# Optimal Reserve Prices in Sequential Auctions with Imperfect Commitment

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**Abstract**

In a sequential auction of perfect substitutes, we analyze the consequences of a seller's incapacity to commit perfectly to a reserve-price schedule. When facing such a seller, the bidders have strong incentives not to reveal during the earlier rounds of the auction any information about their valuations. If the seller observes only the winning bid, as in a sequence of Dutch auctions, there is a symmetric monotone equilibrium in which the seller lowers the reserve price over time, according to the revealed information. The anticipation of lower future reserve prices makes several bidder types abstain from the earlier rounds, even though their valuations exceed the requested price. In addition, because of the restriction in competition, the participating bidders shade their bids sharply. Thus, by failing to commit, the seller suffers a revenue loss. This loss can become more severe if the seller attempts to suppress some of the information revealed in the auction, for example, by learning only whether an item is sold. Finally, if the seller observes all bids, as in a sequence of sealed-bid auctions, there is no symmetric equilibrium in monotone bidding strategies, however small the imperfection of the seller's commitment is. Our results rationalize the adoption of costly commitment or privacy preserving measures, such as the use of a well-established auction house.

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# 1 Introduction

Several multi-unit sales are conducted by means of sequential auctions, carried out either in rapid succession or over long periods of time. For example, wine, art, condominium units, used cars, agricultural products and fish are often auctioned sequentially. On internet auction sites, sellers often auction in a sequence many units of the same consumer product. Furthermore, procurement contracts are also auctioned sequentially, as the need for each project arises. Finally, the radio spectrum auction in the United States, as well as similar spectrum auctions in numerous other countries, was conducted by means of a dynamic procedure.

In the study of such auctions, a typical assumption is that of intertemporal commitment. In particular, the seller is able to commit in a fully credible manner to a specific auction mechanism, or to a sequence of auction mechanisms, via which all sales will be made. The bidders can therefore reveal, in the earlier rounds of the auction process, private information about their valuations, without fearing that the seller will change the rules, for example, alter the reserve price, at their expense.

In reality, however, perfect intertemporal commitment is often infeasible. In many cases, the seller lacks the credibility that this assumption requires.[1] In addition, he may find it too costly to attempt to guarantee the rules of a sale by using an institution like a well-established auction house. Furthermore, even when the seller does commit to a certain selling scheme by means of a contract, he may still try to break the sale rules at a cost, if the revealed information makes it profitable for him to do so.[2] Finally, in several cases in which the seller announces the auction of only few units, possibly because at that time he can guarantee the delivery of only those units, the bidders may try to conceal some of their private information, in anticipation of similar sales in the future.

In this paper, we study the effects of imperfect commitment. In particular, we examine the sequential auction of two identical goods by a seller who can change the reserve price for the second object, after observing the outcome of the first sale. The same group of potential buyers is assumed to be present in both auctions; and each buyer has single-unit demand and a valuation that is persistent over time. The seller is therefore able to take full advantage of any information that the buyers may reveal in the first auction.

The above setting describes problems, for example, in sequential procurement. In such problems, a number of contracts for similar projects, for example, for highway paving, is procured sequentially. Capacity constraints force the competing firms to limit the supply of their services.[3] If the buyer's commitment not to alter the rules pertaining

[1]Typically, this would be a seller with no concerns about reputation.

[2]There are several cases in which the seller tried to break the auction rules in reaction to revealed information. For example, in the sale of the General Motors Building in Manhattan, in 2003, the owners allegedly used their knowledge of the submitted bids so as to bargain with the eventual buyer (who was the last bidder to submit a bid to the auction). For more details, see McAdams and Schwarz [21, 22].

[3]The relevance of such constraints has been documented by Jofre-Bonet and Pesendorfer [15], in their study of highway paving contracts in California.

the future auctions cannot be fully credible, then the issues involved in the design of the optimal procurement process will be identical to the ones we study.

The seller uses a sequence of two first-price auctions[4] with reserve prices $r_1$ and $r_2$, chosen so as to maximize his expected payoff. After the end of the first auction, the seller and the bidders may observe the winning bid, all submitted bids or only whether the first item was sold.[5] The information revelation policy is determined exogenously, prior to the beginning of the sale; in some cases, it can be enforced by the auction format.[6]

We introduce imperfect commitment by restricting the time at which the seller can set the second-period reserve price $r_2$. More specifically, the seller cannot credibly commit at the beginning of the game to any reserve price, or rule for determining the reserve price, for the second auction. Rather, he must choose the reserve price $r_2$ at the beginning of the second period, after the end of the first auction.[7]

The impossibility of intertemporal commitment has important consequences for the bidders' behavior in the first round of the auction. Since the seller cannot restrict the manner in which he will use (in the second round) the information revealed in the first round, the bidders have strong incentives to conceal their valuations. In particular, the non-winning bidders are best-off not submitting any bid.

This incentive to conceal one's private information turns out to be extremely strong. When the seller observes all bids submitted in the first auction, we find that there is no symmetric equilibrium in weakly increasing bidding strategies. The seller's projected use of the revealed information forces the bidders not to shade their bids; as a result, a deviation to non-participation (or to minimal bidding) in the first auction becomes strictly profitable.[8]

A symmetric monotone equilibrium exists only when the non-winning bidders can avoid revealing information; in particular, when the seller observes the first-period winning bid. In this equilibrium, the seller sets a first-period reserve price that allows the sale of the first item, if a bidder with a sufficiently high valuation exists. Subsequently, given the outcome of the first auction, he updates his beliefs (in particular, he obtains a sharper upper bound for the remaining bidders' valuations) and, accordingly, he lowers the reserve price for the second item. Because of the anticipation of a lower future reserve

[4]With only slight modification, our results extend to sequences of other standard auctions. For the case of second-price auctions, see Katsenos [16].

[5]The use of a sequence of two Dutch auctions or two sealed-bid first-price auctions implements respectively the policy of observing only the winning bid or all bids in the first auction. Implementing the policy of observing only whether the first item is sold would require the use of an auctioneer.

[6]For example, a Dutch auction would reveal only the winning bid.

[7]In this setting, therefore, imperfect commitment assumes an extreme form, namely, that of non-commitment. In section 4.4, we show that all intermediate situations, in which the seller is allowed to change the reserve price with some positive probability, are qualitatively identical to non-commitment.

[8]A similar phenomenon occurs in auctions with resale, in which the bidders have incentives to conceal their private information during the auction, so as to gain more from any post-auction resale opportunities. (See Garratt and Tröger [9] and Hafalir and Krishna [11].)

price, several bidder types do not submit any bid in the first auction, even though their valuations exceed the requested price.[9] Consequently, those bidders who participate in the first round shade their bids sharply, knowing that they face limited competition. Both the strategic non-participation and the excessive shading of the submitted bids would be absent, if the auctioneer could commit not to change, in particular, not to lower, the reserve price over time.

Overall, the seller suffers a revenue loss. Although he is able to design the second auction in a better informed manner, and therefore to derive a higher revenue from the sale of the second item than in the case of commitment, he cannot prevent severe losses in the first auction. Thus, the intuition favoring commitment[10] is reaffirmed in the setting of sequential auctions. In particular, the seller would be willing to adopt costly measures to enhance his credibility, for example, he would be willing to pay for the services of a trusted intermediary, like a well-established auction house.

Finally, the seller cannot benefit from restricting partially the information made available to him (as suggested by the result in favor of commitment), in particular, from observing only whether the first item is sold. This informational restriction would not affect the bidders' non-participation decision. On the other hand, it would deprive the seller of the opportunity to design the second auction in a more informed manner. Therefore, it cannot be profitable.

The literature on sequential auctions has paid relatively little attention to the possibility of a strategic auctioneer. Some of this literature has tried to explain the declining-price anomaly[11], a problem in which the auctioneer plays no strategic role. Learning in sequential auctions has been studied by Ortega Reichert [27] and Jeitschko [13]. They concentrate, however, on the manner in which the bidders, rather than the auctioneer, can use the information revealed during a sequential auction. A strategically active auctioneer is present in McAfee and Vincent [24], who study the optimal reserve-price path in a sequence of first- and second- price auctions.[12] In particular, in a setting akin to that of the Coase conjecture, the auctioneer puts the same object for sale repeatedly, until it is sold.[13] At each round he chooses a reserve price according to his (increasingly pessimistic) beliefs about the buyers' valuations. Since the game ends as soon as the object is sold, that is, as soon as a bid is placed, the buyers do not face the problem of

[9]This strategic non-participation decision first appeared in the literature of dynamic bargaining; for example, see Hart and Tirole [12]. For examples of its occurrence in sequential auctions, see McAfee and Vincent [24] and Caillaud and Mezzetti [4]. In our case, we shall remark that it differs from the ratchet effect, despite its resemblance to it. A buyer decides not to participate in the first auction even though the seller cannot use against him, in the future, the information revealed by his bid.

[10]This result was formally established by Stokey [30] and Bulow [3].

[11]Although intertemporal non-arbitrage requires that the expected prices remain constant over time, in practice it has been observed that earlier sales tend to be concluded at higher prices than later ones. For details, see Ashenfelter [1], Jeitschko [14], MacAfee and Vincent [23] and Milgrom and Weber [25].

[12]Sobel and Takahashi [28] have studied a similar problem within the context of dynamic bargaining. Skreta [29] has generalized with respect to the mechanisms that the seller can choose in every period.

[13]Coase [5] conjectured that a durable-good monopolist who is unable to commit to a price schedule over time can have no market power; for more details, see Fudenberg *et al.* [7] or Gul *et al.* [10].

hiding their valuations. Prior to the end of the game, information can be revealed only in a passive manner, by the buyers' refusal to bid for the object at a given reserve price.

The issue of concealing, during the earlier sales, information from the auctioneer has appeared in Caillaud and Mezzetti [4]. In a sequence of two auctions in which the buyers have multi-unit demands and persistent valuations, the bidders face a problem similar to that in our setting. However, because of the multi-unit demands, this problem eventually concerns only the bidder with the highest valuation, all other bidders realizing that they cannot win either of the two auctions.[14] Therefore, it is the format of the English auction that is employed in this setting, as it allows the winning bidder not to reveal his valuation. Our work complements the work of Caillaud and Mezzetti [4] by looking at an environment in which the problem of concealing information is faced by the non-winning bidders.

In both settings, it turns out that relevant information can be revealed only indirectly, through the actions of the bidders that have no further interest in the game. In addition, in both cases, imperfect commitment is costly for the seller. In our setting, however, the seller's loss is the consequence of both the bidders' decisions not to participate and to shade their bids sharply in the first auction.[15]

The issue of commitment in a sequential auction is also studied by Zeithammer [33, 34]. In the presence of production costs, Zeithammer examines whether it can be profitable for the seller to base his future supply decisions on the information revealed during the earlier rounds of the auction. He finds that non-commitment can result in higher revenue than certain simple forms of commitment. Unlike our paper (or Caillaud and Mezzetti [4]), Zeithammer's seller does not face the problem of committing to a future reserve price; he only chooses whether to commit into supplying additional units.

Finally, the problem of deciding how much information to make publicly available in a sequential auction appears in Thomas [31] and Tu [32]. In their settings, however, the auctioneer does not set any reserve prices nor make any decisions during the game. Therefore, the issues involved in their analysis are different than the ones in our problem.

In the next two sections, we present the model describing our problem and we find the optimal reserve prices under commitment. In section 4.1, we analyze the case in which the seller observes only the first-period winning bid. In section 4.2, we show that the seller is worse-off when he observes only whether the first item is sold. In section 4.3, we show that there is no symmetric pure-strategy equilibrium when all bids are observed. In section 4.4, we extend this negative result to a setting in which the seller can change the second-period reserve price with only a small probability. We conclude in section 5.

[14] In fact, on the equilibrium path, if there is a winning bidder in the first auction, then the second auction becomes a bargaining problem between the seller and that bidder.

[15] In Caillaud and Mezzetti [4], the participating bidders are always willing to bid up to their valuation, so as to overbid their competitors. Therefore, any revenue loss for the seller comes directly from the decrease in participation.

## 2 General Model

There is one seller with 2 identical (or equivalent) objects for sale. The seller's valuation for the objects is normalized to zero, so that he can derive no benefit from any object that remains unsold.

The seller faces $N > 2$ potential buyers, indexed by $i = 1, \ldots, N$. Each buyer has single-unit demand and private valuation $v_i \,\epsilon\, [\underline{v}, \bar{v}]$, for $0 \leq \underline{v} < \bar{v}$, which remains constant throughout the game.[16] The valuations are independently drawn, according to a common distribution function $F : [\underline{v}, \bar{v}] \rightarrow [0, 1]$. We assume that the distribution function $F$ is differentiable and that its derivative, $f : [\underline{v}, \bar{v}] \rightarrow \mathbb{R}^+$, has full support. The payoff of buyer $i$, in case he wins one unit, equals his valuation $v_i$ minus the price that he pays for it; otherwise, if he does not win any unit, it equals to zero.

The two objects are allocated to the buyers by means of a sequence of two sealed-bid first- or second-price auctions, conducted in periods (or rounds) $t = 1, 2$. Immediately after the end of the first auction, the seller and the buyers receive information $h(b^1)$ about the bids $b^1 = (b^1_1, \ldots, b^1_N)$ submitted in it. We will examine the extreme cases of no information, $h(b^1) = \emptyset$, and full information, $h(b^1) = b^1$, as well as the intermediate cases of revealing only whether the first item was sold, $h(b^1) \,\epsilon\, \{s, ns\}$, only the winner's bid, $h(b^1) = b^1_w$, and all bids except the highest losing bid, $h(b^1) = b^1 \setminus b^1_{(2,N)}$. In all cases, we assume that the winner of the first auction is informed, possibly privately, about his success; this implies that any other player who submitted a first-round bid can infer his failure.[17]

At the beginning of each auction, the seller can set a reserve price $r_t \in \mathbb{R}^+$ so as to maximize his expected revenue. In particular, the seller must choose $r_2$ after the end of the first auction and after having observed the information $h(b^1)$ that is revealed from it. His strategy, therefore, consists of a reserve price for the first auction and a reserve-price rule for the second auction:[18]

$$r_1 \;\epsilon\; \mathbb{R}^+;$$
$$r_2 \;:\; (\, h(b^1),\, r_1 \,) \;\longmapsto\; r_2 \;\epsilon\; \mathbb{R}^+.$$

We have relaxed the usual assumption of intertemporal commitment by making the auctioneer[19] unable to choose a reserve-price rule $r_2$ at the beginning of the game, in a

[16] For simplicity, we assume no discounting of the second-period payoffs. Our results would not change if a discount factor $\delta \,\epsilon\, (0, 1)$, possibly reflecting the probability with which the second unit becomes available, were introduced; in this case, the seller's loss from non-commitment would be less pronounced.

[17] All these information revelation policies can be implemented through the services of a trusted third party, for example, an auctioneer (who differs from the seller). In addition, a sequential sealed-bid first-price auction in which only the winner's bid is revealed is equivalent to a sequential Dutch auction. In this auction, the price clock is set high, above $\bar{v}$, at the beginning of each round (so that the second item can be sold at a higher price than the first one). Finally, when all first-round bids are revealed, there is obviously no need for a third party.

[18] To simplify the notation, we have allowed $b^1_i$ to take the value of "no-bid" or "abstain".

[19] We use the terms "seller" and "auctioneer" (as well as "buyer" and "bidder") interchangeably.

manner that would allow the bidders to base their first-period behavior upon it.

In each period, every buyer either submits a bid or abstains from the auction. His first-period bid depends on his valuation and the reserve price. His second-period bid, if he does not win the first auction, depends on his valuation, the new reserve price, and the information that has been revealed in the first auction. Therefore, for each information revelation scheme $h(\cdot)$, the strategy of each bidder $i$ consists of the bidding rules

$$\beta_i^1 \,:\, (\,v_i \,|\, r_1\,) \;\longmapsto\; b_i^1 \in \{a\} \cup [r_1, \infty);$$
$$\beta_i^2 \,:\, (\,v_i \,|\, b_i^1,\, h(b^1),\, r_1,\, r_2\,) \;\longmapsto\; b_i^2 \in \{a\} \cup [\,r_2, \infty),$$

where $a$ denotes the action of abstaining from submitting a bid.

The solution concept will be that of perfect Bayesian equilibrium. At each decision node, each player must behave optimally, given the other players' strategies and his beliefs.[20] On the equilibrium path, the players' beliefs are determined by Bayes' rule. Off the equilibrium path, we strengthen the equilibrium concept by requiring that the players cannot infer from an observed action information that the acting player does not have.[21] For example, an off-equilibrium reserve price $r_1$ cannot alter the bidders' beliefs about their opponents' valuations. Finally, we restrict attention to symmetric[22], pure-strategy equilibria, in weakly increasing bidding strategies.

# 3 Optimal Sequential Auctions under Commitment

If there is only one unit for sale, then, according to Myerson [26], the auctioneer will choose the optimal reserve price by considering the bidders' virtual valuation function

$$\psi(v) = v - \frac{1 - F(v)}{f(v)}.$$

We make the standard regularity assumption that the function $\psi(v)$ is increasing. For this assumption to hold, it is sufficient that the hazard rate $\frac{f(v)}{1-F(v)}$ is increasing.

Given the regularity assumption, the auctioneer can maximize his expected revenue by allocating the object by means of any standard auction with reserve price

[20] In particular, non-commitment implies that the optimality of the reserve-price rule $r_2$ is determined by the auctioneer's updated beliefs and not by his initial ones (as when he chooses $r_2$ prior to the beginning of the first auction).

[21] For a formal definition, see Fudenberg and Tirole [8], definition 8.2.

[22] In fact, wherever it is applicable, we impose a stronger symmetry requirement, one that rules out the possibility of using the first-period bids as a labeling device. For example, we will not allow symmetric strategies that prescribe different second-period bidding behavior to the second and the third highest bidders of the first auction.

$$r_0 = \begin{cases} \psi^{-1}(0), & \text{if } \psi(\underline{v}) < 0; \\ \underline{v}, & \text{otherwise.} \end{cases} \quad (1)$$

That is, the auctioneer must exclude all bidders with valuations below $\psi^{-1}(0)$.

If the players receive prior information that their opponents' valuations are bounded above by $\hat{v}$, then they update their beliefs so that they consider the bidders' valuations i.i.d., according to the distribution function $F(\cdot)/F(\hat{v})$ on $[\underline{v}, \hat{v}]$. In this case, the bidders' virtual valuations will be given by the function

$$\psi(v|\hat{v}) = v - \frac{F(\hat{v}) - F(v)}{f(v)}.$$

It is easy to check that the function $\psi(v|\hat{v})$ also satisfies the regularity assumption. Therefore, given the knowledge of the bound $\hat{v}$, the auctioneer maximizes his expected revenue by setting a reserve price

$$r_0(\hat{v}) = \begin{cases} \psi(\cdot \mid \check{v}, \hat{v})^{-1}(0), & \text{if } \psi(\underline{v}|\hat{v}) < 0; \\ \underline{v}, & \text{otherwise.} \end{cases} \quad (2)$$

Notice that the reserve price $r_0(\hat{v})$ is increasing in $\hat{v}$. Finally, for any optimal reserve price $r_0 > \underline{v}$, the condition $\psi(r_0|\hat{v}) = 0$ implies that

$$F(\hat{v}) - F(r_0) = f(r_0)\, r_0, \quad (3)$$

that is, the optimal reserve price $r_0(\hat{v})$ equals to the inverse hazard rate at $r_0(\hat{v})$.

If there are two units for sale, then, because of the single-unit demand and the regularity assumption, the solution to the auctioneer's revenue optimization problem is similar to that of the single-unit case. By Maskin and Riley [20], the optimal selling mechanism takes the form of any standard multi-unit auction with reserve price $r_0 \in [\underline{v}, \bar{v}]$, defined as in the case of a single-unit auction.

If the two units must be allocated sequentially, as in our setting, then the assumptions of risk neutrality and i.i.d. private valuations imply that all allocation-equivalent equilibria of the sequential and the simultaneous auctions are revenue equivalent. Hence, any sequential auction that allocates the two units to the bidders with the two highest valuations, as long as these valuations exceed the price $r_0$, is revenue maximizing.[23]

When the auctioneer can commit to a reserve price $r_2$ at the beginning of the game, then he can implement the optimal outcome by means of a sequential first- or second-price

[23]Notice, in particular, that the optimality extends over sequential auctions in which the second-period reserve price is determined endogenously by the first-period bids, according to a reserve price schedule $r_2 = r_2(b_1^1, ..., b_N^1, r_1)$.

auction, with the same reserve price $r_1 = r_2 = r_0$ in both rounds, in which the bidders learn the first-round winning bid.[24] Finally, this outcome is also implemented, in equilibria resulting in the same behavior but different second-round beliefs, by sequential sealed-bid auctions in which the auctioneer reveals either no information[25] or whether the first round resulted in a sale.

# 4 Sequential Auctions under Non-Commitment

When the seller is unable to commit at the beginning of the game to a reserve price $r_2$ for the second auction, the equilibrium outcome depends on the information revealed in the first round. We treat each case separately.

## 4.1 Revelation of the Winning Bid

When only the highest bid is revealed at the end of the first auction, for example, when the sale is conducted by means of a Dutch auction, the information inferred from the first-round outcome (assuming that the buyers follow monotone strategies) takes the form of an upper bound for the valuations of the remaining bidders.

We show that the following strategies can be part of an equilibrium:

Given a first-period reserve price $r_1 \leq \bar{r}_1$, for a certain threshold $\bar{r}_1$, each bidder $i$ follows a bidding strategy $\beta^1(\cdot \,|\, r_1)$ such that he participates in the auction if and only if his valuation is $v_i \in [\underline{v}(r_1), \bar{v}]$, for some value $\underline{v}(r_1) > r_1$. In addition, in the region of participation, $[\underline{v}(r_1), \bar{v}]$, the strategy $\beta^1(\cdot \,|\, r_1)$ is strictly increasing. Thus, the winning bidder fully reveals his valuation. If $r_1 > \bar{r}_1$, then, in equilibrium, no bidder participates in the first auction.

If the first-period object is sold at a price $\hat{b}^1 = \beta^1(\hat{v} \,|\, r_1)$, corresponding to a winning valuation $\hat{v} \in [\underline{v}(r_1), \bar{v}]$, then the auctioneer and the bidders update their beliefs, so that the remaining bidders' valuations $v_i$ are i.i.d., according to the distribution $F(\cdot)/F(\hat{v})$. The auctioneer sets a new reserve price $r_2$, according to the updated virtual valuation function, and the bidders bid according to the standard first-price auction bidding strategies. Unless $r_1 \leq \underline{v}$, the new reserve price, $r_2$, is strictly lower than the first-period reserve price $r_1$. If the first-period object remains unsold at the reserve price $r_1$, then the same argument applies, with $\hat{v} = \underline{v}(r_1)$ being the revealed upper bound for the bidders' valuations.

[24] The equilibrium strategy profile (and the argument establishing it) is simply a modification of the one described in Krishna [17], Propositions 15.1 and 15.3, so as to account for the reserve price $r_0$.

[25] In the sequential first-price auction, off the equilibrium path (in particular, following deviations to not submitting any bid in the first round), the deviating bidders' second-round bidding strategies must be modified, so as to take into account their failure to infer the number of remaining buyers.

For these strategies to support an equilibrium, a bidder with valuation $\underline{v}(r_1)$ must be indifferent between winning the first-period object at the minimal price $\beta^1(\underline{v}(r_1)\,|\,r_1) = r_1$ and waiting for the second auction, in which the reserve price will be lower. This requires that the bidders shade their second-period bids less than in the first period, so as a bidder with valuation $\underline{v}(r_1)$ will still bid $r_1$.

Notice that any bidder with valuation $v_i \in (r_1, \underline{v}(r_1))$, for a given reserve price $r_1$, does not participate in the first auction. He prefers to wait for the second auction, even if he can buy the first-period object at price $r_1$. This strategic non-participation decision, which also appears in McAfee and Vincent [24] and in Caillaud and Mezzetti [4], is entirely the consequence of the auctioneer's inability to commit not to lower the reserve price. It would not occur, if the auctioneer could commit to a second-period reserve price $r_2 \geq r_1$. In particular, it does not occur in the subgame following a first-period reserve price $r_1 = 0$. Therefore, it does not depend on the bidders' expectation of a smaller number of competing bidders in the future.[26]

We start our formal analysis by investigating the second-period auction. Since we are considering monotone first-period bidding strategies, we can abbreviate the notation for the second-period bidding strategy to $\beta^{2,M}(v_i\,|\,\hat{v}, r_2)$, where $M = N-1$ or $N$ is the number of participating bidders and $\hat{v} \in [\underline{v}, \bar{v}]$ is the upper bound for the participating bidders' valuations revealed in the first auction. The following result, which follows directly from standard arguments regarding symmetric equilibria and optimal reserve prices in first-price auctions, describes the equilibrium behavior in all second-period continuation games.[27]

**Lemma 1**
*Consider a single-unit first-price auction with $M$ bidders, whose valuations are i.i.d. according to the distribution function $F(\cdot)$ on $[\underline{v}, \bar{v}]$. Suppose that the auctioneer and the bidders believe that the unknown valuations are bounded above by the value $\hat{v} \in [\underline{v}, \bar{v}]$. Then, given a reserve price $r_2$, the following symmetric strategy profile constitutes the unique equilibrium of the auction:*

$$\beta^{2,M}(v_i\,|\,\hat{v}, r_2) = \begin{cases} \mathbb{E}[\max\{v_1^{(M-1)}, r_2\}\,|\,v_1^{(M-1)} < v_i\,], & \text{if } v_i \geq r_2,\ v_i \leq \hat{v}; \\ \mathbb{E}[\max\{v_1^{(M-1)}, r_2\}\,|\,v_1^{(M-1)} < \hat{v}\,], & \text{if } v_i \geq r_2,\ v_i > \hat{v}; \\ a, & \text{if } v_i < r_2. \end{cases}$$

*In this auction, it is optimal for the auctioneer to set a reserve price:*

$$r_2(\hat{v}) = \begin{cases} \psi(\cdot\,|\hat{v})^{-1}(0), & \text{if } \psi(\underline{v}\,|\,\hat{v}) < 0; \\ \underline{v}, & \text{otherwise.} \end{cases}$$

[26]This expectation affects only how sharply the bidders shade their bids, not their decision to wait.

[27]Restricting attention to the game described in Lemma 1, notice that the possibility of $v_i > \hat{v}$ does not contradict the bidders' assumed beliefs and, therefore, does not violate the consistency requirement in the definition of equilibrium. We can simply assume that prior to the draw of the privately known valuation $v_i$, each bidder attaches zero probability to the event $v_i > \hat{v}$, for all $i$.

Therefore, if the first-period auction reveals an upper bound $\hat{v}$ for the bidders' valuations, then, in the second period, the auctioneer's optimal reserve price and the bidders' strategies are described by Lemma 1. The possibility of a valuation $v_i > \hat{v}$ corresponds to an off-equilibrium path event, namely, to the case in which bidder $i$ should have won the first-period unit but did not bid according to the prescribed strategy.

Moving backwards, suppose that the auctioneer has set a first-period reserve price $r_1$ and consider a bidder with valuation $v_i \geq \underline{v}(r_1)$, for some value $\underline{v}(r_1) > r_1$ that will be determined later. Suppose that all other bidders follow the strategies $(\beta^1, \beta^2)$, where $\beta^2$ is as in Lemma 1. Furthermore, suppose that the auctioneer follows the strategy $r^2$, again described by Lemma 1.

Then by mimicking a type $\tilde{v}_i > v_i$, bidder $i$ has an expected payoff

$$\begin{aligned}\Pi[\tilde{v}_i; v_i] &= F(\tilde{v}_i)^{N-1} \left[ v_i - \beta^1(\, \tilde{v}_i \,|\, r_1) \right] \\ &+ (N-1)\; [1 - F(\tilde{v}_i)]\; F(v_i)^{N-2} \left[ v_i - \int_{\tilde{v}_i}^{\bar{v}} \beta^{2,N-1}(v_i \,|\, \hat{v}, r_2(\hat{v}))\, \frac{f(\hat{v})}{1 - F(\tilde{v}_i)}\, d\hat{v} \right].\end{aligned}$$

Similarly, by mimicking a type $\tilde{v}_i \in [\underline{v}(r_1), v_i)$, bidder $i$ has an expected payoff

$$\begin{aligned}\Pi[\tilde{v}_i; v_i] &= F(\tilde{v}_i)^{N-1} \left[ v_i - \beta^1(\, \tilde{v}_i \,|\, r_1) \right] \\ &+ (N-1)\; [1 - F(v_i)]\; F(v_i)^{N-2} \left[ v_i - \int_{v_i}^{\bar{v}} \beta^{2,N-1}(v_i \,|\, \hat{v}, r_2(\hat{v}))\, \frac{f(\hat{v})}{1 - F(v_i)}\, d\hat{v} \right] \\ &+ [F(v_i)^{N-1} - F(\tilde{v}_i)^{N-1}] \left[ v_i - \int_{\tilde{v}_i}^{v_i} \beta^{2,N-1}(\hat{v} \,|\, \hat{v}, r_2(\hat{v}))\, \frac{(N-1) F(\hat{v})^{N-2} f(\hat{v})}{F(v_i)^{N-1} - F(\tilde{v}_i)^{N-1}}\, d\hat{v} \right].\end{aligned}$$

The third term corresponds to the possibility in which the first-period object is sold at a price $\hat{b}^1 \in (\beta^1(\tilde{v}_i|r_1), \beta^1(v_i|r_1))$. In this case, the winning bidder reveals the valuation $\hat{v} = \beta^1(\,\cdot\,|\, r_1)^{-1}(\hat{b}^1) \in (\tilde{v}_i, v_i)$. Therefore, in the second auction, with reserve price $r_2(\hat{v})$, bidder $i$ bids $\beta^2[\hat{v} \,|\, \hat{v}, r_2(\hat{v})]$.

In either case, by solving the differential equation that results from the necessary first-order condition at the endpoint $\tilde{v}_i = v_i$ along with the boundary condition $\beta^1(\underline{v}(r_1) \,|\, r_1) = r_1$, we get the bidding function

$$\beta^1(v_i \,|\, r_1) = \frac{1}{G(v_i)} \left[ \int_{\underline{v}(r_1)}^{v_i} \beta^{2,N-1}(v \,|\, v, r_2(v))\; g(v)\; dv \;+\; G[\underline{v}(r_1)]\; r_1 \right], \qquad (4)$$

where $G(v)$ and $g(v)$ denote respectively the distribution and the density of the highest of the competing $N-1$ bidders' valuations.

To determine the threshold value $\underline{v}(r_1)$, consider a bidder with valuation $v_i = \underline{v}(r_1)$. Such a bidder shall be indifferent between winning the first-period object, at the minimal price $\beta^1(\underline{v}(r_1) \,|\, r_1) = r_1$, and abstaining from the first period so as to win the second-period object. This implies that the bidder must pay the same price in either of the two auctions:

$$r_1 \;=\; \beta^{2,N}[\,\underline{v}(r_1) \,|\, \underline{v}(r_1)\,,\; r_2(\underline{v}(r_1))\,]. \tag{5}$$

Since the function $r_1(v) = \beta^{2,N}(v \,|\, v, r_2(v))$ is increasing[28] in the valuation $v$, it follows that the threshold value function $\underline{v}(\cdot) : r_1 \mapsto \underline{v}(r_1)$ is increasing in the reserve price $r_1$. Let $\bar{r}_1$ be the minimal reserve price for which no bidder will participate in the first auction, that is, $\bar{r}_1 = \min\{r_1 : \underline{v}(r_1) = \bar{v}\}$. Then

$$\bar{r}_1 \;=\; \beta^{2,N}(\bar{v} \,|\, \bar{v}, r_2(\bar{v})). \tag{6}$$

Therefore, for all reserve prices $r_1 < \bar{r}_1$, we have $\underline{v}(r_1) \in [r_1, \bar{v})$; and, for all $r_1 \geq \bar{r}_1$, we have $\underline{v}(r_1) = \bar{v}$. Finally, notice that for all reserve prices $r_1 > \underline{v}$, we have $\underline{v}(r_1) < r_1$, implying non-participation for bidders with valuations $v_i \in [r_1, \underline{v}(r_1))$.

The above arguments lead to the following result, regarding the bidders' behavior in the game following a first-period reserve price $r_1$:

**Proposition 2**
*Consider a sequential first-price auction in which the first-round winning bid is revealed. Suppose that the seller has set a reserve price $r_1 \in [\underline{v}, \bar{v}]$. Then there is an equilibrium for the continuation game, in which the strategies $r_2$, $\beta^{2,N}$ and $\beta^{2,N-1}$ are given by Lemma 1 and the strategy $\beta^1$ depends on the value of $r_1$:*

*- If $r_1 < \bar{r}_1$, for the threshold value $\bar{r}_1 = \beta^{2,N}(\bar{v} \,|\, \bar{v}, r_2(\bar{v}))$, the strategy $\beta^1$ is given by*

$$\beta^1(v_i \,|\, r_1) = \frac{1}{G(v_i)}\; [\int_{\underline{v}(r_1)}^{v_i} \beta^{2,N-1}(v \,|\, v, r_2(v))\, g(v)\; dv \;+\; G[\underline{v}(r_1)]\; r_1\,],$$

*for all valuations $v_i \in [\underline{v}(r_1), \bar{v}]$, where $\underline{v}(r_1) \in (r_1, \bar{v})$ is given by the equation*

$$r_1 \;=\; \beta^{2,N}[\,\underline{v}(r_1) \,|\, \underline{v}(r_1),\; r_2(\underline{v}(r_1))\,].$$

*Otherwise, for $v_i \in [\underline{v}, \underline{v}(r_1))$, bidder $i$ abstains from the first-period auction.*

*- If $r_1 \geq \bar{r}_1$, then all bidders abstain from the first-period auction.*

[28] For a function $\Phi(x) = F[f_1(x), f_2(x)]$, we have $\Phi' = F_1\, f_1' + F_2\, f_2'$. Hence, if the derivatives $F_1$, $f_1'$, $F_2$ and $f_2'$ are all positive, then the function $\Phi(x)$ is increasing.

Having described the bidders' behavior, we can now consider the auctioneer's problem of determining the optimal first-period reserve price $r_1^*$. Since there is a bijective relation between a reserve price $r_1 \in [\underline{v}, \bar{r}_1]$ and the participation threshold $v(r_1) \in [\underline{v}, \bar{v}]$, namely,

$$r_1(v) = \beta^{2,N}(v \mid v, r_2(v)),$$

we can think of the auctioneer's problem as one of determining the revenue maximizing first-period participation threshold $v^* = \underline{v}(r_1^*)$.

**Proposition 3**
*The optimal first-period participation threshold $v^*$ in a sequence of two Dutch auctions solves the equation*

$$\frac{1 - F(v)}{f(v)} \; G[r_2(v)] \; \frac{dr_2}{dv}(v) \;\; = \;\; \int_{r_2(v)}^{v} \psi(u) \; g(u) \; du. \tag{7}$$

*The auctioneer always induces participation by a positive measure of bidders' types, that is, $v^* < \bar{v}$. In addition, for distributions $F(\cdot)$ such that $r_0 > \underline{v}$, we have $r_1(v^*) < r_0 < v^*$. Finally, for distributions $F(\cdot)$ such that $r_0 = \underline{v}$, we have $r_1(v^*) = r_0 = v^* = \underline{v}$.*

It is interesting to notice that the auctioneer's choice of the second-period reserve price $r_2 = r_2(b^1, r_1)$ is better informed under non-commitment. Therefore, he expects a greater revenue in the second auction than in the case of commitment to a reserve price $r_1 = r_2 = r_0$. This gain, however, is dominated by the auctioneer's loss in the first period, so that, overall, it is more profitable for him to commit to $r_0$. The two revenues are equal only when $r_0 = \underline{v}$, which occurs when $\psi(\underline{v}) \geq 0$.

There are two reasons for the first-period loss. First, unless $r_0 = \underline{v}$, there is a smaller measure of bidder types participating in the auction. In addition, the participating types bid less aggressively than in the case of commitment. In this respect, the auctioneer's decision in our setting has a stronger effect on the bidders' choice of strategy than that of the auctioneer in Caillaud and Mezzetti [4]. In their setting, because of the use of English auctions and the bidders' multi-unit demands and persistent valuations, each participating bidder always bids up to his valuation.

The following result, which follows directly from the characterization of the optimal selling mechanism under commitment and the failure of the optimal non-commitment mechanism to implement the same allocation, expresses the auctioneer's benefit from committing to a reserve price schedule when this is possible:

**Corollary 4**
*Suppose that the bidders' lowest virtual valuation is $\psi(\underline{v}) < 0$. Then, in a sequential first-price auction in which the first-round winning bid is revealed, the auctioneer's revenue is strictly greater under commitment to reserve prices $r_1 = r_2 = \psi^{-1}(0)$ than in any reserve-price scheme under non-commitment.*

We conclude by applying our results to the case of uniformly distributed valuations.

**Example:**
Suppose that the bidders' valuations are uniformly distributed on [0,1]. In the second round, if the reserve price is $r_2$, the revealed upper bound for the bidders' valuations is $\hat{v}$ and there are $M$ competing bidders, each bidder acts according to the bidding strategy described by Lemma 1:

$$\beta^{2,M}(v_i \,|\, \hat{v}, r_2) \;=\; \begin{cases} \frac{1}{M\, v_i^{M-1}}\,[r_2^M \;+\; (M-1)v_i^M], & \text{if } r_2 \le v_i \le \hat{v}; \\ \frac{1}{M\, \hat{v}^{M-1}}\,[r_2^M \;+\; (M-1)\hat{v}^M], & \text{if } v_i > \hat{v}; \\ a, & \text{if } v_i < r_2. \end{cases}$$

In this auction, it is optimal for the auctioneer to set a reserve price

$$r_2(\hat{v}) \;=\; \frac{1}{2}\,\hat{v}.$$

In the first auction, there will be a positive measure of participating bidder types if and only if the reserve price is $r_1 < \bar{r}_1$, where $\bar{r}_1 \in [0,1]$ is given by the equation (6):

$$\bar{r}_1 \;=\; \beta^{2,N}(1\,|\,1, r_2(1)) \qquad \Longrightarrow \qquad \bar{r}_1 = \frac{N-1+(1/2)^N}{N}.$$

Otherwise, all bidders will wait for the second period.

When $r_1 < \bar{r}_1$, bidders with valuations $v_i \ge \underline{v}(r_1)$ bid according to the strategy

$$\beta^1(v_i\,|\,r_1) \;=\; \frac{1}{v_i^{N-1}} \int_{\underline{v}(r_1)}^{v_i} r_2(v)^{N-1} \;+\; (N-2)v^{N-1}\; dv \;+\; \frac{\underline{v}(r_1)^{N-1}}{v_i^{N-1}}\, r_1,$$

given by equation (4), while bidders with valuations $v_i < \underline{v}(r_1)$ abstain from the auction.

By imposing the indifference condition (5) defining the type $v_i = \underline{v}(r_1)$, we get

$$r_1 \;=\; \beta^{2,N}[\underline{v}(r_1)\,|\,\underline{v}(r_1), r_2(\underline{v}(r_1))] \qquad \Longrightarrow \qquad \underline{v}(r_1) \;=\; \frac{N}{N-1+(1/2)^N}\; r_1 \;=\; \frac{r_1}{\bar{r}_1}.$$

Clearly $\underline{v}(r_1) > r_1$, because of the bidders' anticipation of a lower second-round reserve price. In addition, $\underline{v}(\bar{r}_1) = 1$, according to the definitions (5) and (6) of $\underline{v}(r_1)$ and $\bar{r}_1$.

By using the expressions for $r_2(\bar{v})$ and $\underline{v}(r_1)$ that we derived above, we can simplify the function describing the first-period strategy of a bidder with valuation $v_i \ge \underline{v}(r_1)$:

$$\beta^1(v_i\,|\,r_1) \;=\; \left(\bar{r}_1 - \frac{1}{N}\right) v_i \;+\; \frac{1}{N}\left(\frac{r_1}{\bar{r}_1}\right)^N \frac{1}{v_i^{N-1}}.$$

Finally, suppose that the auctioneer chooses a reserve price $r_1 \leq \bar{r}_1$ corresponding to a participation threshold $v = \underline{v}(r_1) = r_1/\bar{r}_1$. Then his expected payoff will be

$$\begin{aligned} R(v) &= \int_v^1 \left(\bar{r}_1 - \frac{1}{N}\right) v_1^N + \frac{1}{N}\left(\frac{r_1}{\bar{r}_1}\right)^N dv_1 \\ &+ \int_{v/2}^{v} \left(\frac{v}{2}\right)^N + (N-1)\, v_1^N \, dv_1 \\ &+ \int_v^1 \int_{v_1/2}^{v_1} N \left[\left(\frac{v_1}{2}\right)^{N-1} + (N-2)\, v_2^{N-1}\right] dv_2\, dv_1. \end{aligned}$$

The necessary condition (7) from Proposition 3 yields:

$$(1-v)\, v^{N-1} = 2^N \int_{v/2}^{v} (2u-1)\,(N-1)\, u^{N-2}\, du.$$

Therefore, the auctioneer maximizes his expected revenue by inducing a first-period participation threshold or, equivalently, by setting a first-period reserve price

$$v^* = \frac{N\,(2^N-1)}{2\,(N-1)\,(2^N-1)+N} \quad \Longleftrightarrow \quad r_1^* = \frac{N\,(2^N-1)}{2\,(N-1)\,(2^N-1)+N}\, \bar{r}_1.$$

Since $v^* < 1$, the auctioneer is always willing to sell the first-period object. In fact, the first-period reserve price will be $r_1^* < \frac{1}{2}$, the optimal reserve price under commitment. However, since $v^* > \frac{1}{2}$, there are strictly fewer bidder types participating in the first auction than in the optimal sequential auction under commitment.

□

## 4.2 Sale / Non-sale

The revenue comparison in Corollary 4 suggests that a non-commitment seller might be better off acquiring less information about the outcome of the first auction.[29] In this manner, by making his choice of the second-period reserve price less flexible, the seller might eliminate some of the bidders' first-period incentives to wait. However, as the case studied in this section shows, this is not necessarily true.

When the seller observes only whether the first item was sold, the equilibrium strategy profile is slightly different than that of the sequential auction in which the first-period winning bid is revealed.

[29] In particular, if the seller could observe no information about the first round, then the commitment outcome would be replicated.

In the first auction, the seller sets a reserve price $r_1$; in equilibrium, we have $r_1 \geq r_0$. Subsequently, if the first unit is sold, he sets a reserve price $r_2(r_1, s) \equiv r_0$, independently of $r_1$. If the first unit remains unsold, the seller lowers the reserve price to $r_2(r_1, ns) < r_1$, according to his updated beliefs.

Each bidder participates in the first auction if and only if his valuation is sufficiently higher than the reserve price, that is, if and only if $v_i \geq \underline{v}(r_1)$, for some threshold value $\underline{v}(r_1) \geq r_1$. There is a value $\bar{r}_1 \in [r_0, \bar{v}]$ such that for any reserve price $r_1 < \bar{r}_1$ a positive measure of bidder types participates in the first round, i.e., $\underline{v}(r_1) < \bar{v}$; in particular, for $r_1 = \underline{v}$, $\underline{v}(r_1) = \underline{v}$. Otherwise, for $r_1 \geq \bar{r}_1$, no bidder participates in the first round.

For any reserve price $r_1$ such that $\underline{v}(r_1) \geq r_0$, the buyers participating in the first auction shade their bids more sharply than in the case of commitment (to the second-period reserve price $r_2 = r_1$), knowing that they face relatively less competition. In fact, for any $v_i \geq \underline{v}(r_1)$, buyer $i$ bids

$$\beta^1(v_i \,|\, r_1) = \frac{1}{G(v_i)} \left[ \int_{\underline{v}(r_1)}^{v_i} \beta^{2,N-1}(v \,|\, \bar{v}, r_0)\; g(v)\; dv \; + \; G[\underline{v}(r_1)]\; r_1 \right]. \tag{8}$$

Clearly, since $r_0 \geq r_2(v)$ for all $v \in [\underline{v}, \bar{v}]$, the participating bidders bid more aggressively than in the case in which the seller observes the winning bid.

Off the equilibrium path, for any reserve price $r_1$ such that $\underline{v}(r_1) < r_0$, buyers with valuations $v_i \in [\underline{v}(r_1), r_0]$ do not shape their bids much, knowing that if they fail to win the first round they will face a second-period reserve price above their valuation:

$$\beta^1_l(v_i \,|\, r_1) = \frac{1}{G(v_i)} \left[ \int_{\underline{v}(r_1)}^{v_i} v\; g(v)\; dv \; + \; G[\underline{v}(r_1)]\; r_1 \right].$$

On the other hand, buyers with valuations $v_i \geq r_0$ shade their bids sharply:

$$\beta^1_h(v_i \,|\, r_1) = \frac{1}{G(v_i)} \left[ \int_{r_0}^{v_i} \beta^{2,N-1}(v \,|\, \bar{v}, r_0)\; g(v)\; dv \; + \; G(r_0)\; \beta^1_l(r_0 \,|\, r_1) \right].$$

In the second round, each buyer submits a bid if and only if his valuation exceeds the reserve price $r_2$. Following a sale in the first round, buyer $i$ with valuation $v_i \geq r_2$ bids

$$\beta^2(v_i \,|\, b^1_i, s, r_1, r_2) \;=\; \beta^{2,N-1}(v_i \,|\, \bar{v}, r_2),$$

in the manner described in Lemma 1, independently of the reserve price $r_1$ and his bid $b^1_i$ in the first period. Similarly, if the first item has not been sold, buyer $i$ bids

$$\beta^2(v_i \,|\, a, ns, r_1, r_2) \;=\; \beta^{2,N}(v_i \,|\, \underline{v}(r_1), r_2).$$

For any reserve price $r_1 \leq \bar{r}_1$, the lowest participating type, $\underline{v}(r_1)$, is indifferent between acquiring the first unit at price $r_1$ and winning the second auction. Therefore,

$$r_1 \;=\; \beta^2(\underline{v}(r_1)\,|\,a, ns, r_1, r_2(r_1, ns)).$$

The value $\bar{r}_1$ is defined by $\underline{v}(\bar{r}_1) = \bar{v}$; for any reserve price $r_1 > \bar{r}_1$, no buyer is willing to participate in the first auction. Notice that a participation threshold $\underline{v}(r_1)$ results in the same reserve price $r_2$ and subsequent bidding behavior, in the event of non-sale, independently of whether the seller could have observed the first-round winning bid. Therefore, the relation between the reserve price $r_1$ and the participation threshold $\underline{v}(r_1)$ is independent of the seller's information revelation policy.

Finally, the participation threshold $v^*$ corresponding to the optimal first-period reserve price $r_1^*$ is the solution to the equation

$$\frac{1-F(v)}{f(v)}\; G[r_0(v)]\; \frac{dr_0}{dv}(v) \;=\; \int_{r_0}^{v} \psi(u)\; g(u)\; du. \tag{9}$$

The above argument leads to the following result, formally proved in the appendix:

**Proposition 5**
*Under non-commitment, in the sequential first-price auction in which the seller observes only whether the first item was sold, there is a unique symmetric monotone equilibrium, such that the expected revenue is lower than that of the sequential auction in which the seller observes the first-period winning bid.*

As argued above, observing only whether the first item is sold has no effect upon the buyers' incentive to wait for a lower reserve price; it only forces those bidders who participate in the first auction to bid more aggressively. On the other hand, it deprives the seller of the opportunity to set the second-period reserve price in a more informed manner. Thus, for any reserve price $r_1$, since the first-period gains are dominated by the second-period losses, the seller is better off observing the first-period winning bid.[30]

We demonstrate the above results for the case of uniform valuations, as in section 4.1.

**Example:**
Suppose that the bidders' valuations are uniformly distributed on [0,1] and that the auctioneer reveals only whether the first unit was sold. In the second round, the bidders' and the seller's strategies depend only on the number $M \in \{N-1, N\}$ of the buyers that remain in the auction and the revealed upper bound $\hat{v} \in \{\bar{v}, \underline{v}(r_1)\}$, in a manner that is identical to that of the sequential auction in which the winning bid is revealed. Therefore, the seller will set $r_2 = r_2(\hat{v})$ and each buyer $i$ will bid $b_i^2 = \beta^{2,M}(v_i\,|\,\hat{v}, r_2)$.

[30]Recall that when the seller can commit to the optimal reserve price, the information revelation policy has no effect upon his expected revenue, a difference from the case of non-commitment.

In the first period, the threshold price $\bar{r}_1$ determining whether some buyer types may participate in the auction is the same as when the first-round winning bid is revealed. In addition, for any reserve price $r_1 \leq \bar{r}_1$, the lowest participating type is also the same; therefore, $\underline{v}(r_1) = r_1/\bar{r}_1$.

For any reserve price $r_1 \leq \bar{r}_1$ such that $\underline{v}(r_1) \geq r_0$, buyers with valuations $v_i \geq \underline{v}(r_1)$ bid according to the strategy

$$\beta^1(v_i \,|\, r_1) \;=\; \frac{1}{v_i^{N-1}} \int_{\underline{v}(r_1)}^{v_i} r_0^{N-1} \;+\; (N-2)v^{N-1} \; dv \;+\; \frac{\underline{v}(r_1)^{N-1}}{v_i^{N-1}}\, r_1,$$

while buyers with valuations $v_i < \underline{v}(r_1)$ abstain from the auction.

For a reserve price $r_1 \leq \bar{r}_1$ corresponding to a participation threshold $v = \underline{v}(r_1) \geq r_0$, the seller's expected payoff will be

$$\begin{aligned} R(v) \;&=\; \int_v^1 (N-2)\,(v_1^N - v^N) \;+\; N \left(\frac{1}{2}\right)^{N-1} (v_i - v) \;+\; N\, v^N \, \bar{r}_1 \; dv_1 \\ &+\; \int_{v/2}^{v} \left(\frac{v}{2}\right)^N \;+\; (N-1)\, v_1^N \; dv_1 \\ &+\; \int_v^1 \int_{1/2}^{v_1} N \left[ \left(\frac{1}{2}\right)^{N-1} \;+\; (N-2)\, v_2^{N-1} \right] \; dv_2 \, dv_1. \end{aligned}$$

The necessary condition (9) yields:

$$(1-v)\, v^{N-1} \;=\; 2^N \int_{1/2}^{v} (2u-1)\,(N-1)\, u^{N-2} \; du.$$

Solving numerically the resulting $N^{\text{th}}$- degree polynomial, for $N = 6$, yields the optimal participation threshold $v_s^* = 0.631$, which corresponds to the reserve price $r_s^* = 0.528$ and the expected seller payoff $R_s^* = 1.678$. If the seller observed the winning bid, the corresponding values would be $v_1^* = 0.571 < v_s^*$ and $r_1^* = 0.477 < r_s^*$, with an expected payoff $R_1^* = 1.719 > R_s^*$. Therefore, the seller is better off observing the winning bid.

□

## 4.3 Revelation of all Bids

In the absence of commitment, the auctioneer can use in an unrestricted manner any information about the bidders' valuations that the first auction may reveal. For example, if a non-winning bidder is revealed to have a valuation $v_i \geq v_L$, then the auctioneer will set a second-period reserve price $r_2 \geq v_L$. The bidders, therefore, have a strong incentive to conceal, in the first auction, their valuations from the auctioneer. Non-surprisingly, this incentive leads to strong negative results.

**Proposition 6**
*In the sequential first-price auction in which the seller observes all first-period bids, there does not exist any symmetric perfect Bayesian equilibrium in weakly increasing first-period bidding strategies.*

This non-existence result[31] originates from the asymmetric effects of the deviations to $\beta^1(\hat{v}_i)$, for $\hat{v}_i > v_i$, and to $\beta^1(\check{v}_i)$, for $\check{v}_i < v_i$. In particular, a deviation to mimicking a type $\hat{v}_i > v_i$ does not decrease the expected second-period surplus of the bidder; this will still be zero. On the other hand, a deviation to mimicking a type $\check{v}_i < v_i$ does increase the expected second-period surplus of the bidder, as it leads to a lower second-period reserve price. Hence, to avoid deviations to $\check{v}_i < v_i$, the bidders shall shade their first-period bids so much so that the deviation to $\hat{v}_i > v_i$ becomes strictly profitable.

## 4.4 Imperfect Commitment

We consider a variation of the sequential auction that we analyzed in section 4.3, one in which the seller may imperfectly commit, that is, in a non-fully credible manner, not to change the second-period reserve price. In particular, at the beginning of the game, the seller announces a reserve price $r_1 \in [\underline{v}, \bar{v}]$. After the end of the first auction, the seller observes the submitted bids and, with some small probability $\rho > 0$, which reflects the seller's lack of credibility (so, it is commonly known), he can set a new reserve price $r_2 = r_2(b^1, r_1)$ for the second auction; otherwise, with probability $1 - \rho$, we have $r_2 = r_1$.[32] One would hope that for $\rho \approx 0$ there can exist symmetric monotone equilibrium. As the following result shows, this turns out not to be possible.

**Proposition 7**
*Under imperfect commitment, in the sequential first-price auction in which the seller observes all first-period bids, there does not exist any symmetric perfect Bayesian equilibrium in weakly increasing first-period bidding strategies.*

On the other hand, when the seller observes only the first-period winning bid or whether the first item is sold, it is easy to show that there exists a sequential equilibrium demonstrating similar characteristics to the ones of the equilibrium described in the case of non-commitment. Therefore, the two cases, of imperfect commitment and of non-commitment, are qualitatively identical.

[31] Notice that if we adopted a weaker notion of perfect Bayesian equilibrium, one that would not impose any restriction on the players beliefs off the equilibrium path, then an equilibrium would exist in a rather generic manner. In particular, in the equilibrium path, the auctioneer would set $r_1 = \bar{v}$, so that effectively only the second-period auction, with $r_2 = r_0 = \psi^{-1}(0)$, would take place. Off the equilibrium path, for $r_1 < \bar{v}$, we could allow each bidder to believe, in an inconsistent manner, that he does not have the highest valuation; thus, all bidders would abstain from the first auction. Therefore, the auctioneer could not benefit from lowering the reserve price.

[32] We assume that the seller learns his type, whether he can change the reserve price or not, only after the end of the first auction. Hence, his choice of $r_1$ does not convey any information to the bidders.

In conclusion, however small the seller's lack of credibility may be, the bidders will be concerned about not revealing their valuations in the first round. Because of this concern a symmetric, monotone, pure-strategy equilibrium exists only when the seller restricts the amount of information he can observe. In the absence of sufficiently strict legal assurances or other means of establishing credibility[33], the use of a Dutch auction (so that only the winning bid is revealed), or of a single-round, multi-unit auction, is the only manner in which the seller can induce a positive outcome.

# 5 Conclusions

In a sequential auction in which the buyers have single-unit demands and the seller is unable to commit perfectly to the auction rules, we have shown that a perfect Bayesian equilibrium in symmetric monotone strategies exists only when the non-winning bidders can securely hide their valuations, for example, in a sequence of two Dutch auctions. Our result complements Caillaud and Mezzetti [4], where the multi-unit demands, along with the persistent valuations, placed the problem of concealing one's valuation upon the winning bidder and, therefore, forced the use of the English auction. In both settings, information is revealed only indirectly, through the actions of the bidders that do not have any further interest in the game.

Since the seller's use of the first-round information has a negative effect upon his total revenue, we have examined a sequential auction in which less information is revealed, namely, when the seller can observe only whether the first item was sold. However, this policy does not eliminate the bidders' incentives to wait for a lower future reserve price; therefore, it cannot help the seller.

When the non-winning bidders must reveal information about their valuations, such as when a sealed-bid auction is used, there is no symmetric monotone pure-strategy equilibrium, however small the imperfection of the seller's commitment is. In this case, it would be interesting to know the equilibrium bidding behavior. The same problem arises in environments in which all bidders try to hide their valuations, independently of whether they win the first object or not, for example, in a sequential auction with multi-unit demands and non-persistent valuations or decreasing marginal returns. Again, in equilibrium, the bidders need to mix their strategies.

Finally, one could consider the sequential allocation problem in its full generality, by allowing the seller to use a sequence of any single-object selling mechanisms he wishes. In this case, it would be interesting to learn whether the seller can profit from using a first-period mechanism that extracts information from some of the non-winning bidders.

[33]For example, the seller might try to trade though a well-established auction house. One can assume that reputational concerns may be more important for such an institution than for a single individual.

# Appendix: Proof of Results

**Proof of Proposition 2:**
Suppose that $r_1 < \bar{r}_1$. The argument that led to the derivation of the bidding function $\beta^1(v_i \,|\, r_1)$ has shown that no bidder with valuation $v_i \in [\underline{v}(r_1), \bar{v}]$ can profit by deviating unilaterally to bidding $\beta^1(\tilde{v}_i, r_1)$, for $\tilde{v}_i \geq \underline{v}(r_1)$ such that $\tilde{v}_i \neq v_i$.

It remains to show that no bidder with valuation $v_i \geq \underline{v}(r_1)$ can profit by abstaining from the first auction; and that no bidder with valuation $v_i < \underline{v}(r_1)$ can profit by participating in the first auction. This result follows from the indifference of the $\underline{v}(r_1)$-type between abstaining and participating in the first period.

Consider a bidder with valuation $v_i \geq \underline{v}(r_1)$. Since $\Pi[v_i, v_i] > \Pi[\underline{v}(r_1), v_i]$, it suffices to show that the bidder's payoff from abstaining does not exceed his payoff from mimicking the type $\underline{v}(r_1)$. Obviously, these two payoffs differ only if all other bidders' valuations are below $\underline{v}(r_1)$. In this case, in the second period, it is optimal for a bidder with valuation $v_i > \underline{v}(r_1)$ to bid

$$\beta^{2,N}[\,\underline{v}(r_1) \,|\, \underline{v}(r_1),\, r_2(\underline{v}(r_1))\,] \;=\; r_1 \;=\; \beta^1[\underline{v}(r_1) \,|\, r_1].$$

Thus, the bidder's payoff from abstaining at reserve price $r_1 < \bar{r}_1$ is equal to his payoff from acquiring the object at price $\beta^1[\underline{v}(r_1), r_1]$. Hence, it is optimal for the bidder to participate in the first auction with a bid $\beta^1[v_i \,|\, r_1]$.

Conversely, consider a bidder with valuation $v_i < \underline{v}(r_1)$. Since $\Pi[\tilde{v}_i, v_i] < \Pi[\underline{v}(r_1), v_i]$ for all $\tilde{v}_i > \underline{v}(r_1)$, it suffices to compare the bidder's payoff from abstaining with that from mimicking the type $\underline{v}(r_1)$. Since the type $\underline{v}(r_1)$ is indifferent between participating in the first auction and abstaining from it, we can consider, equivalently, the bidder's second-period payoffs from bidding according to his valuation $v_i$ and mimicking the type $\underline{v}(r_1)$. Hence, by Lemma 1, the bidder cannot profit from participating in the first auction.

For reserve prices $r_1 > \bar{r}_1$, no bidder is supposed to participate in the first auction. Therefore any bidder can claim the first-period object at price $r_1$. Since

$$r_1 \;>\; \bar{r}_1 \;=\; \beta^{2,N}(\bar{v} \,|\, \bar{v}, r_2(\bar{v})),$$

claiming the first-period object would be inferior to abstaining from the first auction and mimicking, in the second auction, the type $\bar{v}$. Therefore, by Lemma 1, participation in the first auction cannot be profitable.

□

**Proof of Proposition 3:**
By setting a first-period reserve price $r_1 \in [\underline{v}, \bar{r}_1]$, corresponding to a participation threshold $v = v(r_1) \in [\underline{v}, \bar{v}]$, the auctioneer expects a revenue

$$\begin{aligned} R(v) &= \int_v^{\bar{v}} \beta^1(v_1 \,|\, v)\; Nf(v_1)G(v_1)\, dv_1 \\ &+ \int_{r_2(v)}^{v} \beta^{2,N}[v_1 \,|\, v, r_2(v)]\; Nf(v_1)G(v_1)\, dv_1 \\ &+ \int_v^{\bar{v}} \int_{r_2(v_1)}^{v_1} \beta^{2,N-1}[v_2 \,|\, v_1, r_2(v_1)]\; Nf(v_1)g(v_2)\; dv_2\, dv_1. \end{aligned}$$

The first two integrals correspond to the payment of the buyer with the highest valuation, when he wins either the first or the second of the two auctions, while the third integral corresponds to the payment of the bidder with the second-highest valuation, when he can acquire the second-period object.

By differentiating with respect to the threshold valuation $v$ and substituting the expressions for $\frac{d}{dv}\beta^1(v_1|v)$ and $\frac{d}{dv}\beta^{2,N}[v_1|\, v, r_2(v)]$, we get

$$\begin{aligned} \frac{dR}{dv}(v) &= N\,[1 - F(v)]\; \left[ g(v)\, r_1(v) \,+\, G(v)\, \frac{dr_1}{dv}(v) \right] \\ &- N\,[1 - F(v)]\; g(v)\; \beta^{2,N-1}[v \,|\, v, r_2(v)] \\ &- N\; G[r_2(v)]\; f[r_2(v)]\; r_2(v)\; \frac{dr_2}{dv}(v) \\ &+ N\; [F(v) - F(r_2(v))]\; G[r_2(v)]\; \frac{dr_2}{dv}(v) \\ &- N\; f(v) \int_{r_2(v)}^{v} \beta^{2,N-1}[v_2 \,|\, v, r_2(v)]\; g(v_2)\; dv_2. \end{aligned}$$

Since $\psi(r_2(v) \,|\, v) = 0$ and, therefore, $F(v) - F[r_2(v)] = f[r_2(v)]\, r_2(v)$, as in equation (3), we can simplify this sum by eliminating its third and fourth terms. In addition, by substituting the expressions for $r_1(v)$, $\frac{dr_1}{dv}(v)$ and $g(u)\, \beta^{N-1}[u; v, r_2(v)]$, we get

$$\begin{aligned} \frac{dR}{dv}(v) &= N\,[1 - F(v)\,]\; \left[ g(v)\, v \,+\, G(r_2(v))\, \frac{dr_2}{dv}(v) \right] \\ &- N\,(N-1)\, f(v)\; [1 - F(r_2(v))\,]\, F(r_2(v))^{N-2}\; r_2(v) \\ &- N\,(N-1)\, f(v)\; [1 - F(v)\,] \int_{r_2(v)}^{v} u\; (N-2)\, F(u)^{N-3}\; du \\ &- N\,(N-1)\, f(v) \int_{r_2(v)}^{v} f(v_2) \int_{r_2(v)}^{v_2} u\; (N-2)\, F(u)^{N-3}\; du\; dv_2. \end{aligned}$$

Finally, by integrating the last integral by parts, canceling the opposite-sign terms and substituting the expression $\psi(v) = v - \frac{1-F(v)}{f(v)}$, the derivative becomes

$$\frac{dR}{dv}(v) \;=\; N\,[1 - F(v)\,]\; G(r_2(v))\; \frac{dr_2}{dv}(v) \;-\; N\,f(v) \int_{r_2(v)}^{v} \psi(u)\; g(u)\; du.$$

First, suppose that $r_0 > \underline{v}$. If $v \in [\underline{v}, r_0]$, then, since the function $\psi(v)$ is increasing, we have $\psi(u) < 0$ for all $u \in [\underline{v}, v)$. Therefore, the derivative $\frac{dR}{dv}(v) > 0$, for all $v \in [\underline{v}, r_0]$, implying that $v^* > r_0$. By the Intermediate Value Theorem, since $\frac{dR}{dv}(r_0) > 0$ and $\frac{dR}{dv}(\bar{v}) < 0$, we conclude that there exists a value $v^* \in (r_0, \bar{v})$ for which the auctioneer's revenue function $R(v)$ attains its maximum.

Now, suppose that $r_0 = \underline{v}$. Then, for all $v \in [\underline{v}, \bar{v}]$, we have $r_2(v) = \underline{v}$, implying that $\frac{dR}{dv}(v) < 0$. Hence, in this case, $v^* = \underline{v}$, as asserted.

□

**Proof of Proposition 5:**
The argument showing that the strategy profile that we described forms an equilibrium parallels the one in the proof of Proposition 2. In addition, we can establish the first-order condition (9) by replicating the argument in the proof of Proposition 3.

To compare the expected seller revenues from observing the first-period winning bid, $R^1$, and whether the first item is sold, $R^s$, we will prove a stronger result, namely, that $R^1(v) > R^s(v)$, for all first-period participation thresholds $v = \underline{v}(r_1^1) = \underline{v}(r_1^s) \in [r_0, \bar{v}]$.

From the first-order conditions (7) and (9), it follows that, for all $v \in [r_0, \bar{v}]$,

$$\frac{dR^s}{dv}(v) \;-\; \frac{dR^1}{dv}(v) \;=\; N\,f(v) \int_{r_2(v)}^{r_0} \psi(u)\,g(u)\,du \;\;>\;\; 0.$$

Since $R^1(\bar{v}) = R^s(\bar{v})$, we can conclude that $R^1(v) > R^s(v)$, for all $v \in [r_0, \bar{v})$.

□

**Proof of Proposition 6:**
Suppose that there exists a perfect Bayesian equilibrium $[(\beta_i^1, \beta_i^2)_{i=1}^N, (r_1, r_2)]$ such that the first-period bidding strategies are symmetric and increasing in the valuation $v_i$. To derive a contradiction, it suffices to consider the restriction of the equilibrium to the continuation game following a first-period reserve price $r_1$, such that a positive measure of bidders' types, $[\underline{v}(r_1), \bar{v}]$, participates in the first auction. In particular, in any equilibrium of the continuation game following $r_1 = \underline{v}$, all bidder types must participate to the first auction.

First, we rule out the existence of an equilibrium involving first-period bidding strategies $\beta^1(\cdot \,|\, r_1)$ that are strictly increasing in $[\underline{v}(r_1), \bar{v}]$. Under such a bidding strategy profile, we have perfect revelation of the participating bidders's valuations. Therefore, these bidders expect to make zero profit in the second auction. Hence, on the strategy-realization path, the participating bidders treat the problem as that of a single-period, single-unit auction. According to the symmetric equilibrium of the first-price auction, we must have

$$\beta^1(v_i \,|\, r_1) \;=\; \begin{cases} \mathbb{E}[\max\{v_1^{(N-1)}, r_1\} \,|\, v_1^{(N-1)} < v_i], & \text{if } v_i \geq \underline{v}(r_1); \\ a, & \text{if } v_i < \underline{v}(r_1). \end{cases}$$

By splitting cases, according to the realized type-profile, we claim that each bidder $i$ has a profitable deviation to $\tilde{\beta}^1(v_i \,|\, r_1) \equiv a$. Notice that, along the equilibrium path, the second-period reserve price $r_2$ will be

$$r_2(b_1^1, ..., b_N^1, r_1) \begin{cases} = \; [\beta^1(\cdot \,|\, r_1)]^{-1}(b_2^{(N)}), & \text{if } b_2^{(N)} \neq a; \\ \leq \; r_1, & \text{if } b_2^{(N)} = a. \end{cases}$$

If bidder $i$, with valuation $v_i \in [\underline{v}(r_1), \bar{v}]$, turns out to have the highest valuation, then he will still win an object, in the second auction, at an expected price

$$p_2 \;=\; \mathbb{E}[\max\{v_2^{(N-1)}, r_1\} \,|\, v_1^{(N-1)} < v_i] \;+\; \varepsilon.$$

For sufficiently small $\varepsilon > 0$, the price $p_2$ is smaller than $\beta^1(v_i \,|\, r_1)$, the price bidder $i$ will pay, if he wins the object in the first auction.

If bidder $i$ turns out to have the second-highest valuation, then he will again win an object in the second auction, at an expected price

$$p_2 \;=\; \mathbb{E}[\max\{v_2^{(N-1)}, r_1\} \,|\, v_2^{(N-1)} < v_i < v_1^{(N-1)}] \;+\; \varepsilon.$$

Again, for sufficiently small $\varepsilon > 0$, the price $p_2$ is strictly smaller than $v_i$, the price bidder $i$ would pay in the second auction, if he revealed his valuation

Finally, if bidder $i$ turns out to have the third-highest, or lower, valuation, then he will win no object, as he would do after bidding $\beta^1(v_i \,|\, r_1)$.

We conclude the proof by ruling out the existence of intervals of non-increase in $[\underline{v}(r_1), \bar{v}]$. Suppose that $\beta^1(v_i \,|\, r_1) = b$, for all $v_i \,\epsilon\, [v_L, v_H] \subset [\underline{v}(r_1), \bar{v}]$. If $b > v_L$, then any bidder $i$ with type $v_i \,\epsilon\, V_1 = [v_L, \min\{b, v_H\}]$ is better off bidding $\tilde{b}^1(v_i \,|\, r_1) = v_i$, a bid that avoids winning the object at a price above $v_i$. If $b < v_L$, then there is an $\varepsilon > 0$ sufficiently small such that the deviation to the strategy $\beta^1(v_i \,|\, r_1) = b + \varepsilon$, for all $v_i \,\epsilon\, [v_L, v_H]$ is profitable. Finally, if $b = v_L$, then we can simply apply the argument for $b < v_L$ to the interval $[\frac{1}{2}(v_L{+}v_H), v_H]$. Hence, there cannot exist an interval of non-increase of $\beta^1(\cdot \,|\, r_1)$.

□

**Proof of Proposition 7:**
Suppose that in the second auction, each participating bidder bids his valuation. The reserve price $r_2$ depends on the outcome of the first period and on whether the auctioneer turns out to be credible. It is $r_2 = r_1 = 0$ with probability $1 - \rho$; and $r_2 = \beta^1(\cdot|0)^{-1}(p_1)$, the second-highest valuation revealed in the first period, with probability $\rho$.

If each bidder follows a first-period bidding strategy $\beta^1(v \,|\, 0)$, the expected payoff of a bidder $i$ with valuation $v_i$ who mimics a type $\tilde{v}_i > v_i$ will be

$$\begin{aligned}
\Pi[\tilde{v}_i; v_i] \;\; &= \;\; F(\tilde{v}_i)^{N-1}\, v_i - \int_0^{\tilde{v}_i} \beta^1(v \,|\, 0)(N-1)F(v)^{N-2} f(v)\, dv \\
&+ \;\; (N-1)\,[1 - F(\tilde{v}_i)]\,(1-\rho)\, F(v_i)^{N-2}\, v_i \\
&- \;\; (N-1)\,[1 - F(\tilde{v}_i)]\,(1-\rho) \int_0^{v_i} v(N-2)F(v)^{N-3} f(v)\, dv.
\end{aligned}$$

The necessary first-order condition $\frac{\partial \Pi}{\partial \tilde{v}_i}[v_i; v_i] \leq 0$ yields the inequality

$$F(v_i)^{N-2}\, \beta^1(v_i \,|\, 0) \;\geq\; (1-\rho) \int_0^{v_i} v(N-2)F(v)^{N-3} f(v)\, dv \,+\, \rho\, F(v_i)^{N-2}\, v_i.$$

Therefore, to avoid first-period deviations to mimicking a type $\tilde{v}_i > v_i$, the bidders must bid more aggressively than in the case of perfect commitment.

If bidder $i$ mimics a type $\tilde{v}_i < v_i$, his expected payoff will be

$$
\begin{aligned}
\Pi[\tilde{v}_i; v_i] \;&=\; F(\tilde{v}_i)^{N-1}\, v_i - \int_0^{\tilde{v}_i} \beta^1(v\,|\,0)(N-1)F(v)^{N-2} f(v)\, dv \\
&+\; (N-1)\,[1-F(\tilde{v}_i)]\, F(v_i)^{N-2}\, v_i \\
&-\; (N-1)\,[1-F(\tilde{v}_i)]\,(1-\rho) \int_0^{v_i} v(N-2)F(v)^{N-3} f(v)\, dv \\
&-\; (N-1)\,[1-F(\tilde{v}_i)]\;\rho \int_0^{v_i} \max\{v, \tilde{v}_i\}\,(N-2)F(v)^{N-3} f(v)\, dv.
\end{aligned}
$$

The necessary first-order condition $\frac{\partial \Pi}{\partial \tilde{v}_i}[v_i; v_i] \geq 0$ yields the inequality

$$
\begin{aligned}
F(v_i)^{N-2}\, \beta^1(v_i, 0) \;&\leq\; (1-\rho) \int_0^{v_i} v(N-2)F(v)^{N-3} f(v)\, dv \;+\; \rho\, F(v_i)^{N-2}\, v_i \\
&-\; \rho\; F(v_i)^{N-2}\; \frac{1-F(v_i)}{f(v_i)}.
\end{aligned}
$$

Therefore, to avoid first-period deviations to mimicking a type $\tilde{v}_i > v_i$, the bidders must not bid too aggressively.

Clearly, for any $\rho > 0$, the two necessary conditions cannot hold simultaneously. Hence, as asserted, there cannot exist any symmetric perfect Bayesian equilibrium in increasing first-period bidding strategies.

□

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