

Location of Public Housing*

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Abstract

Existing literature on public housing policy are mainly empirical. This paper attempts to take a preliminary step to investigate the implications of public housing policy in a monocentric city model. We explicitly model the fact that some lower-income people may not be able to get a public housing unit and are forced to compete with higher-income people in the private rental market. We show that both the location and the number of public housing units matter. We also compare the equilibrium outcomes under public housing policy with those under housing vouchers, and discuss when and how vouchers can affect household segregation.

Keywords: Public housing, lottery, housing vouchers, welfare.

JEL Classification: H40, D60, H82, R13

“Yet there are interesting problems that a theory of urban land must consider. There is, for instance, a paradox in American cities: the poor live near the center, on expensive land, and the rich on the periphery, on cheap land. On the logical side, there are also aspects of great interest, but which increase the difficulty of the analysis. When a purchaser acquires land, he acquires two goods (land and location) in only one transaction, and only one payment is made for the combination. He could buy the same quantity of land at another location, or he could buy more, or less land at the same location.” (Alonso, 1960, p.149)

“The public housing units themselves have frequently become slums and hotbeds of crime, especially juvenile delinquency. The most dramatic case was the Pruitt-Igoe public housing project in St. Louis—a massive apartment complex covering fifty-three acres that won an architectural prize for design. It deteriorated to such an extent that part of it had to be blown up. At that point only 600 of 2,000 units were occupied and the project was said to look like an urban battleground. We well remember an episode that occurred when we toured the Watts area of Los Angeles in 1968. We were being shown the area by the man who was in charge of a well-run self-help project sponsored by a trade union. When we commented on the attractiveness of some apartment houses in the area, he broke out angrily: “That’s the worst thing that ever happened in Watts. That’s public housing.” He went on to say, “How do you expect youngsters to develop good character and values when they live in a development consisting entirely of broken families, almost all on welfare?” (Friedman and Friedman, 1990, p.110)

1 Introduction

In contrast to the recommendation of the conventional wisdom of economics, housing market is typically intervened by the government. As in the case of other markets, the forms of real estate market intervention differ significantly across economies. For instance, Priemus (2000) observes that while the United States tend to provide cash subsidy, as college level microeconomics textbooks would typically suggest, the Europe countries seem to be more inclined to directly provide physical structure as a form of social insurance or social welfare, even when the spending are comparable.¹ Laferrere and Blanc (2006) find that in the year 2001, the United States spent slightly above 1.5% of their GDP on public aid on housing, while the counterpart in France is similar, slightly above 1.7% of the GDP. Yet the construction-subsidized rental sector (mainly the habitation a loyer modere, or HLM in short) accommodates 17% of households in France, with less than 2% for the U.S. counterpart. And even within the same country, the public housing program can vary. For instance, Olsen (2003) observes that “... the US has had an enormous number of programs intended to improve the housing of low-income households since the federal government became

¹Among others, see Smeeding et al (1993) for cross-country evidence.

seriously involved in this activity in 1937. There have been many programs (...), each of these programs has typically had a number of variants, and each change in the regulations produces a new program. For example, the public housing program has at least 29 variants.” It also seems that even “cash subsidy” with different institutional details can lead to very different results. For instance, Priemus (2001) finds that, while in the USA, 100% of the additional rent is paid by the tenants; the Netherlands tenants will only pay 25%. The government will be responsible for the rest. Moreover, in the Netherlands, there is no waiting list and the rent subsidy is perceived as a “right.” As a result, the number of applicants rose from 348,000 in 1976, to 922,000 in 1996. In 1998, there were more than a million households receiving a housing allowance, out of more than 6 million households in Netherlands. Thus, housing market intervention can be a very serious issue for the government finance.

These figures also lead to another observation about the government “participation” of the housing market. While such “participations” are often justified by the assertion that the market allocation on housing is inefficient, and government policies are needed to correct for the “market failure,”² it is possible that the government policy itself would introduce a new kind of distortion which may outweigh the original benefit of intervention. Among many other possible distortions, as observed by Banerjee (1997), the number of the subsidized slots is often less than the number of applicants, which inevitably leads to a rationing. Some people will be disappointed at the equilibrium.

Building on the insights of previous research, this paper takes an initial step to evaluate the public housing policy with an explicit consideration of such “lottery nature” of the public housing policy in a spatial general equilibrium model. Following Alonso (1960), the agents in this model care about the location of their residences. Both monetary cost and time cost of transportation are included in the model. And as emphasized by Friedman and Friedman (1990), the households are assumed to concern not only their consumption, but also the quality of education their offspring will

²One common complaint about the housing market is that “housing has become unaffordable, and therefore government intervention is necessary.” It is beyond the scope of this paper to discuss the housing affordability, which is a large literature. Among others, see Quigley and Raphael (2004) for a review on that literature.

receive. The literature has confirmed repeatedly that both the “peer group” effect and the “local public finance” nature of education could be important determinants of the “quality of education” that children would receive, and therefore these features are embedded in the model.³ As a result, the original equilibrium needs not be efficient and leaves room for potential improvement. We then introduce public housing into the model and examine the implications on, for instance, the composition of residents within the community. This paper also examines how the amount of public housing provided, as well as the location choice of the public housing would affect the welfare of different income groups, as well as the welfare of the society as a whole.

Clearly, this paper builds on the insights of many precedents. Due to the space limit, however, we can only refer the interested readers to survey papers such as Nechyba (2006) and the references therein. More recently, Hanushek and Yilmaz (2007) build a spatial general equilibrium model in which the location choice, education quality, etc. are all endogenously determined. This paper builds on their framework and differs in several dimensions. First, this paper explicitly models the public housing, which is absent in Hanushek and Yilmaz (2007). Second, this paper explicitly considers the uncertainty of receiving a public housing unit, and how these agents fate in a distorted housing market. Third, this paper in addition considers that the location of the public housing could affect the aggregate as well as the distribution of resources and utility of agents.

The research strategy of this paper is straightforward. We first build a spatial general equilibrium model with peer group effects in education in line with the literature (such as Nechyba (2006), Hanushek and Yilmaz (2007)). We solve for the benchmark case without any government intervention in the housing market, with a plausible parametrization. We then introduce public housing into the model. We investigate how the introduction of public housing affects the equilibrium sorting of agents, the equilibrium tax rates and rents. We also study the welfare consequences of locating public housing units at different distances from employment opportunities. As a comparison, we also study the case when the government provides housing voucher to the same amount

³Clearly, the literature is too large to be reviewed here. Among others, see Epple and Nechyba (2004), Hanushek and Welch (2006).

of population without providing any physical structure. We find that in fact the location of the public housing matters. Furthermore, the voucher is found to be superior than the public housing policy, even with peer group externality as well as the spatial (and hence the consumption set is not convex), and the local public finance consideration. Thus, the intuition behind basic microeconomic textbook case for “in-cash versus in-kind” case may apply to more general environment than it seems.

The organization of the rest of this paper is as follows: Section 2 develops the theoretical framework. Section 3 discusses the calibration and results of the computable model without government intervention. We discuss our models of housing assistance programs and their results in Section 4. Section 5 concludes.

2 The Model

It may be useful to provide an informal description of the model before going into the details. Households in a monocentric city work at the Central Business District (CBD) and choose residences in the surrounding area. The distance of home to the workplace matters because of pecuniary and time costs of commuting to work. A straight line that goes through the CBD divides the city into two jurisdictions, East (e) and West (w). In each jurisdiction, a local public good, education, is provided for the residents. The provision level in each locality is determined endogenously. Therefore the jurisdictions may differ in their (property) tax and expenditure policies. Agents are allowed to move across jurisdictions and this creates a form of “competition” as emphasized by Tiebout (1956). The quality of education in each district depends on the composition of the households through the “peer group” effect as well as the local spending, which is determined by majority voting.

Households

Households need to choose a neighborhood (East or West, which also determines the school that

the offspring will attend), a location in that neighborhood (r , distance to CBD), amount of land to reside on (s), and consumption level (z). Households have preferences defined over consumption of a composite consumer good z , land consumption (the size of the residential lot) $s > 0$, leisure $l \in [0, 24]$, and quality of education q at the school the offspring attends. Following the literature, each household has one school-age child. The preferences of a household can be represented by the utility function $U(q, s, z, l) = q^\alpha s^\eta z^\gamma l^\delta$. A member of every household supplies labor to earn an exogenously determined hourly wage W . Households differ according to the wages they earn, as well as preferences for education. We name the higher income types *skilled* workers (earning W_S), and the lower income types *unskilled* workers (earning $W_U < W_S$). Skilled workers value education more than unskilled do ($\alpha_S > \alpha_U$).⁴

The city has a dense radial transportation system. The further one lives away from CBD, the higher commuting costs he/she will face. In particular, if a household lives r miles away from CBD, the cost of daily roundtrip commute will be ar dollars (pecuniary cost, $a > 0$) and br hours (time cost, $b > 0$), which converts to bWr dollars given the opportunity cost of time. Let $l \in [0, 24]$ denote the number of non-work or leisure hours. We normalize the price of the composite consumption good to one and denote the unit rent of land (r miles away from CBD) by $R(r)$. Households pay a property tax with rate τ on value of land. The budget constraint of a household is:

$$z + (1 + \tau)R(r)s + Wl = Y(r) = 24W - (a + bW)r. \quad (1)$$

For convenience, we write the budget constraint as if one sells all his time and buys back leisure, both at his hourly wage. The term on the RHS of the above equation can then be interpreted as household income net of transportation costs.⁵

Given the market rent curves $\{R_e(r), R_w(r)\}$ and quality-tax packages $\{(q_e, \tau_e), (q_w, \tau_w)\}$ for

⁴Hanushek and Yilmaz (2007), and Hanushek, Sarpça, and Yilmaz (2007) allow for heterogeneity in tastes for education at every income level. The focus of this study is public housing, and the specification we adopt does not affect related results significantly.

⁵We ignore non-commute transportation costs, and travel within the CBD.

each jurisdiction, the type $i \in \{S, U\}$ household solves the problem:

$$\begin{aligned} \max_{s, z, l, j \in \{e, w\}} U(q, s, z, l) &= q_j^{\alpha_i} s^\eta z^\gamma l^\delta \\ \text{s.t. } z + (1 + \tau_j)R_j(r)s + W_i l &= Y_i(r). \end{aligned} \quad (2)$$

Market Rent Curves and Allocation of Land

To be compatible with the literature, land is assumed to be owned by absentee landlords and auctioned off to the highest bidders. The reservation price of the landlord, R_a , is determined by an alternative use of land, such as agriculture, and is independent of the location. For a given utility level \bar{u} we can find the maximum rent a household is willing to pay per unit of land that is r miles away from CBD by solving the problem $\Psi(r, u, q, \tau) = \max_{s, z, l} \left\{ (Y - z - wl) / ((1 + \tau)s) \mid U(q, s, z, l) = u \right\}$ to obtain the *bid rent function*:

$$\Psi(r, u, q, \tau) = \frac{k^{1/\eta}}{(1 + \tau_j)\omega^{\delta/\eta}} q_j^{\alpha_i/\eta} Y_i(r)^{(\eta+\gamma+\delta)/\eta} u^{1/\eta} \quad (3)$$

where $k = \frac{\eta^\eta \gamma^\gamma \delta^\delta}{(\eta+\gamma+\delta)^{\eta+\gamma+\delta}}$, $i \in \{S, U\}$, and $j \in \{e, w\}$.⁶ At an auction for a particular location r^* , the winner will be the type with the highest bid rent curve at that location. Given the two types in the model, in each jurisdiction there are two bid rent curves.⁷ The *equilibrium rent curve* $R_j(r)$ is the upper envelope of the bid rent curves of two types and the agricultural rent R_a . Because all bid rent curves are convex and decreasing, the equilibrium rent curve $R_j(r)$ will be decreasing up to a distance r_{jf}^* , the *fringe distance*, and will stay constant from that point on. In our computational analysis we check and verify that two different types' bid rent curves can intersect at one point at most. This *single-crossing* property of bid rent curves proves quite useful for our analysis. Households with steeper bid rent curves will locate closer to the CBD. Higher income increases the demand for land consumption and attracts households further away from CBD, but it also increases

⁶For derivations and a detailed discussion of the properties of these bid-rent functions see Hanushek and Yilmaz (2007).

⁷If as a result of a policy, the number of types increases, so will the number of bid-rent curves.

the opportunity cost of commuting time. Our computational models indicate that the first effect dominates.

Our city is a *closed city* in the sense that the population is given exogenously. It will help us to pin down the population density at the equilibrium. A few more notations will clarify this point. Let $L(r)$ represent the land density r miles away from CBD, and $n_j(r)$ the equilibrium density function of the household population in jurisdiction $j \in \{e, w\}$. Suppose in equilibrium the residents of the land at distance r in jurisdiction j are type i households. And suppose that the equilibrium level of utility of the type i agent, $i \in \{S, U\}$ is u_i^* . Then $n_j(r) = \frac{L(r)}{s(r, u_i^*)}$. Let \bar{N}_S, \bar{N}_U denote the populations of the respective types. The population constraint for each type can then be stated as:

$$\int_0^\infty \frac{L(r)}{s_w(r)} I[t_w^*(r) = i] dr + \int_0^\infty \frac{L(r)}{s_e(r)} I[t_e^*(r) = i] dr = \bar{N}_i \quad (4)$$

where $t_j^*(r)$ is a function showing the type of the occupant at distance r in jurisdiction j , and $I[\cdot]$ is an indicator function that takes the value 1 when the condition in brackets is satisfied, and 0 otherwise. The population constraint implicitly assumes the land market clears in each jurisdiction ($\forall r \leq r_{fj}^*, s_j(r)n_j(r) = L(r)$).

Neighborhoods

The construction of neighborhoods in this model also follow closely to the literature, which captures several important features of the U.S. situation. The two neighborhoods differ only in the *quality of education* and *property tax rate* (q_j, τ_j) packages they provide. When choosing a community, households take the tax-expenditure packages as given. They move in and vote for the property tax rate. The households are “myopic” when voting; they do not consider the implications on migration patterns and the composition of neighborhoods. For a type i household most preferred

tax rate $\tau^* = \frac{\alpha_i}{\eta - \alpha_i}$ is the solution to the indirect utility maximization problem:

$$\tau^* = \operatorname{argmax}_{\tau} V(\cdot) = \frac{k}{(1 + \tau_j)^\eta R(r)^\eta \omega^\delta} q_j^{\alpha_i} Y_i(r)^{\eta + \gamma + \delta} \quad \text{s.t. } q_j = \Pi_j E_j \quad (5)$$

and $E_j = \tau_j \bar{R}_j$.

We focus on the stationary equilibrium, which is attained when no one has an incentive to relocate in response to the voting results.

There is one public school in each jurisdiction.⁸ Public schools are neighborhood schools, enrollment is open to residents of the community only. Admission is free, schools are financed by property taxes on residential land. The quality of education $q(\Pi, E)$ in a school is determined by (per-student) instructional expenditures E and the peer quality Π . For a given group of students, an increase in the instructional expenditures increases the quality of education ($\partial q / \partial E \geq 0$).⁹ An equilibrium property of our model is that in each jurisdiction there is a distance r_{fj}^* called the *fringe distance* beyond which no households reside. In each community entire revenue from property taxes is spent on education. Given the equilibrium rent function $R_j(r)$, and equilibrium tax rate τ_j , we can calculate the tax base, and total tax revenues to find the per-student expenditure in the public school system:

$$E_j = \frac{1}{N_j} \tau_j \int_0^{r_{fj}^*} R_j(r) L(r) dr \quad (6)$$

for $j \in \{e, w\}$.

Different groups of students may benefit differently from a given amount of instructional expenditures. That is what the peer quality (or efficiency) component captures ($\partial q / \partial \Pi \geq 0$). Some parents value education more than others, and as a result they may spend more time helping with the kid's homework, provide a nicer study environment at home, be more involved in how schools

⁸If there are multiple schools in the neighborhood, and if one's quality exceeds that of another, students will keep on switching to the better school until quality equalizes. So we consider the neighborhood school system as one big school.

⁹There is a debate on the effectiveness of monetary inputs on student's achievements (Burtless (1996)), however, it is reasonable to assume that households would value an increase in educational expenditures. And if they do, sorting implications would be identical.

operate, etc. Recall that type S agents value education more than type U counterparts, and as a result having more students from type S families may bring in a higher level of positive externality through the peer group effect. The following formulation has been proved to be very tractable and captures the idea that the peer quality is increasing in the proportion of S types:¹⁰

$$\Pi = c_0 + c_1 \exp\left(-c_2 \frac{N_U}{N_S}\right), \quad c_0, c_1, c_2 \geq 0. \quad (7)$$

Clearly, when $c_1 = c_2 = 0$, the peer quality Π becomes a constant.

Definition of Equilibrium

An equilibrium is a set of utility levels for each type $\{u_S^*, u_U^*\}$, market rent curves for each jurisdiction $\{R_e(r), R_w(r)\}$, quality of education and property tax rate pairs $\{(q_e, \tau_e), (q_w, \tau_w)\}$ for each neighborhood, household population distribution functions $\{n_e(r), n_w(r)\}$, and type functions $\{t_e^*(r), t_w^*(r)\}$ that show the equilibrium locations of the occupants at distance r in community j such that:

- Households' choices are determined by solving (2),
- The market rent function $R_j(r)$ in each jurisdiction is determined through a bidding process among different types of households,
- Same types of households obtain the same level of utility regardless of their choices,
- The tax rates in each jurisdiction are determined by majority voting by myopic voters,
- Local governments' budgets balance in each jurisdiction, (6),
- Labor and land markets clear,
- The population constraints (4) hold.

¹⁰Alternative specifications give similar results. See, for example, Hanushek and Yilmaz (2007) and Hanushek, Sarpça, and Yilmaz (2007).

3 Equilibrium without Public Housing

Parameters for the Computational Model

Clearly, the equilibrium of our model is very complicated and closed form solution is not likely. Instead, we turn to numerical methods and as a self-discipline of our work, we specify parameter values to match certain statistics from U.S. cities in 2005. As a starting point, we normalize the sum $\eta + \gamma + \delta$ to 1. The solution to the household problem then gives the optimal budget shares for leisure, consumption, and lot size as δ , γ , and η , respectively. In U.S. average hours of work per week in full time jobs is 40 hours, and average annual earnings of workers are \$30,104 for high school graduates and \$58,114 for college graduates.¹¹ Accordingly, we set the hourly wages for unskilled and skilled types as $W_U = 14$ and $W_S = 27$. In a 168 (= 24 * 7) hour week, 40 hours of work implies a 0.762 budget share for leisure. The data on household expenditures suggest that expenditures on shelter constitute about 20% of the budget of an average household. This implies budget shares of 4.76% for housing and 19% for consumption. There are two possibilities for the most preferred property tax rate according to (5). We set these most preferred tax rates equal to 1.97% (1.04%) for the high (low) valuation types. The average population density in a city with population 1 to 2.5 million is 2901 people per square mile.¹² The utility function parameters consistent with all these are $\alpha_L = 0.014$, $\alpha_H = 0.021$, $\delta = 0.762$, and $\gamma = 0.19$.

We calculate the commuting costs assuming the households drive to work. The pecuniary cost can be calculated based on the cost of owning and operating an automobile. In 2004 pecuniary cost per mile was \$0.56, and we set $a = 1.1$. Assuming the commuting speed in the city is 20 miles per hour, we set $b = 0.13$. We assume 1.5 million households populate the city. When computing the equilibrium, we target for a (endogenous) fringe distance (city radius) around 15 miles in each jurisdiction. The proportion of college graduates in U.S. is about 30%. We expect this proportion to be slightly higher in a city. Hence, we set the proportion of skilled households to 40%. We set

¹¹Current Population Reports 2005, US Census Bureau.

¹²US Census Bureau, Census 2000 Summary File 1.

the parameters of the school quality function to $c_0 = 0.1$, $c_1 = 1.3677$, and $c_2 = 0.05743$ to match some related empirical observations.

Equilibrium

Strictly speaking, the equilibrium contains a spatial distribution of agents as well as of rents. It also include the policy packages in different neighborhoods. Thus, rather than providing all the details, we use figure 1 to provide a visualization (“a map”) of the city. As a complement, table 1 provides some important summary statistics. Several observations are immediate: (1), both household types exist in each community;¹³ (2), low income types choose smaller lots closer to the city center in each neighborhood; (3), There is (partial) Tiebout sorting across neighborhoods.

(Figure 1, Table 1 about here)

The results are intuitive. Costly commuting causes the market rents in each jurisdiction to decrease as one moves away from the CBD. Higher income increases the land demand and attracts households further away from CBD where the land is cheaper. High income also increases the opportunity cost of commuting time, but our results show that this effect is dominated by the former. As a result, in each community lower income U types occupy a semi circle around to the CBD. The S types reside in a semi-ring that surrounds the semi-circle of U types.¹⁴ The outer end of the S semi-ring is the fringe distance, the land beyond which is left for agricultural use.¹⁵

Notice that about seventy percent of high income types live in the same neighborhood. They constitute a fifty five percent majority of the population there. As a result, the taxes are higher (1.97% vs 1.04%). Without loss of generality, we refer to the higher tax neighborhood as *West* school

¹³This is consistent with empirical findings on household sorting (e.g., Davidoff (2005)) as well as results of theoretical studies including Tiebout models such as Epple and Platt (1998), and Tiebout-Urban hybrid models such as Hanushek and Yilmaz (2007).

¹⁴Hanushek and Yilmaz (2007) have obtained a similar heterogenous communities result by allowing preferences for education to vary at different income levels. This paper obtains the same result with variation in income only.

¹⁵This “rings” structure dates back to von Thunen’s model of land use (1826), and is one of the building blocks of Urban Land Use Theory. In our models we have semi-rings instead of full ones because of the jurisdiction boundaries, or the local public goods problem. As a result of this, the widths of rings may differ between neighborhoods. The ordering of households around CBD, however, does not change.

district throughout this paper. Because the two neighborhoods have comparable populations, the higher taxes mean higher educational expenditures per student in West (\$3653 vs \$2027). Also, the peer quality is higher in the West neighborhood, thus quality of education exceeds that in East.¹⁶ This difference in quality of education is capitalized into rents; Rents in West are about twenty-five percent higher on average.¹⁷

4 Public Housing

We consider two housing policies in this section, namely, government provided public housing units, and housing vouchers. These programs are financed by income taxes with rate θ on earnings of all city residents (plus participant contributions in the case of public housing units). Only low-income (type U) households are eligible. The government sets the maximum number of participants N_P exogenously. If the number of program applicants exceed N_P , participants are selected by a lottery. For comparison purposes, we keep the sizes of two programs same.

4.1 Public Housing Units

For the first government policy, we suppose that the government buys the land between r_P and $r_P + w_P$ miles from CBD in one of the neighborhoods. According to Laferrere and Blanc (2006), among others, this seems to be more consistent with the case of France. The rents are determined by the same auction process described above, but now bidders exclude the public housing recipients. For any location between r_P and $r_P + w_P$, the government pays the maximum rent households are willing to pay, had this land been available to them. Government bought land is then divided into equally sized lots s_P and rented out to N_P program participants, both determined exogenously. A program participant household pays a fixed price R_{PSP} as their program contribution, independent

¹⁶The higher tax/expenditure community providing a higher quality of education with higher per-student-expenditure could easily mislead one to overrate the role of expenditures on school quality. Hanushek, Sarpça, and Yilmaz (2007) show the existence of a private sector of education breaks the link between expenditures and school quality.

¹⁷The average rent per acre for S type (U type) households in west is \$2,203 (\$5,416). The average rents in east are \$1,433 and \$3,367.5 for S and U types.

of location of the lot in the band. This policy reduces a public housing unit resident's problem to leisure-consumption choice only:

$$\begin{aligned} \max_{z,l} U(q, s, z, l) &= q_w^{\alpha_U} s_P^\eta z^\gamma l^\delta \quad s.t. \\ z + (1 + \tau)R_P s_P + (1 - \theta)W_U l &= Y_P(r) = 24(1 - \theta)W_U - (a + b(1 - \theta)W_U)r, \end{aligned} \quad (8)$$

whereas others solve the original problem (2) in the presence of income taxes:

$$\begin{aligned} \max_{s,z,l,j \in \{e,w\}} U(q, s, z, l) &= q_j^{\alpha_i} s^\eta z^\gamma l^\delta \quad s.t. \\ z + (1 + \tau_j)R_j(r)s + (1 - \theta)W_i l &= Y_i(r) = 24(1 - \theta)W_i - (a + b(1 - \theta)W_i)r. \end{aligned} \quad (9)$$

taking rents and neighborhood quality-tax pairs as given.

The cost of public housing is financed by income tax revenues and participant contributions. The time constraint of a household give the labor supply as all the time except leisure and commute time: $n = 24 - l - br$. The solution to (9) gives the optimal leisure of household residing at distance r as $l_i^*(r) = \delta \frac{Y_i(r)}{(1-\theta)W_i}$ for $i \in \{S, U\}$. For a household that lives in public housing, optimal leisure is $l_P^*(r) = \frac{\delta}{\delta + \gamma} \frac{Y_P(r) - (1 + \tau)R_P s_P}{(1 - \theta)W_U}$. We can define a $t_j^*(r)$ function that shows the type of the occupant at distance r in jurisdiction j (whether S , or U , or P , i.e., U in public housing). Let $I[.]$ be an indicator function that takes the value 1 when the condition in brackets is satisfied, and 0 otherwise. The government's budget constraint is:

$$\int_{r_P}^{r_P + w_P} R_e(r)L(r)dr - N_P R_P s_P = \theta \sum_{j \in \{e,w\}} \left[\int_0^\infty \frac{L(r)}{s_j(r)} \left(\sum_{i \in \{S,U,P\}} I[t_j^*(r) = i] (24 - l_i^*(r) - br) W_i \right) dr \right] \quad (10)$$

The inside summation in the RHS identifies which household type resides at a particular location and calculates their labor income at equilibrium. The integral calculates the total labor income using the households' density at that location. The outside summation adds labor income of two

communities. A fraction θ of this gives us the income tax revenues. Program's total cost (LHS) is the price of land minus the contributions. Equilibrium now also requires program constraint (10) holds.

We first study a model in which the government provides public housing on the land that is between 4 and 6 miles away from CBD in one of the neighborhoods, say east. This causes most high income types to reside in west, which then becomes the higher rent/tax/school quality neighborhood. In the benchmark model U types lived in this band in east, and the lot sizes ranged from 4,739 to 6,691 sq feet (at 4 and 6 miles away from CBD) with an average lot size of 5,575 sq feet. The average monthly rent was \$3,520 per mile square. Each public housing unit measures 6,970 sq feet now, regardless of its location within the band. This is about twenty-five percent larger benchmark average. This band then accommodates about fifteen percent of all low income types.¹⁸ Some lots may be closer to city center than others, so the equilibrium utility public housing residents may vary depending on their lot location, an outcome of the lottery. A determinant of rent in HUD's public housing program is 10% of monthly income.¹⁹ We set the rent of a subsidized unit to \$239 and the income tax rate to 0.57%, so that utility increase from public housing is equivalent to that from a 10% income subsidy in equilibrium and (10) holds.

Figure 2 and Table 2 present some highlights of the new equilibrium. Sorting is stronger compared to the benchmark. Seventy-five percent of high income types live in West (up from 69% in benchmark) and constitute the majority there. The intuition is straightforward. The public housing policy removes a substantial amount of land from the private market. The non-recipients, who can be the skilled or the "less lucky" unskilled workers, need to compete for the land remaining. However, the composition of the demand changes, and so does the equilibrium price. To fix the idea, let us assume that originally the total land supply is \mathcal{L} . The amount of skilled and unskilled workers are N_S and N_U respectively. Without any public housing policy, the ratio of skilled relative to the unskilled is N_S/N_U . Now the government removes an amount of land $\mathcal{L}_P > 0$, and

¹⁸In France, HLM accommodates about 17% of all households.

¹⁹<http://www.hud.gov/renting/phprog.cfm>.

the remaining amount of land available for the market has decreased to an amount $\mathcal{L} - \mathcal{L}_P$. At the same time, an amount $N_P > 0$ of unskilled workers receive the public housing and essentially “leave” the market. Thus, among those remaining in the market, the ratio of skilled relative to the unskilled becomes $N_S/(N_U - N_P)$, which is clearly larger than the original ratio N_S/N_U . Other things being equal, as the skilled naturally demand more land than the unskilled, such change in the composition of demand would have a tendency to generate a higher rent. But other things are not equal. First, there is a fiscal burden issue here. To finance the public housing project, the government needs to impose taxes and that would discourage the incentive to work. In addition, the public housing residents’ effective rents are below the “natural” market values (which is the “East” in our model). As they only pay property taxes as a proportion of these subsidized rents, it decreases the neighborhood’s property tax revenues and spending on education. Moreover, the skilled workers may “vote on foot” and move to another neighborhood. They have stronger preferences for education. They would prefer to live in West where the marginal effect of a tax dollar on per-student expenditures are higher. As a result, the difference in quality of education is higher between two neighborhoods compared to the benchmark, since both the spending and the peer quality in West (East) are higher (lower) than before. Our numerical implementation of the model simply attempts to capture this chain of effects quantitatively.

(Figure 2, Table 2 about here)

We also find that public housing residents on average decrease their labor supply by about 3 hours a week. This is consistent with the empirical literature that housing subsidies would actually have negative effect on labor supply, and such negative effect tends to be larger under public housing scheme than that under housing voucher system.²⁰

(Table 3 about here)

²⁰Clearly, it is beyond the scope of this paper to review this literature. Among others, see Bingley and Walker (2001), Hulse and Randolph (2005), Olsen et al (2005), and the reference therein.

We calculate the change in household welfare according to types in second row of Table 3. We present the necessary change in rents to provide households their benchmark utility level. A negative number means the household is worse off now, since rents need to be decreased to keep them indifferent to benchmark allocation. The welfare gain to public housing residents is equivalent to a gain from an eighty-six percent decrease in rents. The welfare loss to the rest is equivalent to a 5.5 percent increase in rents. Our overall welfare measure, Aggregate Expected Utility (a normalized sum of household utilities), is lower.

4.2 Location Matters

To further exploit the spatial general equilibrium built in this paper, we conduct additional analysis by comparing the economic outcomes for the public housing units being built at different locations. To facilitate the comparison, we keep the size and the number of subsidized units constant and *vary only* their distance to CBD. Recall that the public housing units are built in the “belt” which are between 4 to 6 miles from the CBD. The other cases are for the public housing to occupy the land between 3 and 5.38 miles, and between 5 and 6.71 miles away from CBD.

When public housing moves from 3 to 4 and then to 5 miles, we see a stronger sorting. More and more S types choose to reside in West, which means larger lots and less population density. Most significant implications are on quality of education and welfare. As public housing moves away from CBD, the education quality gap increases: The quality keeps on increasing in West, and decreasing in East. Both per student expenditure and peer quality affect this, but the role of expenditures is larger. Government provided band houses a fixed number of households (N_P) that pay less than full taxes over subsidized rents, but are fully entitled to educational expenditures. Notice that the area of the band stays constant as we move it around. As this band moves out, and since lot sizes increase in distance from CBD, the band replaces less and less people with public housing residents, increasing the population density in the neighborhood. Rents also decrease in distance to CBD, so the tax revenue from public housing residents decrease too.

The overall welfare decreases as public housing band moves away from its original location (4 miles) in each direction (Table 3, rows 1 and 3). At 4 miles, the public housing unit size is slightly larger than a typical lot at the same location in benchmark model. When we move the public housing units, we keep their sizes same. Now as we move towards CBD, the public housing unit becomes much larger compared to a typical lot at a similar location. As we move away from CBD, it becomes smaller than a typical unit at the neighboring locations. These deviations from optimal lot sizes affect community compositions and local public finance in a way that decreases overall welfare.

4.3 Housing Vouchers

The second policy to be considered is that the government simply redistribute the income tax revenues to the “poor” in the form of housing vouchers. According to Laferrere and Blanc (2006), among others, this seems to be more consistent with the case of the United States. Under this scheme, each of the N_P program participants gets a voucher with amount ν_P . For government budget to balance, income tax revenues must equal the total amount of these vouchers:

$$N_P \nu_P = \theta \sum_{j \in \{e, w\}} \left[\int_0^\infty \frac{L(r)}{s_j(r)} \left(\sum_{i \in \{S, U, P\}} I[l_j^*(r) = i] (24 - l_i^*(r) - br) W_i \right) dr \right]$$

A household’s problem is same as the one in (9), but a voucher recipient’s housing expenditures are those exceeding ν_P only. This is equivalent to creating a third household type (say type P) with the same preferences as type U, and with the kinked budget constraint:

$$z + \max\{0, (1 + \tau_j)R_j(r)s - \nu_P\} + W_U l = Y_P(r) = 24(1 - \theta)W_U - (a + b(1 - \theta)W_U)r. \quad (11)$$

Since household utility increases in lot size, no household will spend less on housing than the voucher amount. Whether the household will spend more depends on the model parameters. The leisure choice of a voucher recipient is $l_P^*(r) = \delta \frac{Y_P(r) + \nu_P}{(1 - \theta)W_U}$ if the household spends on top of the voucher

amount, and $l_P^*(r) = \frac{\delta}{\gamma+\delta} \frac{Y_P(r)}{(1-\theta)W_U}$ otherwise. The land is allocated according to the competitive auction mechanism described in the model section. An additional equilibrium condition is that the program budget (??) holds.

For comparison, we keep the (number of) recipients and the tax rate same as the public housing policy. This implies a housing voucher with amount \$7575 (annual). Again, table 3 provides a summary of the results. Vouchers shift land demand, increasing rents in both neighborhoods. The equilibrium rents, however, are lower than those under public housing policy. Sorting of households is stronger than benchmark, equivalent to the public housing policy levels: Seventy-four percent of high income types live in West (as opposed to 69% in benchmark) and constitute a fifty-six percent majority there. The major cause of this is increase in demand for land in East: All voucher recipients reside in East (where low income types are a majority) because of their weaker preferences for education, and choose lots smaller than they would get under public housing. Figure 3 gives a map of the city, and Table 4 gives some statistics. The quality of education in West is slightly higher than both the benchmark and public housing models, because of both the sorting and higher expenditures. A policy maker concerned with education of the poor may prefer vouchers over public housing: The quality in East, the poorer neighborhood, is higher than that in public housing model.

(Figure 3, Table 4 about here)

Unlike public housing, vouchers do not reduce labor supply of recipients. Also, the fiscal burden problem we described in public housing units section is no concern with vouchers, since every household pay their taxes in full. However, households without vouchers are hurt by the higher rents and income taxes. The equilibrium utility levels of non recipients (both S and U types) are higher than under public housing, but lower than benchmark models. The (average) utility level of voucher recipients falls below public housing utility. We present the change in household welfare according to their types in the last row of Table 3. The welfare gain to voucher recipients is equivalent to a gain from a fifty-nine percent decrease in rents. The welfare loss to the rest is

about 3.3 percent increase in rents, about half of that under public housing policy. Unlike public housing, the change in total welfare (as measured by AEU) is positive under this policy.

4.4 Can Housing Vouchers Alter Household Sorting?

In the previous section, we present several reasons why a housing voucher program may be preferred over a public housing program, given a program size and an income tax rate. Another potential benefit of housing vouchers over public housing is that the former do not impose restrictions on location choice of households. Hence, a policy-maker with concern over the extent of household sorting across neighborhoods may be particularly interested in how the housing vouchers can influence this sorting.

However, in the above analysis, the equilibrium neighborhood compositions under the two programs are almost identical. West community provides higher quality of education at the cost of higher taxes on land consumption. The S type households have stronger preferences (and willingness to pay) for education compared to voucher recipients, so they outbid the voucher recipients on West land away from CBD. On the other hand, U type households without vouchers value proximity to CBD and outbid voucher recipients on West land close to CBD. The U type households demand smaller lots compared to voucher recipients, and therefore are not affected by larger taxes as much as voucher recipients who demand relatively larger lots. As a result, voucher recipients are not observed residing in West.

These observations suggest that whether a voucher program and a public housing program with the same size and income tax rates will have the same impact on household sorting may depend on the combinations of some parameter values. This encourages us to further explore other parameterizations. In particular, we conduct a series of numerical experiments. First, we studied a set of parameterizations in which policy maker increases the number of recipients *without changing the income tax rates*. This of course means a lower voucher amount for every recipient. Second, we studied the effects of increasing tax rates while *keeping the number of recipients the same* as in

the previous section. Neither attempt causes enough increase in bids of voucher recipients in West to overcome the effects summarized in the previous paragraph.²¹ The intuition is straightforward: Increasing number of recipients decreases the number of U types without vouchers, weakening their competition. It also lowers the bids by voucher recipients since the amount of the voucher decreases in the number of recipients. On the other hand, increasing voucher amount for the same number of recipients just help them afford larger lots in East instead of relocating, pushing some U types to West instead.

For those who are interested to kill two birds in one stone, namely providing the housing voucher and at the same time influencing the community composition, we do have some “good news.” More specifically, we find that it is possible to induce voucher recipients residing in West with housing voucher programs *with both larger program size and larger budget*. The increasing population and effective income of voucher recipients may increase their land bids in both neighborhoods and allow them to outbid some households in West. We present the equilibrium of one such model in Table 5 in which 25% of unskilled types receive housing vouchers with amount \$17900, and about one fifth of these recipients live in West. This program is financed by an income tax rate of 1.5%, about twice the income tax rate in the earlier sections. The housing voucher recipients occupy a semi ring between the U types’ semi-circle around CBD and the S types’ semi-ring. The sorting in equilibrium is much weaker than programs we considered earlier. About 45% of U types (with and without vouchers) live in West and constitute 49% of the population. The reduction in sorting comes at the expense of a low quality of education in West, and lower rent differences between neighborhoods. Other aspects of the equilibria remain the same qualitatively, so we skip a detailed discussion here.

(Table 5 about here)

²¹Detailed results are available from the authors.

5 Concluding Remarks

After surveying a vast literature on the housing market and housing policies in the U.S., Green and Malpezzi (2003, p.94) argue that “most economists like vouchers because they are generally more efficient than other programs... But in the United States, political support is generally stronger for programs tied more closely to the consumption of specific goods (housing, food, and medical care) than for income support.” This paper attempts to contribute to the related debate. In particular, this paper explicitly highlights the importance of location of public housing on equilibrium outcomes such as rents, neighborhood compositions, schooling opportunities, labor supply decisions, and social welfare. We explain the channels through which such location effects work. Using a rich general equilibrium model that combines land use theory with Tiebout framework, we provide a comparison of public housing and housing vouchers policies, and discuss several reasons why vouchers may be preferred over subsidized units. Our findings are consistent with previous “in kind vs. cash” discussions.

In addition, we also find that public housing policy tends to discourage labor supply, especially for the unskilled workers who reside in public housing, as some empirical literature has suggested. This seems to strengthen the “in cash rather than in kind” position even further.

Several directions for extension seem to be natural. First, this paper only considers a monocentric city. Future research may allow for private school as well as multi-centric city and re-examine the desirability of alternative public housing policies. Second, city population does not stay constant in practice. What policy is more desirable in an environment with stochastic population flow? This seems to be another topic worth exploring. Third, recent research in urban economics emphasize the importance of dynamics. Future research should consider a dynamic extension for the public housing policy analysis. These research efforts would definitely deepen our understanding and enrich the debate on the government involvement in the housing market.

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Table 3: Welfare Changes

Policy	Type P	Type U	Type S	AEU
Public Housing at 3 miles	77.48	-5.61	-5.33	-0.69
Public Housing at 4 miles	85.75	-5.34	-5.58	-0.30
Public Housing at 5 miles	88.76	-5.42	-6.29	-0.54
Vouchers	59.01	-3.31	-3.29	0.54

Figure 1: Benchmark Model

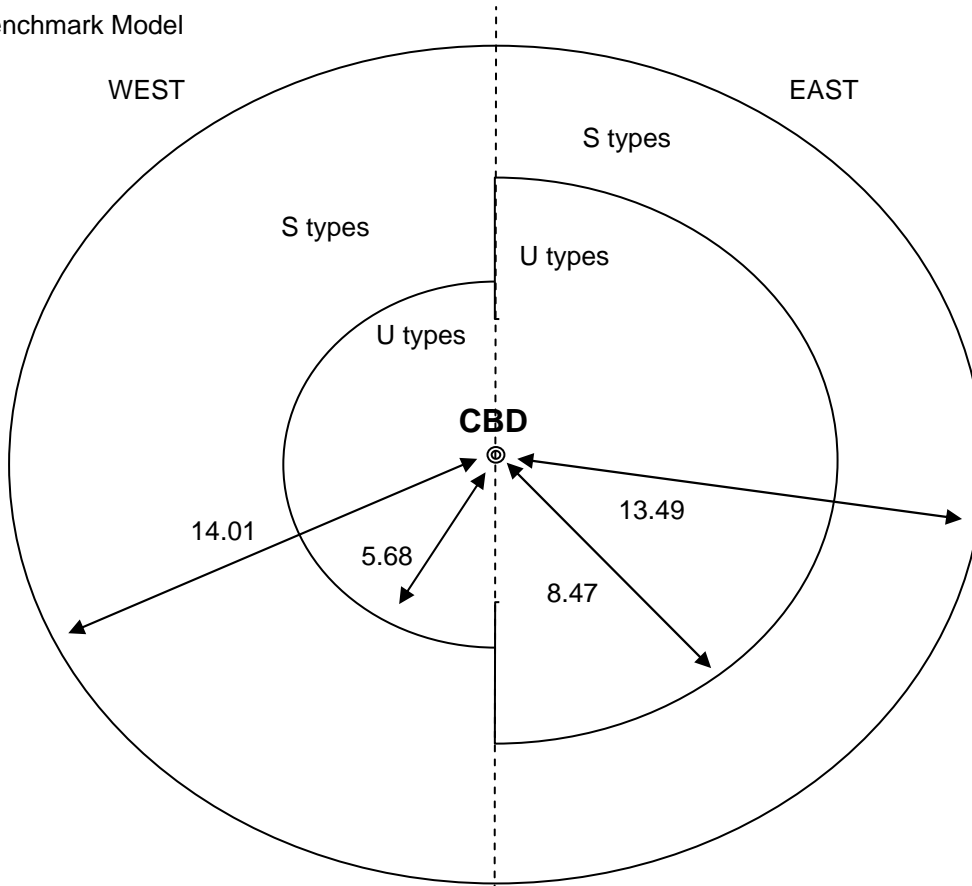


Table 1: Benchmark Model

	west	east
ave mo rent per acre	2729	2194
tax rate	1.97	1.04
quality of education	14	7
exp per pupil	3653	2027
ave monthly rent	2729	2194
rent S pays	2204	1433
rent U pays	5416	3368
density of S per mi ²	1616	1058
density of U per mi ²	7417	4665
distribution of types across neighborhoods		
S	69%	31%
U	42%	58%
neighborhood population breakdown		
S	53%	26%
U	47%	74%

Figure 2: Public Housing Model

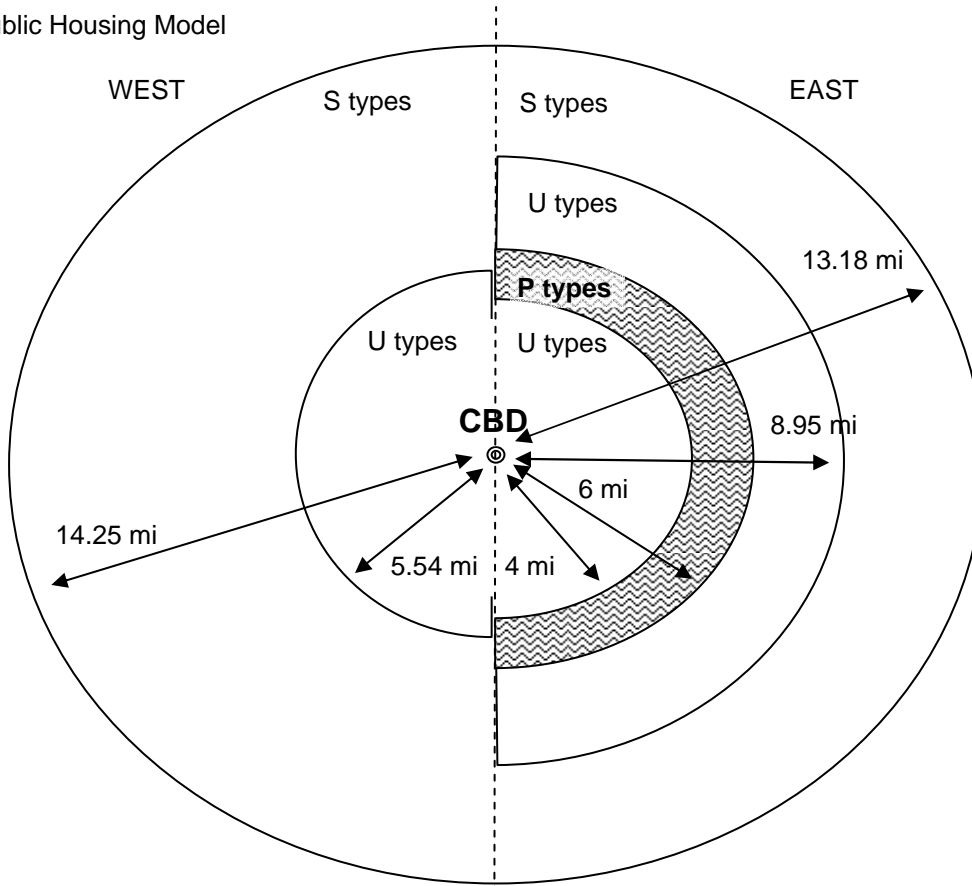


Table 2: Public Housing Model

	west	east
ave mo rent per acre	2764	1955
tax rate	1.97	1.04
quality of education	14.15	6.02
exp per pupil	3677	1806
distribution of types across neighborhoods		
S	75%	25%
U	41%	59% (14% in public housing)
neighborhood population breakdown		
S	55%	22%
U	45%	78% (19% in public housing)

Figure 3: Voucher Model

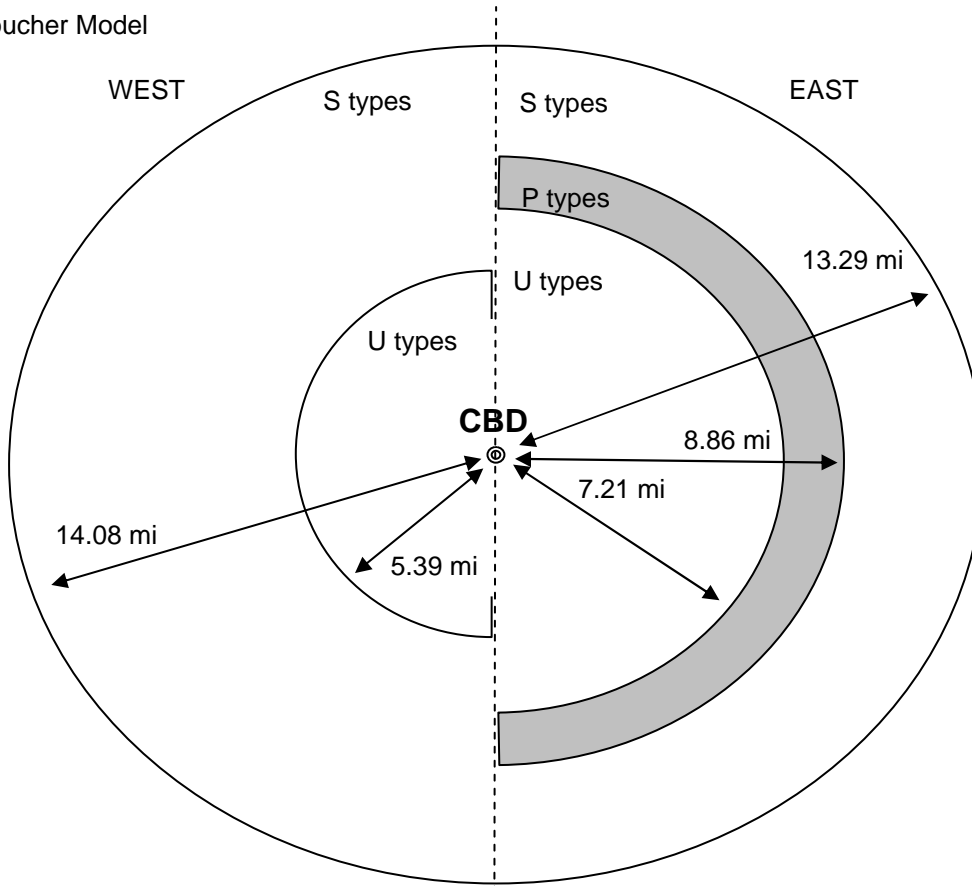


Table 4: Voucher Model

	west	east
ave mo rent per acre	2741	2186
tax rate	1.97	1.04
quality of education	14.25	6.56
exp per pupil	3699	1963
distribution of types across neighborhoods		
S	74%	26%
U	39%	61% (14% in public housing)
neighborhood population breakdown		
S	56%	22%
U	44%	78% (18% in public housing)

Figure 4: Desegregation via Vouchers

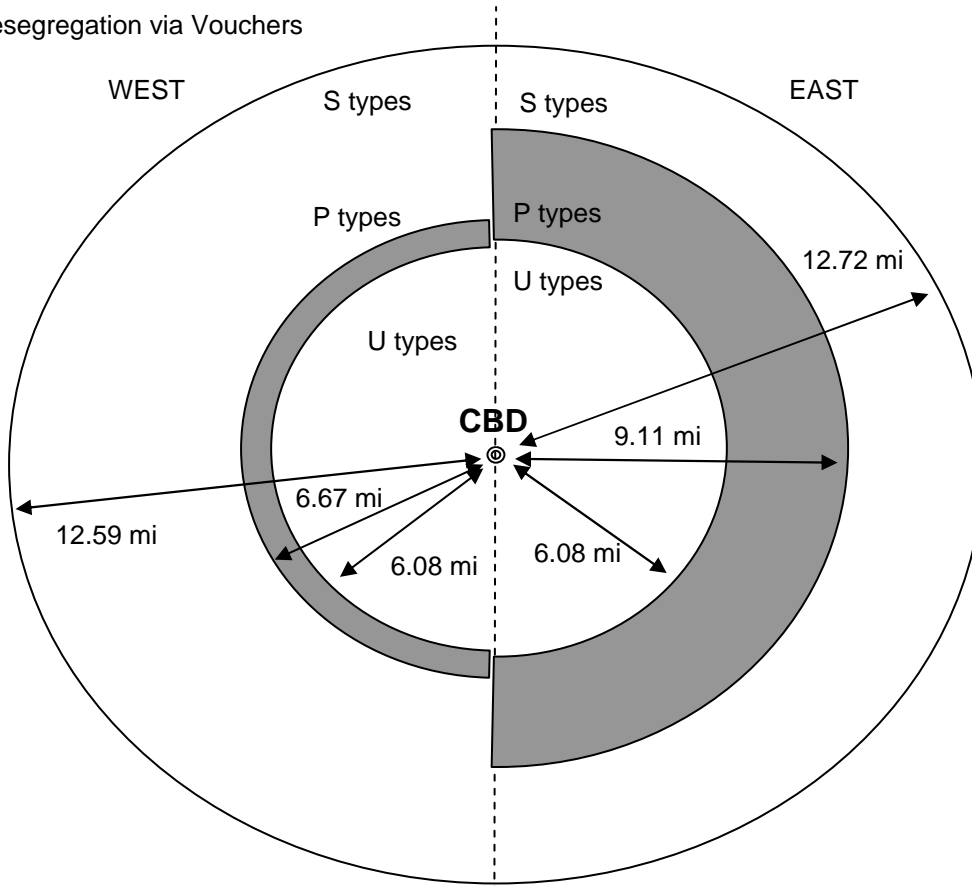


Table 5: Desegregation via Vouchers

	west	east
ave mo rent per acre	2553	2123
tax rate	1.97	1.04
quality of education	10.28	6.3
exp per pupil	2697	1795
distribution of types across neighborhoods		
S	68%	32%
U (4.7% vouchers)	45%	55% (20% vouchers)
neighborhood population breakdown		
S	50%	28%
U (5% vouchers)	50%	72% (26% vouchers)