.05	−0.06
	future real ER	0.04	0	−0.04	−0.06	−0.07	−0.07	−0.07
No. of Home Bonds		**0.33**	**0.78**	**1.30**	**1.50**	**1.66**	**1.74**	**1.76**
	due to:							
	human capital	1.46	0.78	−0.10	−0.49	−0.39	−1.06	−1.20
	real ER	−0.14	0	0.2	0.30	0.38	0.45	0.49
	future real ER	−0.99	0	1.2	1.70	2.11	2.35	2.48

without investment ($\sigma = 2$)

Statistic		$\phi = 0.8$	$\phi = 1$	$\phi = 1.5$	$\phi = 2$	$\phi = 3$	$\phi = 5$	$\phi = 10$
Share of Home Equity		**1.10**	**0.95**	**0.73**	**0.60**	**0.44**	**0.28**	**0.11**
	due to:							
	human capital	0.80	0.61	0.32	0.16	−0.03	−0.21	-0.39
	real ER	−0.05	−0.04	−0.03	−0.03	−0.02	−0.02	−0.01
	future real ER	−0.15	−0.11	−0.06	−0.03	−0.01	−0.01	0.01
No. of Home Bonds		**0.12**	**0.67**	**1.50**	**1.99**	**2.55**	**3.13**	**3.75**
	due to:							
	human capital	−2.28	−1.56	−0.49	0.12	0.80	1.47	2.13
	real ER	0.35	0.33	0.30	0.28	0.26	0.23	0.21
	future real ER	2.05	1.90	1.70	1.59	1.50	1.43	1.41

Table 4

without investment ($\sigma = 2, \phi = 1.5$)

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**−14.52**	**−3.10**	**−0.31**	**0.47**	**0.73**	**0.87**	**0.96**
	due to:							
	human capital	0.05	0.31	0.33	0.32	0.32	0.34	0.40
	real ER	−1.63	−0.50	−0.18	−0.07	−0.03	−0.01	−0.00
	future real ER	−13.45	−3.40	−0.96	−0.28	−0.06	0.05	0.06
No. of Home Bonds		**1.35**	**1.72**	**1.67**	**1.58**	**1.50**	**1.38**	**1.13**
	due to:							
	human capital	0.01	−0.05	−0.16	−0.32	−0.49	−0.78	−1.37
	real ER	0.15	0.22	0.26	0.28	0.30	0.31	0.32
	future real ER	1.20	1.54	1.58	1.62	1.70	1.86	2.19

without investment ($\sigma = 2, \phi = 1.5$)

Statistic		$\gamma = 0.5$	$\gamma = 1$	$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3.5$	$\gamma = 5$	$\gamma = 10$
Share of Home Equity		**0.47**	**0.69**	**0.71**	**0.73**	**0.74**	**0.75**	**0.76**
	due to:							
	human capital	0.44	0.34	0.33	0.32	0.31	0.31	0.31
	real ER	−0.03	−0.03	−0.03	−0.03	−0.03	−0.03	−0.03
	future real ER	−0.44	−0.11	−0.08	−0.06	−0.04	−0.03	0.02
No. of Home Bonds		**1.51**	**1.62**	**1.56**	**1.50**	**1.45**	**1.43**	**1.40**
	due to:							
	human capital	−0.49	−0.54	−0.51	−0.49	−0.47	−0.46	−0.45
	real ER	0.30	0.29	0.29	0.30	0.30	0.30	0.30
	future real ER	1.70	1.87	1.78	1.70	1.62	1.59	1.55

7.7 Model with Investment Shocks and without Price Rigidities(ignore for now)

7.7.1 with short term bonds

7.7.2 with long term bonds

7.8 Model with Real Bonds

7.8.1 with Short Term Bonds

7.8.2 with Long Term Bonds

7.9 Figures

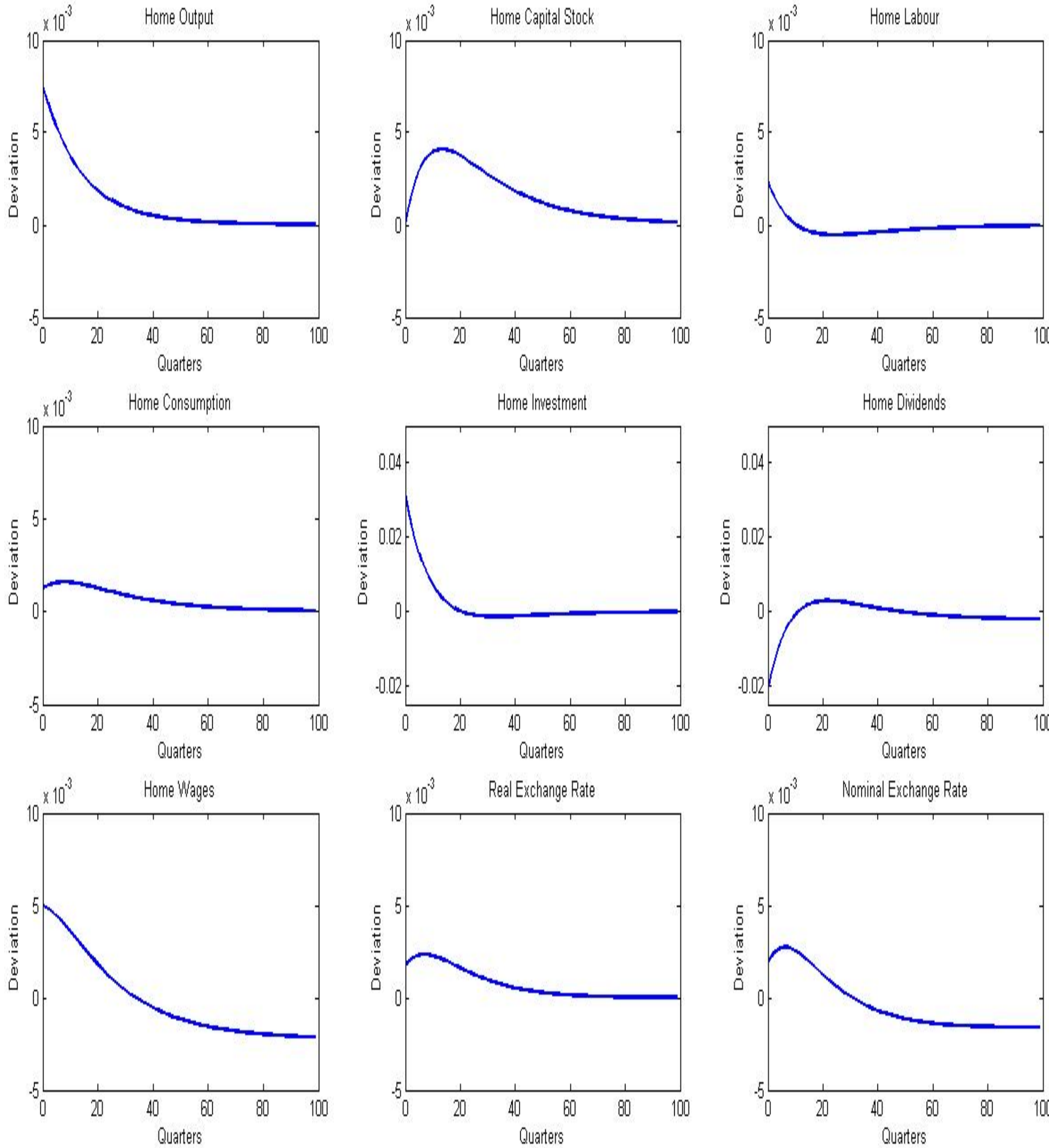

Figure 1: Impulses Response to a relative Home Shock to Productivity in the Model without Nominal Rigidities

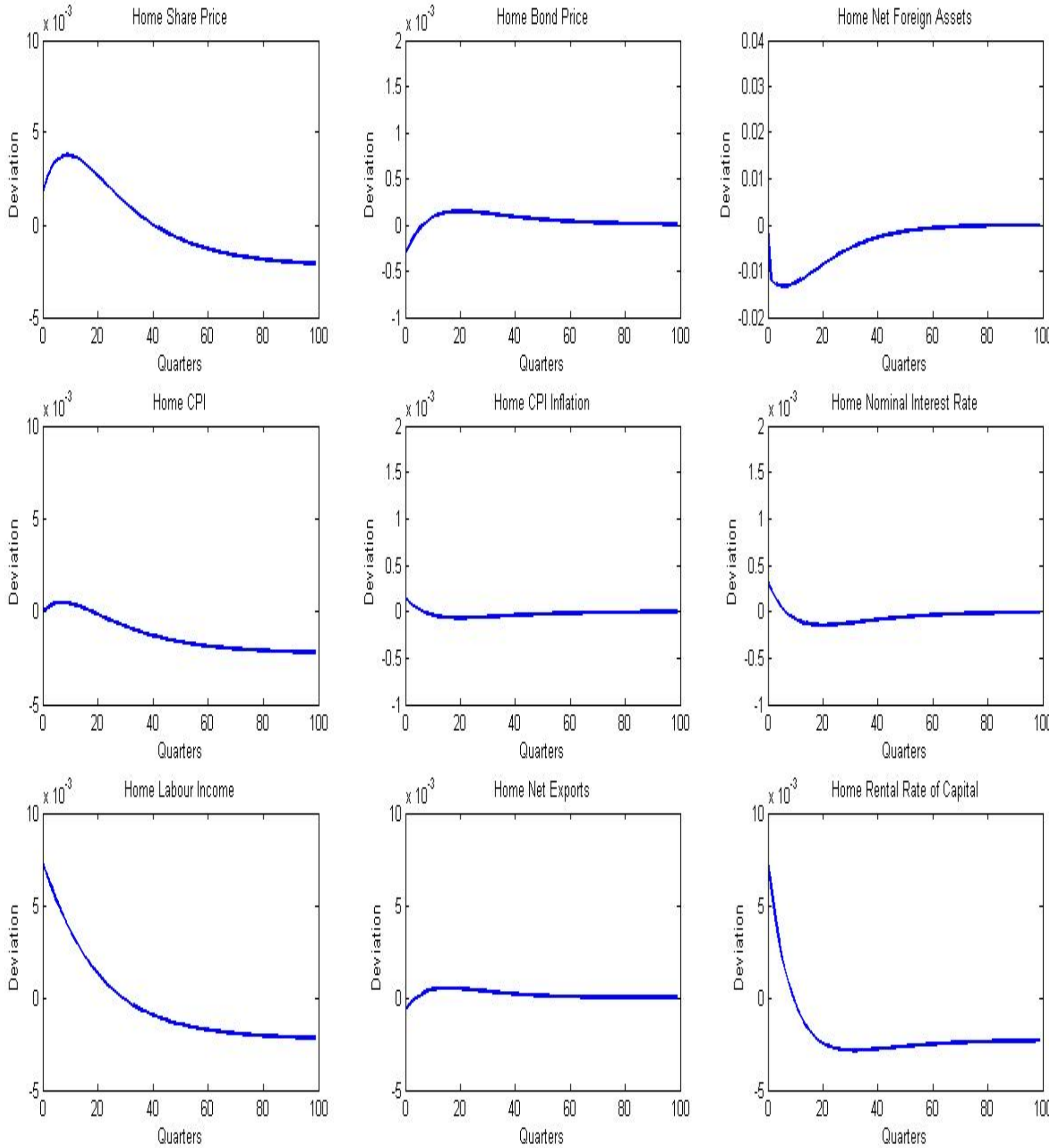

Figure 2: Impulses Response to a relative Home Shock to Productivity in the Model without Nominal Rigidities

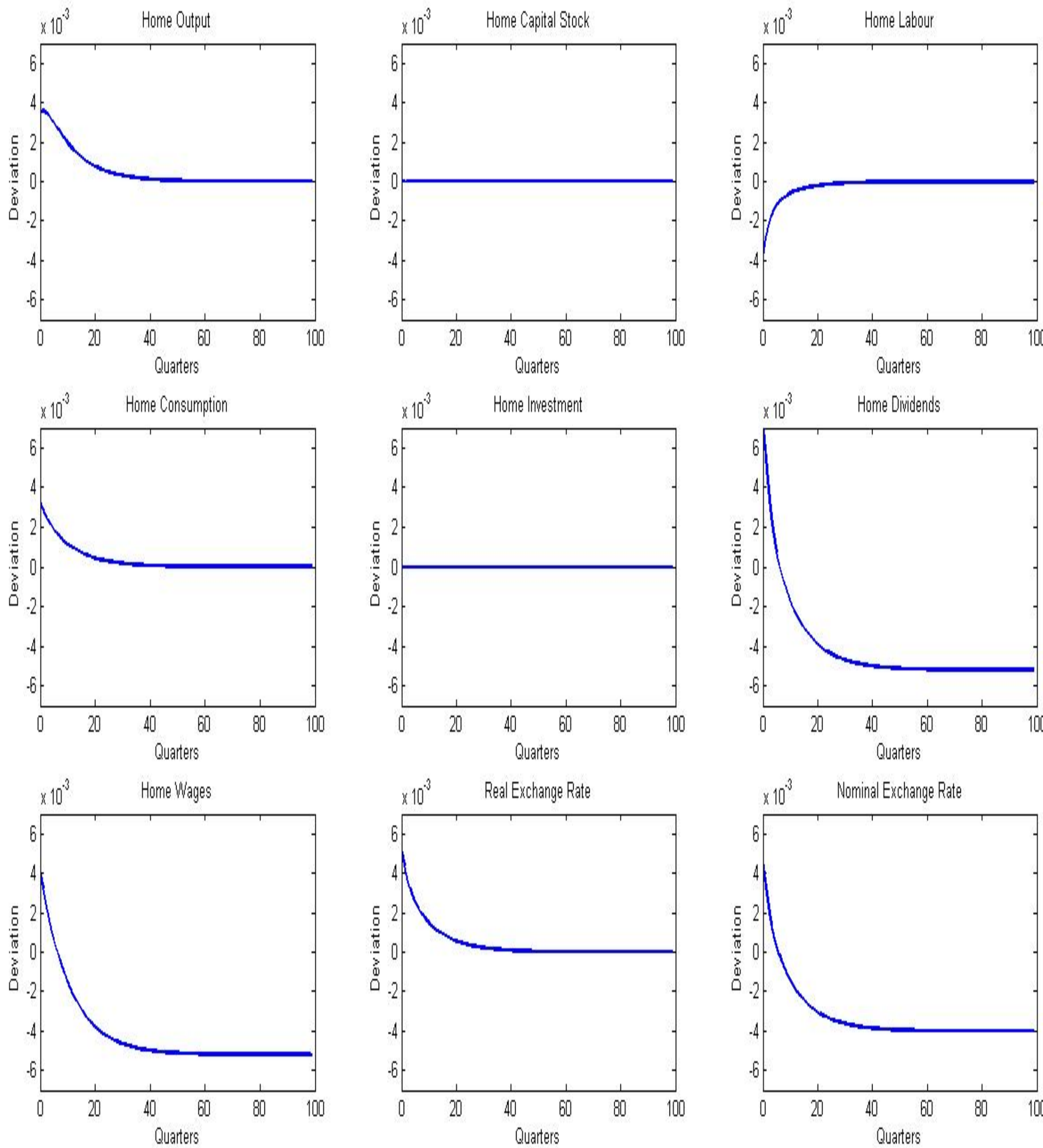

Figure 3: Impulses Response to a relative Home Shock to Productivity in the Model without Investment

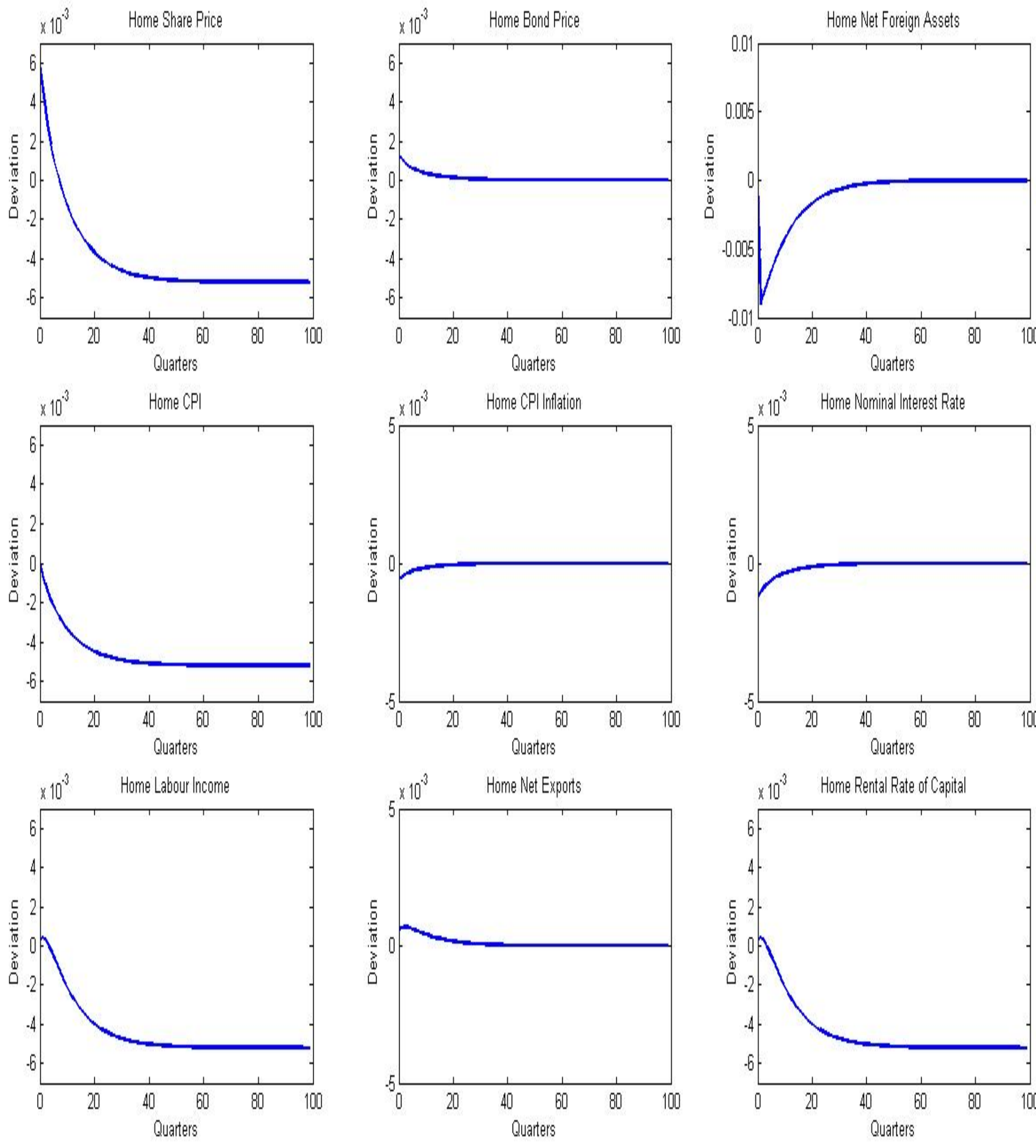

Figure 4: Impulses Response to a relative Home Shock to Productivity in the Model without Investment

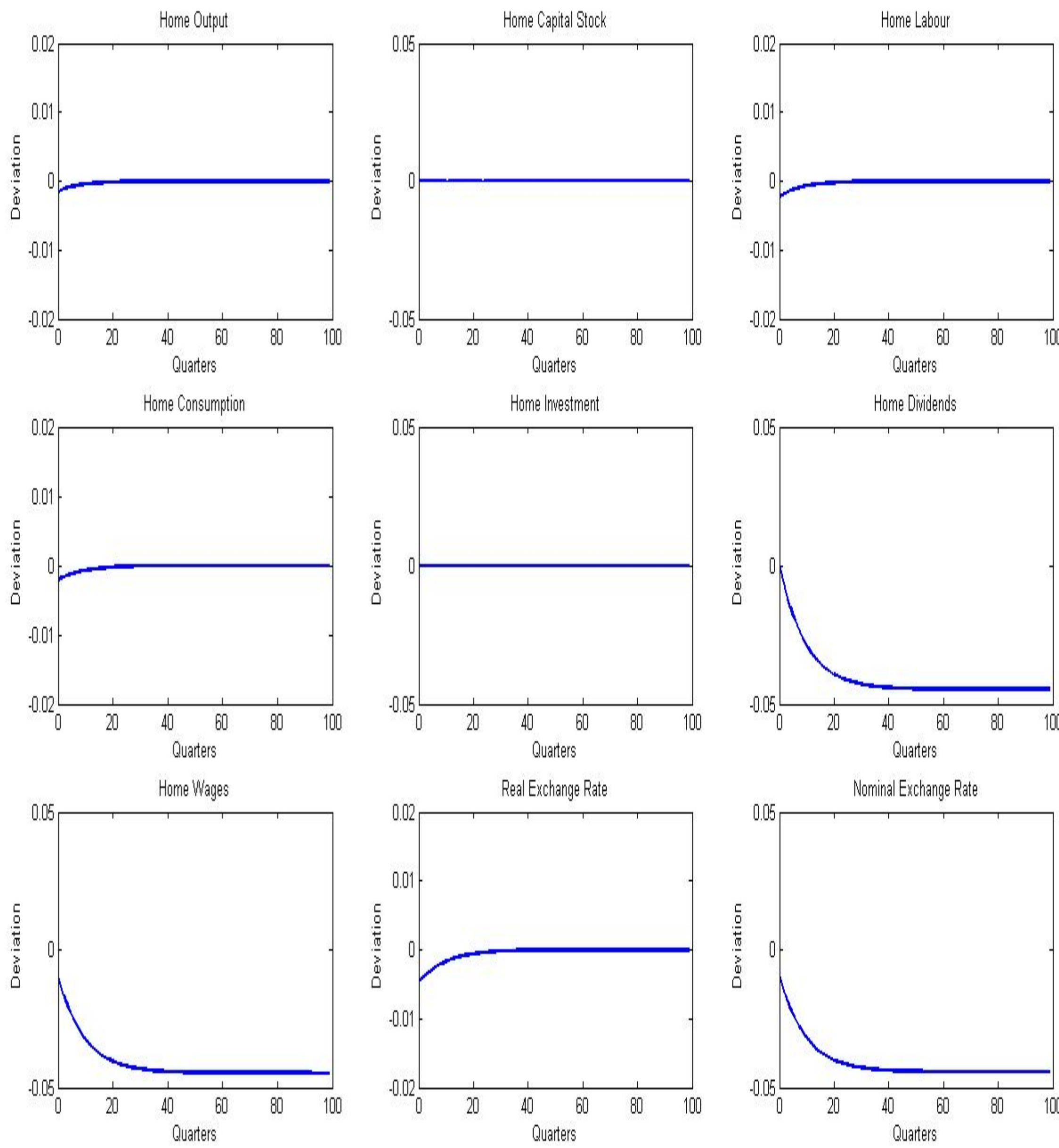

Figure 5: Impulses Response to a relative Home Shock to the Interest Rate in the Model without Investment

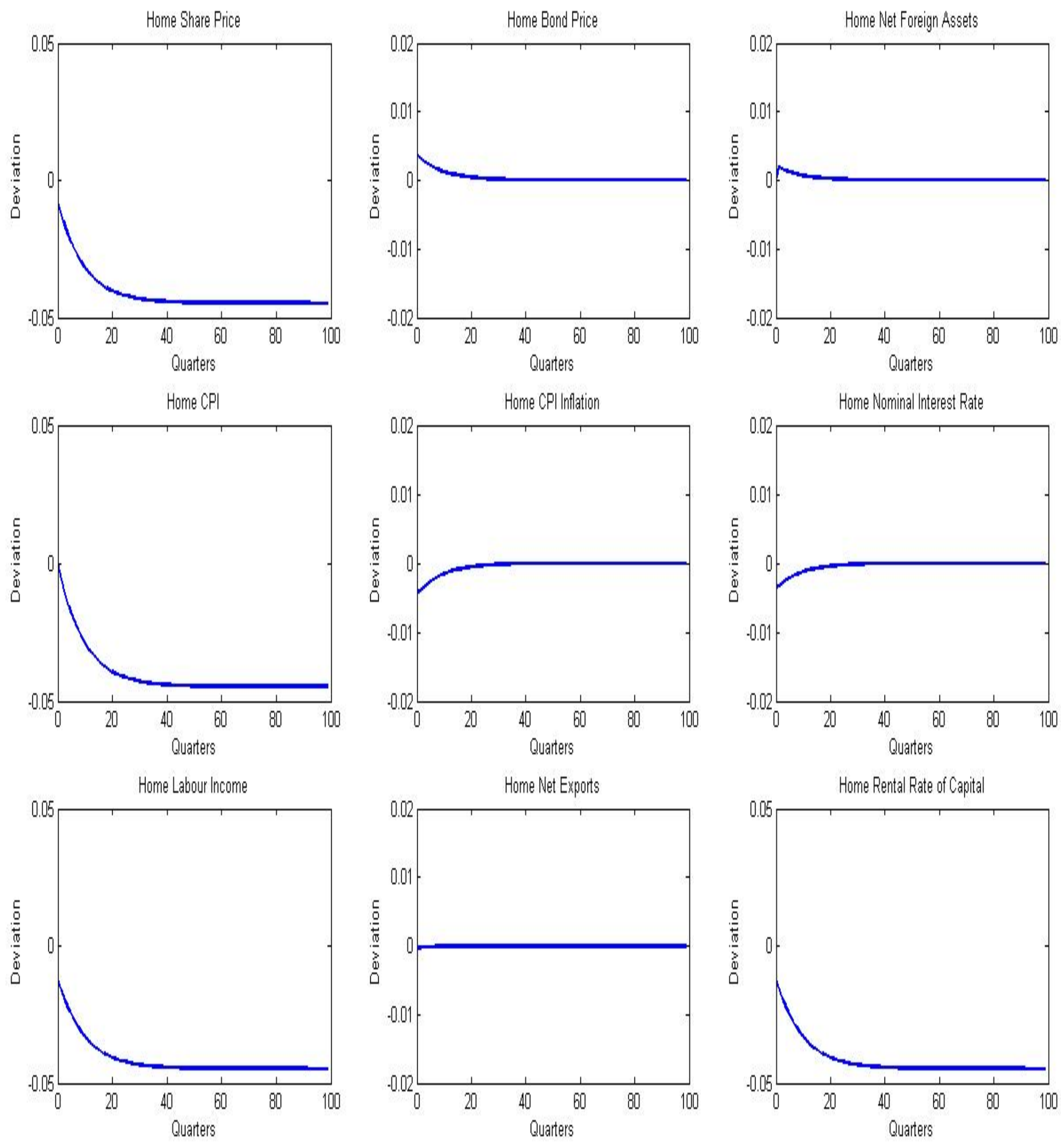

Figure 6: Impulses Response to a relative Home Shock to the Interest Rate in the Model without Investment

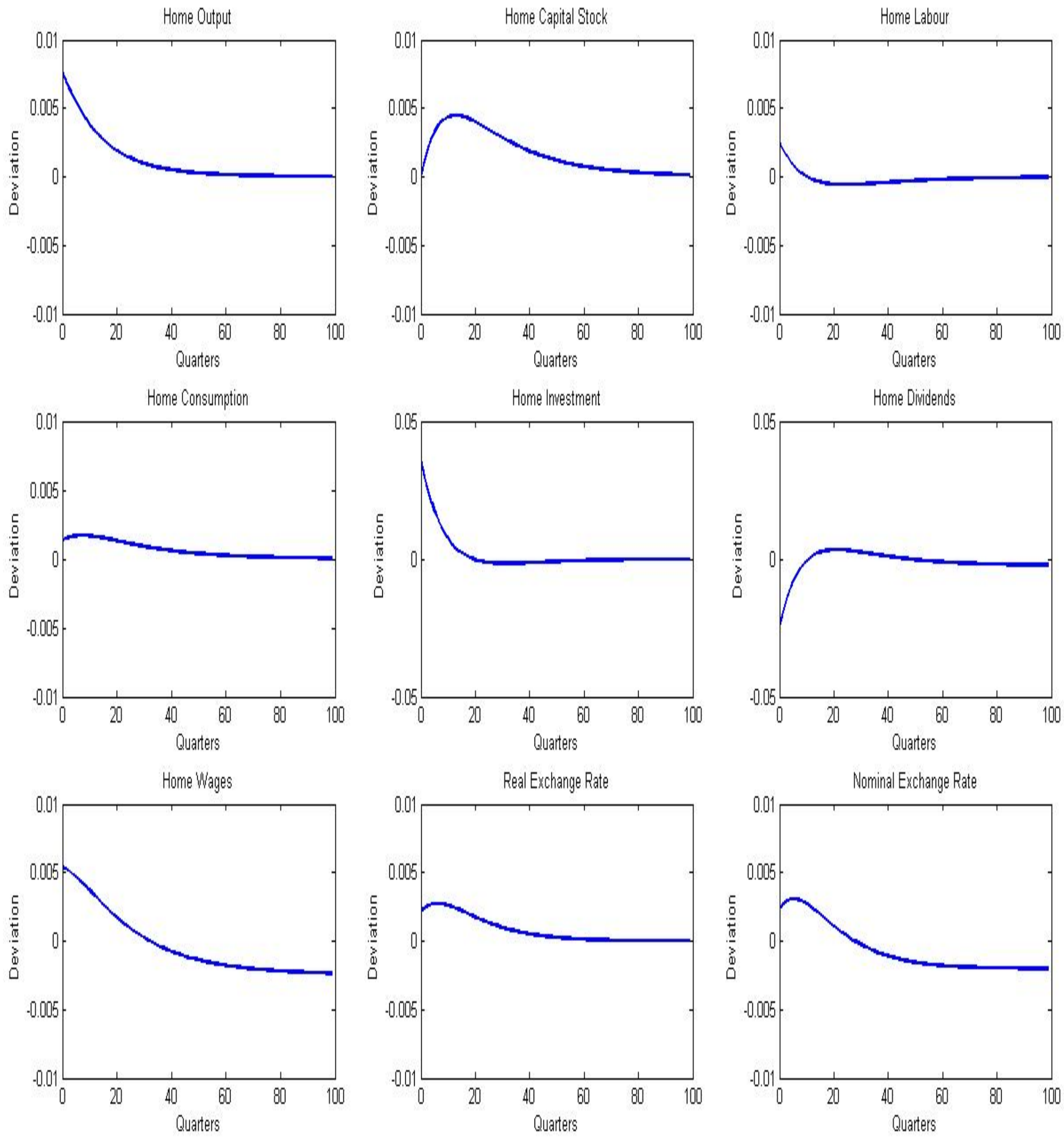

Figure 7: Impulses Response to a relative Home Shock to Productivity in the Model with Nominal Rigidities and Investment

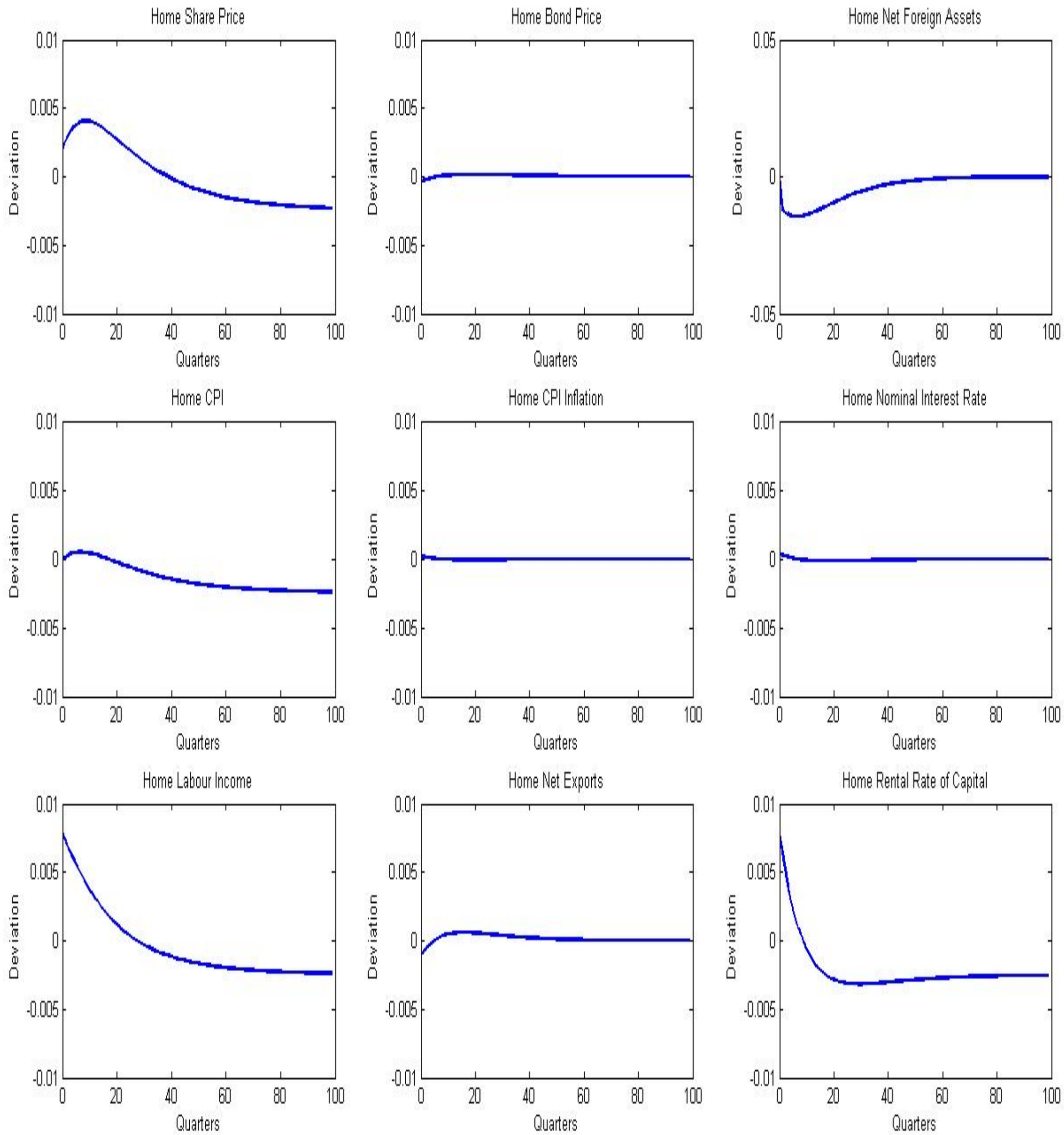

Figure 8: Impulses Response to a relative Home Shock to Productivity in the Model with Nominal Rigidities and Investment

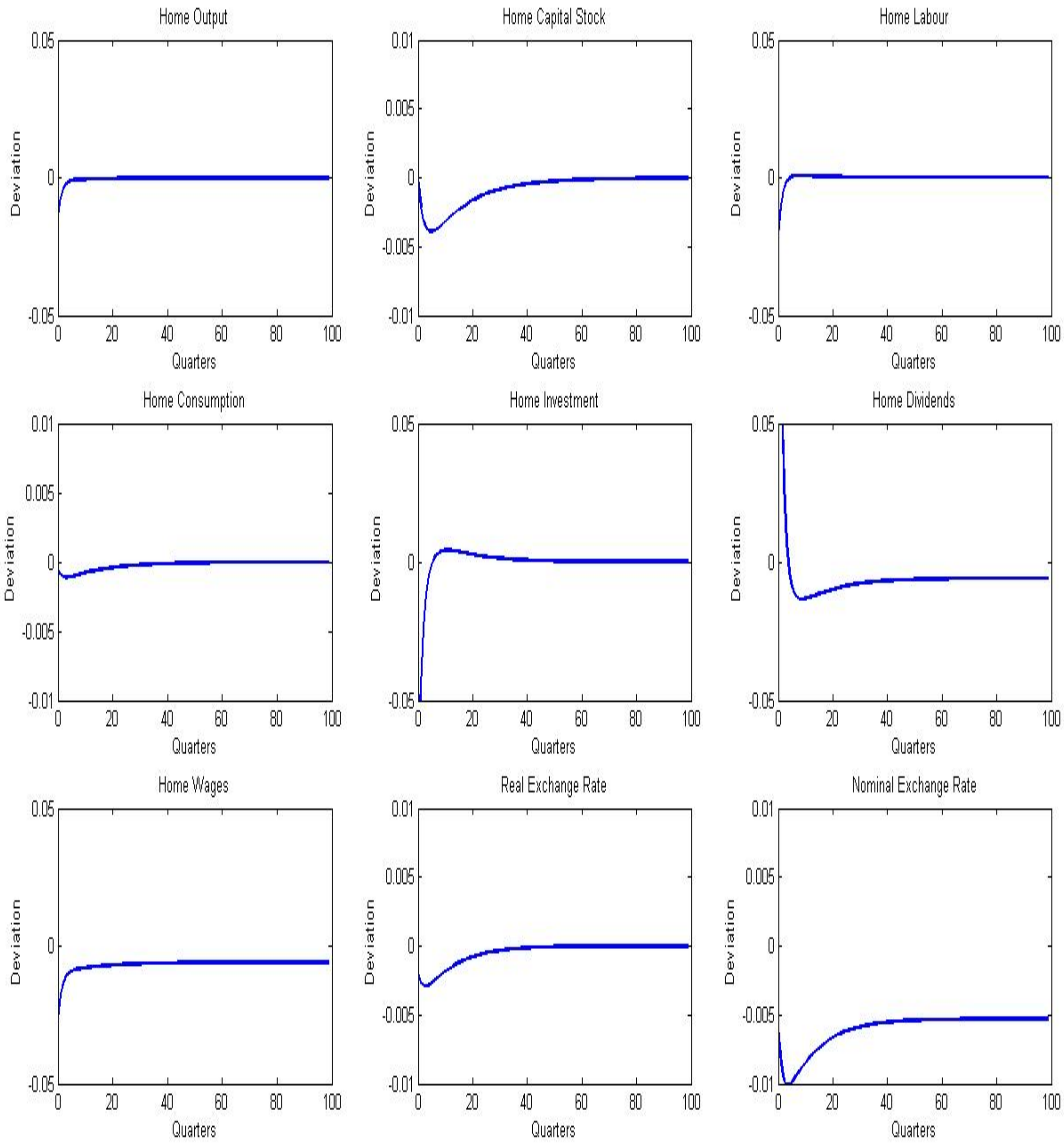

Figure 9: Impulses Response to a relative Home Shock to the Interest Rate in the Model with Nominal Rigidities and Investment

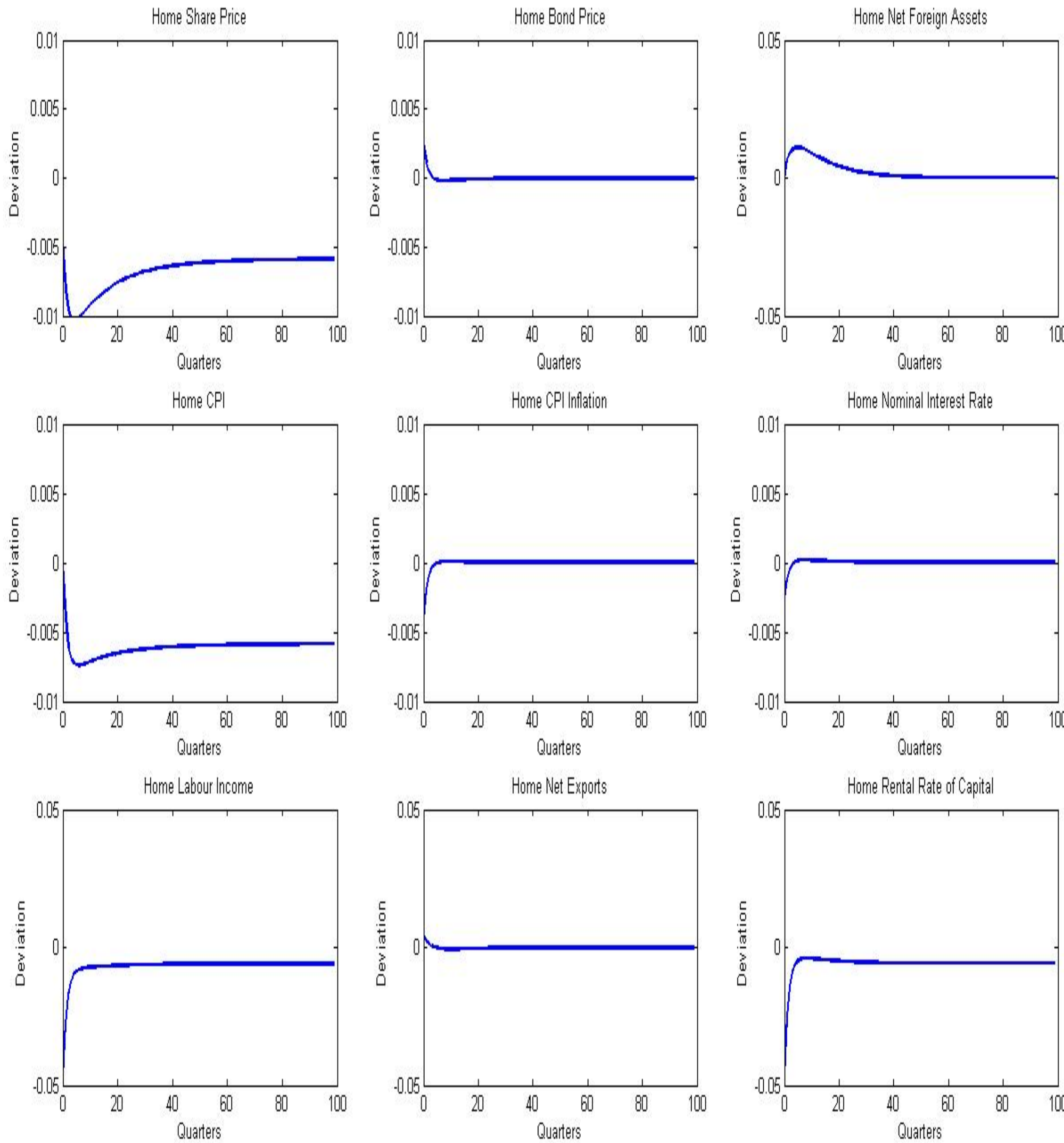

Figure 10: Impulses Response to a relative Home Shock to the Interest Rate in the Model with Nominal Rigidities and Investment

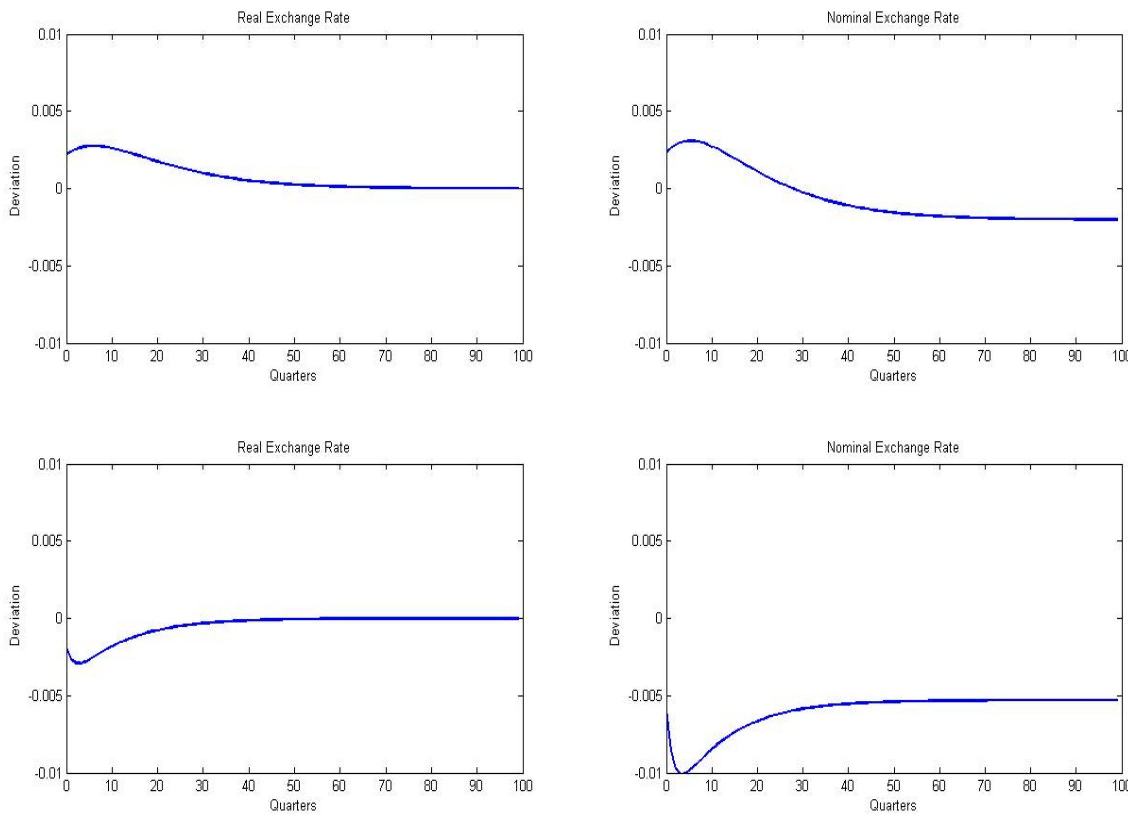

Figure 11: Impulses Response of Real and Nominal Exchange Rates

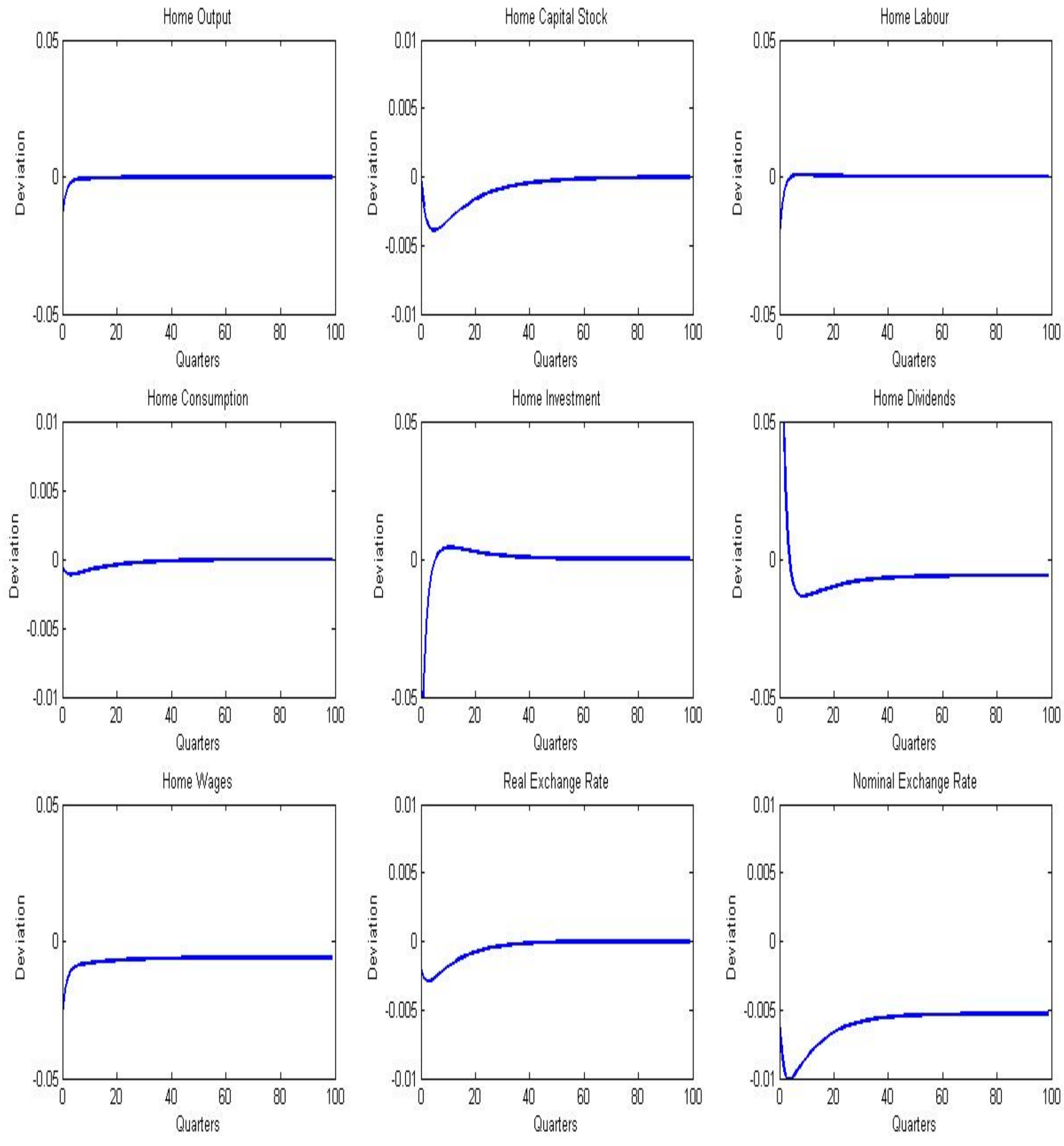

Figure 12: Impulses Response to a relative Home Shock to Investment Efficiency in the Model with Nominal Rigidities and Investment

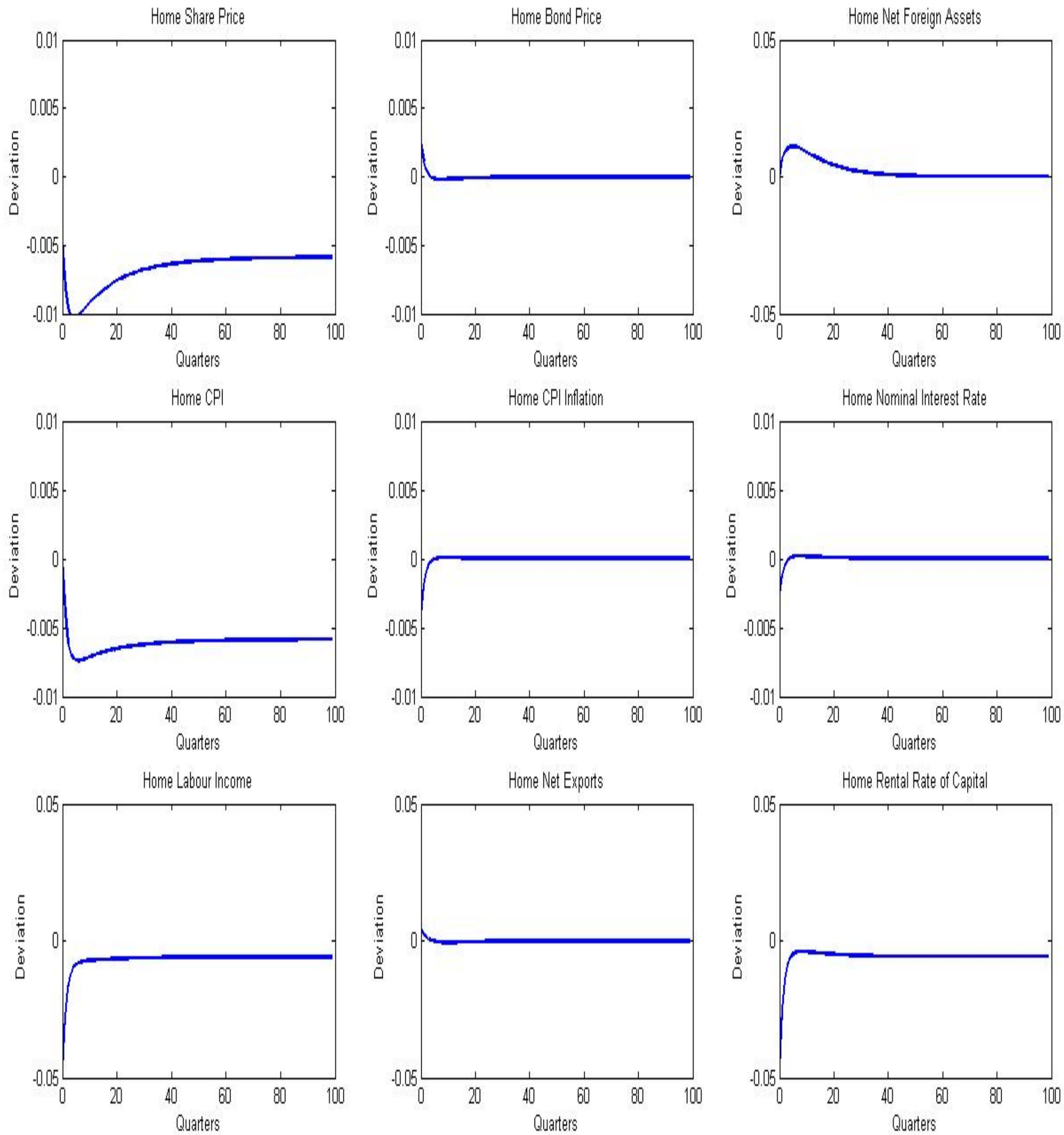

Figure 13: Impulses Response to a relative Home Shock to Investment Efficiency in the Model with Nominal Rigidities and Investment

Table 1a

with sticky prices and capital accumulation: changing σ

Statistic		$\sigma = 0.8$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 3$	$\sigma = 5$	$\sigma = 10$
Share of Home Equity		**0.37**	**0.12**	**−0.30**	**−0.55**	**−0.85**	**−1.13**	**−1.36**
	due to:							
	human capital	−0.60	−0.38	−0.01	0.22	0.48	0.72	0.93
	real ER	0.02	0	−0.03	−0.05	−0.08	−0.10	−0.12
	future real ER	0.46	0	−0.75	−1.22	−1.75	−2.25	−2.66
Bond Pos./ GDP		**3.93**	**6.30**	**10.25**	**12.67**	**15.47**	**18.07**	**20.26**
	due to:							
	human capital	8.37	6.30	2.85	0.73	−1.74	−4.03	−5.96
	real ER	−0.14	0	0.27	0.45	0.68	0.90	1.09
	future real ER	−4.30	0	7.12	11.48	16.54	21.20	25.13

7.10 Tables

Table 1b

with sticky prices and capital accumulation: changing ϕ

Statistic		$\phi = 0.8$	$\phi = 1$	$\phi = 1.5$	$\phi = 2$	$\phi = 3$	$\phi = 5$	$\phi = 10$
Share of Home Equity		**1.84**	**0.68**	**−0.55**	**−1.06**	**−1.62**	**−1.81**	**−2.00**
	due to:							
	human capital	4.20	2.27	0.22	−0.64	−1.60	−1.94	−2.30
	real ER	−0.14	−0.10	−0.05	−0.03	−0.01	−0.01	−0.00
	future real ER	−2.72	−2.00	−1.22	−0.89	−0.51	−0.37	−0.21
Bond Pos./ GDP		**−6.79**	**2.70**	**12.67**	**16.70**	**20.97**	**22.33**	**23.49**
	due to:							
	human capital	−31.86	−16.04	0.73	7.65	15.26	17.87	20.50
	real ER	1.17	0.82	0.45	0.30	0.15	0.10	0.06
	future real ER	23.89	17.92	11.48	8.75	5.57	4.36	2.93

Table 2a

with sticky prices and capital accumulation: changing θ

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**2.01**	**2.07**	**2.33**	**4.27**	**−0.55**	**0.80**	**1.02**
	due to:							
	human capital	0.48	0.49	0.51	0.70	0.22	0.35	0.37
	real ER	0.03	0.03	0.04	0.11	−0.05	−0.01	0.00
	future real ER	0.99	1.05	1.28	2.97	−1.22	−0.04	0.15
Bond Pos./ GDP		**−0.10**	**−0.51**	**−2.00**	**−12.44**	**12.67**	**4.97**	**2.62**
	due to:							
	human capital	−0.02	−0.10	−0.35	−1.55	0.73	−0.53	−2.07
	real ER	−0.00	−0.01	−0.06	−0.39	0.45	0.22	0.19
	future real ER	−0.07	−0.39	−1.60	−10.50	11.48	5.28	4.50

Table 2b

with sticky prices and capital accumulation: Correlations

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**2.01**	**2.07**	**2.33**	**4.27**	**−0.55**	**0.80**	**1.02**
	Corr. with:							
	human capital	−1.00	−0.99	−0.88	−0.70	−0.68	−0.52	−0.24
	real ER	−1.00	−0.99	−0.90	−0.80	−0.85	−0.78	−0.65
	future real ER	−1.00	−1.00	−0.98	−0.99	−1.00	−1.00	−0.93
Bond Pos./ GDP		**−0.10**	**−0.51**	**−2.00**	**−12.44**	**12.67**	**4.97**	**2.62**
	Corr with:							
	human capital	−0.39	−0.42	−0.47	−0.61	−0.76	−0.91	−1.00
	real ER	−0.37	−0.43	−0.52	−0.72	−0.91	−1.00	−0.92
	future real ER	−0.38	−0.49	−0.71	−0.96	−1.00	−0.78	0.07

Table 3

with sticky prices and capital accumulation: changing γ

Statistic		$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$
Share of Home Equity		**−0.57**	**−0.55**	**−0.55**	**−0.55**	**−0.54**	**−0.54**	**−0.54**
	due to:							
	human capital	0.21	0.22	0.22	0.22	0.22	0.22	0.22
	real ER	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05
	future real ER	−1.22	−1.22	−1.22	−1.22	−1.22	−1.22	−1.22
Bond Pos./ GDP		**12.80**	**12.67**	**12.67**	**12.67**	**12.67**	**12.67**	**12.67**
	due to:							
	human capital	0.85	0.73	0.69	0.67	0.66	0.65	0.65
	real ER	0.46	0.45	0.45	0.45	0.45	0.45	0.45
	future real ER	11.49	11.48	11.48	11.48	11.48	11.47	11.47

Table 4a

with long term bonds: changing θ

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**1.99**	**1.98**	**1.97**	**1.93**	**1.85**	**1.81**	**0.47**
	due to:							
	human capital	0.48	0.47	0.44	0.41	0.36	0.25	0.80
	real ER	0.03	0.03	0.03	0.03	0.03	0.03	−0.04
	future real ER	0.98	0.98	0.99	0.99	0.96	1.03	−0.79
Bond Pos./ GDP		**−0.01**	**−0.04**	**−0.17**	**−0.42**	**−0.89**	**−2.72**	**13.46**
	due to:							
	human capital	−0.00	−0.01	−0.03	−0.05	−0.05	0.27	−10.62
	real ER	−0.00	−0.00	−0.00	−0.01	−0.03	−0.12	1.00
	future real ER	−0.01	−0.04	−0.13	−0.36	−0.81	−2.87	23.08

Table 4b

with long term bonds: Correlations

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**1.99**	**1.98**	**1.97**	**1.93**	**1.85**	**1.81**	**0.47**
	Corr. with:							
	human capital	−1.00	−0.99	−0.88	−0.70	−0.68	−0.68	−0.27
	real ER	−1.00	−0.99	−0.90	−0.80	−0.85	−0.91	−0.73
	future real ER	−1.00	−1.00	−0.98	−0.99	−1.00	−1.00	−0.96
Bond Pos./ GDP		**−0.01**	**−0.04**	**−0.17**	**−0.42**	**−0.89**	**−2.72**	**13.46**
	Corr with:							
	human capital	0.00	−0.12	−0.11	−0.27	−0.47	−0.58	−0.48
	real ER	0.00	−0.04	−0.16	−0.41	−0.69	−0.85	−0.85
	future real ER	−0.01	−0.05	−0.40	−0.80	−0.96	−0.99	−0.87

Table 5a

with long term bonds: changing γ

Statistic		$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$
Share of Home Equity		**1.63**	**1.85**	**1.98**	**2.03**	**2.07**	**2.11**	**2.11**
	due to:							
	human capital	0.35	0.36	0.36	0.36	0.36	0.36	0.36
	real ER	0.03	0.03	0.04	0.04	0.04	0.04	0.04
	future real ER	0.76	0.96	1.08	1.13	1.17	1.20	1.21
Bond Pos./ GDP		**−0.72**	**−0.89**	**−0.99**	**−1.03**	**−1.07**	**−1.10**	**−1.10**
	due to:							
	human capital	−0.05	−0.05	−0.05	−0.06	−0.06	−0.06	−0.06
	real ER	−0.03	−0.03	−0.04	−0.04	−0.04	−0.04	−0.04
	future real ER	−0.65	−0.81	−0.90	−0.94	−0.98	−1.00	−1.00

Table 5b

with long term bonds: Correlations

Statistic		$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$
Share of Home Equity		**1.63**	**1.85**	**1.98**	**2.03**	**2.07**	**2.11**	**2.11**
	Corr. with:							
	human capital	−0.84	−0.67	−0.65	−0.72	−0.88	−0.99	−1.00
	real ER	−0.95	−0.85	−0.78	−0.80	−0.91	−0.99	−1.00
	future real ER	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
Bond Pos./ GDP		**−0.72**	**−0.89**	**−0.99**	**−1.03**	**−1.07**	**−1.10**	**−1.10**
	Corr with:							
	human capital	−0.74	−0.46	−0.26	−0.17	−0.16	−0.48	−0.74
	real ER	−0.89	−0.68	−0.43	−0.30	−0.21	−0.49	−0.74
	future real ER	−0.98	−0.96	−0.88	−0.76	−0.54	−0.56	−0.77

Table 6

Changing Calibration

Statistic		$\alpha = 0.75$	$\varepsilon = 20$	$\beta = 0.995$	$\delta = 0.05$	$\rho_M = 0.95$	$\rho_M = 0.5$	ρ_M
Share of Home Equity		**0.08**	**−0.68**	**−1.42**	**0.34**	**−0.64**	**−0.82**	−
	Corr. with:							
	human capital	0.31	0.20	2.27	0.28	0.15	0.43	
	real ER	−0.04	−0.06	−0.07	−0.03	−0.06	−0.05	−
	future real ER	−0.70	−1.32	−1.91	−0.41	−1.23	−1.70	−
Bond Pos./ GDP		**7.31**	**14.36**	**20.41**	**5.07**	**13.09**	**14.06**	**2**
	Corr with:							
	human capital	−0.03	1.00	2.13	0.26	1.07	−0.35	−
	real ER	0.34	0.50	0.58	0.24	0.48	0.45	
	future real ER	7.00	12.86	17.71	4.57	11.54	13.96	

Table 7a

with investment shocks and short term bonds: changing θ

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**1.54**	**1.54**	**1.53**	**1.50**	**1.41**	**1.40**	**1.55**
	due to:							
	human capital	0.41	0.41	0.41	0.40	0.38	0.35	0.19
	real ER	−0.00	−0.00	−0.00	−0.00	−0.01	−0.00	−0.00
	future real ER	0.63	0.63	0.62	0.61	0.53	0.55	0.97
Bond Pos./ GDP		**3.49**	**3.42**	**3.31**	**3.21**	**2.60**	**2.90**	**2.95**
	due to:							
	human capital	0.54	0.46	0.34	0.17	−0.11	−0.41	−2.03
	real ER	0.20	0.20	0.20	0.20	0.22	0.20	0.19
	future real ER	2.75	2.75	2.77	2.83	2.49	3.12	4.79

Table 7b

with investment shocks and short term bonds: correlations

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**1.54**	**1.54**	**1.53**	**1.50**	**1.41**	**1.40**	**1.55**
	corr. with:							
	human capital	−0.99	−0.99	−0.99	−1.00	−1.00	−0.99	−0.99
	real ER	0.45	0.45	0.45	0.45	0.45	0.46	0.73
	future real ER	−0.84	−0.84	−0.82	−0.80	−0.80	−0.76	−0.63
	bond returns	−0.34	−0.33	−0.35	−0.34	−0.34	−0.40	−0.41
Bond Pos./ GDP		**3.49**	**3.42**	**3.31**	**3.21**	**2.60**	**2.90**	**2.95**
	corr with:							
	human capital	0.18	0.20	0.24	0.29	0.34	0.51	0.99
	real ER	−0.99	−0.99	−0.99	−0.99	−0.99	−1.00	−1.00
	future real ER	−0.22	−0.23	−0.25	−0.28	−0.30	−0.29	−0.45

Table 8a

with investment shocks and short term bonds: changing γ

Statistic		$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$
Share of Home Equity		**1.30**	**1.45**	**1.53**	**1.56**	**1.58**	**1.60**	**1.61**
	due to:							
	human capital	0.37	0.38	0.39	0.39	0.39	0.39	0.39
	real ER	−0.01	−0.00	−0.00	−0.00	−0.00	−0.00	−0.00
	future real ER	0.44	0.57	0.64	0.67	0.69	0.71	0.71
Bond Pos./ GDP		**3.38**	**3.12**	**2.99**	**2.95**	**2.91**	**2.88**	**2.87**
	due to:							
	human capital	0.03	−0.02	−0.04	−0.04	−0.05	−0.05	−0.05
	real ER	0.21	0.20	0.20	0.20	0.19	0.19	0.19
	future real ER	3.15	2.94	2.83	2.80	2.76	2.73	2.73

Table 8b

with investment shocks and short term bonds: correlations

Statistic		$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$
Share of Home Equity		**1.30**	**1.45**	**1.53**	**1.56**	**1.58**	**1.60**	**1.61**
	corr. with:							
	human capital	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
	real ER	0.45	0.46	0.47	0.45	0.45	0.45	0.46
	future real ER	−0.81	−0.79	−0.79	−0.78	−0.78	−0.78	−0.79
	bond returns	−0.16	−0.35	−0.42	−0.42	−0.44	−0.45	−0.46
Bond Pos./ GDP		**3.38**	**3.12**	**2.99**	**2.95**	**2.91**	**2.88**	**2.87**
	corr. with:							
	human capital	0.15	0.36	0.43	0.43	0.45	0.46	0.48
	real ER	−0.95	−0.99	−1.00	−1.00	−1.00	−1.00	−1.00
	future real ER	−0.44	−0.29	−0.23	−0.24	−0.23	−0.21	−0.19

Table 9a

with investment shocks and long term bonds: changing θ

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**1.90**	**1.89**	**1.86**	**1.82**	**1.74**	**1.89**	**0.87**
	due to:							
	human capital	0.47	0.46	0.44	0.42	0.38	0.28	0.55
	real ER	0.02	0.02	0.02	0.02	0.02	0.03	−0.04
	future real ER	0.92	0.91	0.91	0.89	0.85	1.08	−0.14
Bond Pos./ GDP		**−5.82**	**−5.71**	**−5.55**	**−5.43**	**−5.38**	**−11.63**	**13.06**
	due to:							
	human capital	−0.90	−0.78	−0.57	−0.30	0.03	1.64	−8.97
	real ER	−0.34	−0.34	−0.34	−0.34	−0.34	−0.78	0.85
	future real ER	−4.59	−4.60	−4.64	−4.79	−5.07	−12.49	21.18

Table 9b

with investment shocks and long term bonds: correlations

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**1.90**	**1.89**	**1.86**	**1.82**	**1.74**	**1.89**	**0.87**
	corr. with:							
	human capital	−0.99	−0.99	−0.99	−1.00	−1.00	−0.99	−0.73
	real ER	0.45	0.45	0.46	0.45	0.47	0.47	0.63
	future real ER	−0.85	−0.84	−0.83	−0.81	−0.79	−0.76	−0.41
	bond returns	0.77	0.77	0.77	0.76	0.77	0.90	0.53
Bond Pos./ GDP		**−5.82**	**−5.71**	**−5.55**	**−5.43**	**−5.38**	**−11.63**	**13.06**
	corr. with:							
	human capital	−0.66	−0.67	−0.70	−0.73	−0.77	−0.94	0.19
	real ER	0.91	0.92	0.92	0.92	0.92	0.81	−0.32
	future real ER	−0.32	−0.30	−0.28	−0.24	−0.22	−0.39	−0.99

Table 10a

with investment shocks and long term bonds: changing γ

Statistic		$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3.5$	$\gamma = 5$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$
Share of Home Equity		**1.46**	**1.74**	**2.27**	**3.69**	**−0.43**	**0.80**	**0.86**
	due to:							
	human capital	0.37	0.38	0.38	0.36	0.43	0.41	0.41
	real ER	0.00	0.02	0.05	0.14	−0.14	−0.05	−0.05
	future real ER	0.58	0.85	1.35	2.69	−1.22	−0.06	−0.00
Bond Pos./ GDP		**−2.21**	**−5.38**	**−16.12**	**−49.55**	**50.62**	**21.49**	**20.16**
	due to:							
	human capital	−0.02	0.03	0.20	0.72	−0.85	−0.39	−0.37
	real ER	−0.14	−0.34	−1.06	−3.29	3.39	1.45	1.36
	future real ER	−2.06	−5.07	−15.26	−46.98	48.07	20.44	19.17

Table 10b

with investment shocks and long term bonds: correlations

Statistic		$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3.5$	$\gamma = 5$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$
Share of Home Equity		**1.46**	**1.74**	**2.27**	**3.69**	**−0.43**	**0.80**	**0.86**
	corr. with:							
	human capital	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
	real ER	0.47	0.45	0.47	0.46	0.46	0.46	0.48
	future real ER	−0.81	−0.79	−0.78	−0.78	−0.78	−0.78	−0.78
	bond returns	0.57	0.76	0.92	0.99	0.97	0.81	0.78
Bond Pos./ GDP		**−2.21**	**−5.38**	**−16.12**	**−49.55**	**50.62**	**21.49**	**20.16**
	corr. with:							
	human capital	−0.56	−0.56	−0.56	−0.56	−0.56	−0.56	−0.56
	real ER	0.99	0.92	0.77	0.60	0.25	−0.15	−0.18
	future real ER	0.02	−0.21	−0.48	−0.66	−0.90	−1.00	−1.00

Table 11a

with equity price shocks and short term bonds: changing θ

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
Share of Home Equity		**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	due to:							
	human capital	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	future real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	equity shock	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Bond Pos./ GDP		**6.76**	**6.61**	**6.33**	**5.93**	**5.44**	**4.70**	**3.05**
	due to:							
	human capital	3.02	2.90	2.67	2.34	1.96	1.18	−2.01
	real ER	0.18	0.18	0.18	0.18	0.18	0.18	0.19
	future real ER	6.57	6.49	6.34	6.12	5.87	5.59	4.96
	equity price	−3.02	−2.96	2.86	−2.72	−2.58	−2.26	−0.09

Table 11b

with equity price shocks and short term bonds: changing γ

Statistic		$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$
Share of Home Equity		**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	due to:							
	human capital	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	future real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	equity shock	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Bond Pos./ GDP		**5.04**	**5.44**	**5.60**	**5.72**	**5.80**	**5.85**	**5.85**
	due to:							
	human capital	2.05	1.96	1.93	1.91	1.90	1.89	1.89
	real ER	0.17	0.18	0.19	0.19	0.19	0.19	0.19
	future real ER	5.52	5.87	6.02	6.12	6.18	6.23	6.23
	equity shock	−2.71	−2.58	−2.53	−2.50	−2.47	−2.46	−2.46

Table 12a

with equity price shocks and long term bonds: changing γ

Statistic		$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.65$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.9$
Share of Home Equity		**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.0**
	due to:							
	human capital	0.00	0.00	0.00	0.00	0.00	0.00	0.0
	real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.0
	future real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.0
	equity shock	0.50	0.50	0.50	0.50	0.50	0.50	0.5
Bond Pos./ GDP		**−29.68**	**−28.00**	**−25.24**	**−21.82**	**−18.36**	**−81.67**	**12.9**
	due to:							
	human capital	−13.27	−12.28	−10.65	−8.62	−6.63	−20.48	−8.5
	real ER	−0.79	−0.77	−0.72	−0.67	−0.62	−3.25	0.8
	future real ER	−28.85	−27.48	−25.26	−22.53	−19.84	−20.48	21.0
	equity price	13.37	12.65	11.50	10.11	8.82	39.60	0.3

Table 12b

with equity price shocks and long term bonds: changing γ

Statistic		$\gamma = 1.5$	$\gamma = 2.15$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$
Share of Home Equity		**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	due to:							
	human capital	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	future real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	equity shock	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Bond Pos./ GDP		**−4.34**	**−18.36**	**−228.89**	**36.81**	**22.82**	**18.39**	**18.00**
	due to:							
	human capital	−1.77	−6.63	78.92	12.27	7.46	5.93	5.80
	real ER	−0.15	−0.62	−7.64	1.22	0.76	0.65	0.61
	future real ER	−4.76	−19.84	−245.86	39.37	24.34	19.59	19.17
	equity price	2.36	8.82	104.57	−16.21	−9.83	−7.81	−7.63